

Possibility of long-distance heat transport in weightlessness using supercritical fluids

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Abstract

Heat transport over large distances is classically performed with gravity or capillarity driven heat pipes. We investigate here whether the "piston effect", a thermalization process that is very efficient in weightlessness in compressible fluids, could also be used to perform long distance heat transfer. Experiments are performed in a modeling heat pipe (16.5 mm long, 3 mm inner diameter closed cylinder), with nearly adiabatic polymethylmethacrylate walls and two copper base plates. The cell is filled with H₂ near its gas-liquid critical point (critical temperature: 33K). Weightlessness is achieved by submitting the fluid to a magnetic force that compensates gravity. Initially the fluid is isothermal. Then heat is sent to one of the bases with an electrical resistance. The instantaneous amount of heat transported by the fluid is measured at the other end. The data are analyzed and compared with a 2-D numerical simulation that allows an extrapolation to be made to other fluids (e.g. CO₂, with critical temperature 300 K). The major result is concerned with the existence of a very fast response at early times that is only limited by the thermal properties of the cell materials. The yield in terms of ratio: injected/transported heat power does not exceed 10-30% and is limited by the heat capacity of the pipe. These results are valid in a large temperature domain around the critical temperature.

I. INTRODUCTION

Heat transport over large distances is classically performed by the latent heat transport. Conduction is effective only on short distances because of the slowness of the heat diffusion process. In the 1990s, a new way of the heat transfer in very compressible fluids has been evidenced. This phenomenon, which is particularly important under weightlessness, has been called piston effect [1–3]. When heating a fluid confined in a container, a thin hot boundary layer (HBL) of thickness δ_1 forms close to the heating wall [4]. This layer expands and at the same time compresses adiabatically the rest of the fluid. As a result, a spatially uniform heating of the bulk fluid is achieved in a very short time.

The heat flux is limited by the temperature gradient in the heating wall and in the fluid HBL. If the other side of the sample, located at a distance L from the heating wall, is colder, a cold boundary layer (CBL) of length δ_2 will simultaneously form at the corresponding wall, see Fig. 1c below. In between, the temperature is homogeneous. A heat transport can thus be performed in both HBL and CBL, connected by the bulk fluid that acts as a kind of thermal short circuit. While such a "heat pipe" does not work in the stationary regime (the piston effect concerns only transient behavior), it could serve to quickly transfer heat.

Studies of transient heat flux in presence of piston effect have already been performed in ^3He under Earth's gravity in a flat sample of large aspect ratio (thickness 1 mm, diameter 57 mm) perpendicular to gravity, for Rayleigh numbers below and above the convection threshold [5] and in CO_2 under weightlessness [6]. Here, in order to evaluate whether this process can be used to transport heat on large distances under weightlessness, experiments are performed under the conditions of magnetic gravity compensation in a cylindrical cell with the largest length that preserves magnetic compensation (aspect ratio 0.182). The cell is filled with H_2 near its gas-liquid critical point (critical temperature $T_c = 33$ K).

II. EXPERIMENTAL SETUP

Weightlessness conditions are achieved by using magnetic forces to compensate the gravity. The magnetic gravity compensation technique is recently reviewed in [7]. The HYLDE (Hydrogen Levitation DEvice) situated at CEA/Grenoble has been used here. It is described in details in [8, 9].

A. The cell

The fluid is confined in a cylinder of $L = 16.5$ mm long, 3 mm inner diameter and 4 mm outer diameter made of polymethylmetacrylate (PMMA = Plexiglas, denoted below with the subscript PA) (Fig. 1). The cylinder axis coincides with the magnetic coil axis. PMMA has been chosen because it is transparent and has a low thermal conductivity ($k_{PA} = 0.125 \text{ Wm}^{-1}\text{K}^{-1}$ at 33K) and the specific heat of the walls is low ($C_{PA} = 180 \text{ J kg}^{-1}\text{K}^{-1}$ at 33K). With a mass density of $1.15 \cdot 10^3 \text{ kgm}^{-3}$, the thermal diffusivity is $D_{PA} = 6.04 \cdot 10^{-7} \text{ m}^2\text{s}^{-1}$, corresponding to the characteristic diffusion time $t_{PA} = L^2/D_{PA} = 450 \text{ s}$. The PMMA cylinder is sealed with stainless steel rings to two parallel electrolytic copper (thermal conductivity at 33K is $1130 \text{ Wm}^{-1}\text{K}^{-1}$) blocks. They are in thermal contact with the helium bath provided by thermal conductors. The temperatures of the blocks are measured with Cernox® thermistors. They are made of a composite that provides a very weak resistance variation with the magnetic field. Their time constant is of the order of 20 ms. Stainless steel is a thermal insulator at these low temperatures (thermal conductivity is $3.37 \text{ Wm}^{-1}\text{K}^{-1}$). The mass of the upper copper block (hot part), called the “head”, is $m = 2.37 \text{ g}$. The other block, the “base” (a colder part) is kept at constant temperature T_B by a closed loop temperature regulation, the proportional integral differential device (PID). The temperature control accuracy is $\pm 1 \text{ mK}$ at 33 K and the working temperature range is 15-40 K.

The cell can be filled *in situ* with pure pressurized H_2 through a capillary. This capillary is closed by an H_2 ice plug (H_2 solidification temperature is 14 K), whose formation is provided locally by a thermal conductor in contact with the helium bath. The cell is observed by light transmission. Parallel light is directed with mirrors from outside the anticryostat to the sample. The beam is parallel to the axis of the cylindrical cell and is deflected by mirrors to the camera outside the anticryostat. The whole set-up is situated in a vacuum chamber.

At room temperature and at equilibrium, H_2 is actually a mixture of three volumes of orthohydrogen (parallel spins of H nuclei, o- H_2) with one volume of para hydrogen (antiparallel spins, p- H_2). This gas is called normal hydrogen, n- H_2 . When H_2 is cooled, the o- H_2 -p- H_2 equilibrium is shifted and almost all o- H_2 gradually transforms to p- H_2 (more precisely, 96% of p- H_2 at equilibrium at 30K). The exact critical pressure, temperature and density of p- H_2 are $T_c = 32.976 \text{ K}$, $p_c = 1.2928 \text{ MPa}$, $\rho_c = 31.426 \text{ kgm}^{-3}$ [8]. The cell is filled at critical density ρ_c by checking that the gas-liquid meniscus does not displace in the

cell while approaching T_c from below. The precision is of the order of ± 0.5 mm which corresponds to the $\pm 3\%$ uncertainty of the critical density. The precision of the determination of T_c is within 1 mK. This value shifts with the conversion o- H_2 - p- H_2 with a time constant of order 50 hours [8]. Then the experiments close to T_c are performed more than 2 days after the beginning of the runs with the value T_c checked before and after these runs.

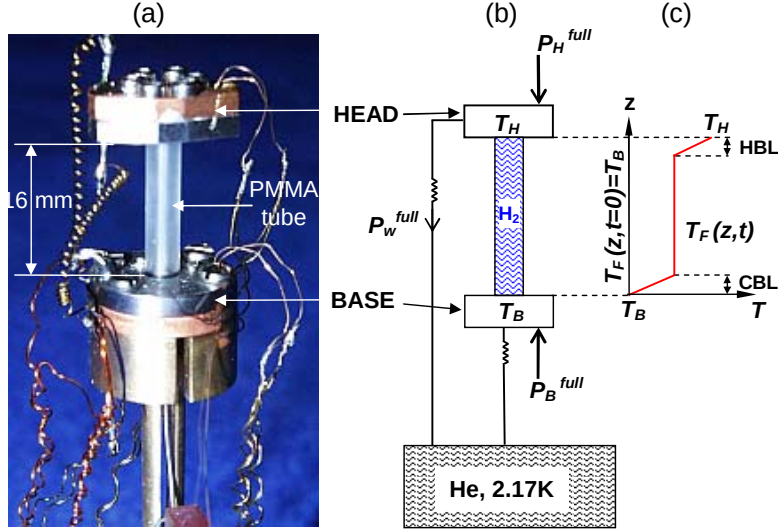


FIG. 1. (Color online) The experimental piston effect heat pipe. (a): photo. (b) heat exchange model. Both the head and the base are connected to the helium bath via heat conductors (thin wires). The heat power P_w^{full} leaks from the head to the bath, see section III A 2. The controlled heat power is supplied to the head (P_H^{full}) and to the base (P_B^{full}). (c) Schematic instantaneous fluid temperature variation provided by the piston effect. The cold (CBL) and hot (HBL) boundary layers are indicated.

B. Measurement procedure

Initially, the base, the head, the fluid and the cell walls are at the same temperature T_0 . The head temperature is maintained constant by sending a constant power (~ 5 mW) that compensates the heat losses through the wiring (Fig. 1). It is important to precisely calibrate the head and base temperature sensors. We took benefit of the strong convective flows that form a few mK to T_c under gravity in the presence of the slightest temperature gradients. Head (T_H) and base (T_B) temperatures are assumed to be equal when, under Earth gravity,

the convection disappears. Then the magnetic field is switched on and further measurements are performed at weightlessness.

The electric power P_B^{full} is supplied to the cell base by the temperature regulation that maintains T_B constant. $P_B^{full}(t = 0) \approx 200$ mW when the head and the base are at the same temperature. The power P_H^{full} is supplied to the head. P_B^{full} , P_H^{full} , T_H and T_B are recorded throughout the experiment.

At $t = 0$, the power $P_0 = 7.5$ mW is sent during $\Delta t = 300$ s in the head by increasing in a single step the power P_H^{full} . This value corresponds to 1 kW/m². It has been chosen because (1) it is the smallest value that provides enough precision in the data; (2) this power is close to the maximum power that can be extracted by the cell base temperature regulation. The initial temperature T_0 remains the reference for the base temperature regulation PID (i.e. $T_B = T_0$ within the precision of the PID). During the experiment, the temperature of the base varies only slightly, of the order of 6 mK (Fig. 2). The integration time of the PID is typically 10 s. This value thus corresponds to the temporal resolution of the transient heat flux measurements.

The heat power P_B transferred by the cell is determined as

$$P_B(t) = P_B^{full}(t = 0) - P_B^{full}(t). \quad (1)$$

The power $P_B(t)$ is thus limited by the value $P_B^{full}(t = 0)$.

The measurements were initially carried out in the empty cell to evaluate the heat conduction by the PMMA walls. The cell is then filled and measurements are performed at different temperatures T_0 .

C. Measurement results

In Fig. 2 are shown typical evolutions of the transmitted power P_B and head temperature when the pipe contains H₂ initially at $T_0 = T_c + 5$ mK. The corresponding curves for the empty (under vacuum) cell are shown for comparison. In this latter case, heat is transmitted by the PMMA pipe walls only. One notes that (i) T_H is lower when the pipe is filled, which agrees with the higher value of P_B , (ii) the dynamics of P_B at small time is quite different with empty and filled cells. This is even clearer from Fig. 3 where the typical small time behavior is reported for different temperatures. After the 10 s reaction time of the thermal

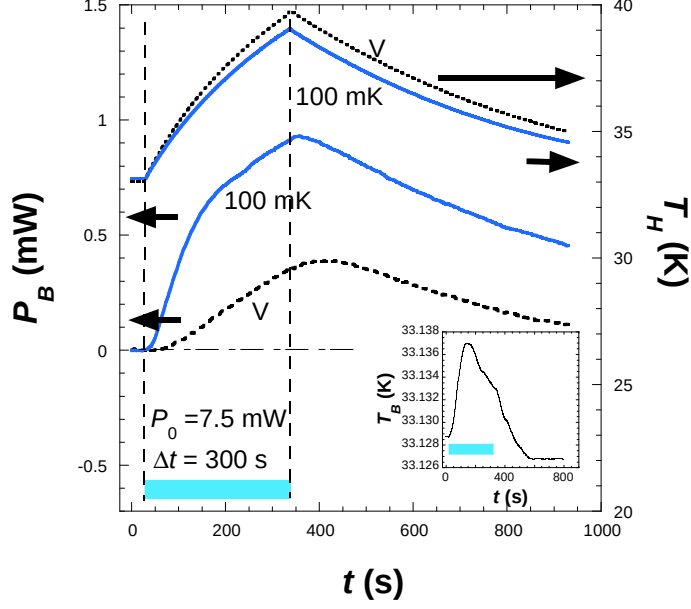


FIG. 2. (Color online) Typical measurements of the transmitted heat flux (P_B), head temperature (T_H) and base temperature (T_B , in the insert) when a 7.5 mW power is injected into the head during about 300 s (shown with a bar). Solid lines: filled cell, $T_0 = T_c + 100$ mK; dotted lines: empty cell (under vacuum).

regulation, P_B increases linearly. In contrast, the heat flux in the empty pipe remains close to zero for long time. The difference in behavior highlights two different mechanisms of heat transfer (i) thermal diffusion in the walls of the empty pipe where the heat flux is zero before the front of heat diffusion reaches the base, and (ii) piston effect in the filled pipe. In the latter case, the dynamics (and the yield, see below) increases when nearing T_c . The behavior of $P_B(t)$ at $T_0 < T_c$ (i.e. when both liquid and gas phases are present in the pipe) is similar to that at $T_0 > T_c$.

The measured time evolution of the transmitted power P_B is plotted at different temperatures above and below T_c in Fig. 4. One notes that the behavior at $T_0 < T_c$ is similar to the one at $T_0 > T_c$, even very close to T_c . The yield defined as $Y = P_B/P_0$ attains at its maximum about 10%. The yield for the filled pipe is always larger than for the empty pipe. At large times, the pipe walls contribute to the heat transfer, as seen in Fig. 4, where the rise of P_B for the filled cell is roughly parallel to the empty cell curve. The maximum heat flux (P_B^M) is then always reached at the end of the heating period.

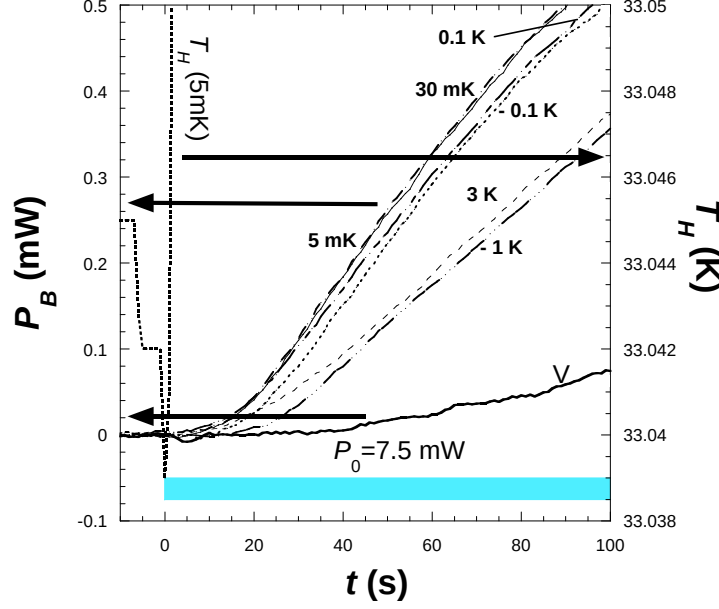


FIG. 3. Small time evolution of the transmitted power (P_B) for several temperature distances $T_0 - T_c$, both above and below the critical point. The head temperature evolution (dotted line) is also shown to precisely detect the onset of heating. V: empty cell; $T_0 - T_c = 5$ mK (solid line), 30 mK (broken line, long dashes), 0.1 K (double broken line), 3 K (broken line, short dashes), -0.1 K (dotted line), -1 K (dotted-dashed line).

In order to determine what are the relative contributions of the cell walls and the fluid, it is necessary to analyze the process in great detail. This is performed in the section III. It is found that the wall contribution is much smaller than in the empty cell due to the rapid thermalization of the bulk fluid that reduces the temperature gradient in the pipe walls.

It is possible to infer the "initial" slope dP_B/dt from Fig. 3. Heat is sent into the cell head at $t = 0$ and, after 10 to 15 s, P_B grows linearly in time during at least 20–30 s. One thus define as initial slope the value of dP_B/dt as measured between 27 s and 47 s. During this period, the contribution of the transient conduction by the pipe walls is negligible. The temperature dependence of this slope is reported in Fig. 5a for two experimental runs. The data are similar above and below T_c . The slope increases when going to T_c and saturates for $|T_0 - T_c| \leq 0.3$ K. The maximum heat flux (at 300 s) P_B^M (Fig. 4) is plotted with respect to temperature in Fig. 5b for two different runs. Data between both runs show a systematic difference of order 15% that we attribute to the uncertainty of filling of the cell at critical

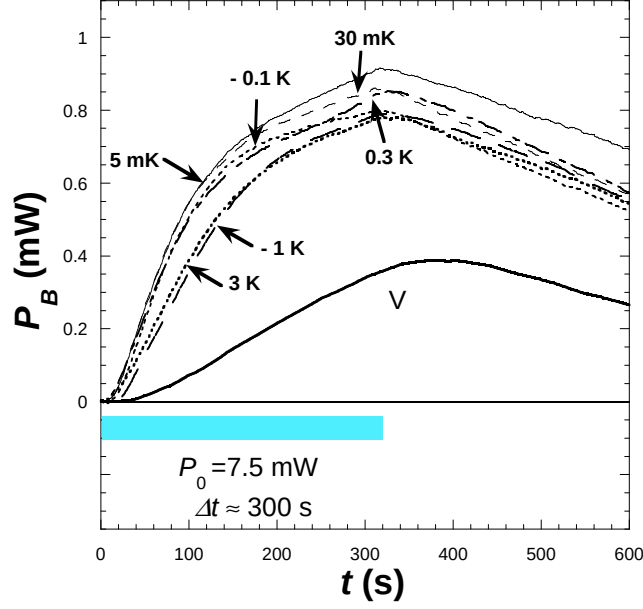


FIG. 4. Measured transient of the transmitted power P_B for several temperatures above and below the critical point. The time interval during which the power is injected is shown with a bar. V: empty pipe; Other curves: filled cell; $T_0 - T_c = 5$ mK (solid line), 30 mK (broken line, long dashes), 0.1 K (double broken line), 3 K (broken line, short dashes), -0.1 K (dotted line), -1 K (dotted-dashed line).

density ($\pm 3\%$). They, however, exhibit the same behavior, similar to dP_B/dt , with an increase when going to T_c and saturation for $|T_0 - T_c| \leq 0.3$ K.

III. 2D NUMERICAL SIMULATION AND ANALYSIS

Several important points influence the choice of a numerical method for the simulation.

(i) 1D numerical simulations are relatively easy to perform. However, in the present study, a 1D simulation would not take into account an important effect, the heat exchange with the lateral walls. Lateral walls are heated by the bulk fluid that is quickly thermalized by the piston effect. This wall influence limits the fluid temperature rise and thus the value of the output heat flux. At least a 2D numerical simulation is necessary to take this influence into account.

(ii) The near-critical fluid introduces a nonlinearity in our system. Its simulation thus requires an iterative algorithm. On each iteration a complete set of temperature values at

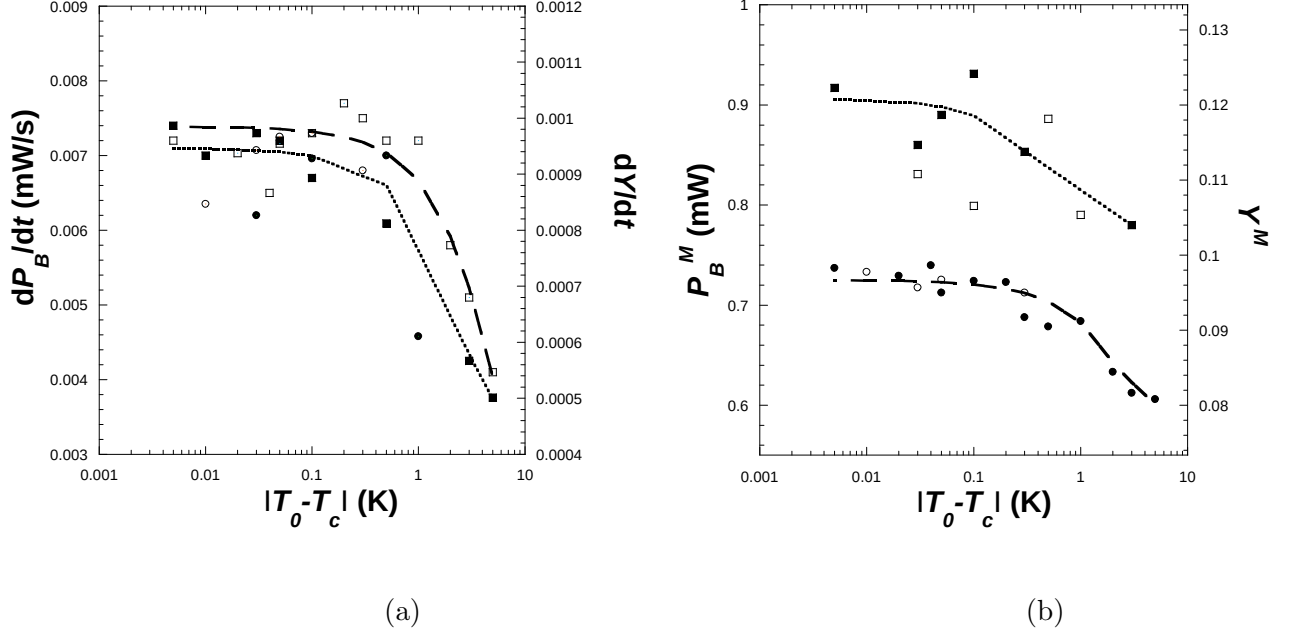


FIG. 5. (a) Measured transient of the transmitted heat flux initial rise dP_B/dt (see text) with respect to $|T_0 - T_c|$. Run 1: circles, run 2: squares. $T_0 > T_c$: black circles; $T_0 < T_c$: open circles. The dotted (broken) line is the run 1 (2) smoothing of $T_0 > T_c$ data. (b): Fluid maximum heat transmission P_B^M (see text) with respect to $|T_0 - T_c|$. Run 1: circles, run 2: circles. $T_0 > T_c$: filled circles; $T_0 < T_c$: open circles. The dotted (broken) line is the run 1 (2) smoothing of $T_0 > T_c$ data.

each node point should be determined. It means that (i) in practice, the equation of state (EOS), i.e. the relation between fluid temperature T , pressure p and density ρ , must be solved at each iteration, node point and time step. It involves an enormous total number of EOS treatments. At the same time, the system is driven quite far from the critical point, the relative temperature distance can be as large as $(T - T_c)/T_c \sim 0.15$. This means that (ii) any simple model EOS (parametric [10], van der Waals, linearized [11], etc.) would not work well. A real EOS for H_2 needs to be used in the simulation. The available hydrogen EOS [12, 13], is the modified Benedict-Webb-Rubin (MBWR) EOS that includes dozens of terms. The requirements (i) and (ii) make the computation time prohibitively long when the full hydrodynamic description of the supercritical fluid (as in [11]) is used.

Recently, some of us [10] suggested a fast calculation method that results in accurate heat transfer calculations wherever the impact of the fluid advection on the heat transfer can be neglected. It makes use of the Boundary Element Method (BEM) [14] that permits a reduction by one of the dimensionality of the problem: only the boundary surfaces needs

to be meshed. Here we use this method to simulate this problem in microgravity where the fluid convection can be neglected.

A. Problem statement

We aim to model the amount of heat transferred to the base from the head of the cell while the temperature of the base T_B remains constant. Initially, the temperature of the whole cell is equal to T_0 .

The cell consist of several parts: (i) Cell base (denoted by B index), (ii) Cell head (denoted by H index), (iii) PMMA tube (denoted by PA index), (iv) Fluid (denoted by F index). These parts need to be modelled separately.

1. Cell base model

The cell base is made of electrolytic copper and is assumed isothermal thanks to its very high thermal conductivity. Its temperature T_B is controlled by the temperature regulation and assumed to be constant(= T_0) during each experiment. In reality, the temperature regulation device (see above section II B) exhibits an integration time $t_I \simeq 10$ s that characterizes the time over which it averages the actual base temperature data T_B in order to maintain the base temperature at its reference value T_0 . We take into account this regulation "inertia" indirectly by integrating over t_I the function $P_B(t)$ obtained in the simulation.

The power P_B transferred to the base is the main output of the simulation, to be compared with the experiment. It is a sum of the heat power that arrives from the cell head by the tube walls and by the fluid column:

$$P_B = -k_{PA} \int_{(S_{PA})} \left. \frac{\partial T_{PA}}{\partial \vec{n}} \right|_B dS - k_F \int_{(S_F)} \left. \frac{\partial T_F}{\partial \vec{n}} \right|_B dS \quad (2)$$

where k denotes the thermal conductivity and the integration is performed over the cross section areas of the PMMA tube S_{PA} and of the fluid S_F (see Fig. 6); the unit normal vector $\vec{n}$ is directed inside the base. Since T_B is assumed to be constant in time, the heat power that leaks to the He bath remains constant and P_B given by (2) corresponds to the measured quantity (1).

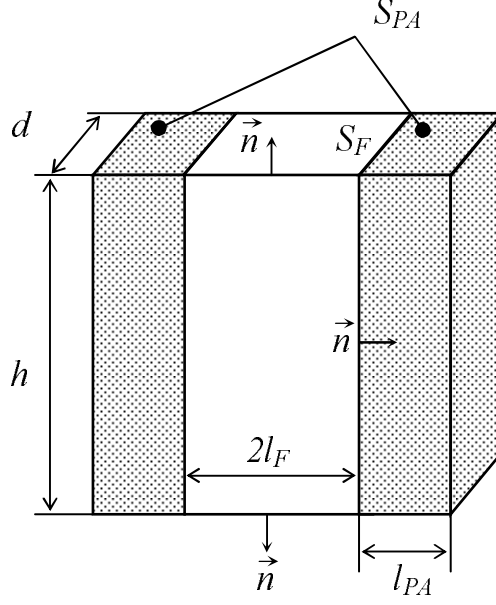


FIG. 6. Scheme of the cell representation in 2D (fluid layer sandwiched between the PMMA walls).

2. Cell head model

The main part of the head is made of electrolytic copper that can be assumed isothermal thanks to its very high thermal conductivity at this temperature. The other part is made of stainless steel, which conducts heat much worse than copper (ratio of thermal conductivities steel/copper is 0.003, see section II A). However, because of its very small thickness, slow temperature evolution and a good thermal contact with copper, it can also be assumed isothermal.

The power balance of the head can then be written in the form

$$P_H^{full} = k_{PA} \int_{(S_{PA})} \frac{\partial T_{PA}}{\partial \vec{n}} \Big|_H dS + k_F \int_{(S_F)} \frac{\partial T_F}{\partial \vec{n}} \Big|_H dS + C_H \frac{dT_H}{dt} + P_w^{full}(T_H), \quad (3)$$

where P_H^{full} is the controlled heat power supplied to the head. The first two terms at the r. h. s. correspond to the heat power transferred to the tube walls and the fluid respectively. The unit normal vector $\vec{n}$ is chosen to be directed towards the head. The third term reflects the heat inertia of the head, the head total heat capacity (measured in J/K) being denoted by C_H . The power $P_w^{full} = P_w^{full}(T_H)$ is the heat leak due to the copper wire that connects the head to the He bath at temperature $T_{He} = 2.17$ K. Due to the high thermal diffusivity of the copper, the temperature distribution in the wire can be assumed to follow immediately

the temperature of the head. This implies that

$$P_w^{full}(T) = \frac{S_w}{l_w} \int_{T_{He}}^T k_{Cu}(T) dT, \quad (4)$$

where $S_w = 0.05 \text{ mm}^2$ and $l_w = 20 \text{ cm}$ are the section area and the length of the wire. Since the copper thermal conductivity k_{Cu} does not exhibit a strong temperature dependence between 33K and 43K (where the experiments are carried out), $P_w(T_H)$ can be approximated by the linearized expression

$$P_w^{full}(T_H) \simeq P_w^{full}(T_0) + \frac{S_w}{l_w} k_{Cu}(T_0) (T_H - T_0). \quad (5)$$

Note that the constant power $P_w^{full}(T_0)$ needs to be supplied to the head both before (to maintain $T_H = T_0$) and during the experiment (as a part of P_H^{full}) to compensate the heat leaks. In the following, we will denote $P_H(t) = P_H^{full}(t) - P_w^{full}(T_0)$. It represents the *excess* of the supplied power with respect to its value at $t = 0$. In the above experiment, P_H was switched on at $t = 0$, kept at a constant level P_0 during the time Δt , and then switched off.

3. PMMA walls model

To estimate correctly the heat exchange between the fluid and the tube, it is important that the heat exchange area $2\pi R_i h$ be reproduced correctly in the simulated 2D configuration. The fluid layer is supposed to be sandwiched between two plain walls (see Fig. 6) so that the PMMA-fluid contact area is $2dh$. This condition defines the depth $d = \pi R_i$ of the sandwich. The effective fluid layer half-thickness is then $l_F = S_F/2d = R_i/2$. The effective wall thickness is $l_{PA} = S_{PA}/2d$. The heat flux and temperature are continuous at the PMMA-fluid contact area, where the normal is directed inside the PMMA:

$$\begin{aligned} k_{PA} \frac{\partial T_{PA}}{\partial \vec{n}} &= k_F \frac{\partial T_F}{\partial \vec{n}}, \\ T_{PA} &= T_F. \end{aligned} \quad (6)$$

Since the cell is in vacuum and the radiative heat transfer is negligible because of the low temperature, the zero flux boundary condition can be defined at the external surface of the cell PMMA walls. The isothermal conditions $T_{PA} = T_H$ and $T_{PA} = T_0$ are defined at the areas of contact PMMA-head and PMMA-base respectively. Notice that while T_0 remains constant, T_H is a function of time. Its evolution is calculated by using the Eqs. (4-5).

The heat conduction problem has trivial initial conditions

$$\begin{aligned}\frac{\partial \varphi}{\partial t} &= D_{PA} \nabla^2 \varphi, \\ \varphi|_{t=0} &= 0,\end{aligned}\tag{7}$$

where D_{PA} is the thermal diffusivity of the PMMA. It has to be solved for the reduced temperature $\varphi = T_{PA}(\vec{x}, t) - T_0$ (where $\vec{x}$ is the position vector) to find the variation of the temperature T_{PA} inside the PMMA. The parameters k_{PA} and D_{PA} are assumed to be constant.

4. Fluid model

Due to the symmetry, only half of the cell through a cut along the axis needs to be simulated, the zero radial flux being imposed at the cell axis. The approach [10] was used in the simulations. We summarize it below.

The fluid temperature satisfies the equation

$$\frac{\partial T_F}{\partial t} = D_F \nabla^2 T_F + g(t),\tag{8}$$

with

$$g(t) = \left(1 - \frac{c_v}{c_p}\right) \frac{d\bar{T}}{dt},\tag{9}$$

where $\bar{T} = \bar{T}(t)$ is the average fluid temperature (not to be confused with space- and time-dependent *local* fluid temperature T_F), $c_v(c_p)$ is the specific heat of the fluid at constant volume (pressure), and D_F is the fluid thermal diffusivity. The initial condition for T_F is

$$T_F|_{t=0} = T_0.\tag{10}$$

One can note that the term $g(t)$ is only relevant near the critical point where $c_p \gg c_v$. Since the cell is closed, the mean fluid density ρ in the cell remains constant. The average temperature $\bar{T}$ is defined using the fluid EOS

$$\Lambda(p, \rho, \bar{T}) = 0.\tag{11}$$

where p is the fluid pressure. To obtain realistic results in a wide temperature interval (in the experiments, the temperature varied within 20% from T_c), we have to use a realistic EOS. Only the quite complicated NIST MBWR EOS [12, 13] is available for parahydrogen. It is

unfortunately precise only relatively far from the critical point, typically for $T_0 - T_c > 0.5$ K. Discrepancies between simulation and experiments can be then expected very close to the critical point because the critical divergence of some parameters will not be properly accounted for in the simulation.

Several methods are available to simulate the thermal behavior of the cell, see [5, 15, 16]. However, with such a complicated EOS, only the approach [10] developed by some of us can be used, the others being prohibitively time consuming. The temporal evolution of the average temperature is given directly by the equation first derived in [3], which replaces the more complicated pressure evolution determination:

$$\frac{d\bar{T}}{dt} = \frac{1}{\rho v c_v} \int_A k_F \frac{\partial T_F}{\partial \vec{n}} dA, \quad (12)$$

where $v = S_F h$ is the total fluid volume. The integration is performed over the whole fluid boundary A . $\vec{n}$ is the outward normal to A . Instead of the local values for the fluid parameters (c_v , c_p , k_F , ...) we use everywhere their time-dependent spatially averaged counterparts defined. The EOS (11) is used to determine the average density ρ and the average temperature $\bar{T}$. The fluid thermal conductivity k_F is determined with a correlation implemented in the NIST REFPROP data base [13]. Eq. (12) have to be complemented with the initial condition

$$\bar{T}|_{t=0} = T_0. \quad (13)$$

5. Reduction of the fluid problem

Two more steps must be performed before the numerical procedure can be applied. First, it is convenient to introduce a new ψ variable instead of T_F :

$$\psi(\vec{x}, t) = T_F(\vec{x}, t) - T_0 - \int_{T_0}^{\bar{T}} \left(1 - \frac{c_v}{c_p}\right) d\bar{T}. \quad (14)$$

Eq. (8) reduces then to the equation

$$\frac{\partial \psi}{\partial t} = D_F \nabla^2 \psi. \quad (15)$$

One notes that since all the fluid parameters depend on the time-dependent average temperature $\bar{T}$, the fluid thermal diffusivity can be written as

$$D_F = D_d f(\bar{T}) \quad (16)$$

and is time dependent. In this equation, the function f is non-dimensional and on the order of unity and D_d is a constant. To eliminate this time dependence from D_F in Eq. (15), one makes a second step by introducing a new variable τ defined by the equations

$$\begin{aligned}\frac{d\tau}{dt} &= f(\bar{T}), \\ \tau|_{t=0} &= 0.\end{aligned}\tag{17}$$

Since $\bar{T}$ is a function of t only, the initial value of the problem is fully defined. The substitution of Eq. (17) into Eq. (15) results in a linear diffusion problem with a constant diffusion coefficient D_d

$$\begin{aligned}\frac{\partial\psi}{\partial\tau} &= D_d\nabla^2\psi, \\ \psi|_{\tau=0} &= 0.\end{aligned}\tag{18}$$

B. Numerical implementation

A linear 2D heat diffusion problem can be solved by BEM. Below we outline this method for the problem (18) that is equivalent to the following integral equation

$$D_d \int_0^\tau d\tau' \int_A \left[G(\vec{x} - \vec{x}', \tau - \tau') q(\vec{x}', \tau') - \psi(\vec{x}', \tau') \frac{\partial_x G(\vec{x} - \vec{x}', \tau - \tau')}{\partial \vec{n}} \right] d_{x'} A = \frac{1}{2} \psi(\vec{x}, \tau),\tag{19}$$

where $q(\vec{x}, \tau) = \partial\psi(\vec{x}, \tau)/\partial\vec{n}$ and G is the Green function for the infinite space:

$$G(\vec{x}, \tau) = \frac{1}{4\pi\tau D_d} \exp\left(-\frac{|\vec{x}|^2}{4\tau D_d}\right).\tag{20}$$

Eq. (19) is solved by breaking the integration contour A into straight Boundary Elements (BE's). On each of them ψ and q are assumed to be constant and equal to their values at the node point $\vec{x}_i$ in the middle of the i -th BE. The time interval $(0, \tau)$ is broken into the intervals (τ_{f-1}, τ_f) on which ψ and q are assumed to be constant and equal to ψ_{fi} and q_{fi} at the i -th BE. Eq. (19) then reduces to the set of linear algebraic equations

$$\sum_{f=1}^F \sum_{j=1}^N (q_{fj} G_{ij}^{Ff} - \psi_{fj} H_{ij}^{Ff}) = \psi_{Fi}/2,\tag{21}$$

where $i = 1 \dots N$. H_{ij} and G_{ij} are the BEM coefficients:

$$\begin{aligned}G_{ij}^{Ff} &= D_d \int_{\tau_{f-1}}^{\tau_f} dt \int_{A_j} G(\vec{x}_i - \vec{x}, \tau_f - t) d_x A, \\ H_{ij}^{Ff} &= D_d \int_{\tau_{f-1}}^{\tau_f} dt \int_{A_j} \frac{\partial G(\vec{x}_i - \vec{x}, \tau_f - t)}{\partial \vec{n}_x} d_x A.\end{aligned}\tag{22}$$

Their calculation is discussed in detail in [14]. The $2N$ variables $\{q_{Fi}, \psi_{Fi}\}$ ($i = 1 \dots N$) are related via N more equations that correspond to the boundary conditions given at each of the N BE's. The total number of equations is then $2N$ and corresponds to the total number of the unknowns. This set of equations should be solved sequentially for each time moment τ_F , where F varies from 1 to the maximum required value.

The calculation algorithm needs to be iterative to find $\bar{T}$:

1. $\bar{T}(t_{F-1})$ is taken as the initial guess for $\bar{T}(t_F)$ (T_0 is used for $F = 1$).
2. The fluid parameters are calculated for $\bar{T}(t_F)$ using the EOS; τ_F is calculated with Eqs. (17).
3. The coupled problems (7) and (18) are solved by BEM, the heat flux distribution over the fluid boundary is found.
4. It is used to determine the corrected value of $\bar{T}(t_F)$ with Eq. (12).
5. The steps 2–4 are repeated until the convergence is achieved.
6. The steps 1–5 are repeated for the next time moment.

Since the summation in Eqs. (21) has to be performed over all $f = 1 \dots F$, the described BEM formulation has a disadvantage of an increase of the calculation volume with F . On the other hand, the algorithm is very stable, which permits to choose a relatively large time step with practically no loss in accuracy.

C. Adjustment of the cell parameters

First, one needs to check if the model parameters correspond to the actual experimental values. The test consists in comparing the simulation and the experiments in an empty cell of both the evolution of the heat flux and head temperature. This simulation appears to be very sensitive to the precise value of the heat capacity of the head $C_H = m_H c_H$, where m_H and c_H are the mass and the specific heat of the head. The pieces of the head are made of stainless steel and copper, the specific heats of which at 35K being close. For simplification, we will take that of copper ($c_H = 42.6$ J/kg/K [17]). A good agreement between the calculation and the experiment (Fig. 7) is found for $m_H = 6$ g. Taking into

account that the copper part weight is 2.4 g and the stainless steel pieces are about of the same size, this m_H value is quite reasonable.

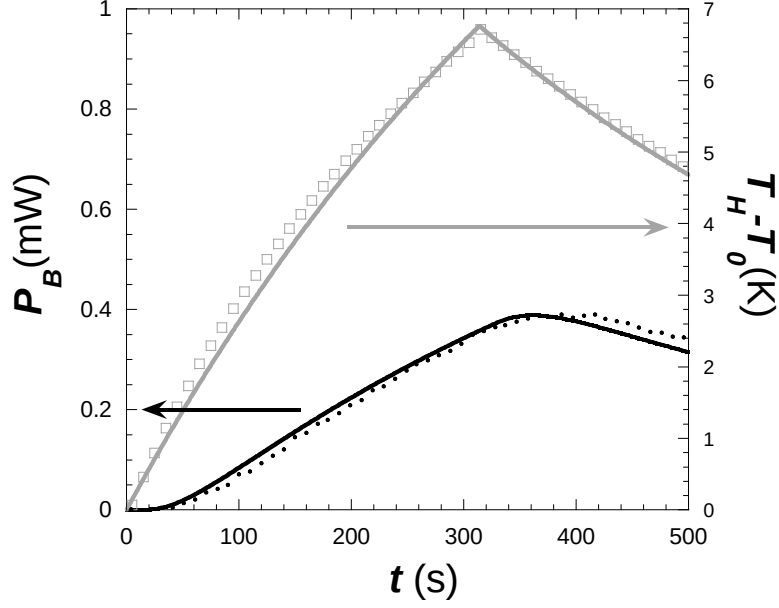


FIG. 7. Empty pipe: comparison between the simulation (lines) and the experiments (characters) for 7.5mW injected power. $T_0 = 33\text{K}$. Black line and dots: transmitted power P_B . Gray line and squares: head temperature T_H .

D. Some analytical results

1. Small time asymptotics

No heat exchange of the fluid with the tube walls is assumed in this section in order to obtain analytical results. At small times, all fluid parameters can be approximated by their values at the initial temperature, i.e. that of the base T_0 . The temperature profile in the fluid is sketched in Fig. 1c. It shows three regions: hot and cold diffusion boundary layers (HBL and CBL) that form near the extremities of the cell and a nearly isothermal bulk. The thickness of both HBL and CBL can be estimated with the expression $\delta \sim \sqrt{D_F t}$ where D_F is the fluid thermal diffusivity. This allows the main parameter, i.e. the transferred power to be found in the form

$$P_B \approx S_F k_F (T_{bk} - T_B) / \delta, \quad (23)$$

where T_{bk} means the temperature in the bulk of the fluid and k_F is the fluid thermal conductivity. $S_F (= \pi R_i^2)$ is the fluid cross-section area. Unless stated otherwise, $T_B = T_0$ and is constant. Similarly to Eq. (23), the transferred to the fluid power is

$$P_H \approx S_F k_F (T_H - T_{bk}) / \delta. \quad (24)$$

The bulk of the fluid remains isothermal, which means that $\Delta T_{bk} = 0$. By making use of the Eqs. (8, 12) one obtains

$$\frac{dT_{bk}}{dt} \approx \left(1 - \frac{c_v}{c_p}\right) \frac{P}{\rho v c_v}, \quad (25)$$

where $v = S_F h$ is the total fluid volume, $c_v(c_p)$ is the specific heat of the fluid at constant volume (pressure) and P is the total heat supplied to the fluid per unit time.

Two kinds of model problems are considered below for the sake of comparison: a constant heat flux boundary condition at the fluid-head interface, the numerical solution for which is analyzed in [10], and a more realistic problem where the heat capacity of the head controls the head temperature evolution. As previously, it is assumed that T_B is equal to the initial fluid temperature T_0 .

2. Constant heat flux boundary condition

In this case the P value is constant and one obtains from Eqs. (23, 25) the following small time asymptotics :

$$T_{bk} - T_0 \approx \left(\frac{1}{c_v} - \frac{1}{c_p}\right) \frac{P}{\rho v} t, \quad (26)$$

$$P_B \approx \left(\frac{c_p}{c_v} - 1\right) \frac{P}{h} \sqrt{t D_F}. \quad (27)$$

One notices a steep rise of the output power P_B with time [10] with $dP_B/dt \rightarrow \infty$ when $t \rightarrow 0$.

3. Heat capacity-limited regime

Let us assume that the contributions of the heat fluxes going both into the fluid and into the PMMA walls are negligible in the head heat balance Eq. (3). One can then obtain from Eqs. (3, 5)

$$P_H = C_H \frac{dT_H}{dt} + \frac{S_w}{l_w} k_{Cu} (T_H - T_0). \quad (28)$$

Since all the parameters in this equation are constant, one can infer from it the time evolution of T_H ,

$$T_H = T_0 + \frac{P_H \tau_r}{C_H} \left[1 - \exp \left(-\frac{t}{\tau_r} \right) \right], \quad (29)$$

where $\tau_r = l_w C_H / (S_w k_{Cu}) \approx 600$ s is the relaxation time. It appears that this regime applies in our case. Indeed, Fig. 8 below shows a behavior similar to (29) with a relaxation time of the order of several hundreds of seconds. This implies that if P_H were switched on for a long time in our experiment, T_H would relax to the value $T_0 + P_H \tau_r / C_H \approx 50$ K at the time scale of several τ_r . Note that this behavior is completely independent of the fluid.

The small time asymptotics ($t \ll \tau_r$) is linear

$$T_H - T_0 \approx \frac{P_H}{C_H} t. \quad (30)$$

The total heat supplied to the fluid can be approximated by the following expression (see Eqs. 23,24) :

$$P = P_H - P_B \approx S_F k_F (T_H - T_0) / \delta. \quad (31)$$

Using it and Eq. (25) one obtains the small time asymptotics

$$T_{bk} - T_0 \approx \left(\frac{c_p}{c_v} - 1 \right) \frac{P_H}{h C_H} \sqrt{D_F} t^{3/2}. \quad (32)$$

Eq. (23) then results in the linear small time behavior for P_B :

$$P_B \approx \left(\frac{c_p}{c_v} - 1 \right) \frac{P_H S_F k_F}{h C_H} t. \quad (33)$$

A comparison of Eq. (27) with Eq. (33) shows that the small time behavior of both T_H and P_B depends strongly on the boundary condition imposed at the solid-fluid boundary. While the slope dP_B/dt is infinite for the constant heat flux injection, it remains finite for the heat injector of finite heat capacity. This conforms to the experimental data. For the sample at critical density, dP_B/dt ought to increase as $\left(\frac{c_p}{c_v} \right) k_F \sim \left(\frac{T_0 - T_c}{T_c} \right)^{-0.57}$ when approaching T_c . In the experiments (Fig. 5a), a slope increase is indeed observed when approaching T_c . The increase is followed however by a saturation of the slope for $T_0 - T_c < 0.3$ K. This saturation cannot be attributed to a substantial change of the fluid properties with temperature in the bulk fluid as only the early times are concerned. As a matter of fact, this saturation is not observed in the simulation (see Fig. 9a below) where the temperature inhomogeneity is considered rigorously. We thus attribute the saturation to a slightly off-critical density of the cell.

E. Simulation results

1. Temperature dependence and scaling

A simulation is compared to the experiment performed at (i) a large distance from the critical point $T_0 - T_c = 3$ K (Fig. 8a) and (ii) at a short distance $T_0 - T_c = 50$ mK (Fig. 8b). One notes that the simulation represents relatively well the temperature evolution of the head with, however, some systematic discrepancies. These differences are also found in the transmitted flux, somewhat smaller in the simulation far from T_c , corresponding to a larger rise in T_H , and larger close to T_c , corresponding to a smaller rise in T_H . We attribute these discrepancies to the EOS which is not supposed to be correct in the close vicinity of the critical point.

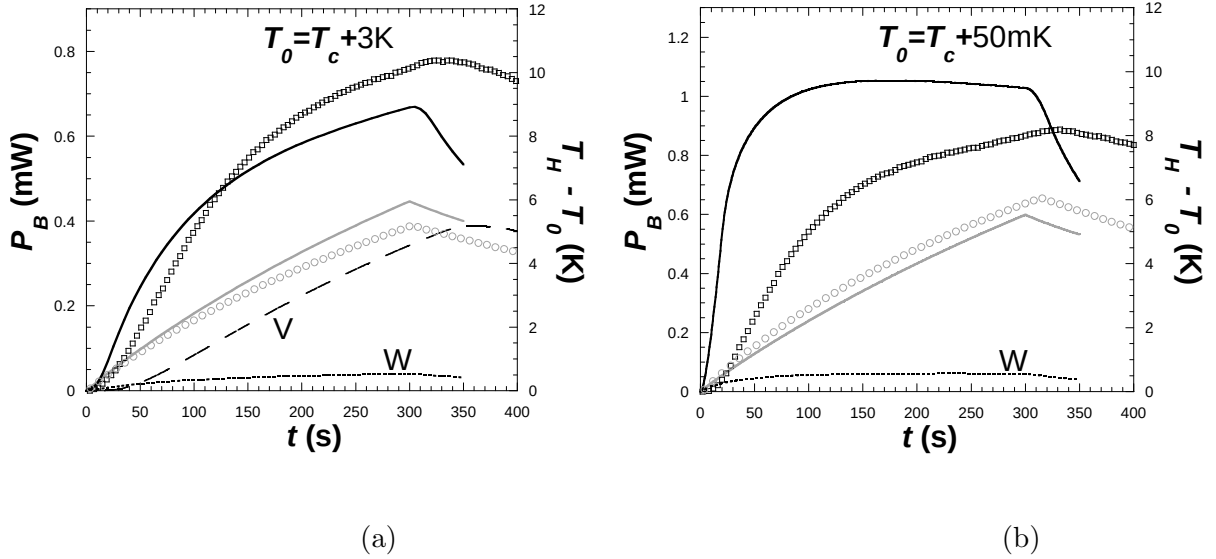


FIG. 8. Comparison between simulation and experiments concerning the evolution of the transmitted heat flux P_B (simulation: solid black line; experiment: squares) and head temperature $T_H - T_0$ (simulation: solid gray line; experiment: circles). Dashed line (V): contribution of the wall conduction for the empty cell (the same as the black curve in Fig. 7), to be compared with the dotted line (W) that represents the contribution of the wall conduction in the filled cell. (a) $T_0 = T_c + 3$ K. (b) $T_0 = T_c + 50$ mK.

For the sake of comparison, the quantities P_B^M and dP_B/dt for both the experiments and the simulations, obtained as an average slope between 27 s and 47 s (as in section II C) have also been plotted in Fig. 9. dP_B/dt evolves strongly in this time range for $T - T_c < 0.2$ K

and for this reason is not plotted. However, it is clear that the initial slope dP_B/dt grows as $T - T_c$ decreases. The saturation of P_B^M near T_c is probably related to the change of the fluid properties with temperature in the bulk fluid that occur at large times. The agreement is fair if one considers the differences in the two experimental runs and the fact that the EOS apparently overestimates the divergence of the compressibility close to T_c .

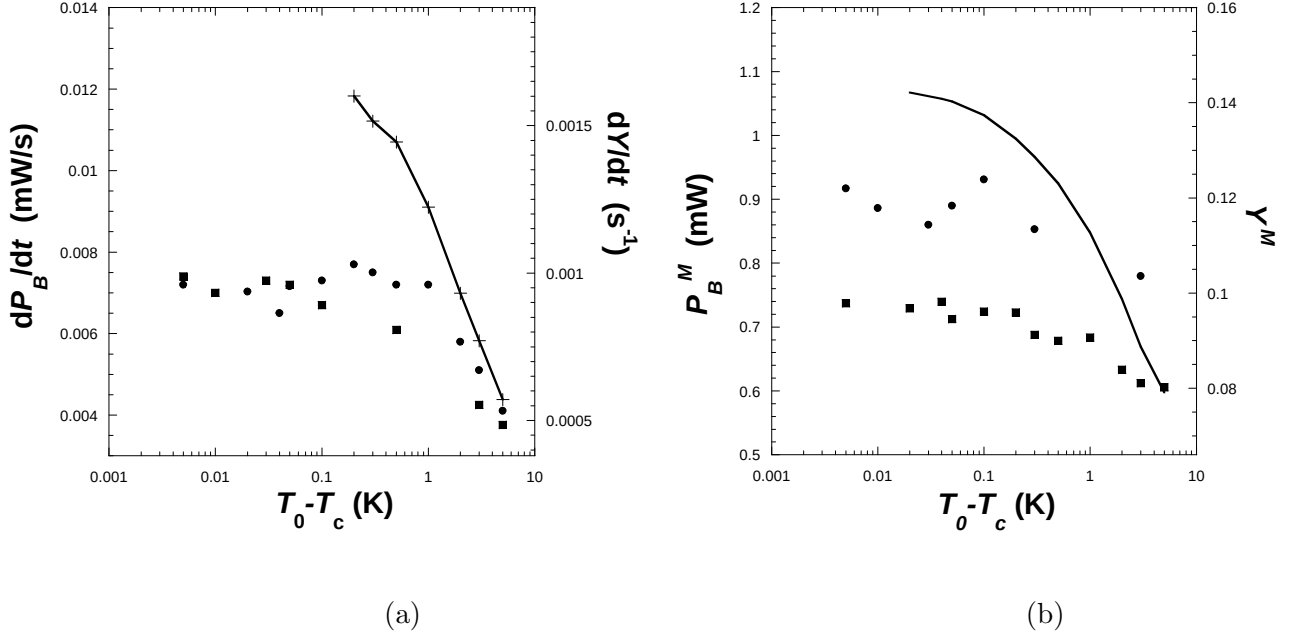


FIG. 9. Comparison (log-log plots) between simulation and experiments concerning the $T - T_c$ behavior of: (a) the transmitted heat flux initial rise dP_B/dt and reduced yield rise dY/dt (see text). Simulations: solid line and crosses. Experiments: Run 1: circles, run 2: squares. (b): maximum heat transmission P_B^M and Y^M (see text). Simulations: line. Experiments: Run 1: circles, run 2: squares.

2. Length dependence

We consider now the length dependence of the flux yield. The efficiency of the bulk heating produced by the Piston effect is inversely proportional to the fluid volume v , see Eq. (25). According to Eq. (33), P_B (and thus Y) should be inversely proportional to h at small times. Although it is obvious that the maximum of yield should decrease with increasing pipe length, it is difficult to obtain the analytical dependence on h at long evolution times and a numerical simulation is necessary. The simulations are carried out for a single temperature

$T_c + 3$ K and different multiples of 16.5 mm: ($\times 2$: 33 mm; $\times 5$: 82.5 mm; $\times 10$: 165 mm). The results are reported in Fig. 10. The maximum yield is seen to decrease according to a law close to logarithmic $Y^M = Y_0 \ln(h_M/h)$, with $Y_0 = 0.0267 \pm 0.0006$ and $h_M = 475 \pm 23$. The reason for such a law is unclear.

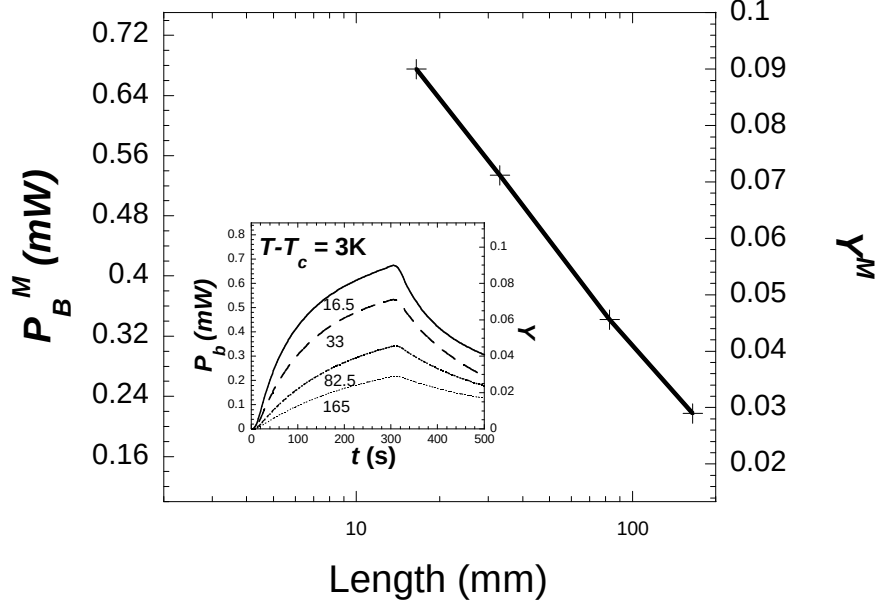


FIG. 10. Variation at $T_c + 3$ K of the maximum heat flux P_B^M and corresponding yield Y^M with respect to cell length (semi-log scale) for 7.5 mW injected power. In the insert is shown the evolution for the different lengths.

3. Generalization to CO_2

The simulation realized for the same parameters can be applied to another fluid, CO_2 , whose critical temperature is $T_c = 304$ K. The restricted cubic parametric EOS [18] has been used to describe CO_2 . For the sake of comparison, the material parameters of the cell solid cell components were chosen to be the same as for the experimental cell at 30 K. The simulation is carried out for 4 temperature values ($T_0 = T_c + 5$ K, 3 K, 1 K, 0.5 K), which correspond to about the same reduced temperatures $(T_0 - T_c)/T_c$ as H_2 at $T_c + 0.5$ K, 0.3 K, 100 mK and 50 mK. The results are shown in Fig. 11. The yield is larger with respect to that of H_2 . In particular, it is on order 30% at $T_0 - T_c = 1$ K, whereas it is about 10% for H_2 at $T_0 - T_c = 0.11$ K, which corresponds to the same reduced temperature (0.033). This can

be explained by the fact that the same power causes a smaller rise in reduced temperature so that the ratio: (fluid heat capacity)/(head heat capacity) is larger for CO₂ than for H₂.

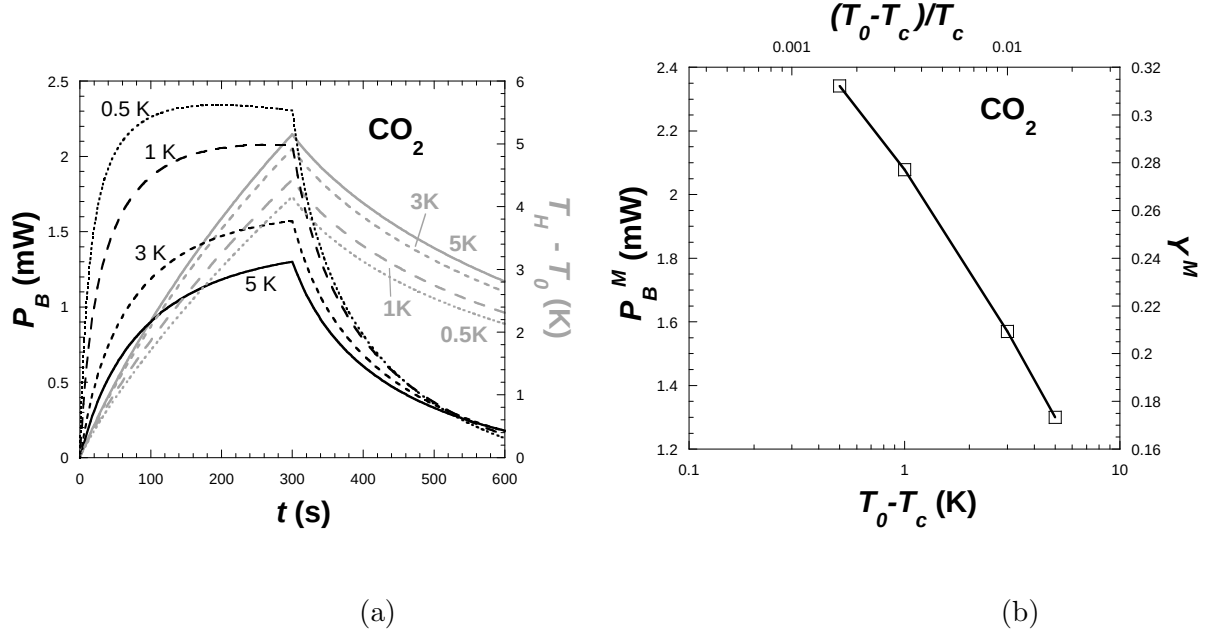


FIG. 11. (a) CO₂ heat pipe simulation for 7.5 mW injected power. (a) Evolution of the transmitted heat flux P_B (black curves) and head temperature $T_H - T_0$ (gray curves) for different base temperatures: $T_0 - T_c = 5$ K (solid line); 3 K: broken line, short dashes; 1 K: broken line, long dashes; 0.5 K: dotted line). (b): Semi-log plot of the maximum heat flux P_B^M and corresponding yield Y^M with respect to $T_0 - T_c$ (lower abscissa) or to the reduced temperature $(T_0 - T_c)/T_c$ (upper abscissa).

IV. CONCLUSION

This study aimed to determine whether the piston effect in near critical fluids could be used to transport heat at large distance, as heat pipes do. The yield defined as the ratio of the injected and transported heat power, does not exceed 10-30%. It also decreases with the tube length (nevertheless, with a slow logarithmic decay). An interesting transport property, however, has been found. Any change in injected power is transmitted very rapidly, with a rate that is only limited by the heat capacity of the cell. Although the experiments (and most simulations) were performed with H₂, the conclusions are applicable to any other fluid. In particular to CO₂, which was studied numerically. This fast heat change transmission

remains valid in a large temperature range around the critical point (10 K in H_2 , 100 K in CO_2). This means that a tube filled with such (appropriately chosen according to the temperature range) fluid can be used as a thermal link installed in parallel with a conventional heat pipe. Such a tube filled with fluid is an efficient thermal link during fast changes of the applied power.

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