Superconductivity

S2634: Physique de la matière condensée & nano-objets

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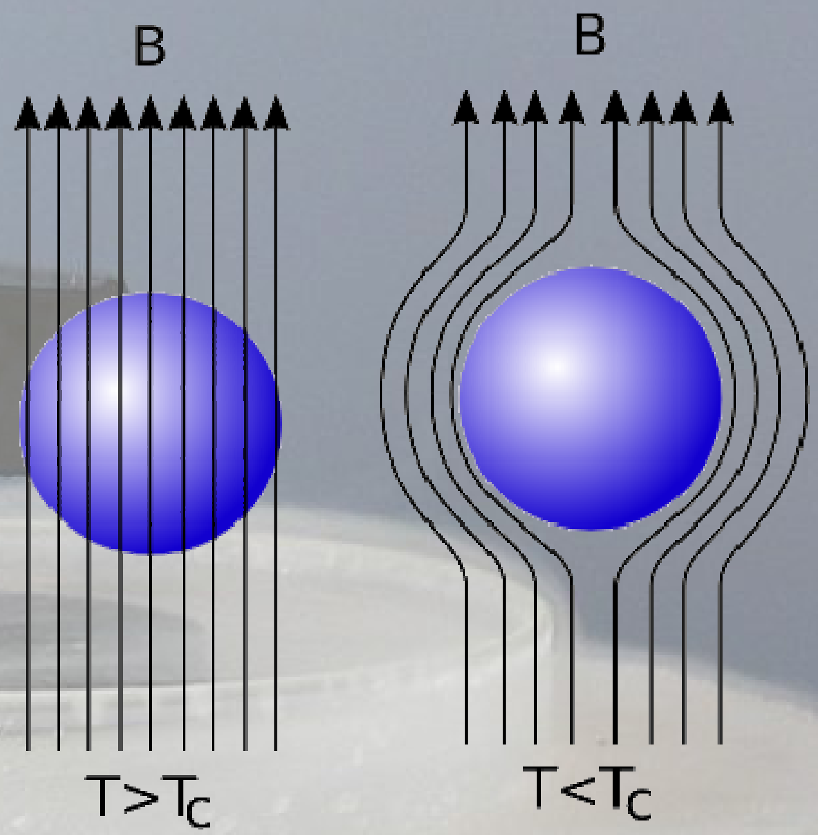
What is superconductivity?

Superconductivity

- Superconductivity generally occurs at very low temperatures.
- In this state the materials have exactly zero electrical resistivity.
- As the material drops below its superconducting critical temperature, magnetic fields within the material are (totally or partially) expelled.
- Superconductivity occurs in a wide variety of materials, including simple elements like tin and aluminium, various metallic alloys and some heavily-doped semiconductors.

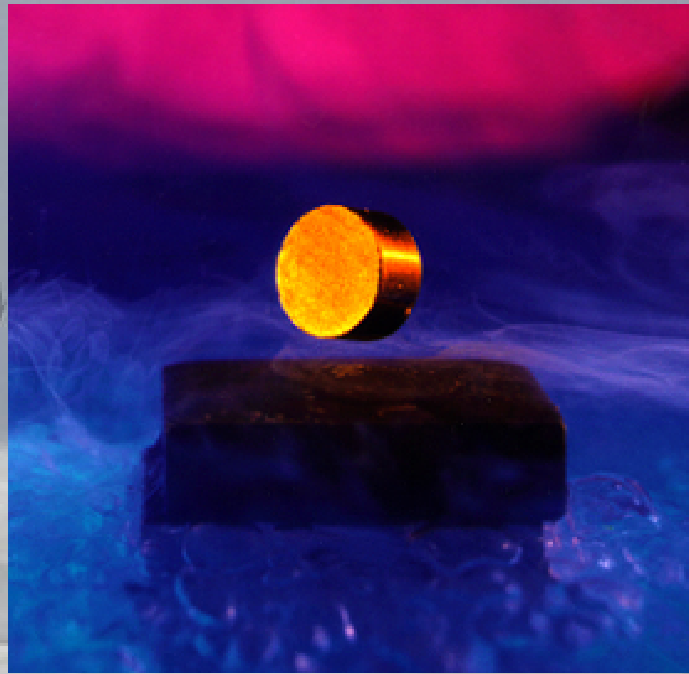


Meissner effect

- When a superconductor is placed in a weak external magnetic field, the field penetrates the superconductor only at a small distance, called the London penetration depth, decaying exponentially to zero within the bulk of the material.

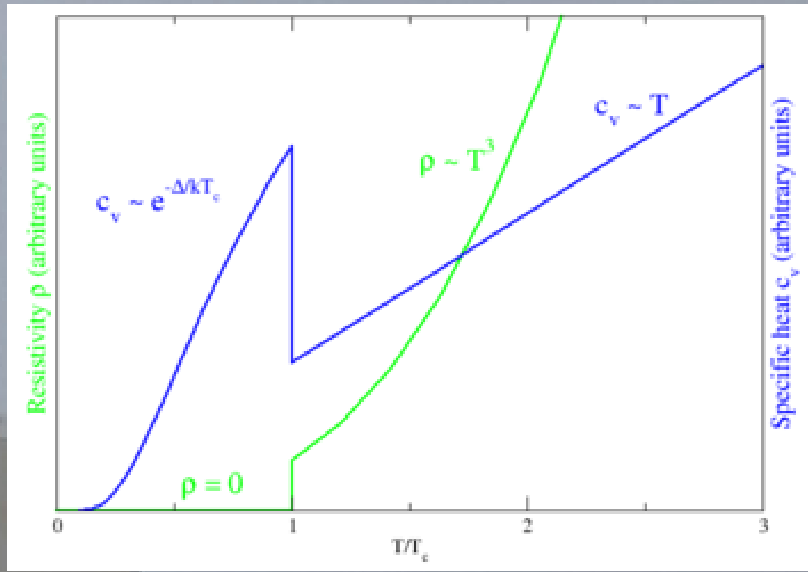
$$\chi\big|_{bulk} = -1$$

- This is called the Meissner effect, and is a defining characteristic of superconductivity.
- For most superconductors, the London penetration depth is on the order of 100 nm.

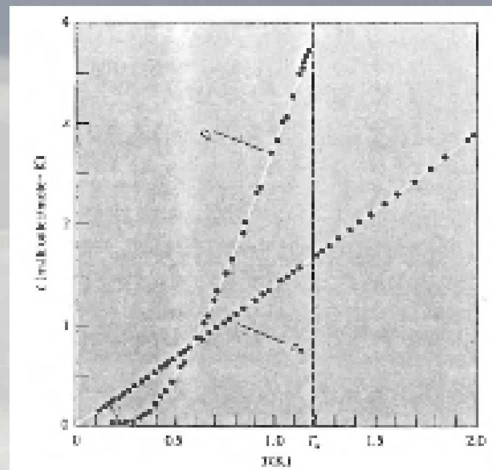


Phase transition

- In superconducting materials, the phase transition appear when the temperature T is lowered below a critical temperature T_c. The value of this critical temperature varies from material to material
- Superconductors usually have critical temperatures below 20 K (down to less than 1 K)
- Cuprate superconductors can have much higher critical temperatures. Mercury-based cuprates have been found with critical temperatures in excess of 130 K

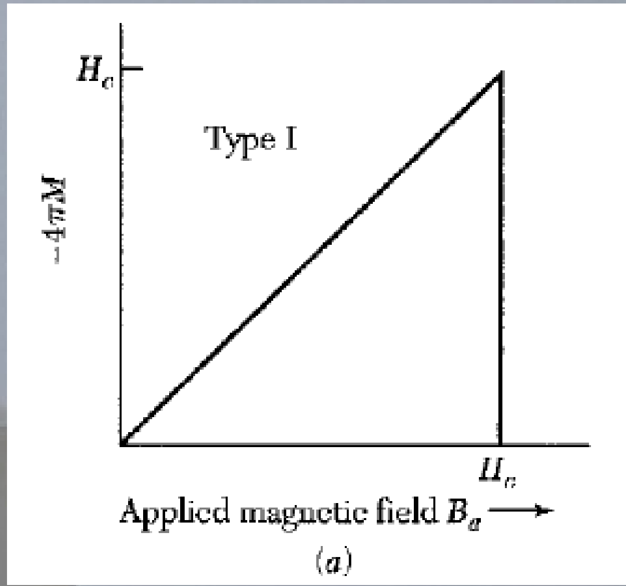


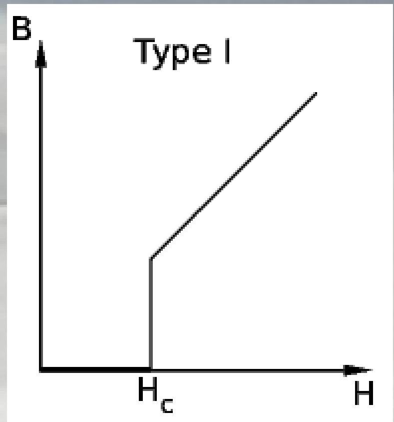
$BaPb_{0.75}Bi_{0.25}O_3$	$T_c = 12$ K	[BPBO]
$La_{1.85}Ba_{0.15}CuO_4$	$T_c = 36$ K	[LBCO]
$YBa_2Cu_3O_7$	$T_c = 90$ K	[YBCO]
$Tl_2Ba_2Ca_2Cu_3O_{10}$	$T_c = 120$ K	[TBCO]
$Hg_{0.8}Tl_{0.2}Ba_2Ca_2Cu_3O_{8.33}$	$T_c = 138$ K	



Type I superconductors

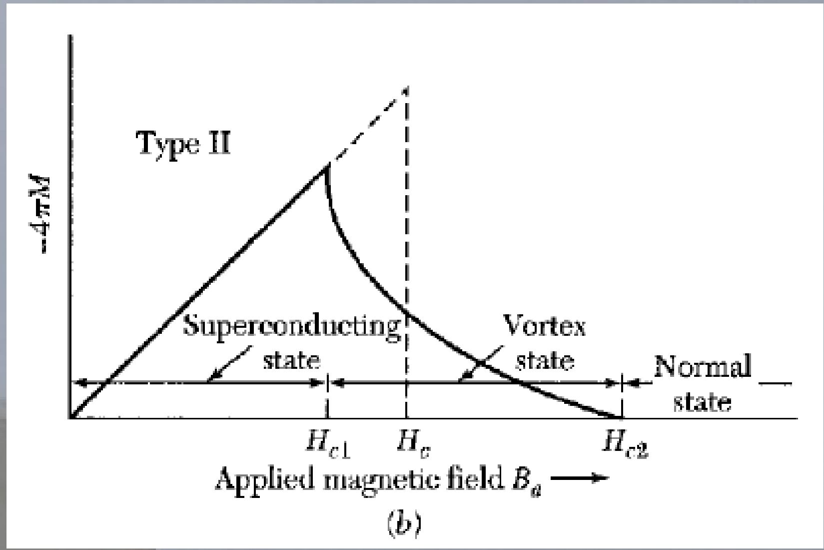
- Type I superconductors are superconductors that cannot be penetrated by magnetic flux lines (complete Meissner effect). As such, they have only a single critical temperature at which the material ceases to superconduct, becoming resistive. Elementary metals, such as aluminium, mercury and lead behave as typical Type I superconductors below their respective critical temperatures

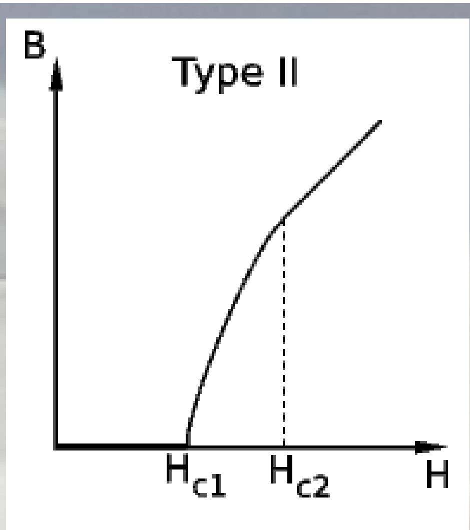




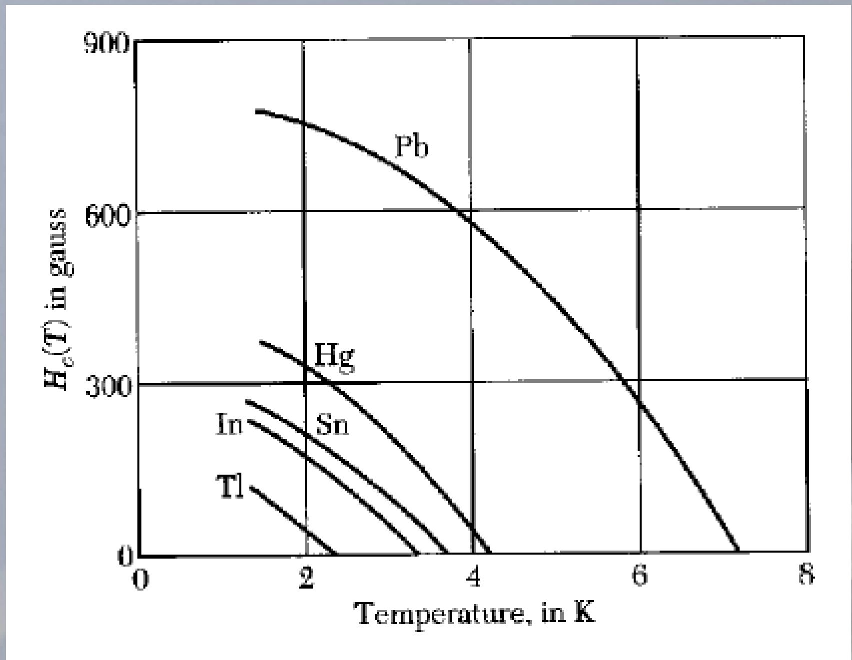
Type II superconductors

- A Type-II superconductor is a superconductor characterised by its gradual transition from the superconducting to the normal state within an increasing magnetic field
- Typically they superconduct at higher temperatures and magnetic fields than Type-I superconductors. This allows them to conduct higher currents
- Type-II superconductors are usually made of metal alloys or complex oxide ceramics





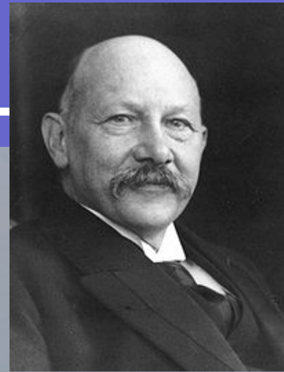
Critical Field vs Transition Temperature



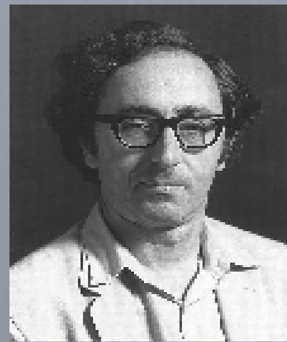
$$\frac{H_c(T)}{H_c(0)} \approx 1 - \left(\frac{T}{T_c}\right)^2$$

History of Superconductivity

- 1911 - Discovery of superconductivity by Heike Kamerlingh Onnes, while studying the resistance of solid mercury at cryogenic temperatures using liquid helium.
- 1933 - Meissner and Ochsenfeld discover the Meissner Effect.
- 1950 - Ginzburg-Landau theory.
- 1957 - Complete microscopic theory of superconductivity proposed by Bardeen, Cooper, and Schrieffer (1972 Nobel Prize winners in Physics).
- 1962 – Josephson Effect



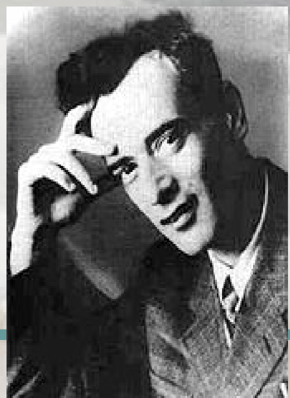
Kamerlingh Onnes



Josephson

Ochsenfeld

Meissner



Landau

Ginzburg

London theory (1935)

- Macroscopic theory

- Fundamental facts :
  - When $T<T_c$ supercurrent flows without resistance
  - **B**=0 inside the superconductor

London equation

- The electrons are freely accelerated by the electric field :

$$m\frac{d\mathbf{v}_s}{dt} = -e\mathbf{E}.$$

- Supercurrent is given by : $\mathbf{j} = -e\mathbf{v}_s n_s,$

- Therefore : $\frac{d}{dt}\mathbf{j} = \frac{n_s e^2}{m}\mathbf{E}.$

- Substituting the previous into the equation for Faraday's law :

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t},$$

- We obtain a first equation :

$$\frac{\partial}{\partial t}\left(\nabla \times \mathbf{j} + \frac{n_s e^2}{mc}\mathbf{B}\right) = 0.$$

London equation

- But this does not immediately imply **B**=0.
  Let us assume that : $\nabla \times \mathbf{j} = -\frac{n_s e^2}{mc}\mathbf{B},$
- Using : $\nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{j}.$
- We find :

$$\nabla^2\mathbf{B} = \mathbf{B}/\lambda_L^2 \ .$$

which is seen to account for the Meissner effect, since it does not allow a spatially uniform solution

- We call $\lambda_L = (mc^2/4\pi n q^2)^{1/2}$ the penetration depth, which is of the order of 100nm

Landau-Ginzburg theory (1950)

- Phenomenological theory
- Based on Landau's theory of second order phase transitions
- Describes the phenomenon when T~Tc
- The idea is to find an expression for the free energy introducing a so called "order parameter" $|\psi|^2 = n_s/2.$

Expression of the free energy

- Free energy in a metal with cubic symmetry when there is no electrmagnetic field :

$$F = F_n + \int \left\{ \frac{\hbar^2}{4m} |\nabla\psi|^2 + a|\psi|^2 + \frac{b}{2}|\psi|^4 \right\} dV.$$

- When there is an electromagnetic field, jauge invariace imposes :

$$F = F_{n0} + \int \left\{ \frac{\mathbf{B}^2}{8\pi} + \frac{\hbar^2}{4m} \left| \left(\nabla - \frac{2ie}{\hbar c}\mathbf{A} \right) \psi \right|^2 + a|\psi|^2 + \frac{b}{2}|\psi|^4 \right\} dV$$

- Next we have to minimize this expression. We perform variations with respect rot are 3 parameters : $\psi, \psi^*, \text{ and } \vec{A}$

Landau-Ginzburg equations

- First equation:

$$\frac{1}{4m}\left(-i\hbar\nabla-\frac{2e}{c}\mathbf{A}\right)^2\psi+a\psi+b|\psi|^2\psi=0$$

- Second equation:

$$\mathbf{j}=-\frac{ie\hbar}{2m}(\psi^*\nabla\psi-\psi\nabla\psi^*)-\frac{2e^2}{mc}|\psi|^2\mathbf{A}$$

- We recall Ampère's law:

$$\text{rot}\,\mathbf{B}=\frac{4\pi}{c}\mathbf{j},$$

- When the field is weak, $|\psi|^2=-\frac{a}{b}$, and we find the London equation:

$$\nabla\times\mathbf{j}=-\frac{n_s e^2}{mc}\mathbf{B},$$

Coherence length

- We can express the London penetration depth in terms of *a* and *b* :

$$\delta = \left[\frac{mc^2 b}{8\pi e^2 |a|} \right]^{1/2}$$

- In GL theory, another parameter describing a superconductor states appears : **the coherence length**. It marks the fluctuation of ψ :

$$\xi(T) = \frac{\hbar}{2(m|a|)^{1/2}}$$

- The ratio : $\varkappa = \dfrac{\delta(T)}{\xi(T)}$

determines whether a superconductor behaves as type I or type II (when it is less or greater than $1/\sqrt{2}$ respectively).

BCS theory

- BCS: microscopic theory of superconductivity
  - Requires a net attractive interaction between electrons in the neighborhood of the Fermi surface – Cooper pairs
  - Cooper pair wave functions are singlets with a symmetric, translationally invariant orbital part.
  - By a variational calculation, the spatial range of the wavefunction is shown to be several orders of magnitude larger than the average spacing between electrons.
  - Central feature: an energy gap exists between the energy of the cooper pairs in the BCS electronic ground state and the “single particle” excited states.
- BCS theory is widely applicable and tremendously predictive
  - From He^3 in its condensed state to type I and type II metallic and high-temperature superconductors
  - Critical field, thermal properties, and most magnetic properties are consequences of the energy gap.

BCS theory: Quantitative Predictions

- Transition temperature: $k_B T_c = 1.13\hbar\omega e^{-1/N_0 V_0}$
- Energy gap: $\Delta(0) = 2\hbar\omega e^{-1/N_0 V_0}$
- Specific Heat:

$$\frac{\Delta(0)}{k_B T_c} = 1.76.$$

$$\frac{c_s}{\gamma T_c} = 1.34\left(\frac{\Delta(0)}{T}\right)^{3/2} e^{-\Delta(0)/T}$$

- Magnetic flux through a superconductor ring is quantized, and the flux quantum (fluxoid) is given by:

$$\Phi_0 = 2\pi\hbar c/2e \cong 2.0678 \times 10^{-7} \text{ gauss cm}^2$$

- Duration of persistent currents "practically infinite" (in most cases the expected time required for one fluxoid to escape is calculated to be larger than the age of the universe)

BCS theory: Flux quantization

- GL formula for the superconducting current density:

$$\mathbf{j} = -\frac{e}{2m}\left[\psi^*\left\{\left(\frac{\hbar}{i}\nabla + \frac{2e}{c}\mathbf{A}\right)\psi\right\} + \left\{\left(\frac{\hbar}{i}\nabla + \frac{2e}{c}\mathbf{A}\right)\psi\right\}^*\psi\right]$$

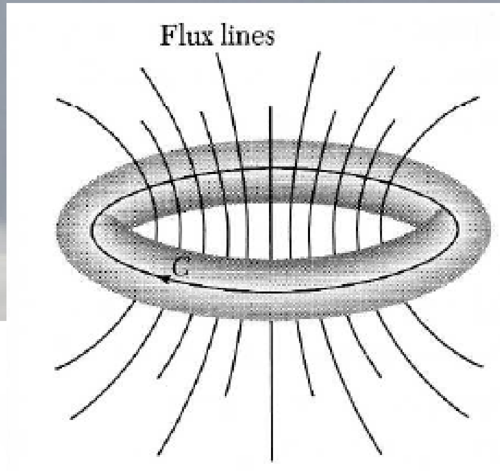
- We suppose that the spatial variation of ψ is contained in its phase:

$\psi = |\psi|e^{i\phi}$ Therefore: $\mathbf{j} = -\left[\frac{2e^2}{mc}\mathbf{A} + \frac{e\hbar}{m}\nabla\phi\right]|\psi|^2$

$$0 = \oint \mathbf{j}\cdot d\boldsymbol{\ell} = \oint\left(\frac{2e^2}{mc}\mathbf{A} + \frac{e\hbar}{m}\nabla\phi\right)\cdot d\boldsymbol{\ell}.$$

$$\int \mathbf{A}\cdot d\boldsymbol{\ell} = \int \nabla\times\mathbf{A}\cdot d\mathbf{S} = \int \mathbf{B}\cdot d\mathbf{S} = \Phi,$$

$$\oint \nabla\phi\cdot d\boldsymbol{\ell} = \Delta\phi = 2\pi n. \Rightarrow |\Phi| = \frac{nhc}{2e} = n\Phi_0.$$



BCS theory: Type II SupC's

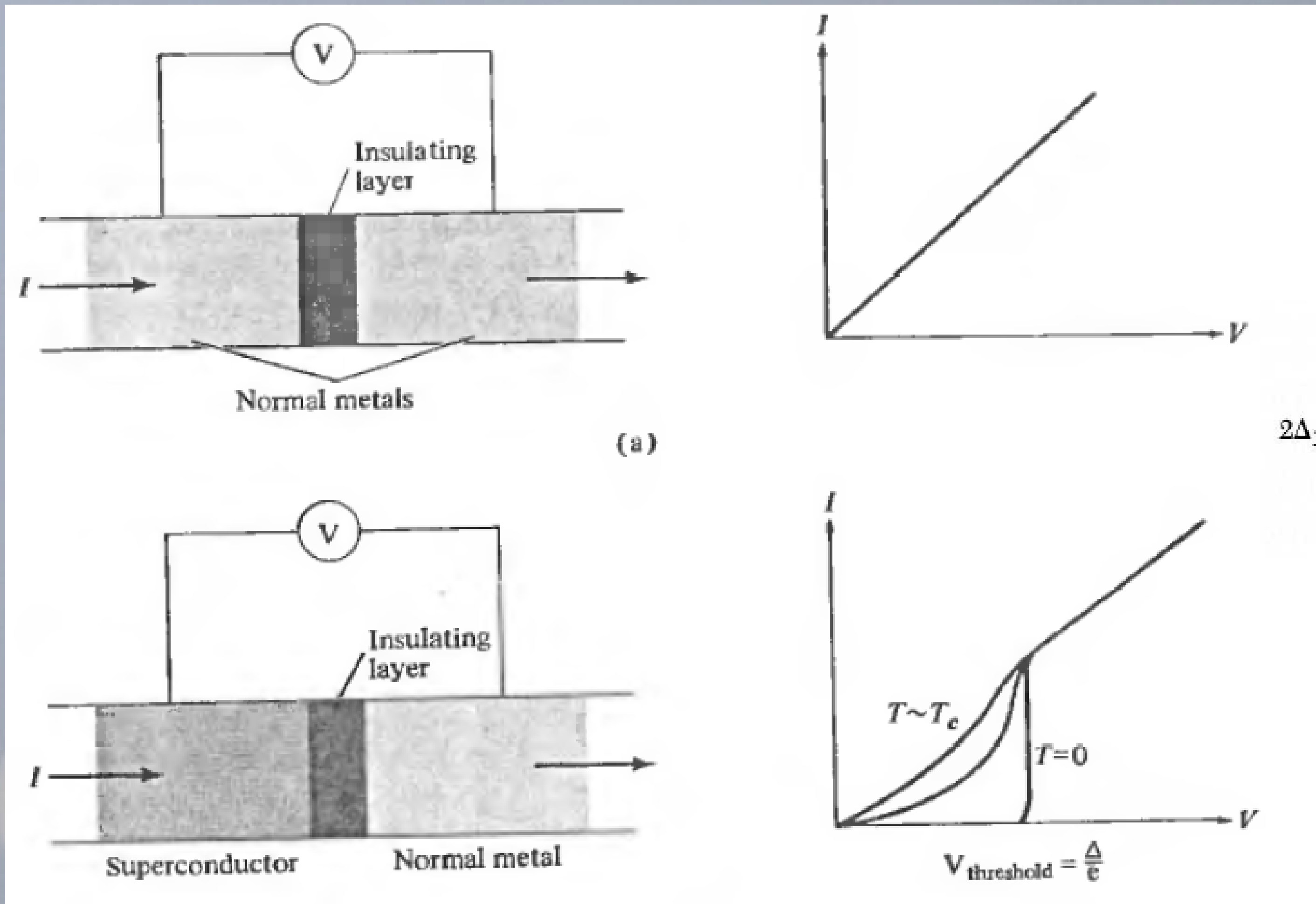
- How do we explain the existence of type II superconductors?
- Theory of type II superconductors developed by G&L + Abrikosov & Gorkov
- Vortex State : Stable configurations of a superconductor in a magnetic field with rods or plates in the normal state surrounded by the superconducting region.
- From this model (which is verified experimentally) one can estimate the lower and upper critical fields.

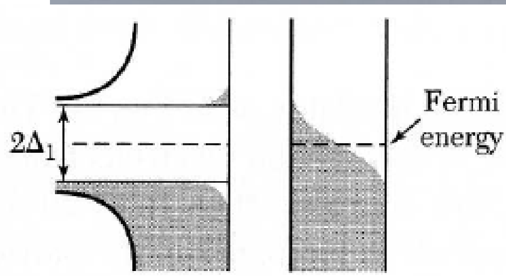
$$H_{c1} \simeq \Phi_0/\pi\lambda^2 \qquad H_{c2} \approx \Phi_0/\pi\xi^2$$

- Finally, from thermodynamical stability considerations:

$$(H_{c1}H_{c2})^{1/2} \approx H_c$$

BCS Theory:
Single particle tunelling



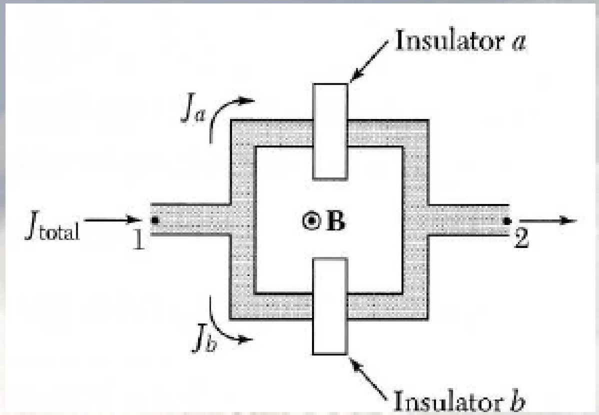


BCS Theory: Josephson tunelling

- Semiconductors separated by an insulator junction
- Due to electron-pair interaction accross the junction
  - **DC Josephson Effect :** DC current accross the junction with no applied voltage.
  - **DC Josephson Effect :** A DC voltage gives rise to current oscillations accross the junction.

$$\omega = 2eV/\hbar$$

  - **Macroscopic quantum interference:** Total current shows interference effects as a function of the applied magnetic flux. Used to design sensitive magnetometers.

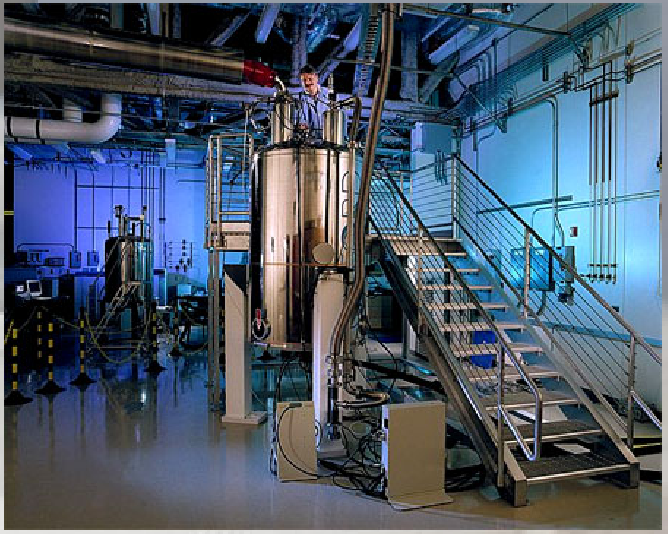


Current maxima at :

$$e\Phi/\hbar c = s\pi \ , \quad s = \text{integer}$$

Applications

- Superconductors are used in NMR machines
- Mass spectrometers
- The beam-steering magnets used in particle accelerators
- Superconductors are used to build Josephson junctions, which are the building blocks of SQUIDs the most sensitive magnetometers known.
- The large resistance change at the transition from the normal to the superconducting state is used to build thermometers in cryogenic micro-calorimeter photon detectors.
- Josephson junction can be used as a photon detector



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Questions?