Inhomogeneous quantum turbulence in thermal counterflow

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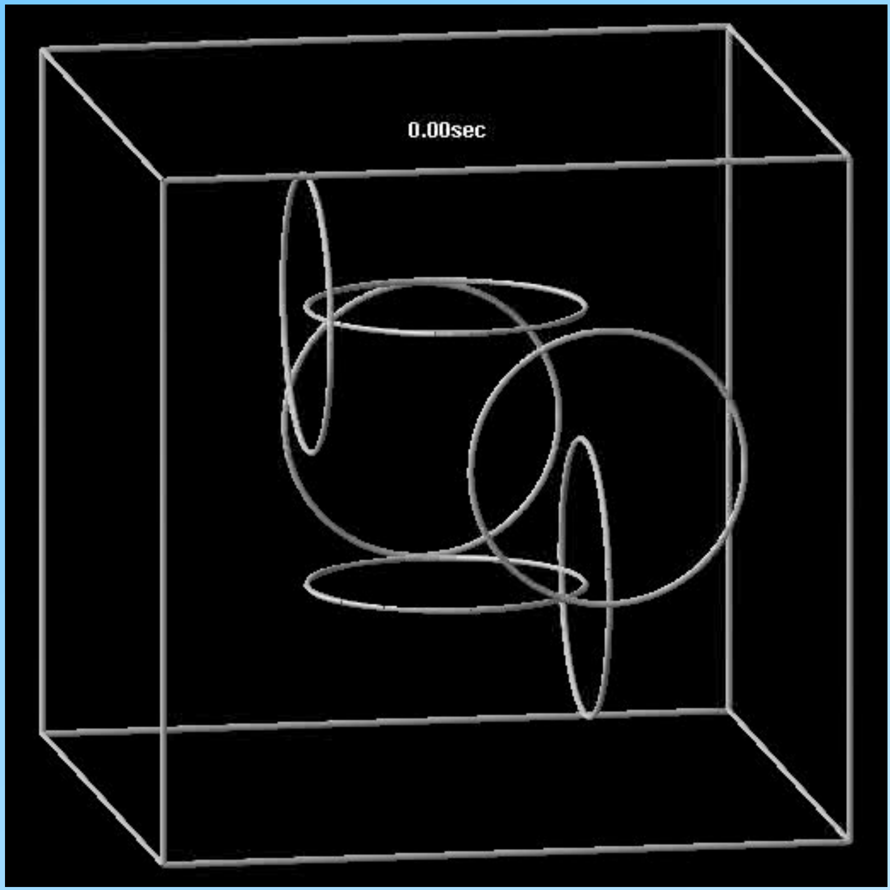
Thanks to Wei Guo, W. F. Vinen, Y. Tsuji

1. Introduction
2. Previous simulation for the homogeneous normal fluid flow (1980's-2010)
3. Recent visualization experiments (2006-)
4. The new simulation for the inhomogeneous normal fluid flow (2013-)

Main messages of my talk (1)

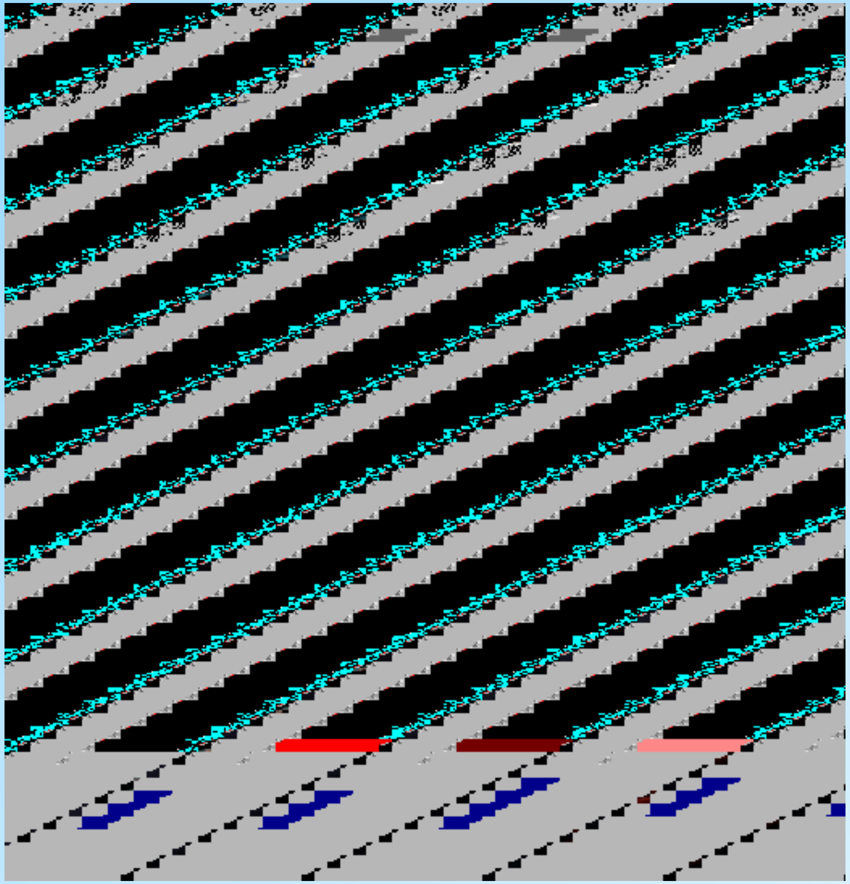
We revealed inhomogeneous QT in a square channel.

Homogeneous QT



H. Adachi, S. Fujiyama, M. Tsubota, Phys. Rev. B81, 104511(2010) .

Inhomogeneous QT

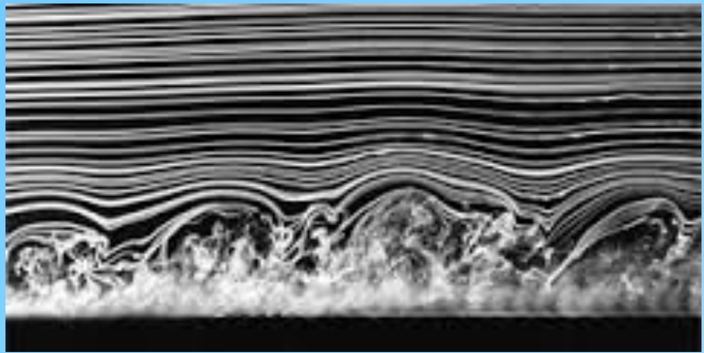


S. Yui, M. Tsubota, Phys. Rev. B91, 184504(2015).

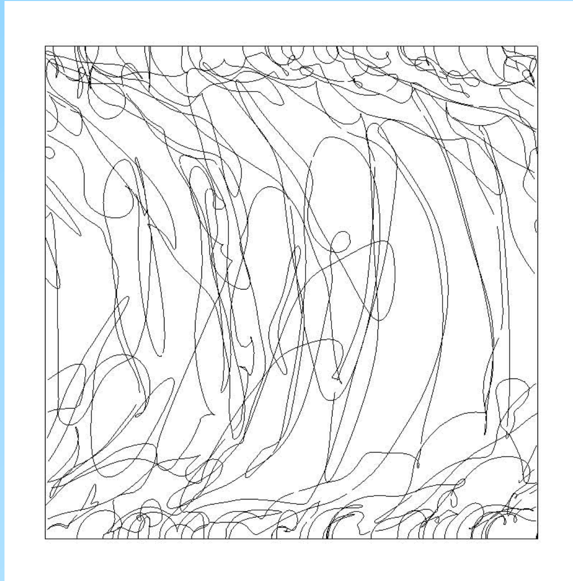
Main message of my talk (2)

S. Yui, K. Fujimoto, M. Tsubota,
arXiv:1508.01347

We study boundary layer of QT and found the well-known log-law.



y



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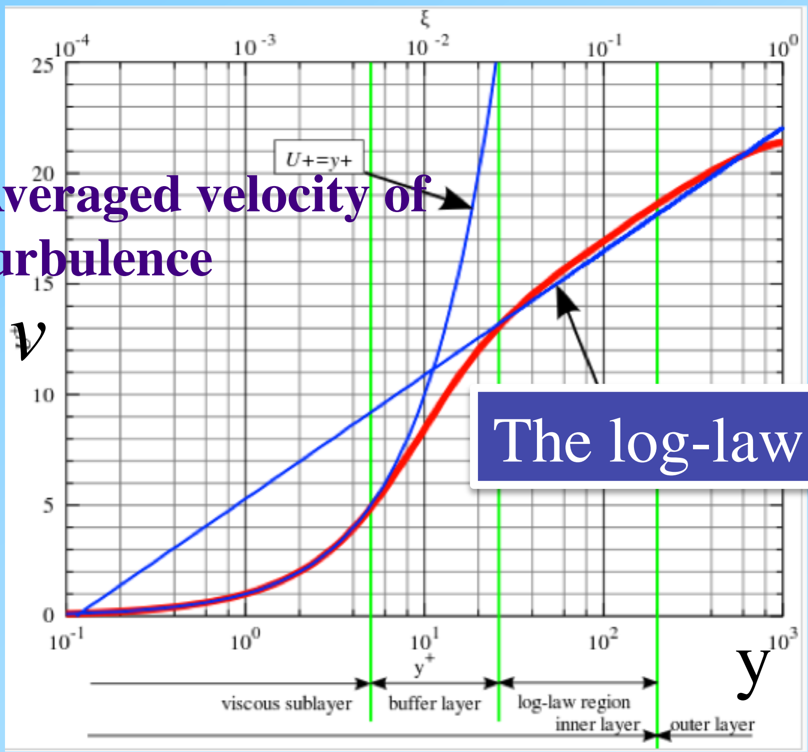


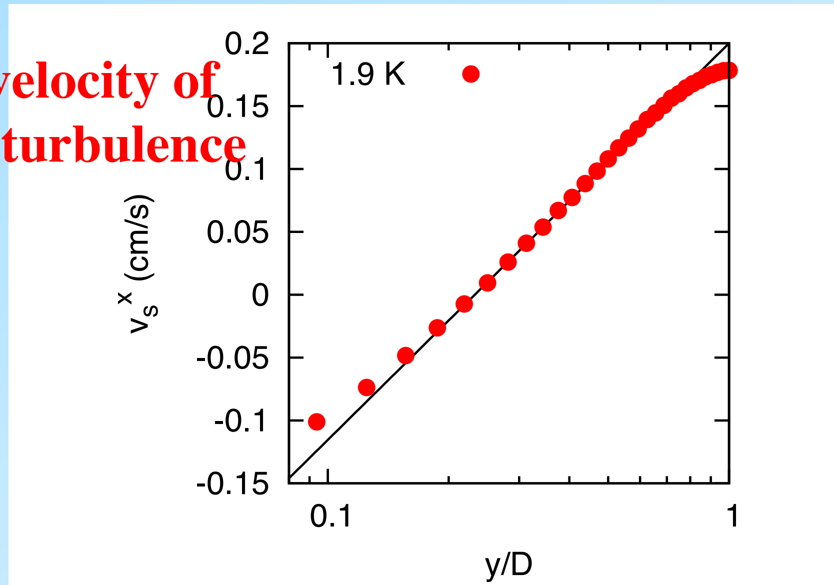
Averaged velocity of turbulence

v

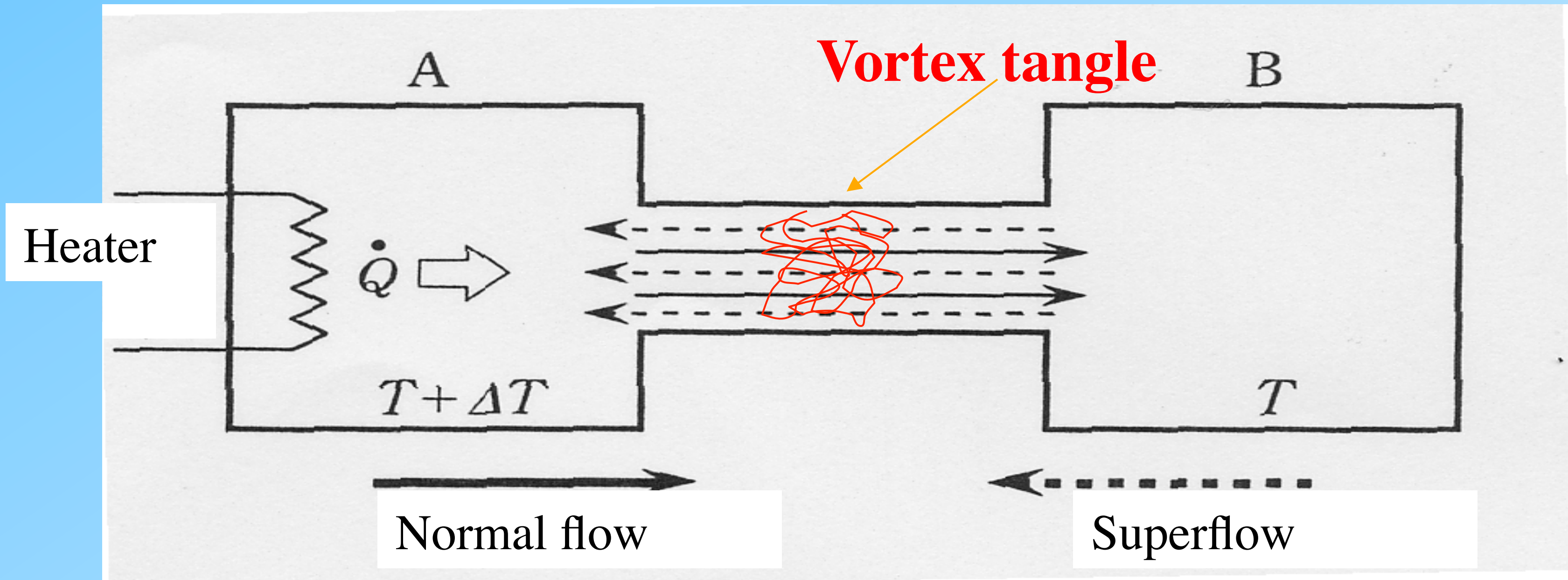
y

The log-law

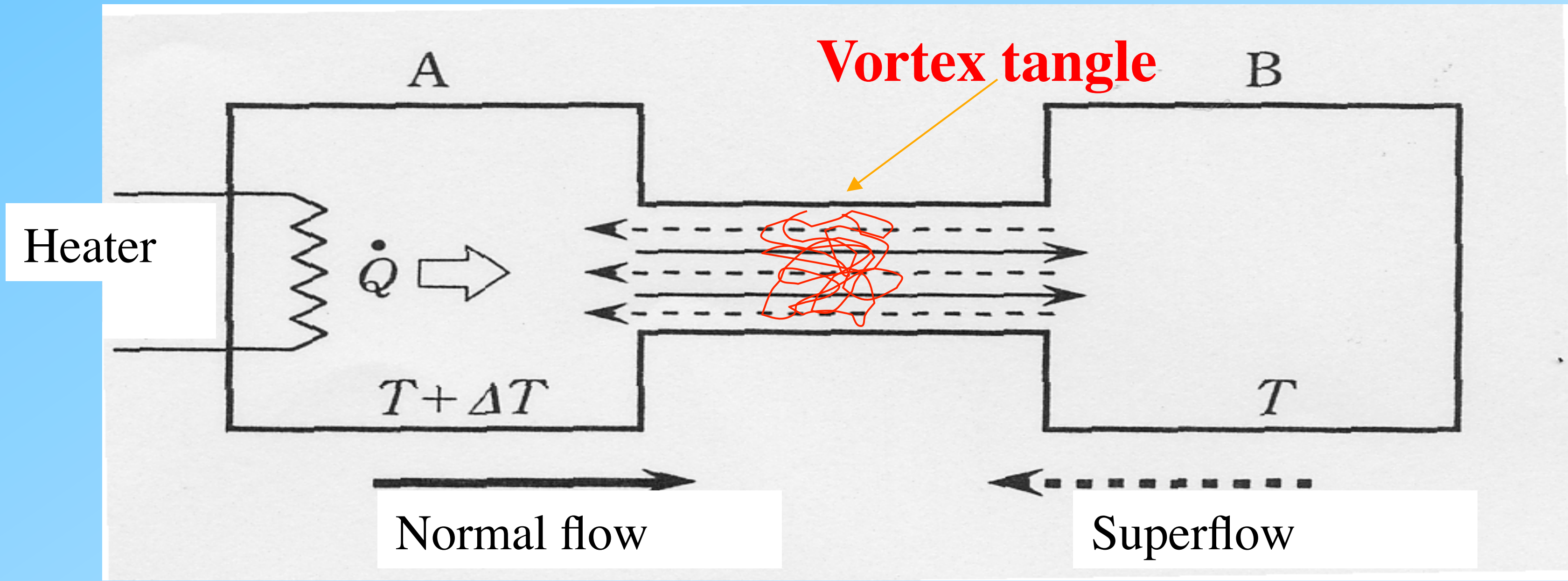
Averaged velocity of superfluid turbulence



1. Introduction



QT has been long studied chiefly in thermal counterflow.

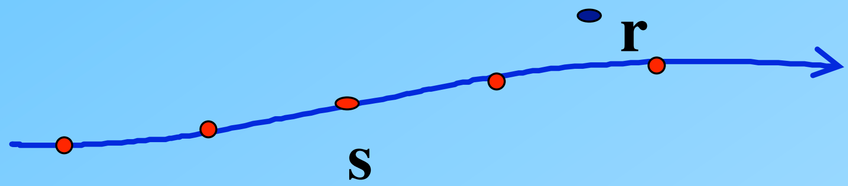


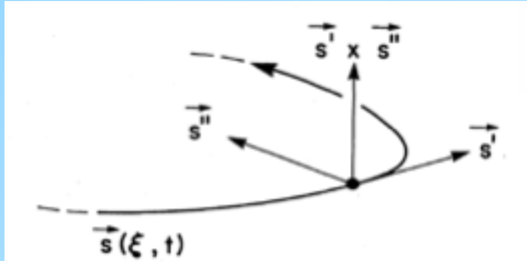
1980s **K. W. Schwarz** Phys. Rev. B38, 2398 (1988)

Performed a direct numerical simulation of the three-dimensional dynamics of quantized vortices and succeeded in quantitatively explaining the observed temperature difference *ΔT* .

2. Previous simulation for the homogeneous normal fluid flow

Vortex filament model (VFM)





A vortex makes the superflow of the Biot-Savart law, and moves with this local flow. At a finite temperature, the mutual friction should be considered.

$$\dot{\mathbf{s}}_0 = \frac{\beta}{4\pi}\mathbf{s}' \times \mathbf{s}'' + \frac{\kappa}{4\pi}\int_L' \frac{(\mathbf{s}_1 - \mathbf{r}) \times d\mathbf{s}_1}{|\mathbf{s}_1 - \mathbf{r}|^3} + \mathbf{v}_{s,a}(\mathbf{s})$$

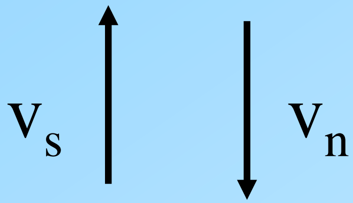
$$\dot{\mathbf{s}} = \dot{\mathbf{s}}_0 + \alpha\,\mathbf{s}' \times (\mathbf{v}_n - \dot{\mathbf{s}}_0) - \alpha'\mathbf{s}' \times [\mathbf{s}' \times (\mathbf{v}_n - \dot{\mathbf{s}}_0)]$$

The approximation neglecting the nonlocal term is called the LIA(Localized Induction Approximation).

$$\dot{\mathbf{s}}_0 = \frac{\beta}{4\pi}\mathbf{s}' \times \mathbf{s}'' + \mathbf{v}_{s,a}(\mathbf{s})$$

Simulation for the homogeneous normal fluid flow

Simulation under LIA



K. W. Schwarz, Phys. Rev. B38, 2398 (1988).
-Obtained a statistically steady state by the vortex filament model (VFM) under the localized induction approximation (LIA).

Periodic boundary conditions for all three directions

Schwarz's simulation(1) PRB38, 2398(1988)

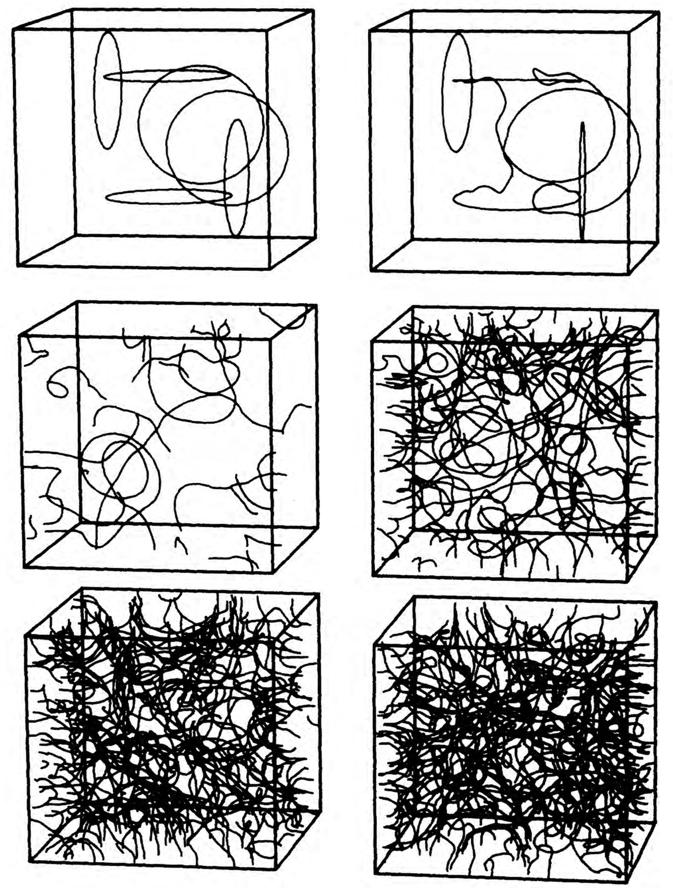


FIG. 4. Case study of the development of a vortex tangle in a real channel. Here, $\alpha=0.10$, corresponding to a temperature of about 1.6 K, and $v_{s,0}=75$ into the front face of the channel section shown. Upper left: $t_0=0$, no reconnections; upper right; $t_0=0.0028$, three reconnections; middle left: $t_0=0.05$, 18 reconnections; middle right: $t_0=0.20$, 844 reconnections; lower left: $t_0=0.55$, 12 128 reconnections; lower right: $t_0=2.75$, 124 781 reconnections.

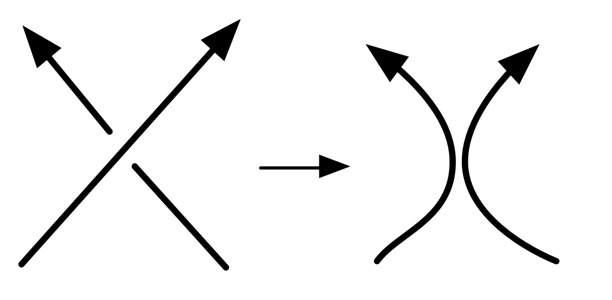
Schwarz simulated the counterflow turbulence by the vortex filament model and obtained the statistically steady state.

However, this simulation had nontrivival serious problems.

1. Vortex reconnections were modeled artificially.

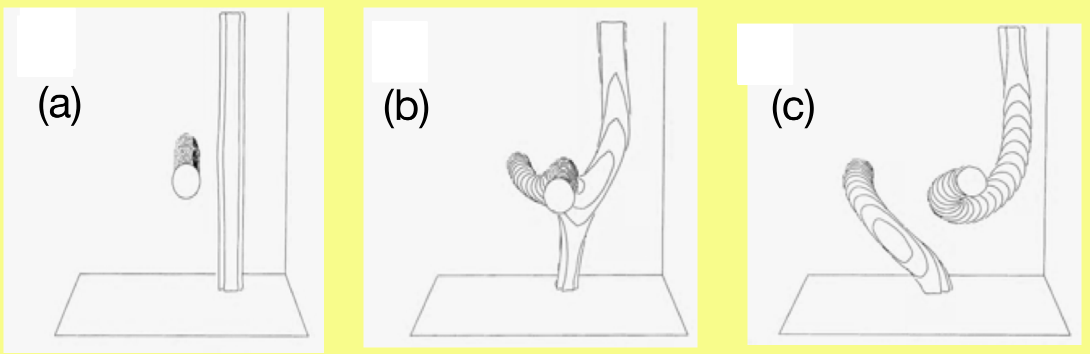
Reconnection of quantized vortices (1)

In the field of classical fluid dynamics, vortex reconnections are believed to occur by the viscous diffusion of vorticity.



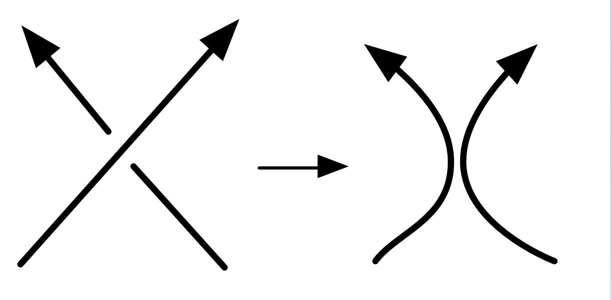
Can quantized vortices reconnect? The vortex filament model (VFM) cannot answer the question.

The simulation of the Gross-Pitaevskii model shows reconnection. J. Koplik and H. Levin, PRL71, 1375 (1993)



Reconnection of quantized vortices (2)

Reconnections in VFM are modeled with an algorithmical procedure. However, this procedure is more or less arbitrary.



Schwarz reported that the statistically steady state is independent of the detail of the procedure.

K. W. Schwarz, PRB38, 2398 (1988)

Schwarz’s simulation(2) PRB38, 2398(1988)

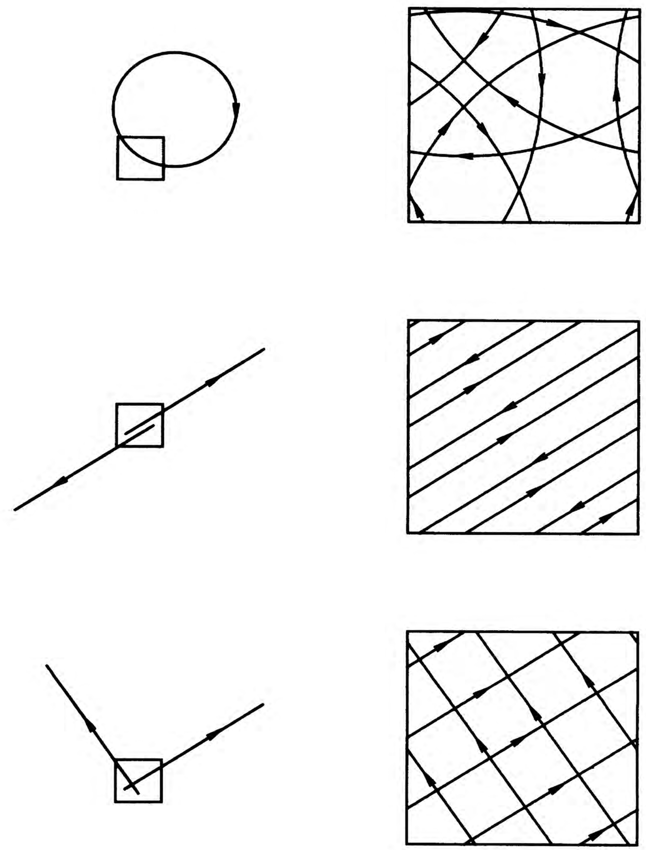


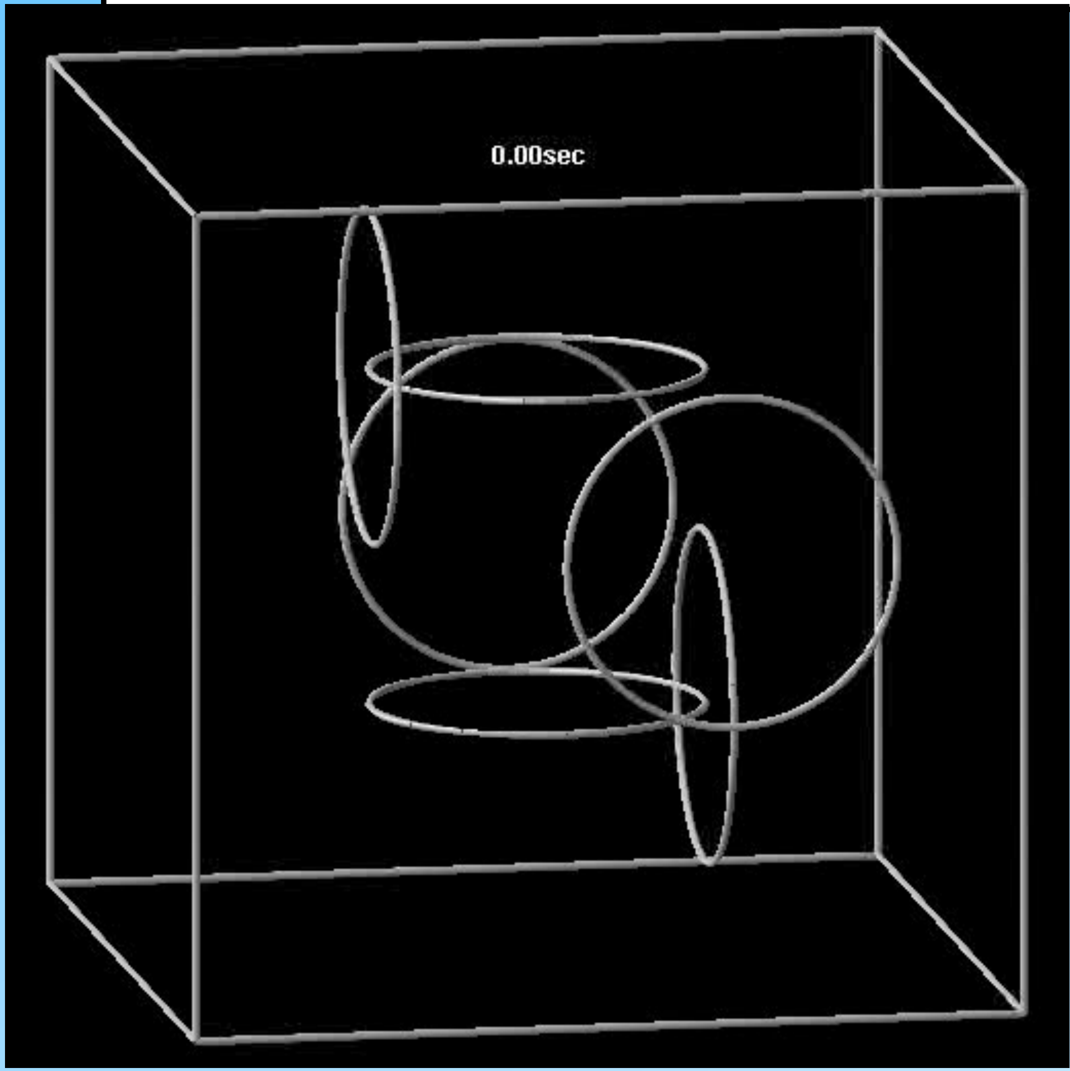
FIG. 8. Mapping of various vortex configurations into the computational volume, showing the appearance of the unit cell when all space is filled by the repetition of these objects. The end points of the lines represent equivalent points in the unit cell. Top row: closed loops; middle row: parallel infinite lines characteristic of a dead-end fluctuation; bottom row: infinite lines after randomizing procedure designed to reestablish three-dimensional behavior. The illustrations are intended to be purely schematic.

However, this simulation had nontrivial serious problems.

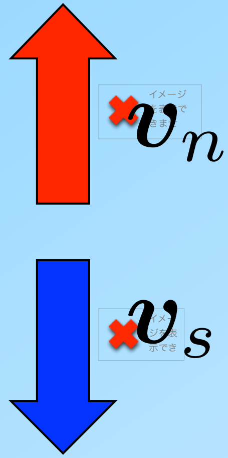
2. All calculation was performed by the LIA.

→ He used an artificial mixing procedure in order to obtain the steady state.

Simulation by the **full Biot-Savart law**



H. Adachi, S. Fujiyama, M. Tsubota, Phys. Rev. B81, 104511(2010) .



BOX (0.1cm)^3 $T = 1.6$ K

Vns = 0.367cm/s

Periodic boundary conditions for all three directions

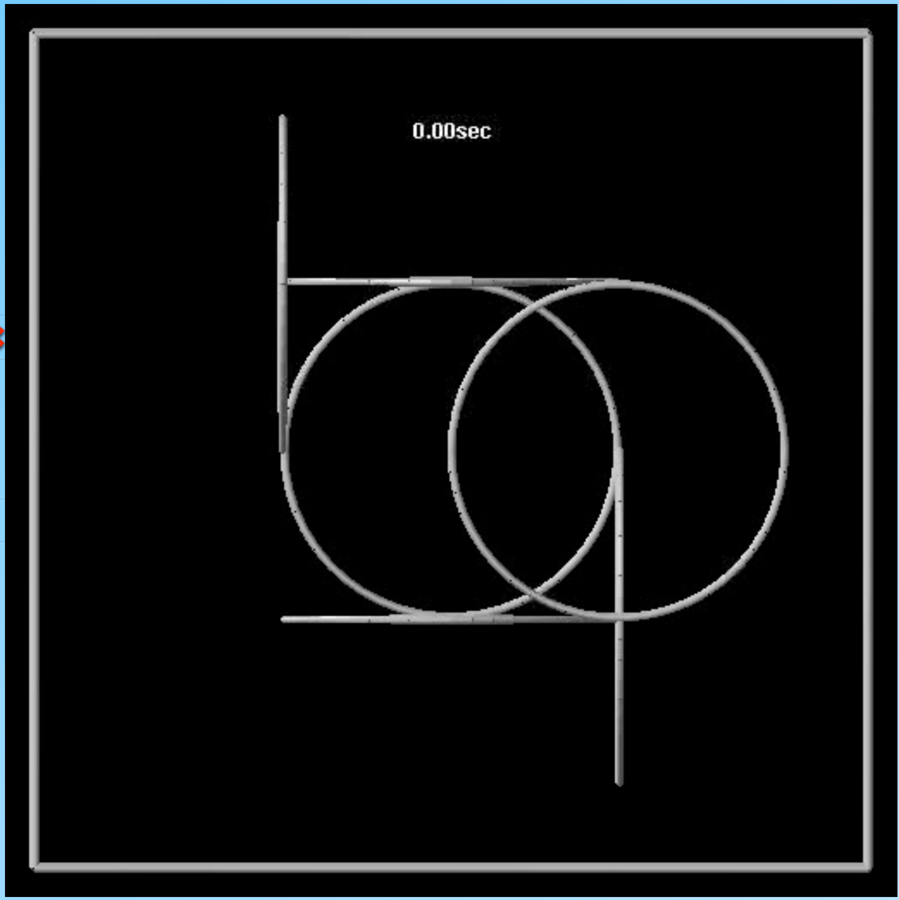
The statistically steady states were obtained without the artificial mixing procedure.

Comparison between LIA and full Biot-Savart

Full Biot-Savart $T = 1.6$ K LIA

$\boldsymbol{v}_n$

$\boldsymbol{v}_s$



We need intervortex interaction.

Vortices become anisotropic, forming layer structures.

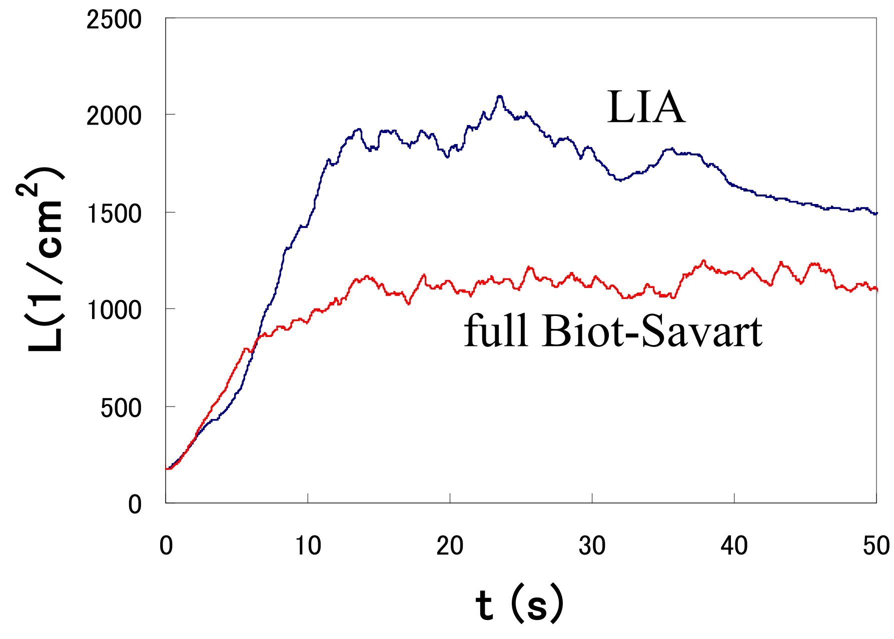
Comparison between the LIA and full BS calculation

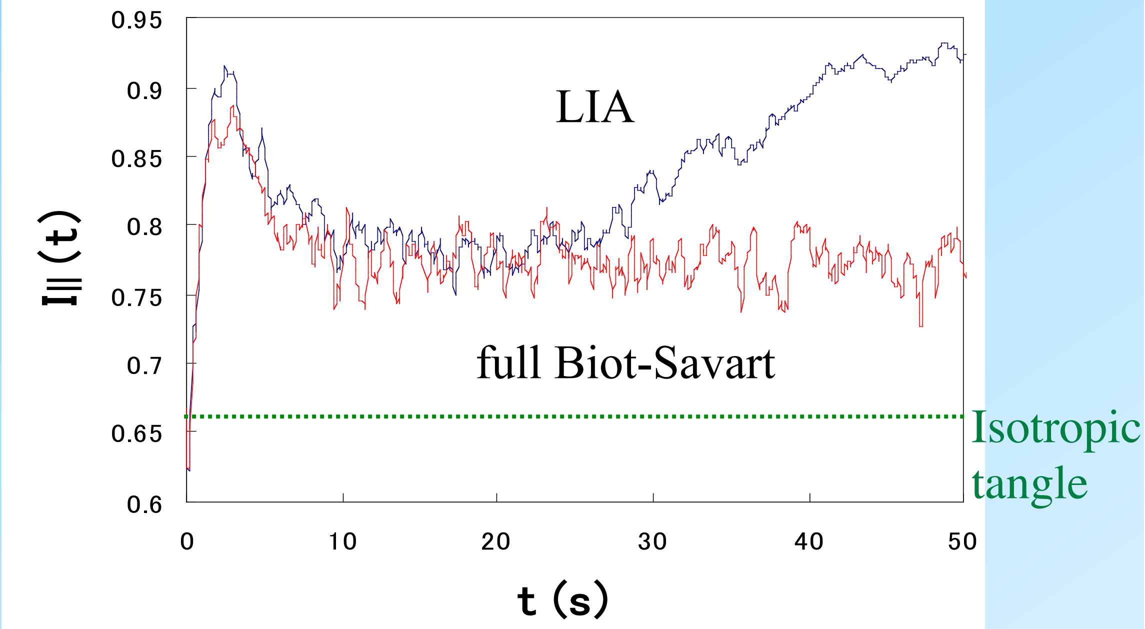
Vortex Line Density

$$L = \frac{1}{\Omega}\int_{\mathcal{L}} d\xi$$

Anisotropic parameter

$$I_{||} = \frac{1}{\Omega L}\int_{L}[1-(\mathbf{s}'\cdot\hat{\mathbf{r}}_{||})^2]d\xi$$





The LIA calculation is quite different from the full Biot-Savart one.

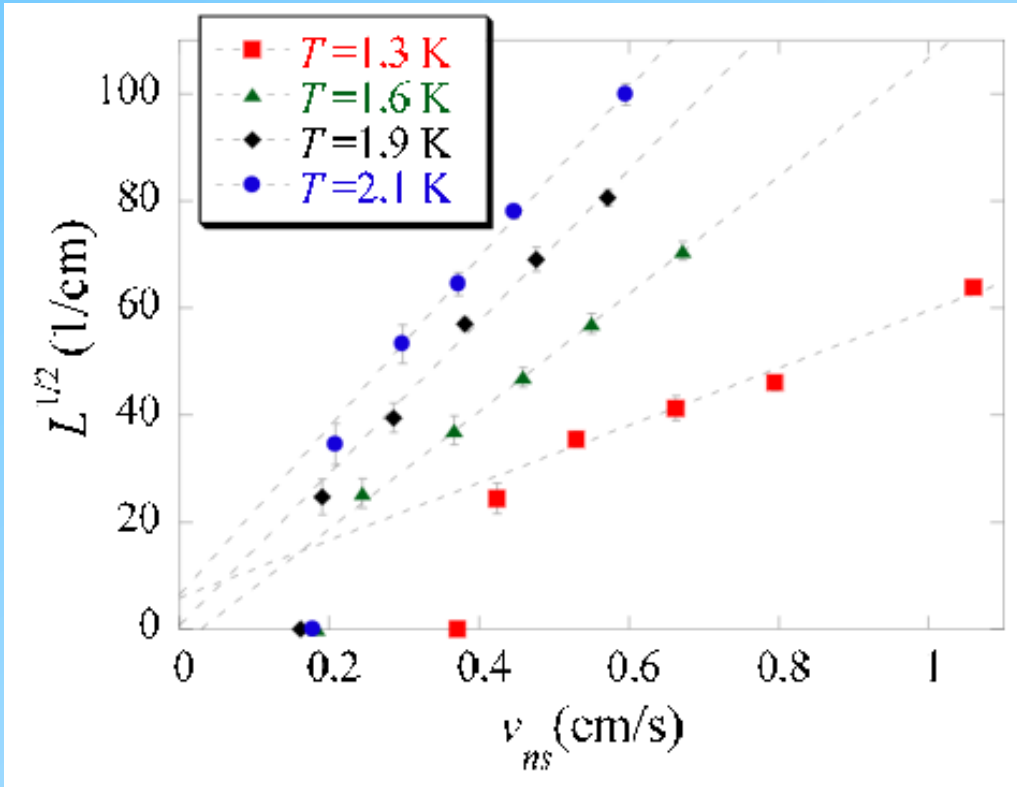
The LIA is not good.

Quantitative comparison with observations

An important criterion of the steady state is to obtain

$$L^{1/2} = \gamma v_{ns}$$

L: Vortex density, v_{ns}:relative velocity in counterflow



	γ (s/cm^2) Our calculation	γ (s/cm^2) Experiment
1.3 K	54	59
1.6 K	109	93
1.9 K	140	133
2.1 K	157	(154)

Childers and Tough, Phys. Rev. B13, 1040 (1976)

The parameter γ agrees with the experimental observation quantitatively.

3. Recent visualization experiments (2006-)

Visualizing quantized vortices and the profile of the normal fluid flow

Tallahassee, Maryland, Prague

Tracer particles

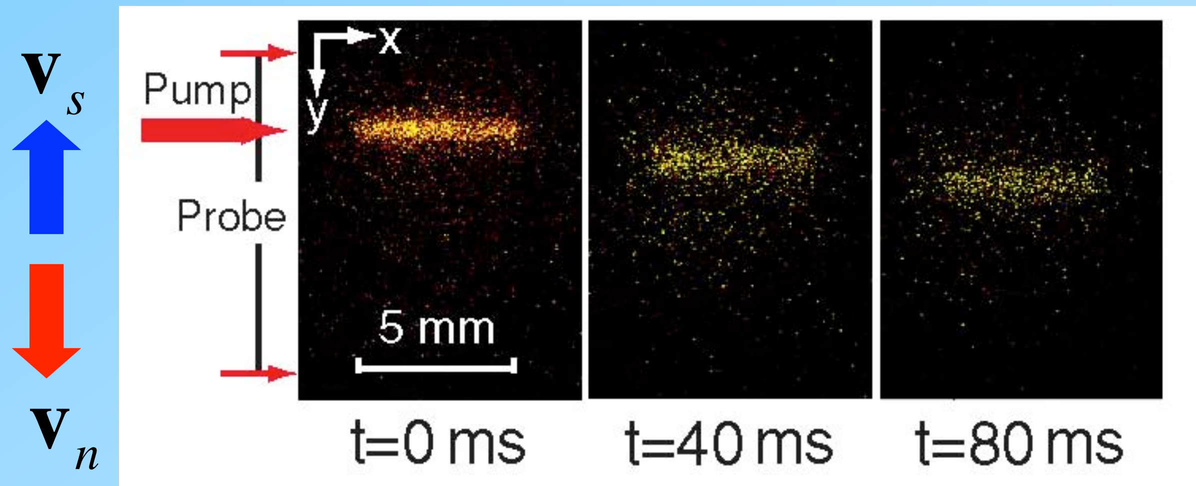
- Hydrogen particles μ m
- Metastable He_2^* molecules nm

Visualization using metastable He_2^* molecules reveals the profile of the normal fluid flow.

W. Guo, S. B. Cahn, J. A. Nikkel, W. F. Vinen, D. N. McKinsey, Phys. Rev.Lett. 105, 045301(2010)

Metastable He_2^* molecules

- Excited by laser light. The lifetime is 13s.
- The size is 1nm.
- They are not trapped by vortices above 1K, following the normal flow.



1.95K Square channel

When the heat flux is large, He_2* molecules obey turbulent diffusion.

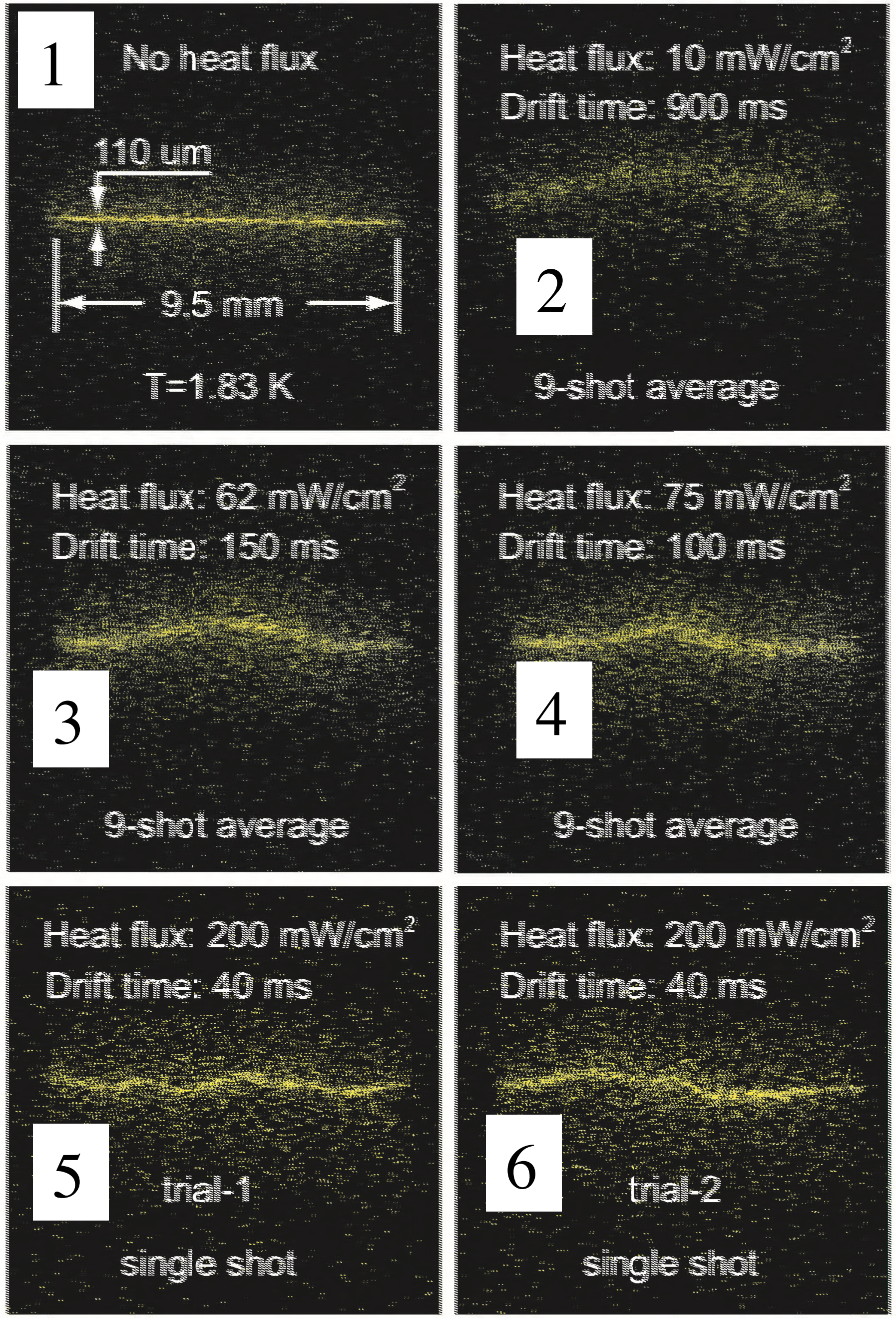
--> The normal fluid is turbulent.

Marakov *et al*. observed a novel profile of the normal fluid flow, namely the tail-flattened flow.

A. Marakov, J. Gao, W. Guo, S. W. Van Sciver, G. G. Ihas, D. N. McKinsey, W. F. Vinen, Phys. Rev. B 91, 094503(2015).

1 No heat flux → No flow
2 Poiseuille flow
3, 4 Tail-flattened flow
5, 6 Turbulence

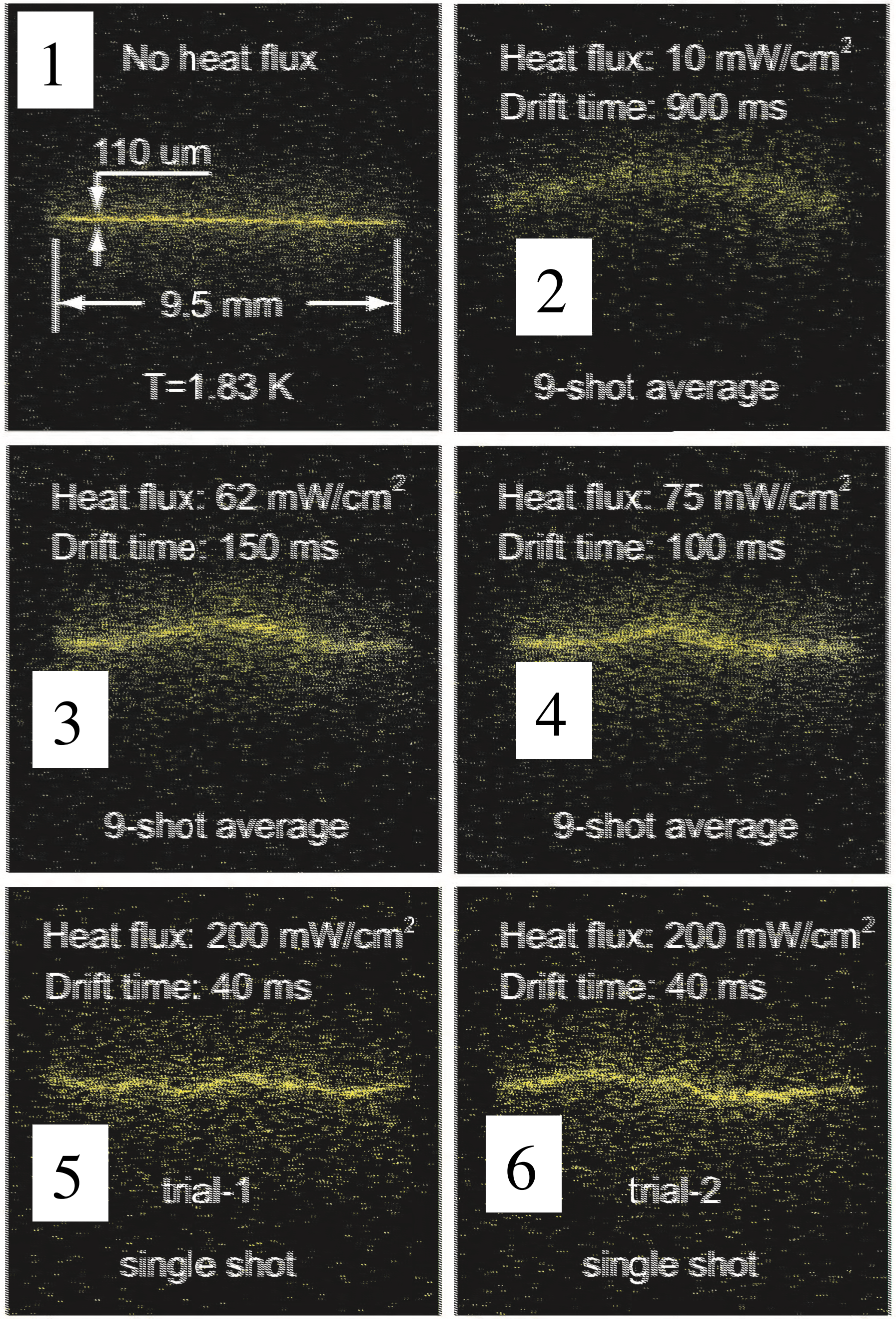
Such tail-flattened flow has never been observed even in a classical fluid.



Marakov *et al*. observed a novel profile of the normal fluid flow, namely the tail-flattened flow.

Importance of this work

1. Effect of the normal fluid
2. Inhomogeneous turbulence affected by the channel walls



4. The new simulation for the inhomogeneous normal fluid flow

4-1. Counterflow quantum turbulence of He-II in a square channel: Numerical analysis with nonuniform flows of the normal fluid

S. Yui and M. Tsubota, Phys. Rev. B91, 184504 (2015): arXiv: 1502.06683

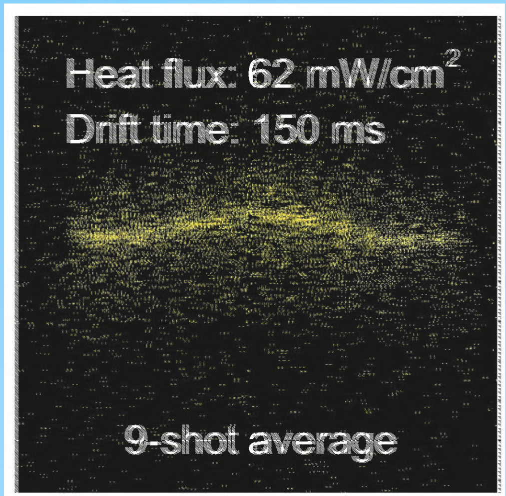
4-2. Logarithmic velocity profile (the log-law) of quantum turbulence of superfluid ^{4}He

S. Yui, K. Fujimoto and M. Tsubota, arXiv:1508.01347

What is lacking in the previous simulations?

Most previous numerical works suppose

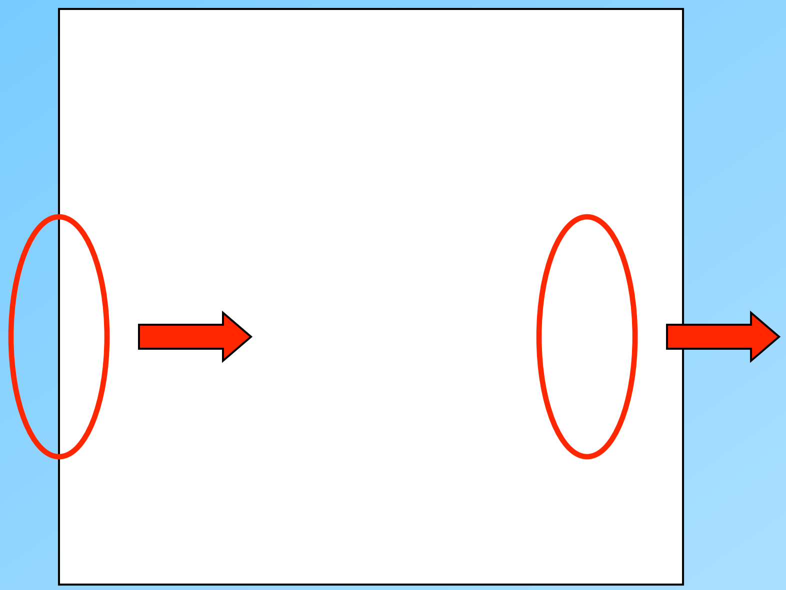
- Periodic boundary for all three directions
- Prescribing the homogeneous profile of the normal fluid



In order to understand these phenomena, we should suppose solid boundary condition in a channel and couple the superfluid and the normal fluid properly.

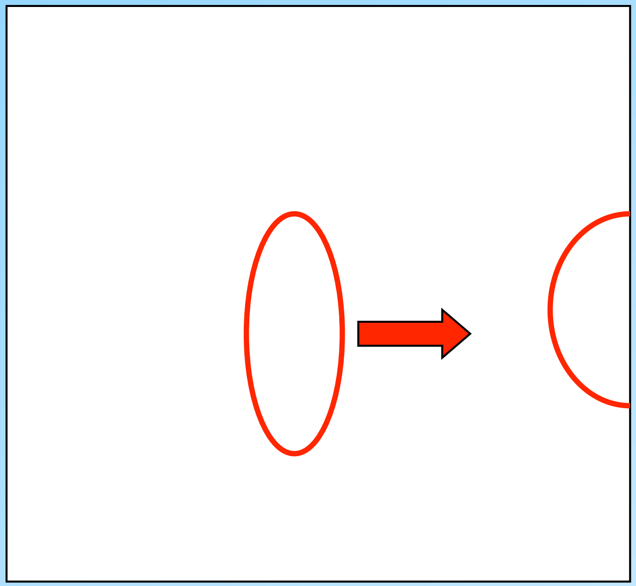
Difference between solid- and periodic boundary conditions

Periodic



A vortex ring that comes out of the right enters the system from the left again.

Solid boundary



A vortex ring moving to the right reconnects with the solid wall.

--> Solid walls can work as an absorber for vortices.

Full formulation of the two-fluid model

D. Kivotides, PRB76, 054503(2007)

Superfluid --> VFM

Normal fluid --> Navier-Stokes equation

$$\frac{\partial \mathbf{v}_n}{\partial t} + \left(\mathbf{v}_n \cdot \nabla\right)\mathbf{v}_n = -\frac{1}{\rho}\nabla p + \nu \nabla^2 \mathbf{v}_n - \frac{\nu_*}{\zeta^3}\int_{L \cap V_\zeta} d\xi \, \mathbf{s}' \times \left(\mathbf{v}_n - \dot{\mathbf{s}}\right)$$

$$+ \frac{\nu_{**}}{\zeta^3}\int_{L \cap V_\zeta} d\xi \, \mathbf{s}' \times [\mathbf{s}' \times \left(\mathbf{v}_n - \dot{\mathbf{s}}\right)]$$

Both ways

VFM ⇄ Normal fluid

Full formulation of the two-fluid model

D. Kivotides, PRB76, 054503(2007)

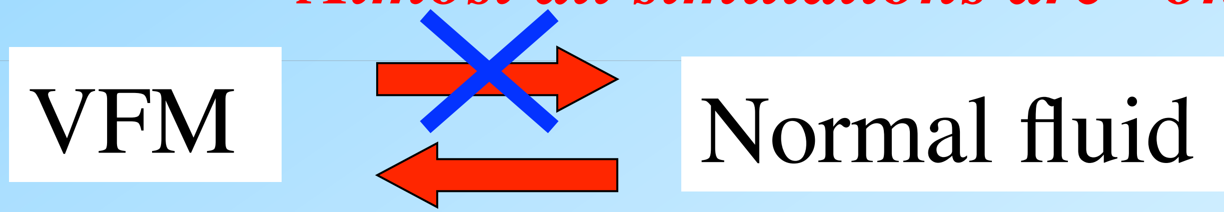
Superfluid --> VFM

Normal fluid --> Navier-Stokes equation

$$\frac{\partial \mathbf{v}_n}{\partial t} + (\mathbf{v}_n \cdot \nabla)\mathbf{v}_n = -\frac{1}{\rho}\nabla p + \nu \nabla^2 \mathbf{v}_n - \frac{\nu_*}{\zeta^3}\int_{L \cap V_\zeta} d\xi\, \mathbf{s}' \times (\mathbf{v}_n - \dot{\mathbf{s}})$$

$$+ \frac{\nu_{**}}{\zeta^3}\int_{L \cap V_\zeta} d\xi\, \mathbf{s}' \times [\mathbf{s}' \times (\mathbf{v}_n - \dot{\mathbf{s}})]$$

Almost all simulations are "one way".

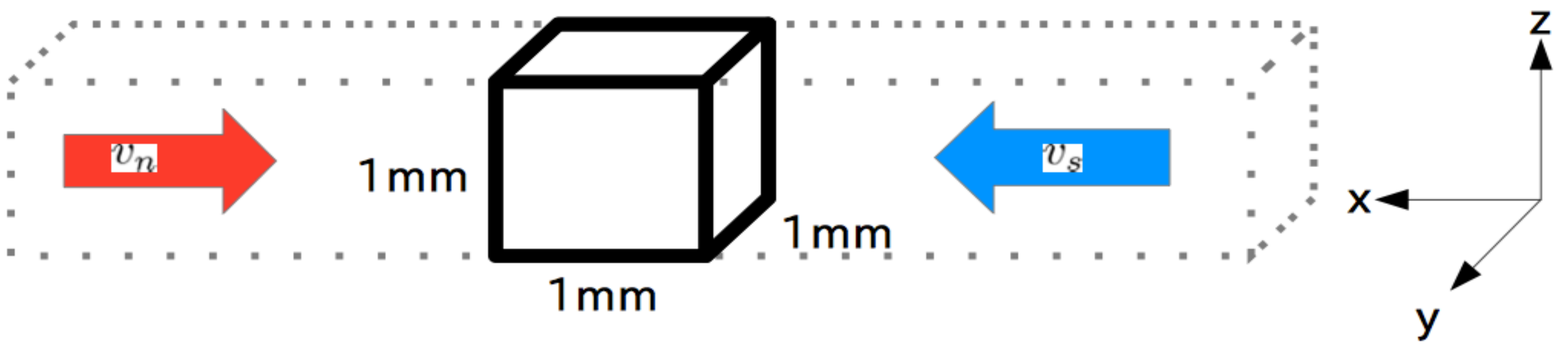


4-1. Counterflow quantum turbulence of He-II in a square channel: Numerical analysiswith nonuniform flows of the normal fluid

S. Yui and M. Tsubota,
Phys. Rev. B91, 184504 (2015)

- Square cross section 1mm ×1mm
- Computational volume is 1mm ×1mm × 1mm
- Periodic B. C. along the x-axis, and solid smooth B. C. for other walls.
- T= 1.3K, 1.6K and 1.9K

Full Biot-Savart calculation

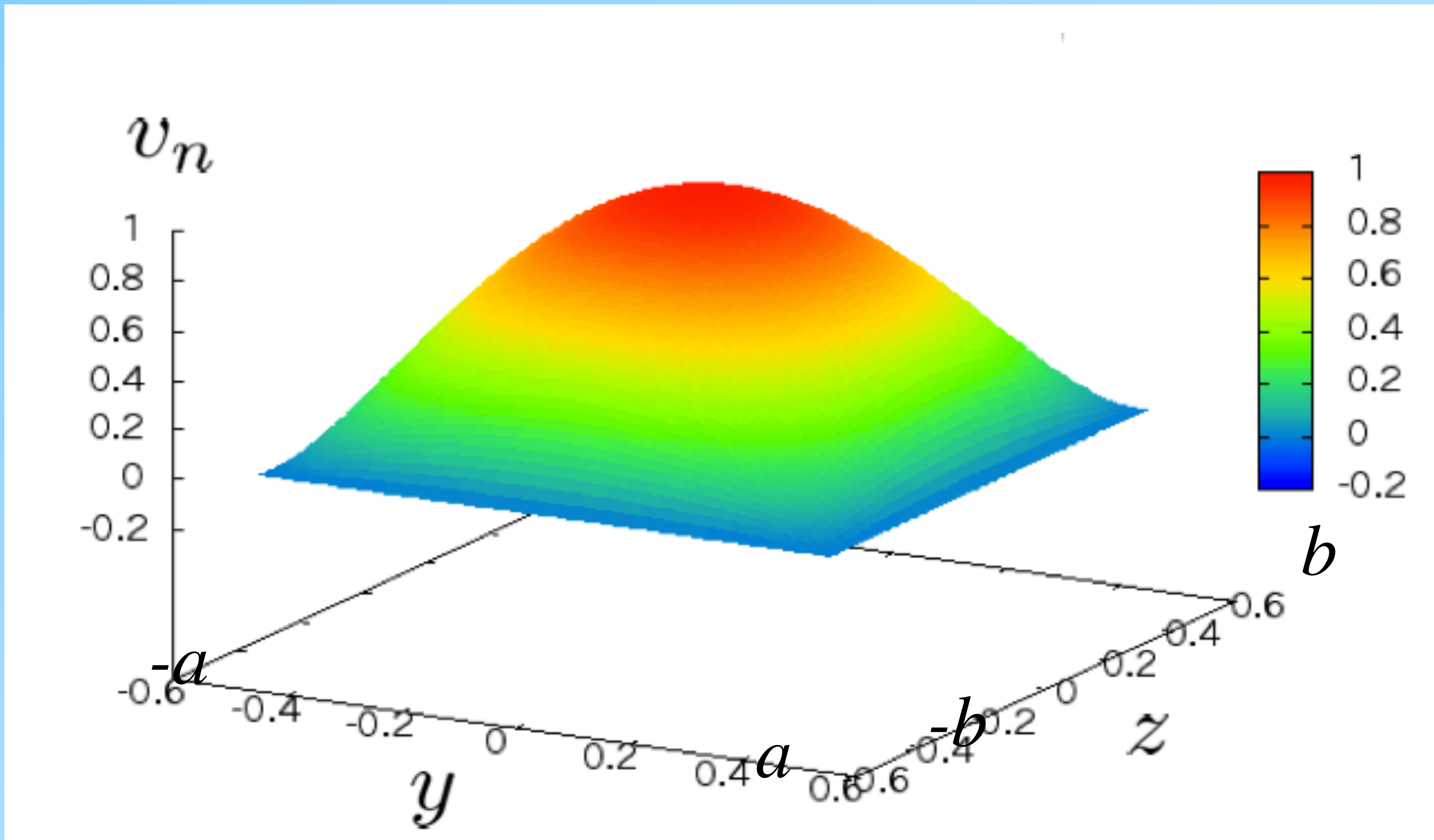


(1) Poiseuille flow in a rectangular channel

For the cross section $(-a < y < a, -b < z < b)$

$$v_n(y,z) = u_0 \sum_{i=1,3,5,\cdots}^{\infty} (-1)^{\frac{i-1}{2}} \left[1 - \frac{\cosh(i\pi z/2a)}{\cosh(i\pi b/2a)}\right] \frac{\cosh(i\pi y/2a)}{i^3}$$

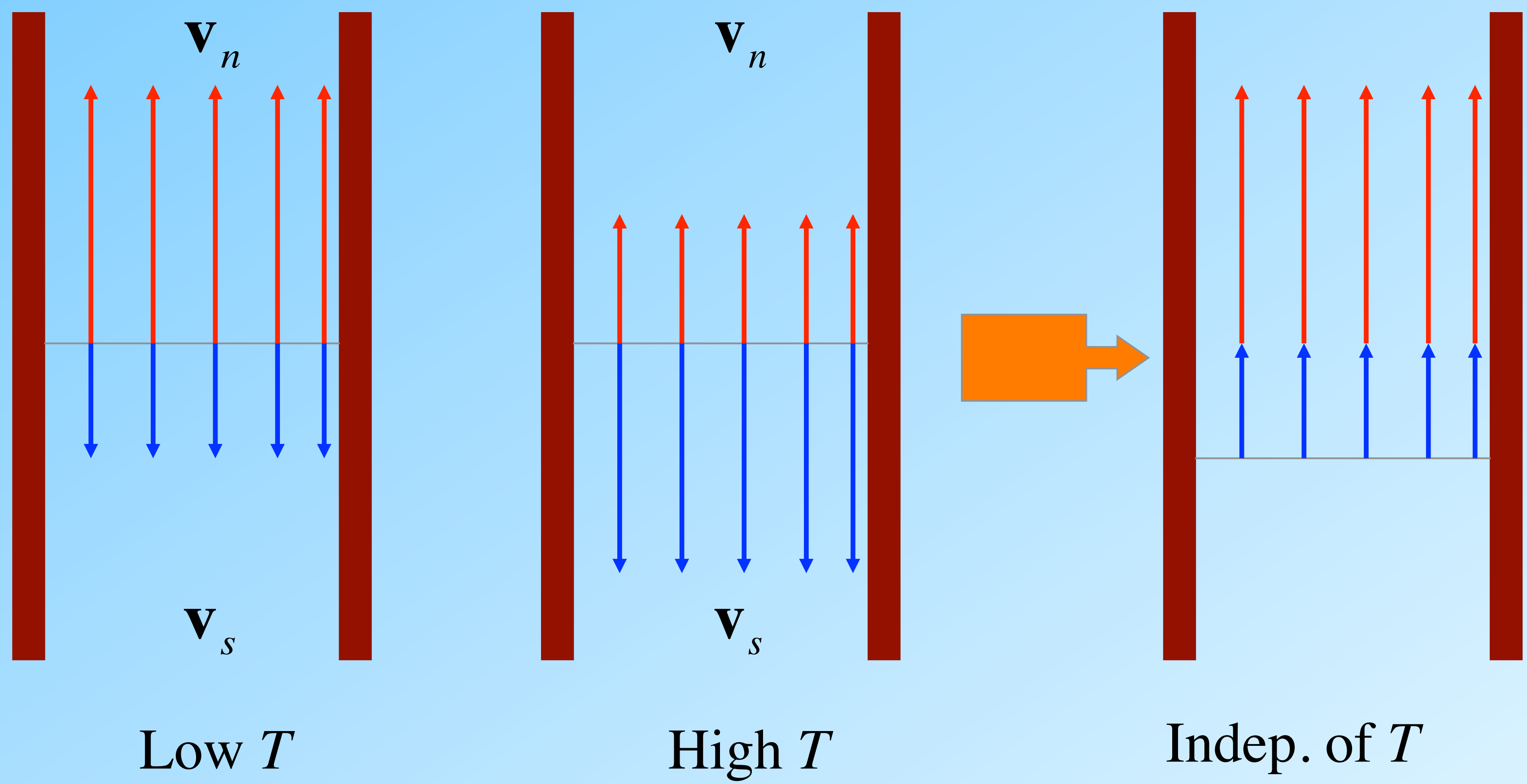
The Handbook of Fluid Dynamics, edited by R. W. Johnson (CRC, Boca Raton,1998)



Inhomogeneous normal flow causes an interesting effect.

The case of uniform normal flow $\rho_s v_s + \rho_n v_n = 0$ → $v_s = -\frac{\rho_n}{\rho_s} v_n$

In a laboratory frame $\boldsymbol{v}_{ns} = \boldsymbol{v}_n - \boldsymbol{v}_s \propto W$

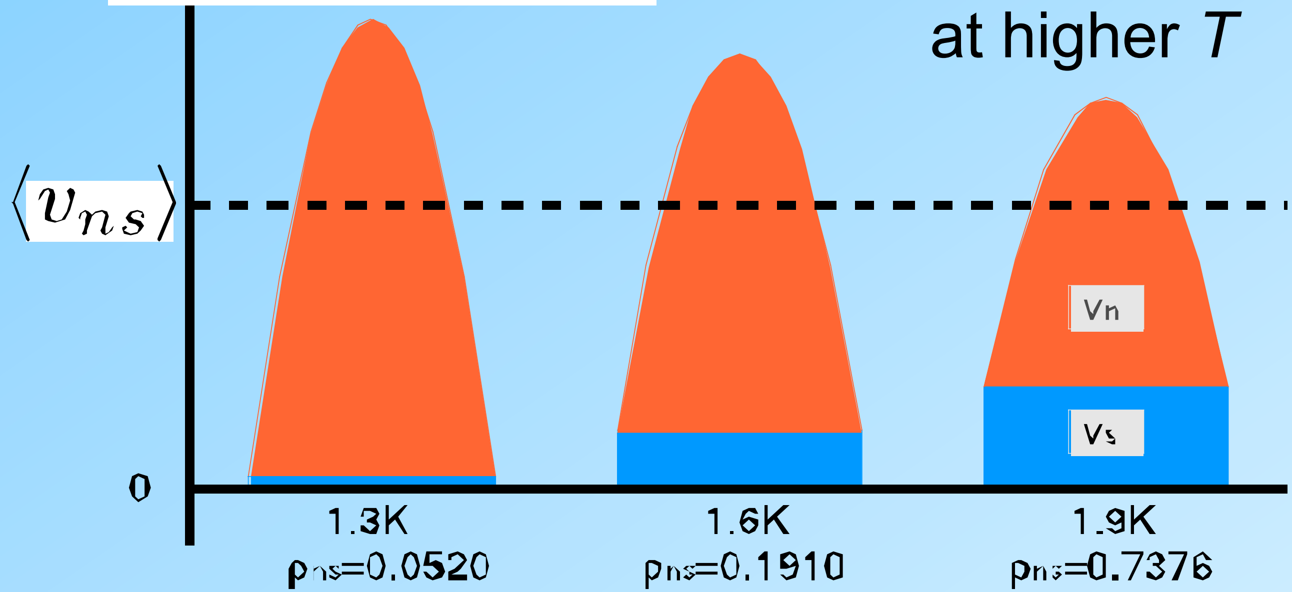


When the normal flow is Poiseuille-like, $\mathbf{V}_{ns}$ depends on temperature.

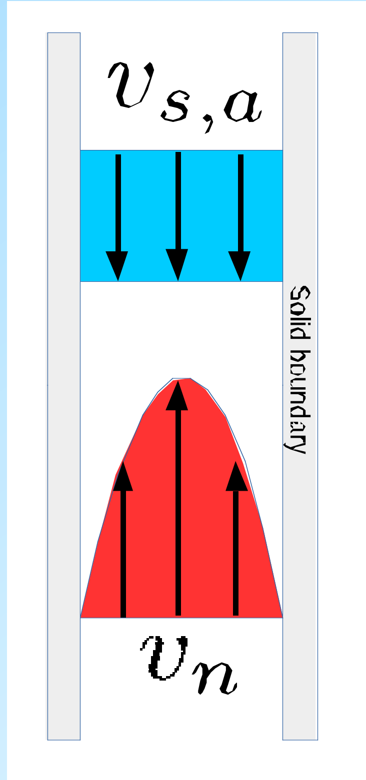
$$\int (\rho_s \boldsymbol{v}_{s,a} + \rho_n \boldsymbol{v}_n)\, d\boldsymbol{S} = 0$$

$$\boldsymbol{v}_{s,a} = -\frac{\rho_n}{\rho_s}\overline{\boldsymbol{v}_n}, \qquad \overline{\boldsymbol{v}_n} = \frac{\int \boldsymbol{v}_n d\boldsymbol{S}}{\int d\boldsymbol{S}}$$

$$\boldsymbol{v}_{ns}(\boldsymbol{r}) = \boldsymbol{v}_n(\boldsymbol{r}) - \boldsymbol{v}_{s,a}$$



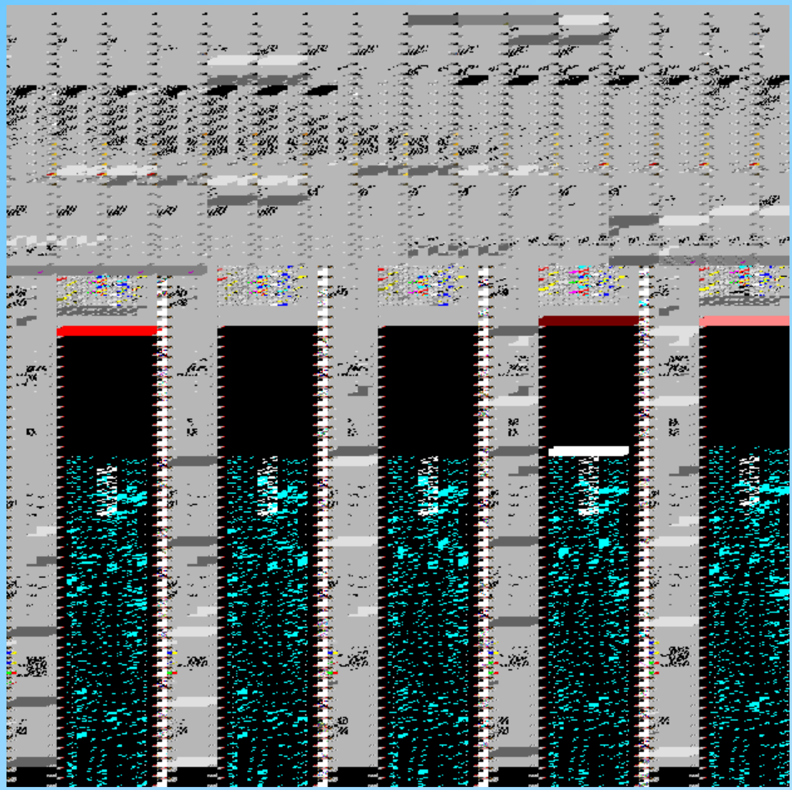
More uniform at higher *T*



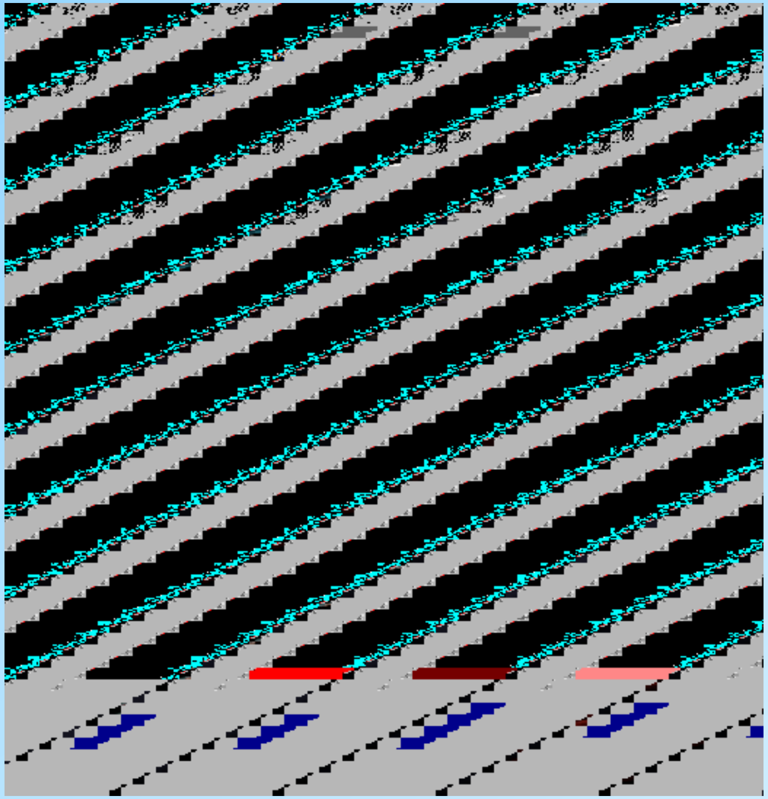
Vortex tangle in a square channel

$\otimes v_{ns}$ 1.4cm/s

T=1.3K

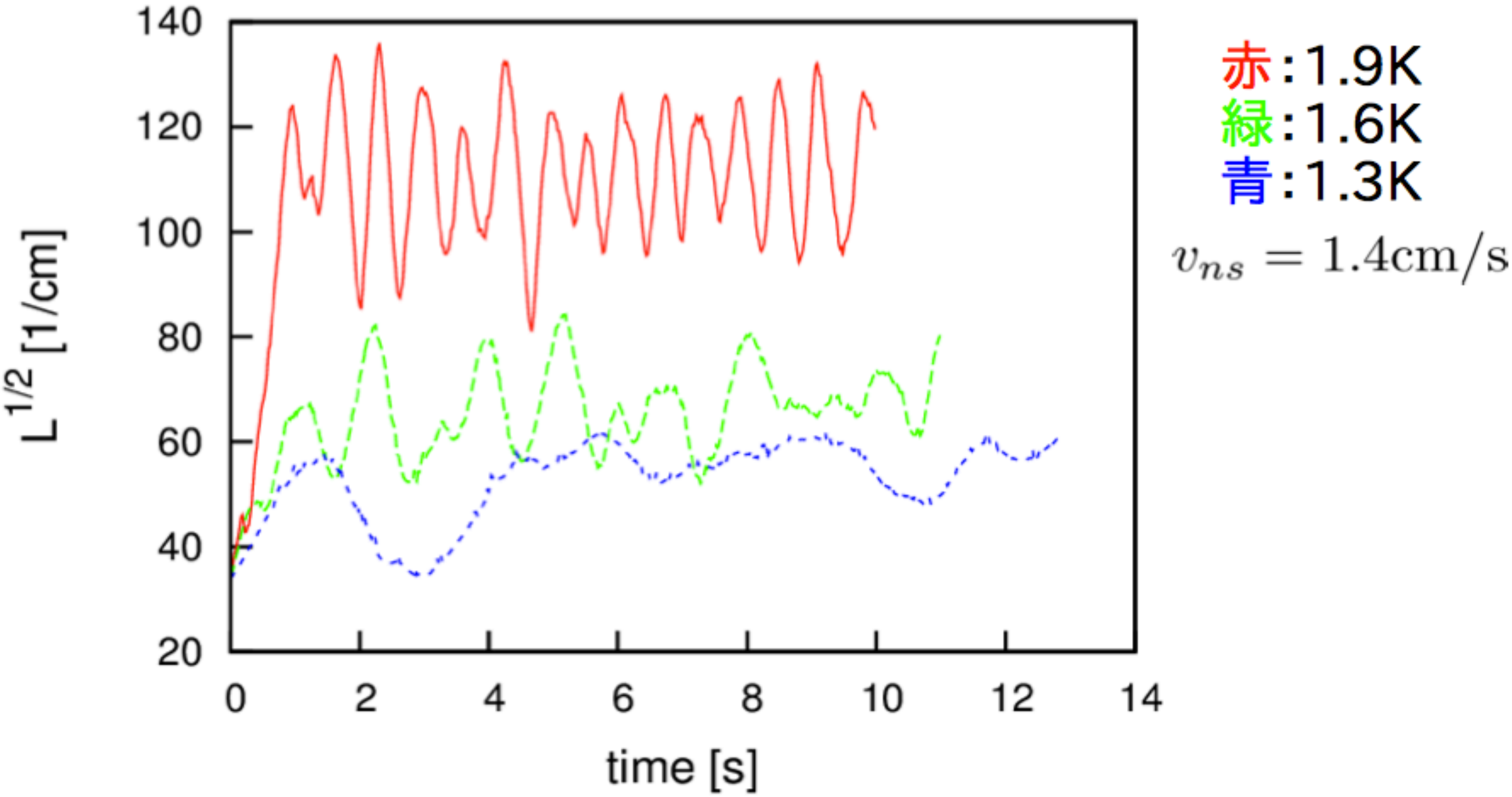


T=1.9K



1. Vortices expand from the center toward the walls, trapped by the walls.
2. Vortices are denser near the walls than the center.
3. At higher temperatures, the strong mutual friction grows the vortices fast and dense.

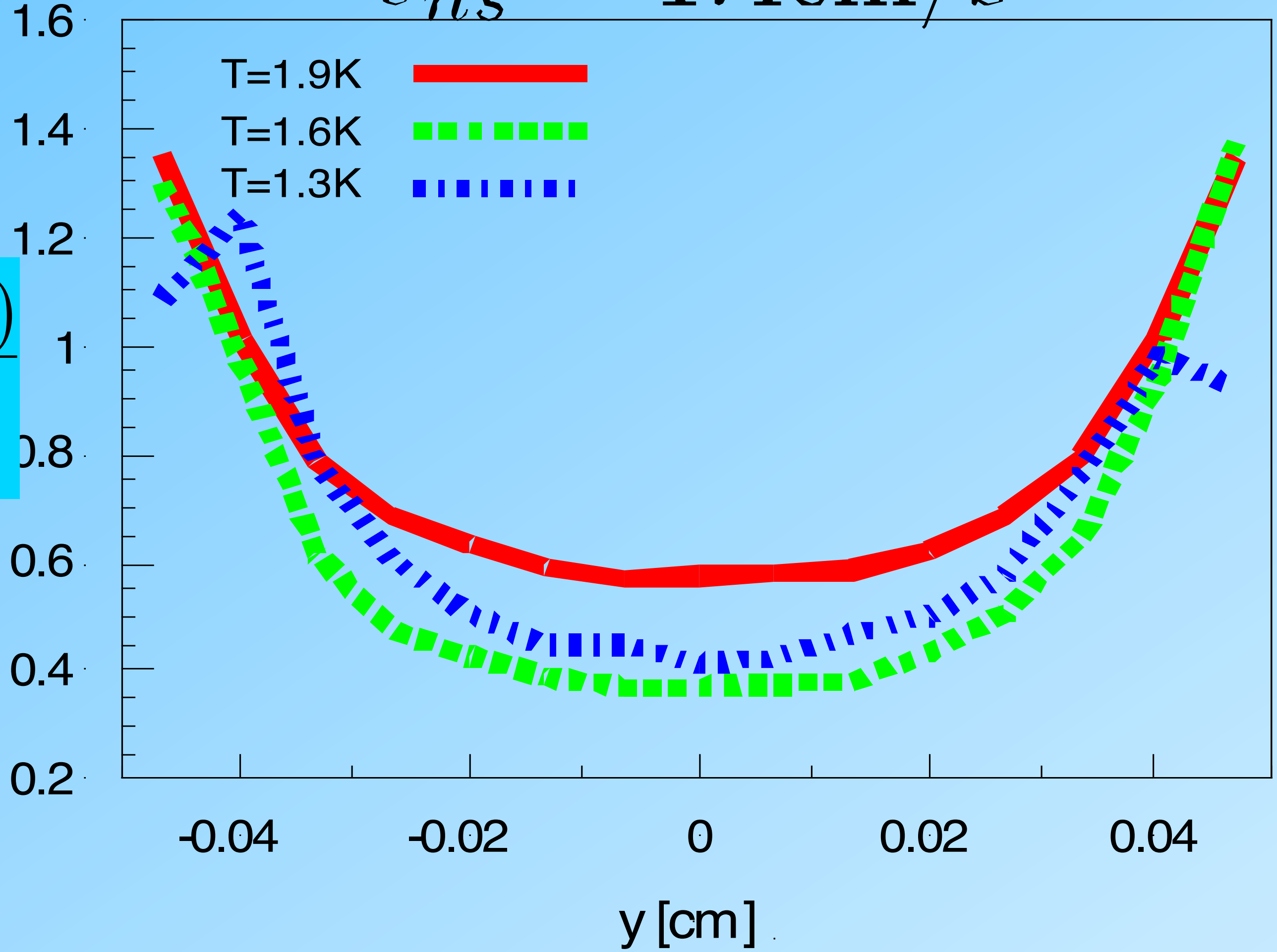
Statistically steady states are obtained despite large fluctuation.



$$L = \frac{1}{\Omega} \int d\xi$$: Vortex line density

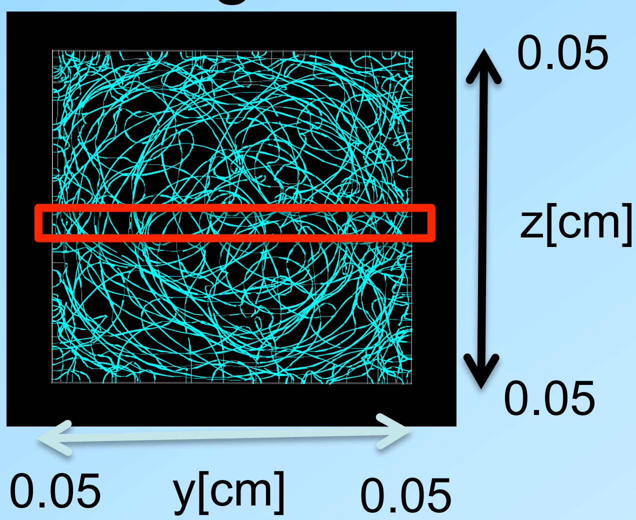
Local vortex line density

$$v_{ns} = 1.4\mathrm{cm/s}$$



$\frac{L(y)}{\bar{L}}$

Along z=0 cm

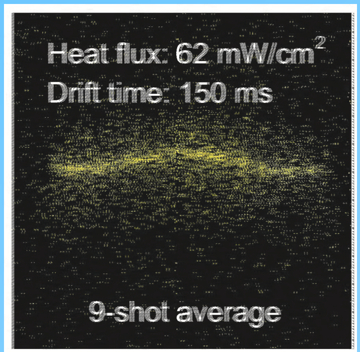


$L(y)$: Local density

$\bar{L}$: Average of $L(y)$

Vortices are dense near the walls.

(2) Tail-flattened flow in a rectangular channel

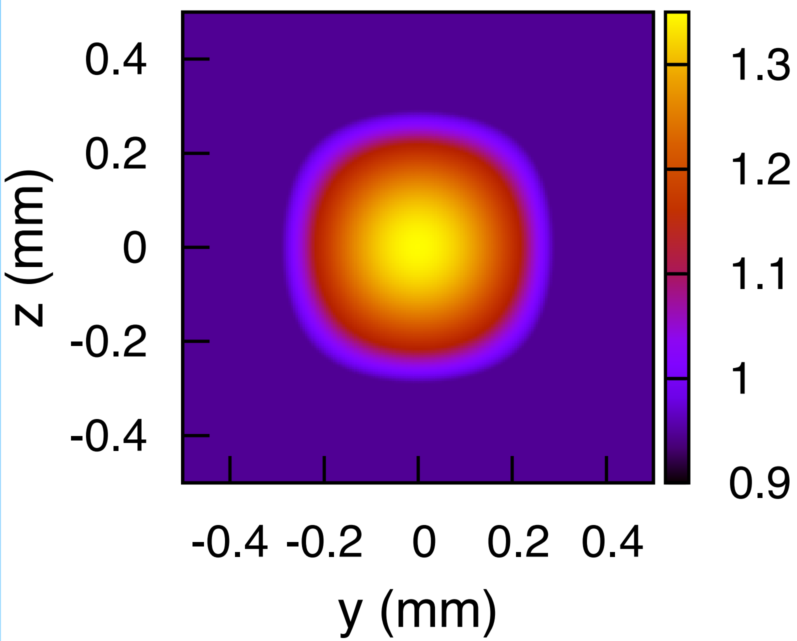


How to make the flow profile?

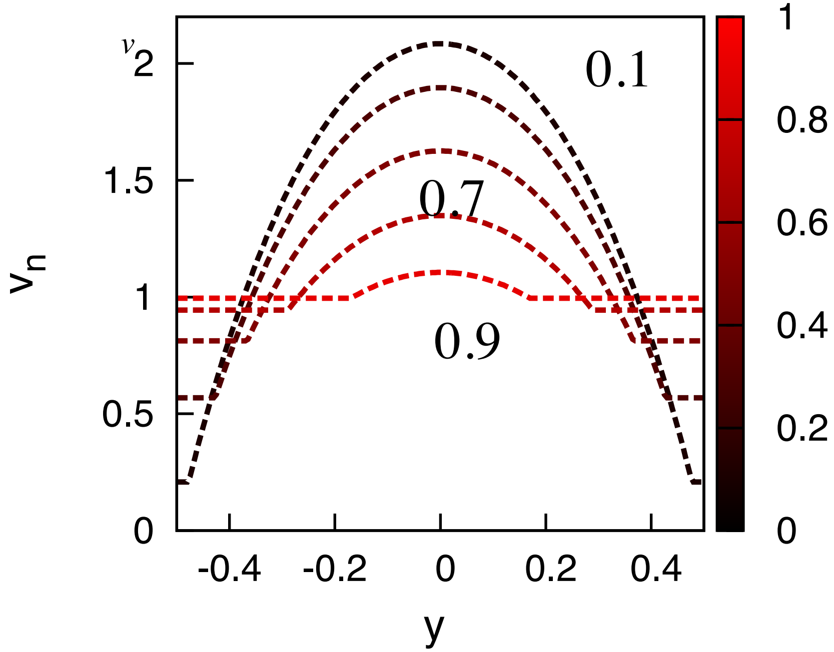
Combining the Poisuille flow $v_n^p(\mathbf{r})$ and the flat flow $v_n^p(0)$

$$v_n(\mathbf{r}) = u_0 \max[v_n^p(\mathbf{r}),\ h v_n^p(0)]$$

h=0.7



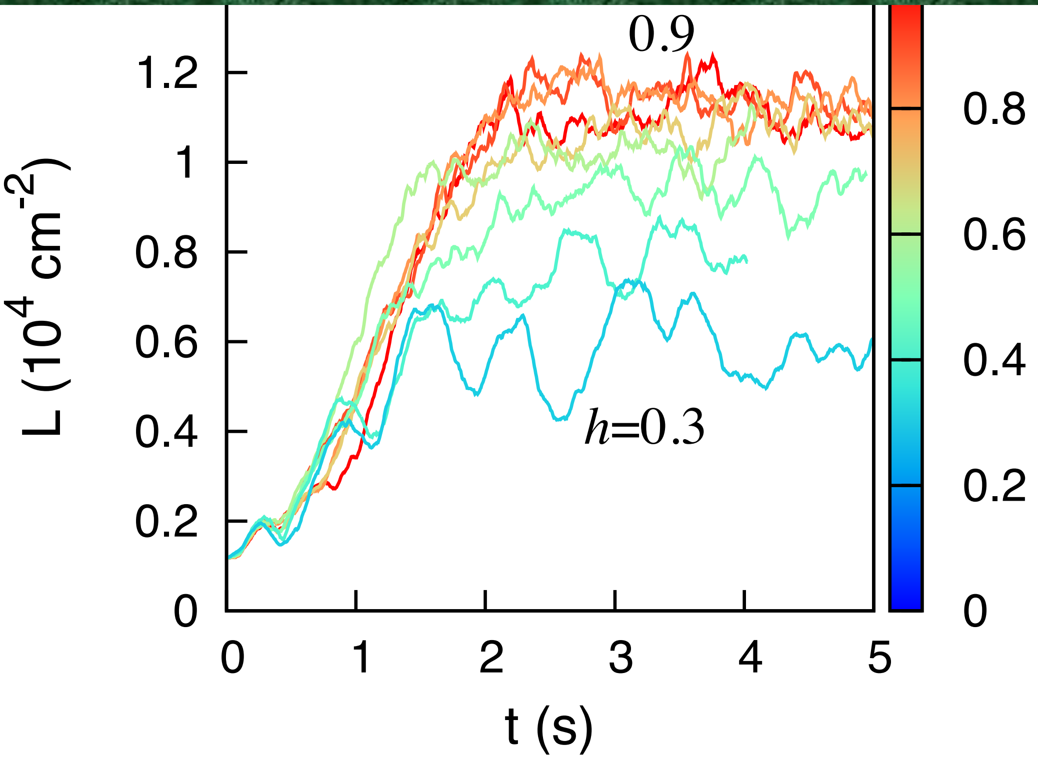
Parameter *h*

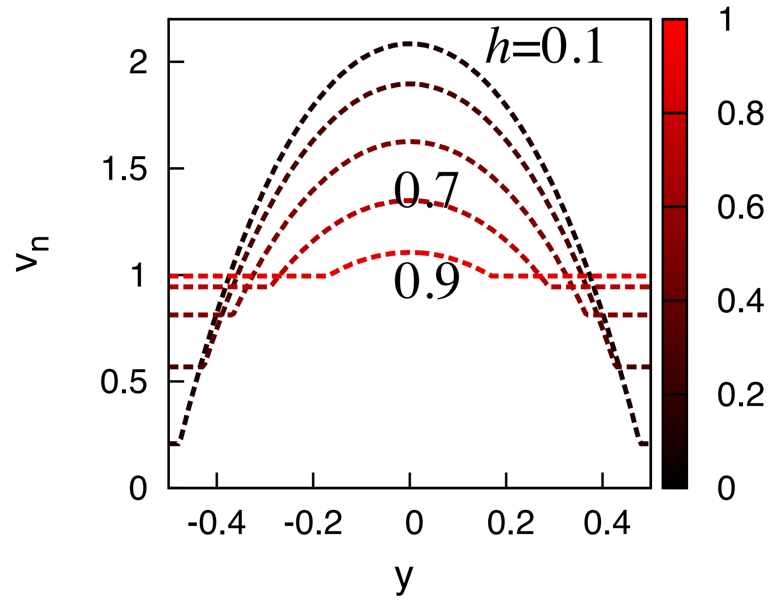


Increasing *h* makes the flow profile more uniform.

Time development of the line length density *L* for different values of *h* *T*=1.9K, v_{ns}=0.5 cm/s

All the following calculations are performed for *h*=0.7.

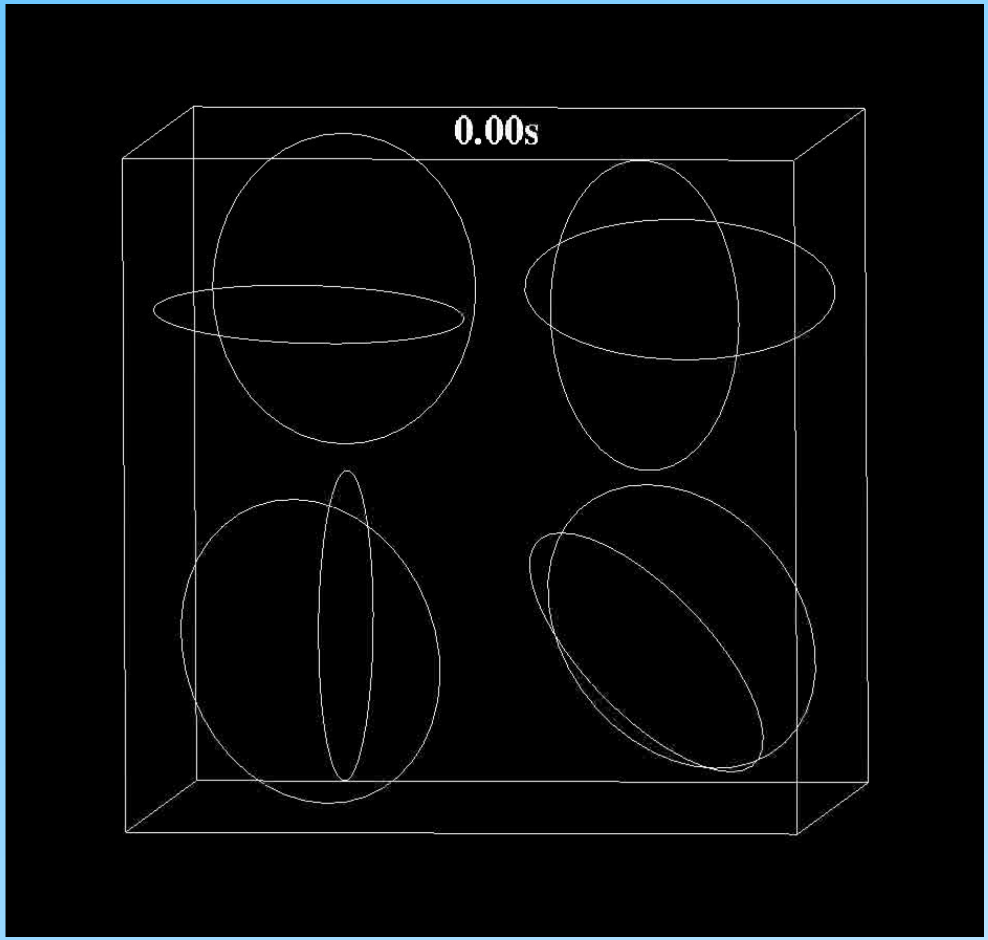




The behavior of *L* is saturated for *h* >0.8, almost equivalent to the case of the uniform normal fluid flow.

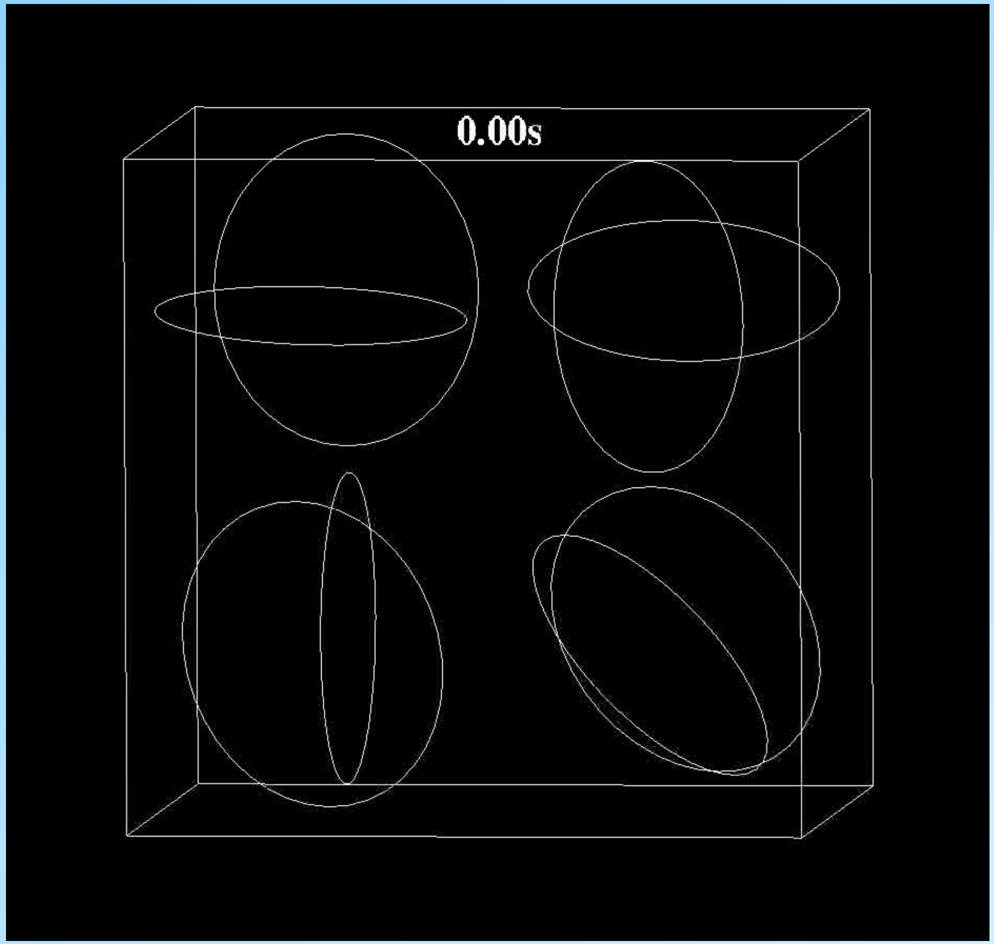
Tail-flattened flow with h=0.7

T=1.9K, v_{ns}=0.5cm/s



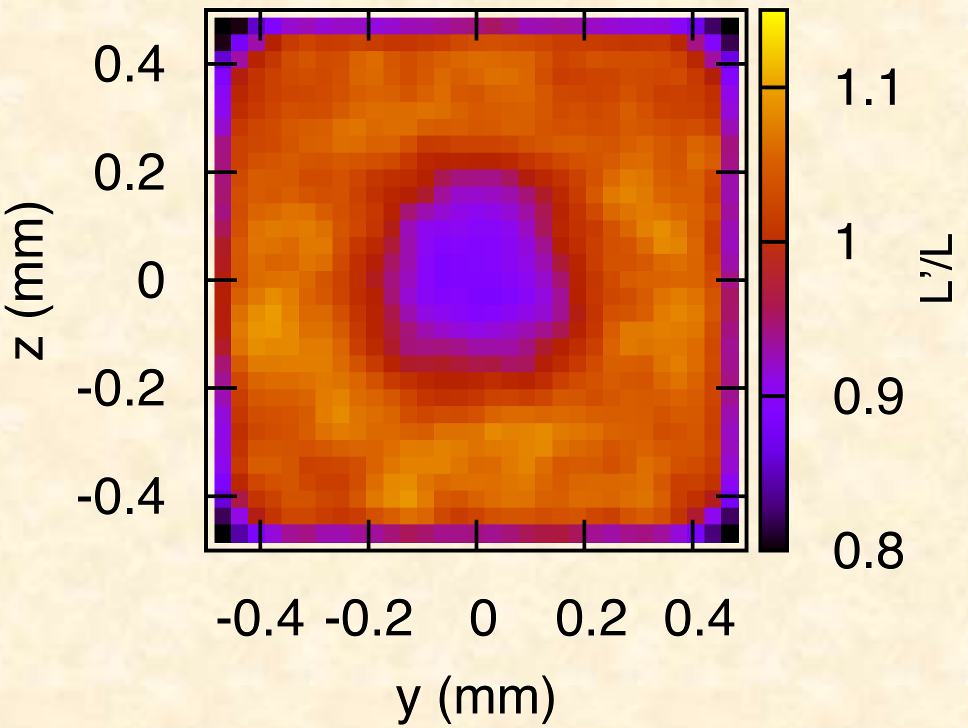
Poiseuille flow

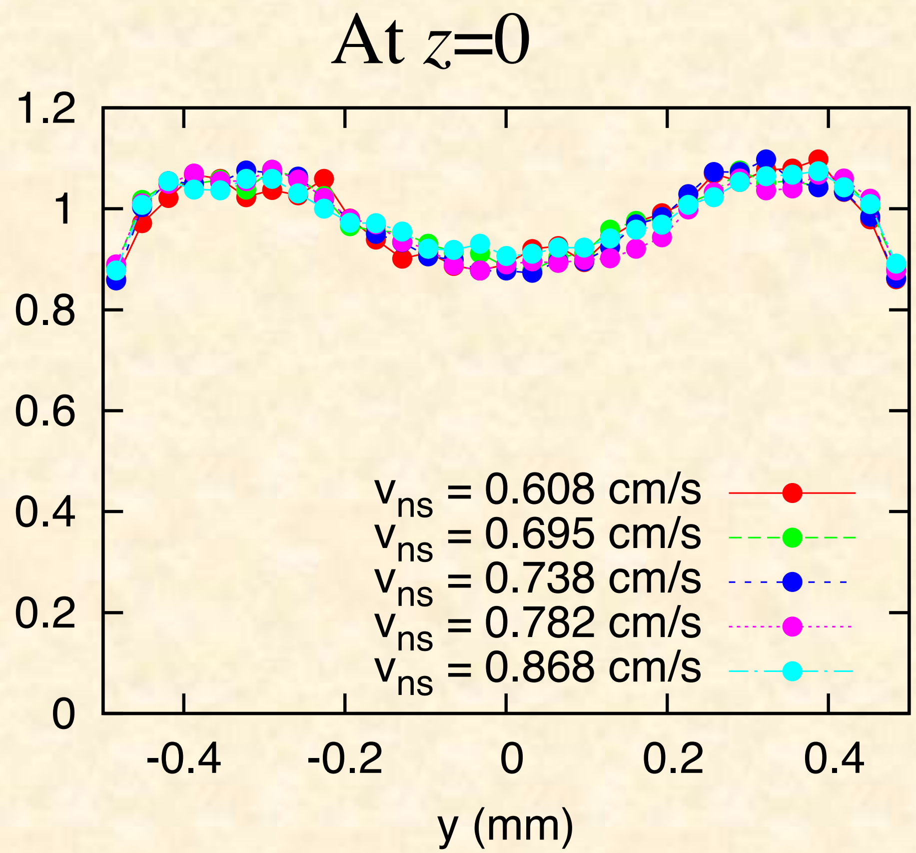
T=1.9K, v_{ns}=0.7cm/s



Vortex tangle is more homogeneous in tail-flattened flow than in Poiseuille flow.

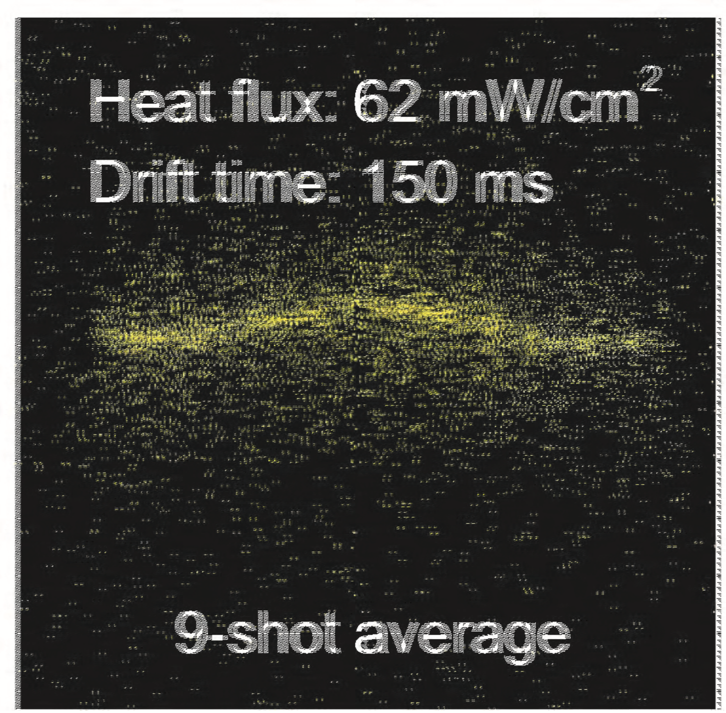
Distribution of the line length density $L(y, z)$ for the tail-flattened flow





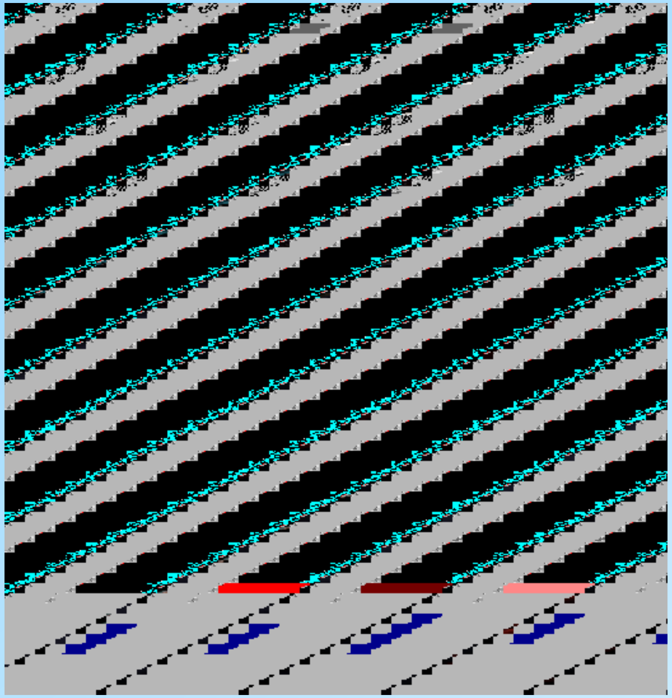
The distribution is uniform compared with that of the Poiseuille flow.

What causes the tail-flattened flow?



v_{ns}

Vortex tangle made under the Poiseuille flow



$\otimes v_{ns}$

If we turn on the mutual friction from vortices to normal fluid, the Poiseuille profile may be changed.

How dose the superfluid component mimic the normal fluid component through the mutual friction?

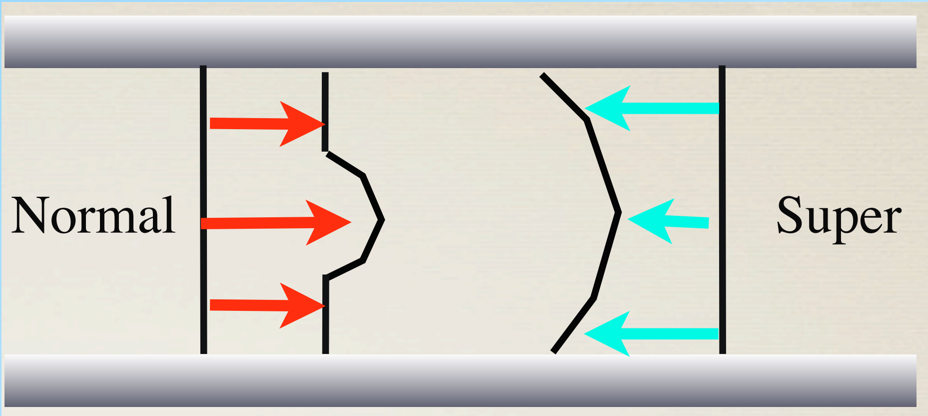
This interest appears in many contexts of superfluid hydrodynamics.

Cf. W. F. Vinen, Phys. Rev. B61, 1410 (2000)

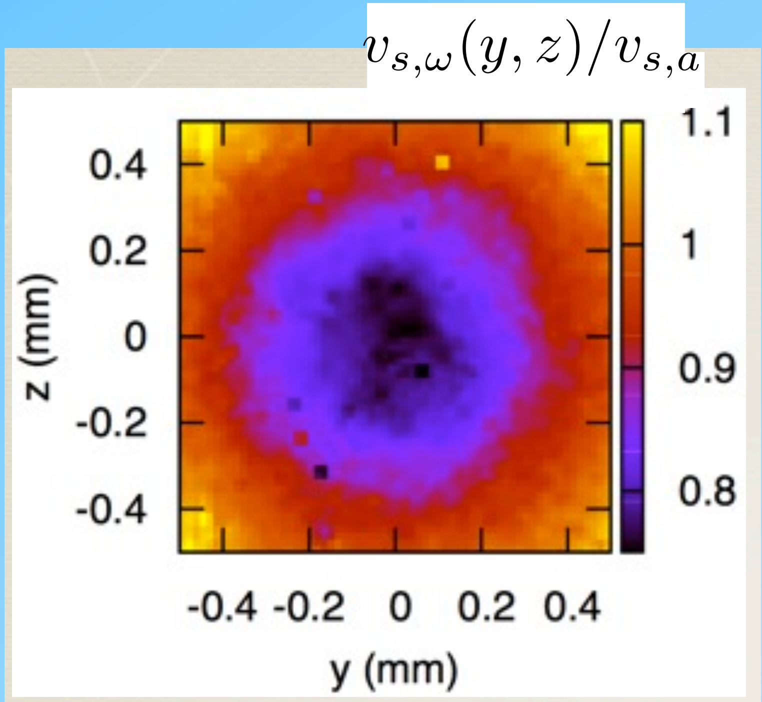
S. R. Stalp, L. Skrbek, R.J. Donnelly, Phys. Rev. Lett. 82, 4831 (1999)

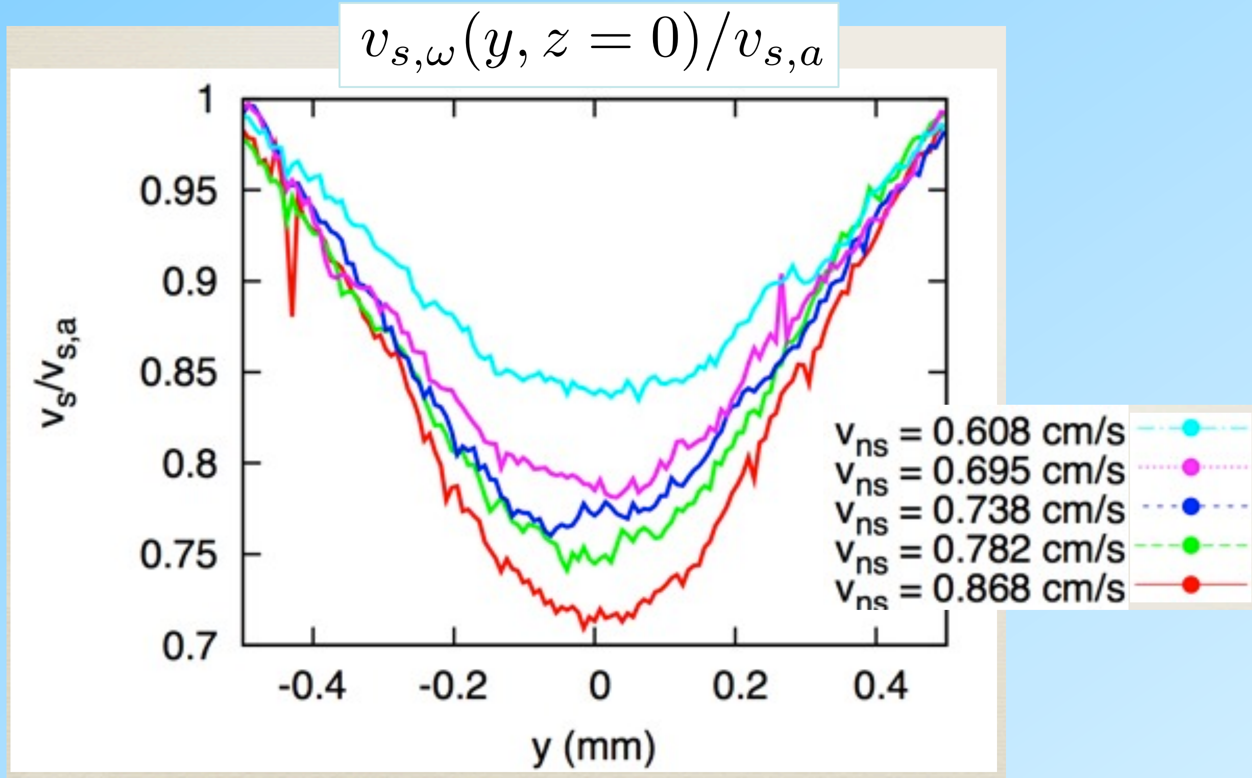
W. F. Vinen, W. Guo, in this workshop

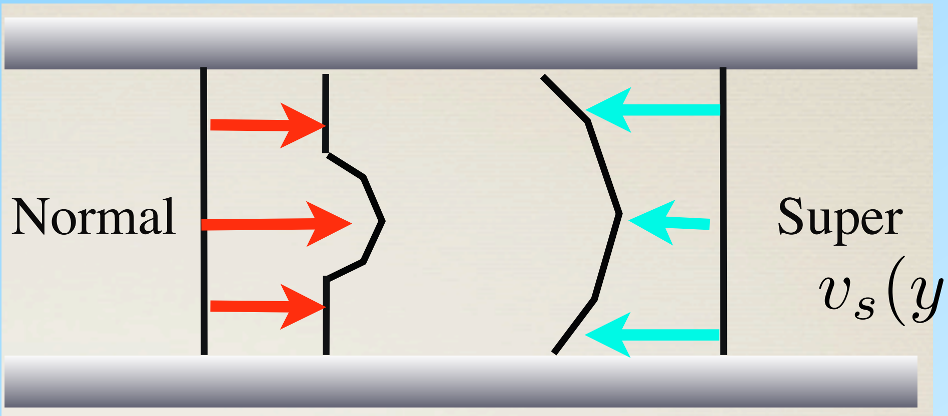
We investigated this issue in our situation.



Superfluid velocity field $v_{s,\omega}(y,z)$ created by the vortex tangle





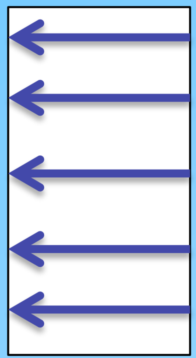


Superfluid velocity by the tangle is along v_n, depressing $v_{s,a}$.

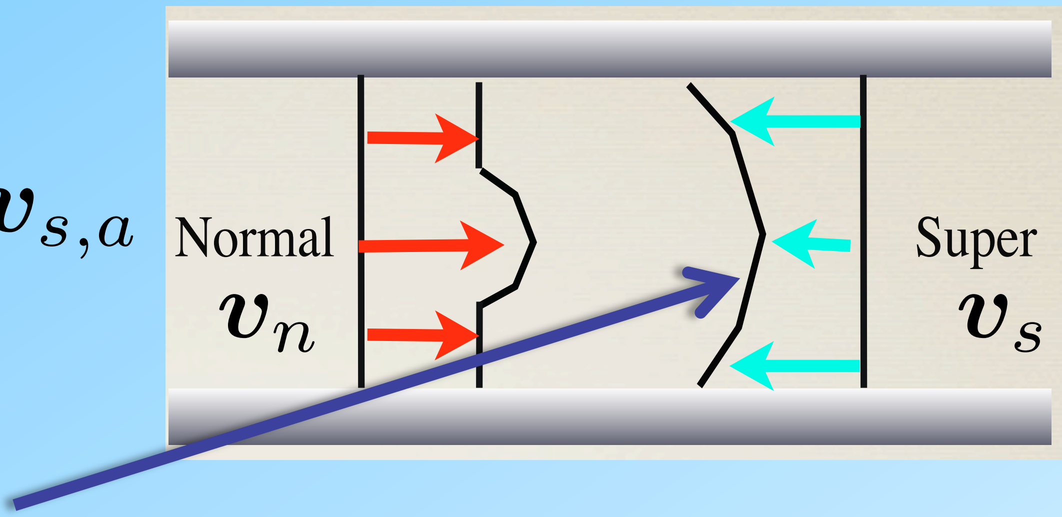
$$v_s(y,z) = v_{s,a} + v_{s,\omega}(y,z)$$

Superfluid velocity

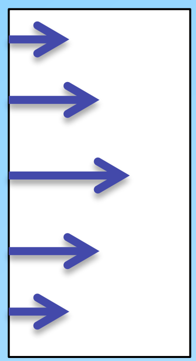
$$\boldsymbol{v}_s = \boldsymbol{v}_{s,a} + \boldsymbol{v}_{s,vortex}$$



Applied velocity $\boldsymbol{v}_{s,a}$



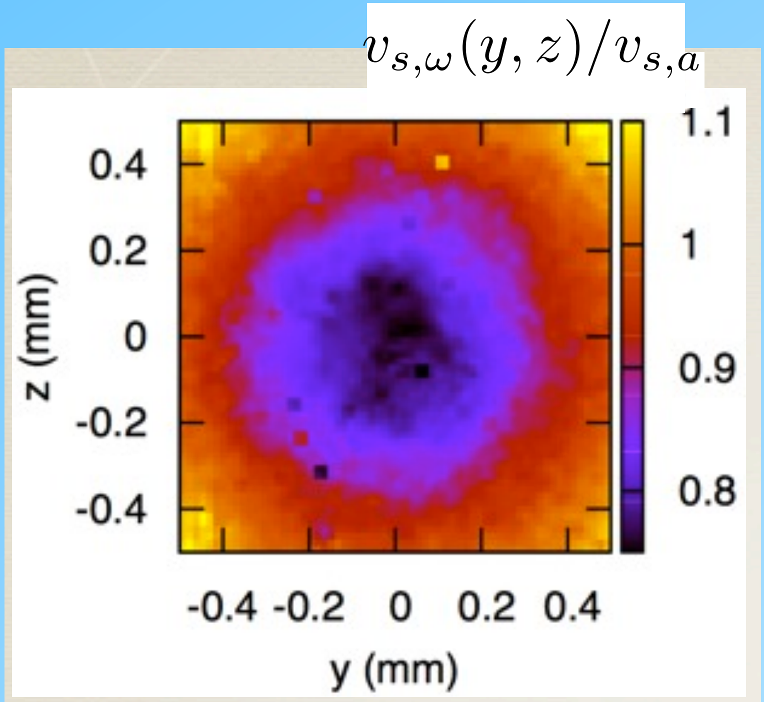
+

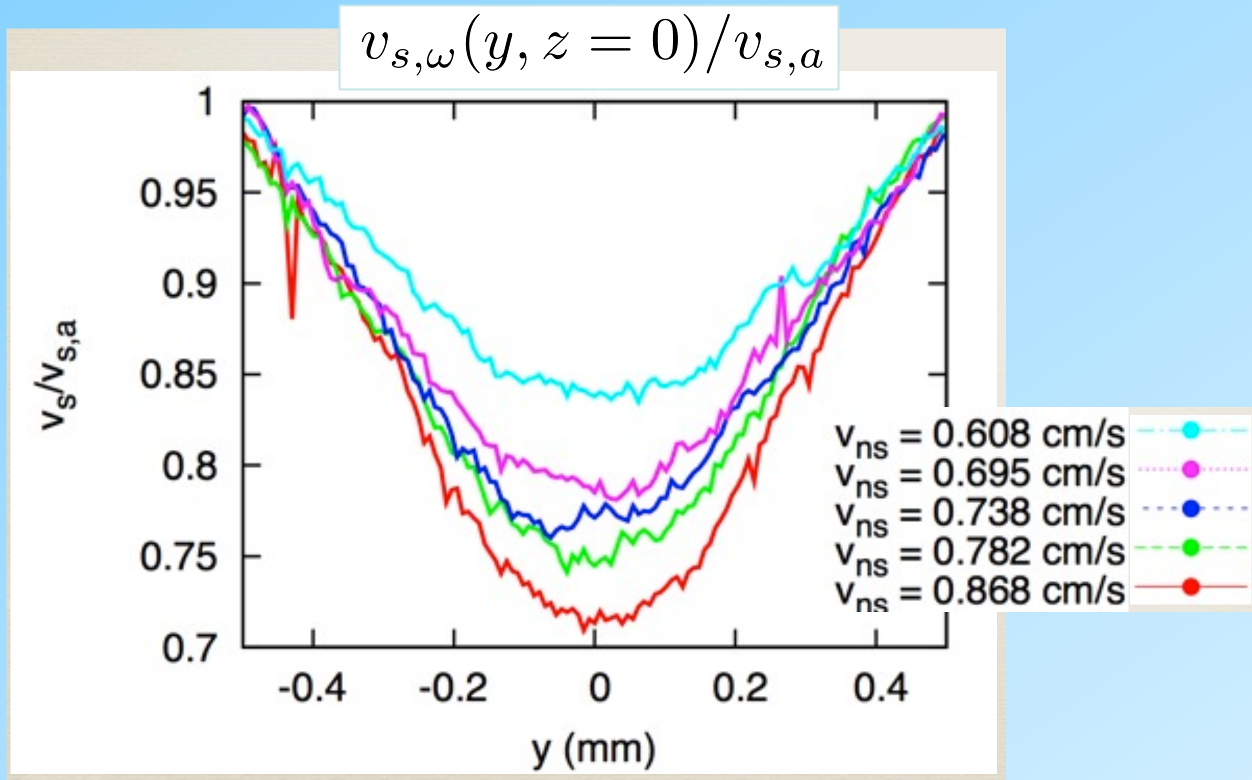


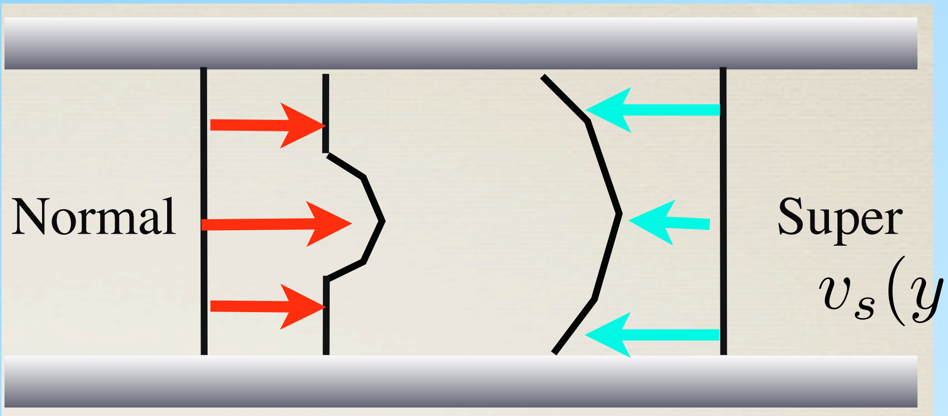
Velocity made by the vortex tangle

$$\boldsymbol{v}_{s,vortex} = \frac{\kappa}{4\pi} \int_{\mathcal{L}} \frac{(\boldsymbol{s}_1 - \boldsymbol{r}) \times d\boldsymbol{s}_1}{|\boldsymbol{s}_1 - \boldsymbol{r}|^3}$$

Superfluid velocity field $v_{s,\omega}(y, z)$ created by the vortex tangle







Superfluid velocity by the tangle is along v_n, depressing $v_{s,a}$.

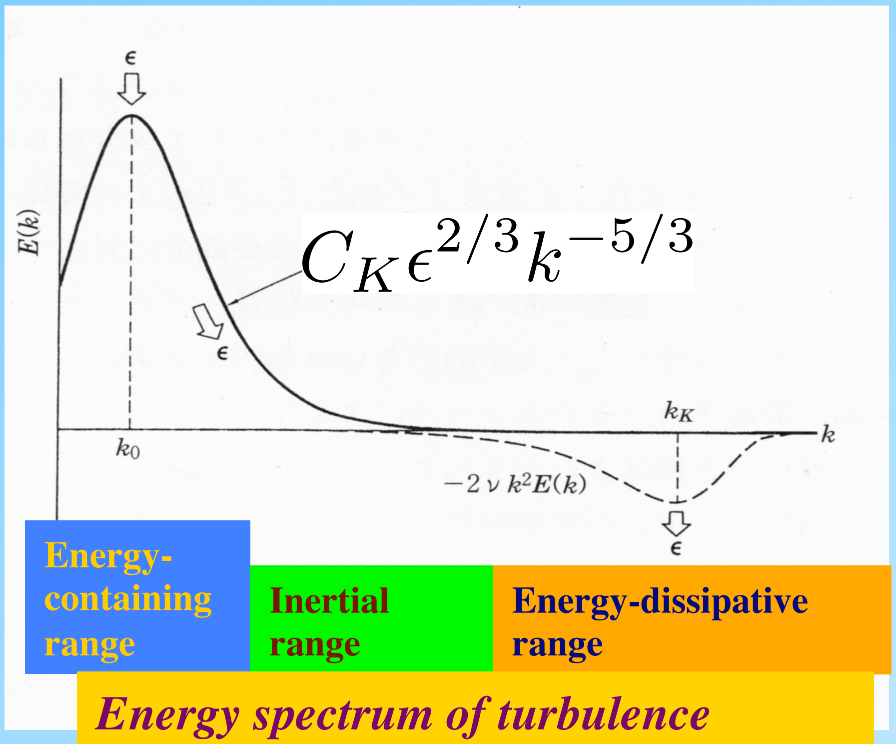
$$v_s(y, z) = v_{s,a} + v_{s,\omega}(y, z)$$

4-2. Logarithmic velocity profile of quantum turbulence of superfluid ^{4}He

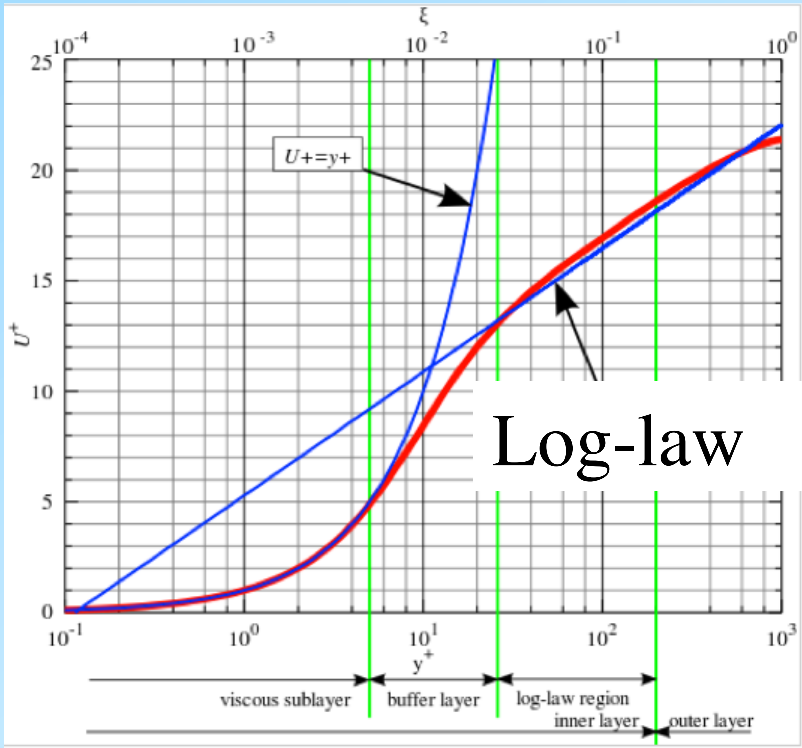
S. Yui, K. Fujimoto and M. Tsubota, arXiv:1508.01347

Two well-known statistical laws in classical turbulence

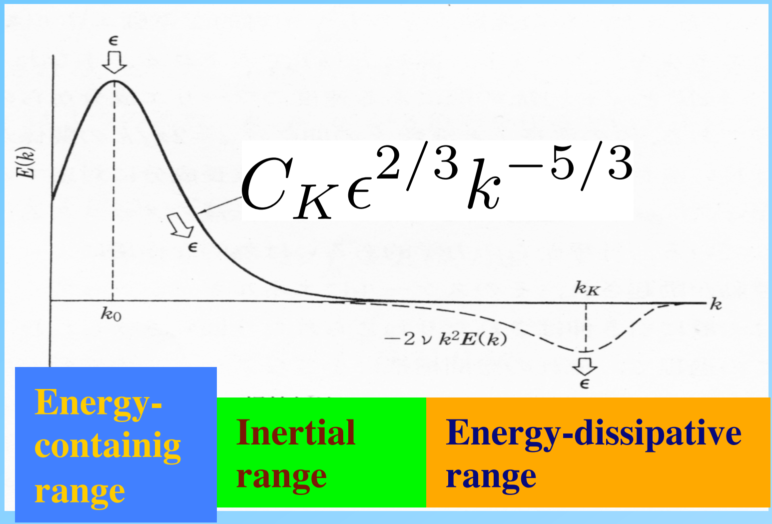
Kolmogorov -5/3 law in the bulk



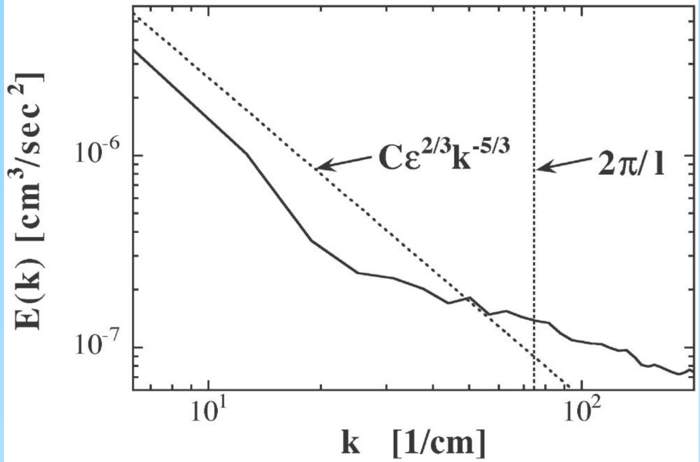
Log- law near walls



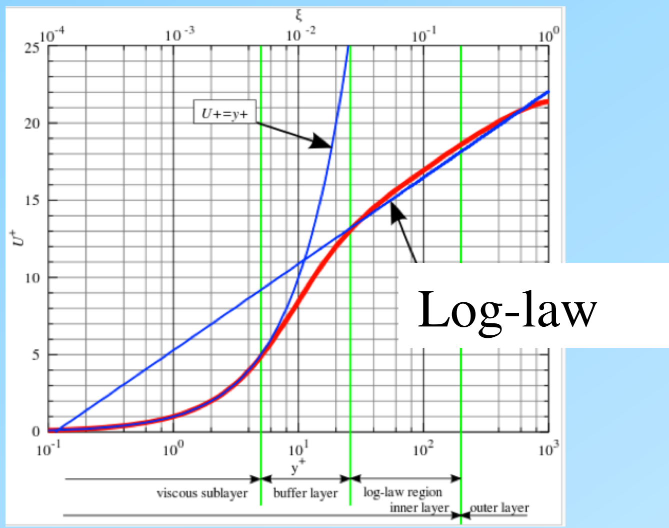
Classical turbulence vs Quantum turbulence



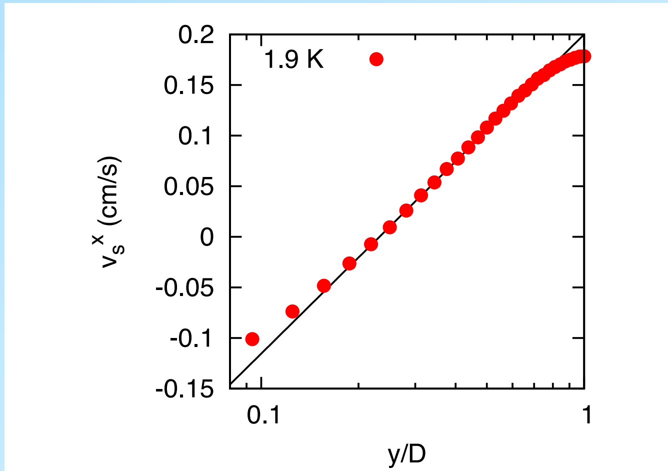
Kolmogorov -5/3 law in the bulk



T. Araki, M. Tsubota and S. K. Nemirovskii, PRL89, 145301(2002)

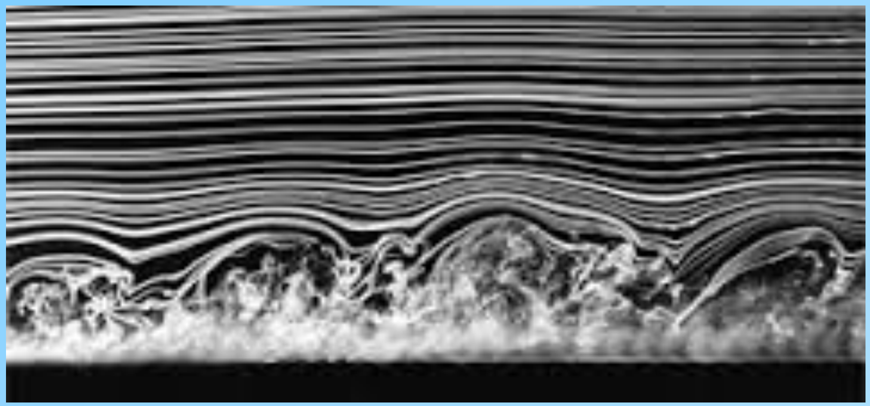


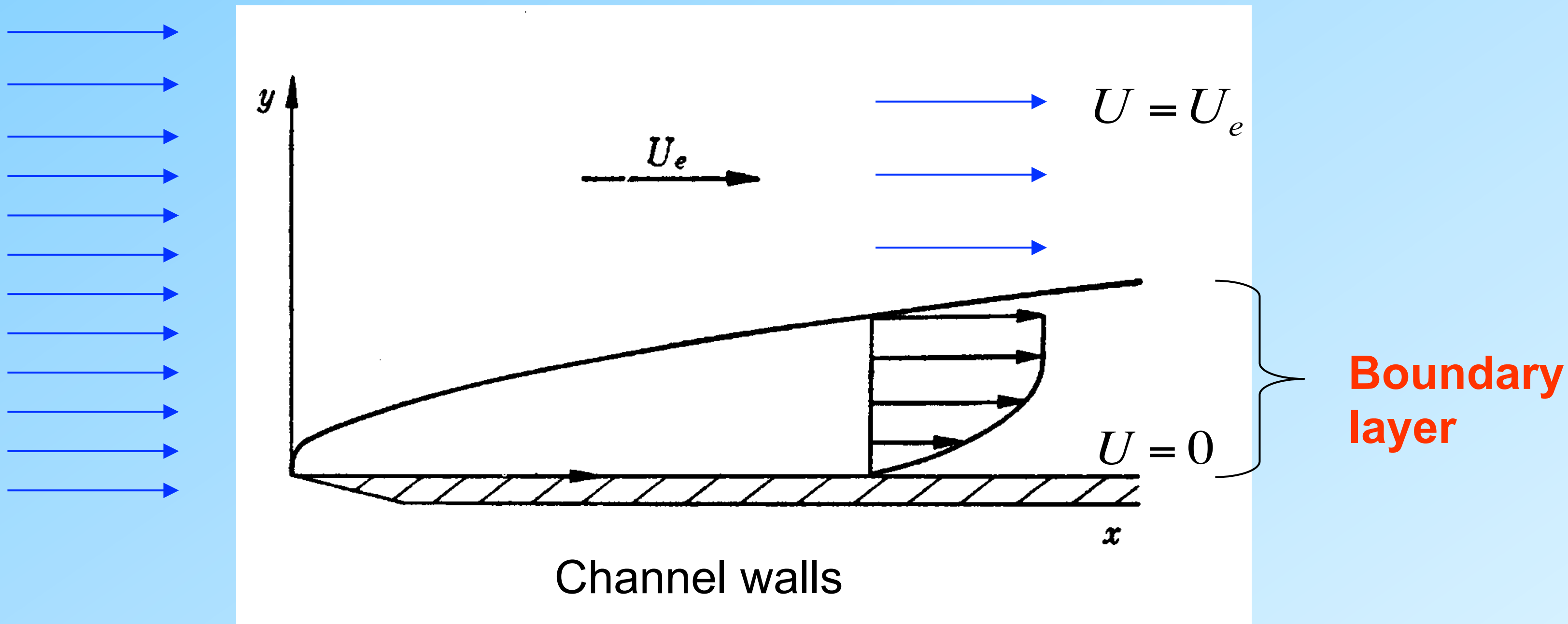
Log- law near walls



S. Yui, K. Fujimoto and M. Tsubota, arXiv.1508.01347

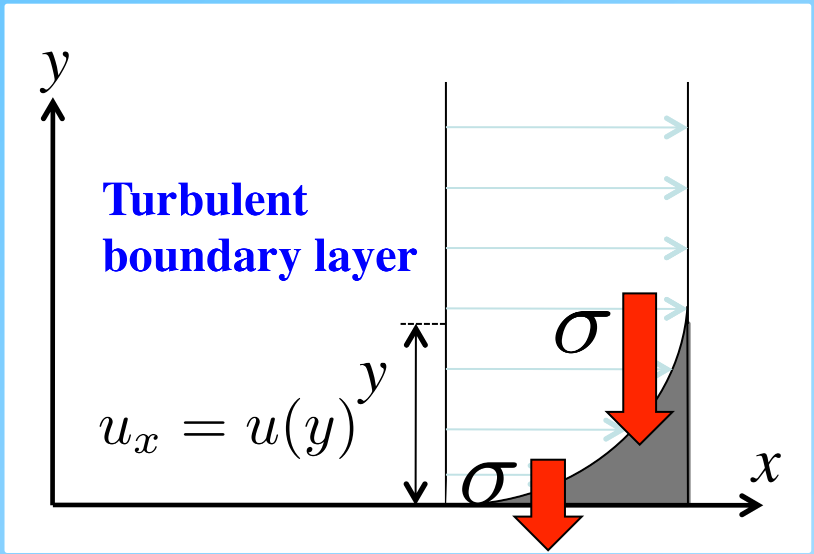
Turbulent boundary layer in a classical fluid





How to derive the log-law (1)

cf. Landau-Lifshitz: Fluid Mechanics



Averaged velocity $u_x = u(y), u_y = u_z = 0$

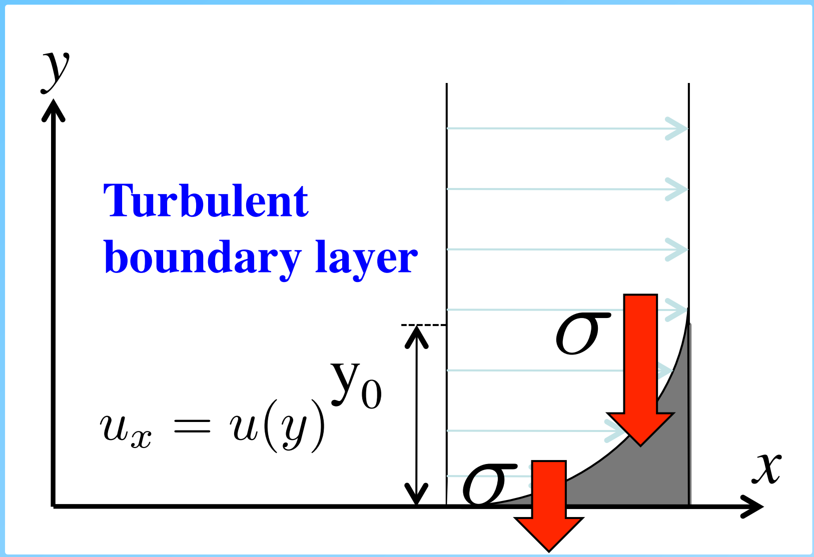
- Viscosity is not available except near the walls..
- Constant momentum flux σ (Reynolds stress) flows from the bulk to the walls.
- σ dissipates by the viscosity near the walls.

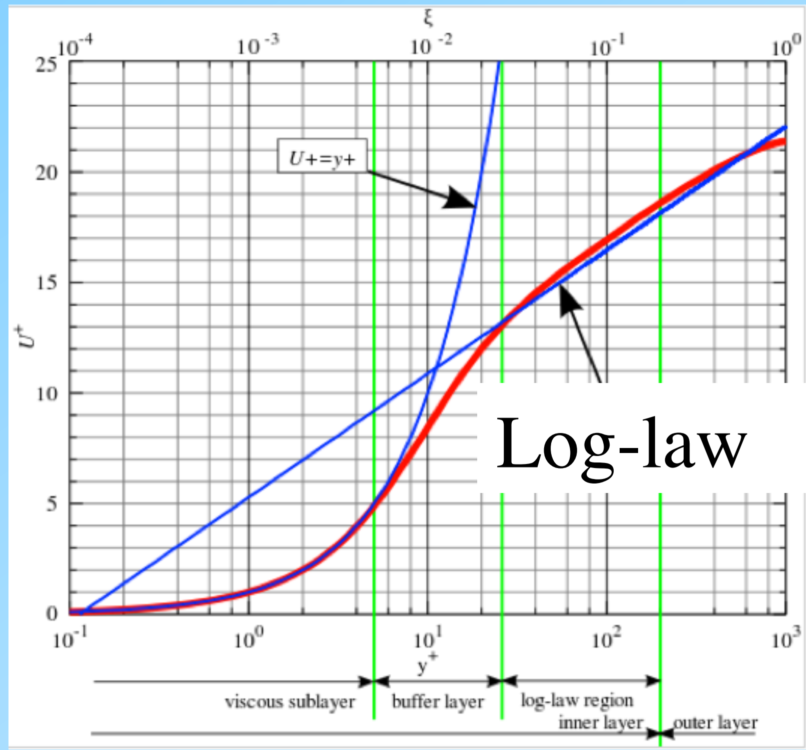
du/dy is determined by the fluid density ρ , momentum flux σ , distance *y*.

Dimension $[du/dy] = 1/T, [\rho] = M/L^3, [\sigma] = M/(L \cdot T^2), [y] = L$

$$\frac{du}{dy} = \frac{\sqrt{\sigma/\rho}}{by}, \qquad b = 0.417$$:Karman constant

How to derive the log-law (2)





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locity

What determines the width y_0 of the boundary layer ?

$$Re = \frac{v_* y_0}{\nu} \sim 1 \qquad \rightarrow \qquad y_0 \sim \frac{\nu}{v_*}$$

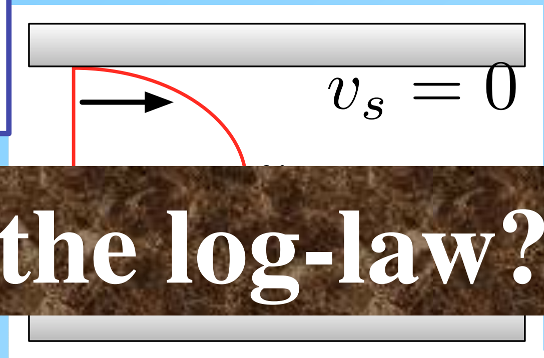
By some considerations,

$$\frac{u}{v_*} = \frac{1}{b} \log\left(\frac{y}{y_0}\right)$$

Quantum-turbulent boundary layer

A.W. Baggaley, S Laizet, Phys. Fluids 162, 354 (2011)
A. W. Baggaley, S. Laurie, JLTP178, 35(2014)

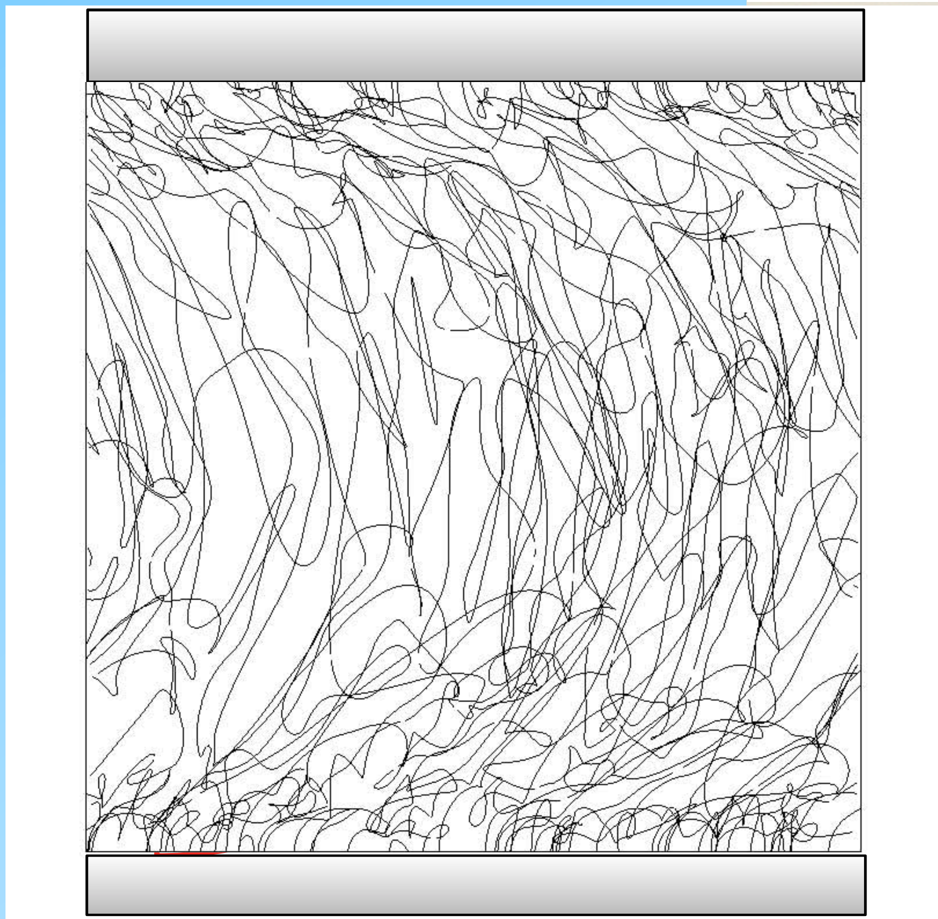
Pure normal flow between two parallel plates

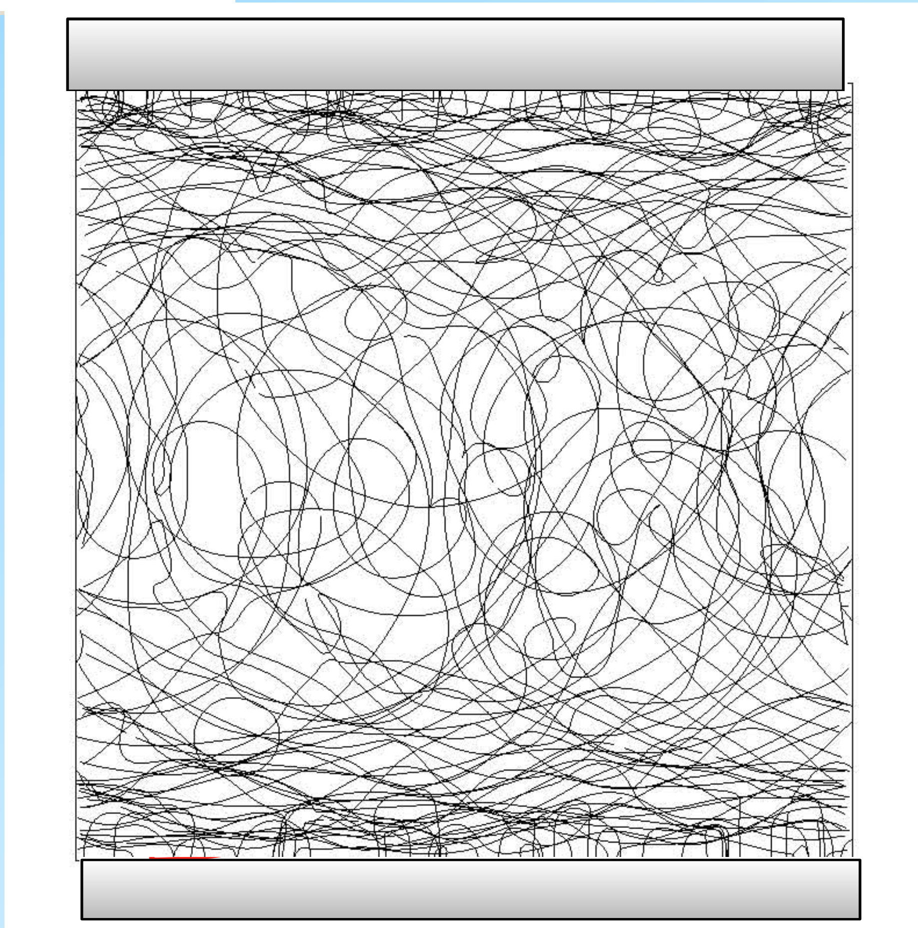


T=1.9K
$\overline{v_n}$ =0.9cm/s

How about the log-law?

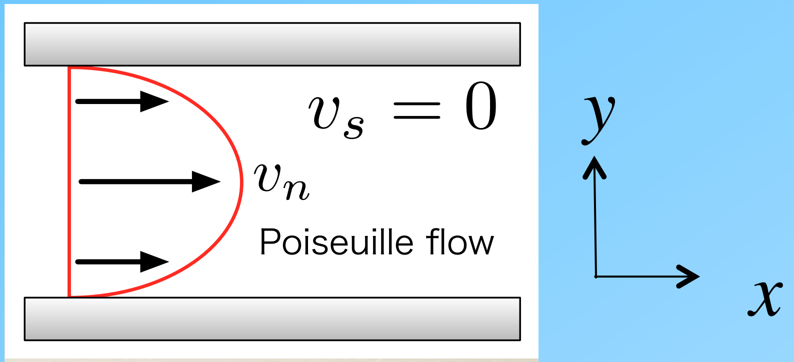
$\otimes\, v_n$



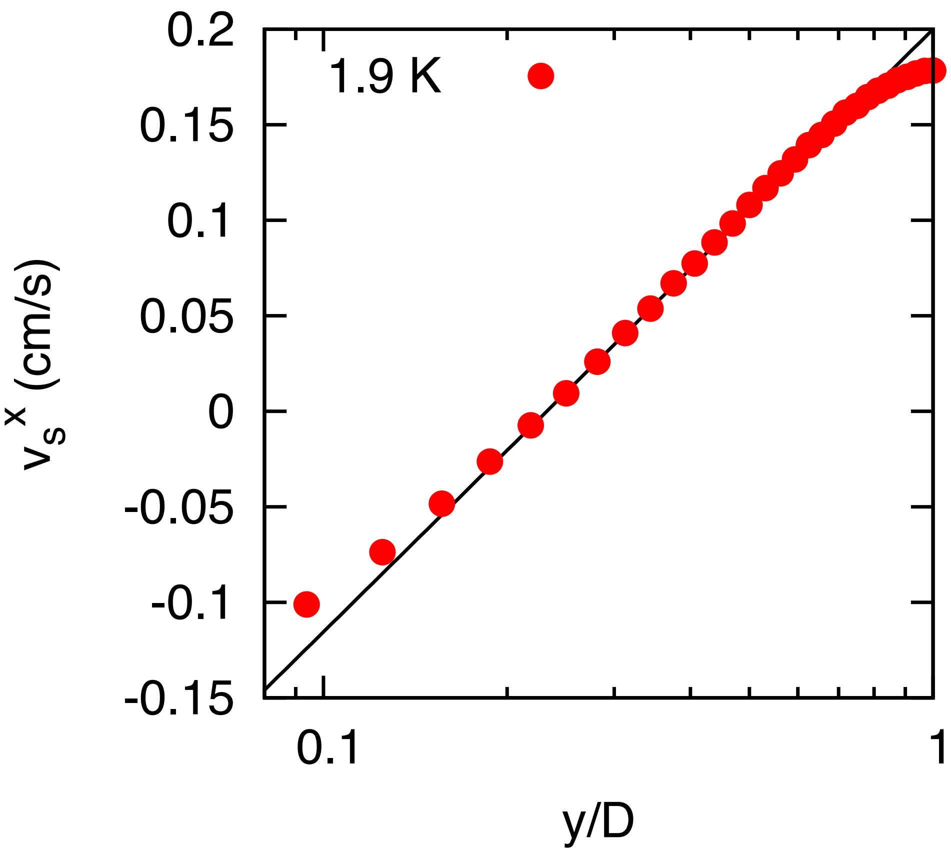


Quantum-turbulent boundary layer

$$\boldsymbol{v}_s = \frac{\kappa}{4\pi}\int_{\mathcal{L}} \frac{(\boldsymbol{s}_1 - \boldsymbol{r}) \times d\boldsymbol{s}_1}{|\boldsymbol{s}_1 - \boldsymbol{r}|^3}$$



The averaged velocity of turbulent superfluid flow obeys the log-law !



Temperature dependence of the log-law

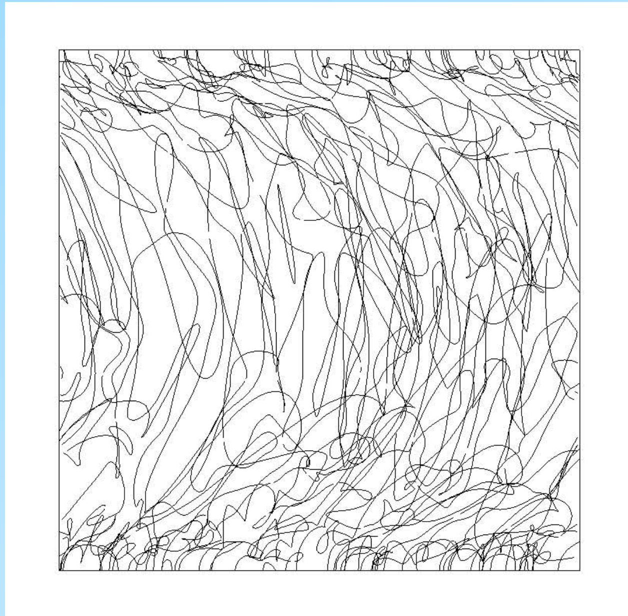
$\overline{v_n}$ =0.9cm/s

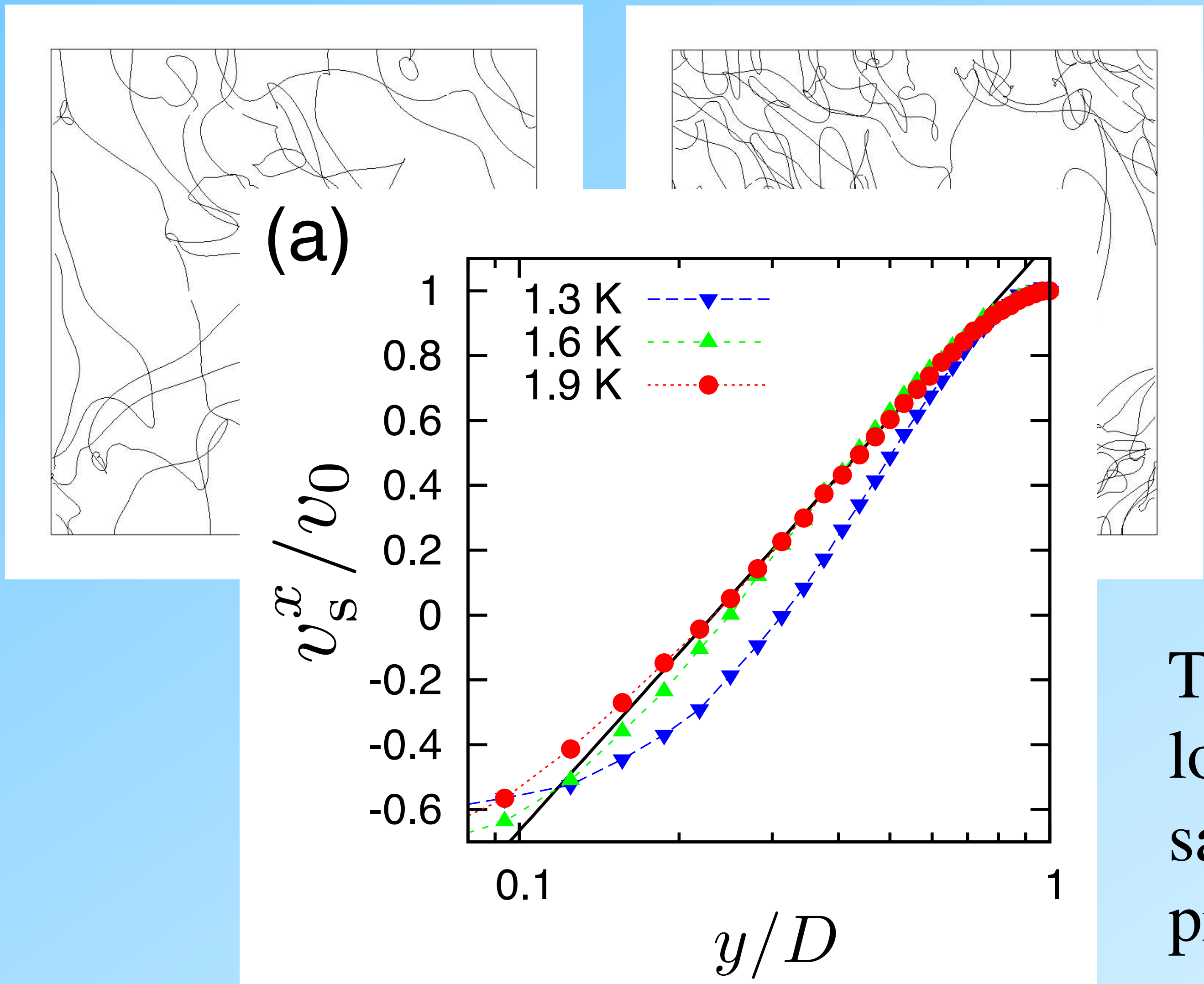
→ v_n

1.3K

1.6K

1.9K

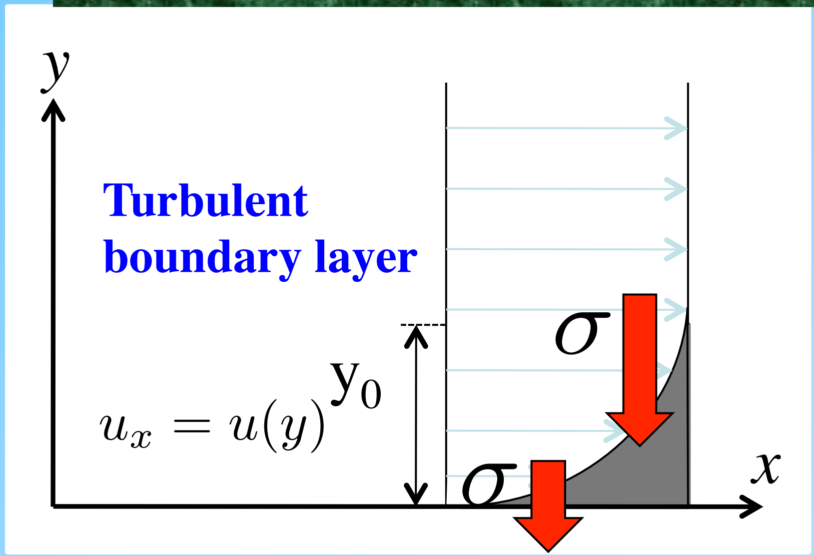


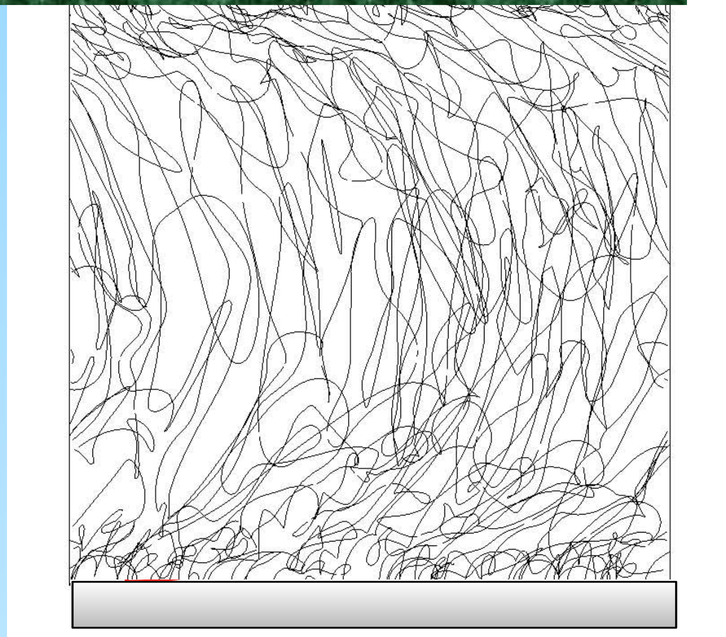


The dilute vortices at low temperature do not satisfy the log-law properly.

Every behavior of quantum turbulence comes from the dynamics and the configuration of quantized vortices, so does even the log-law.

Can we understand the log-law from the behavior of quantized vortices?





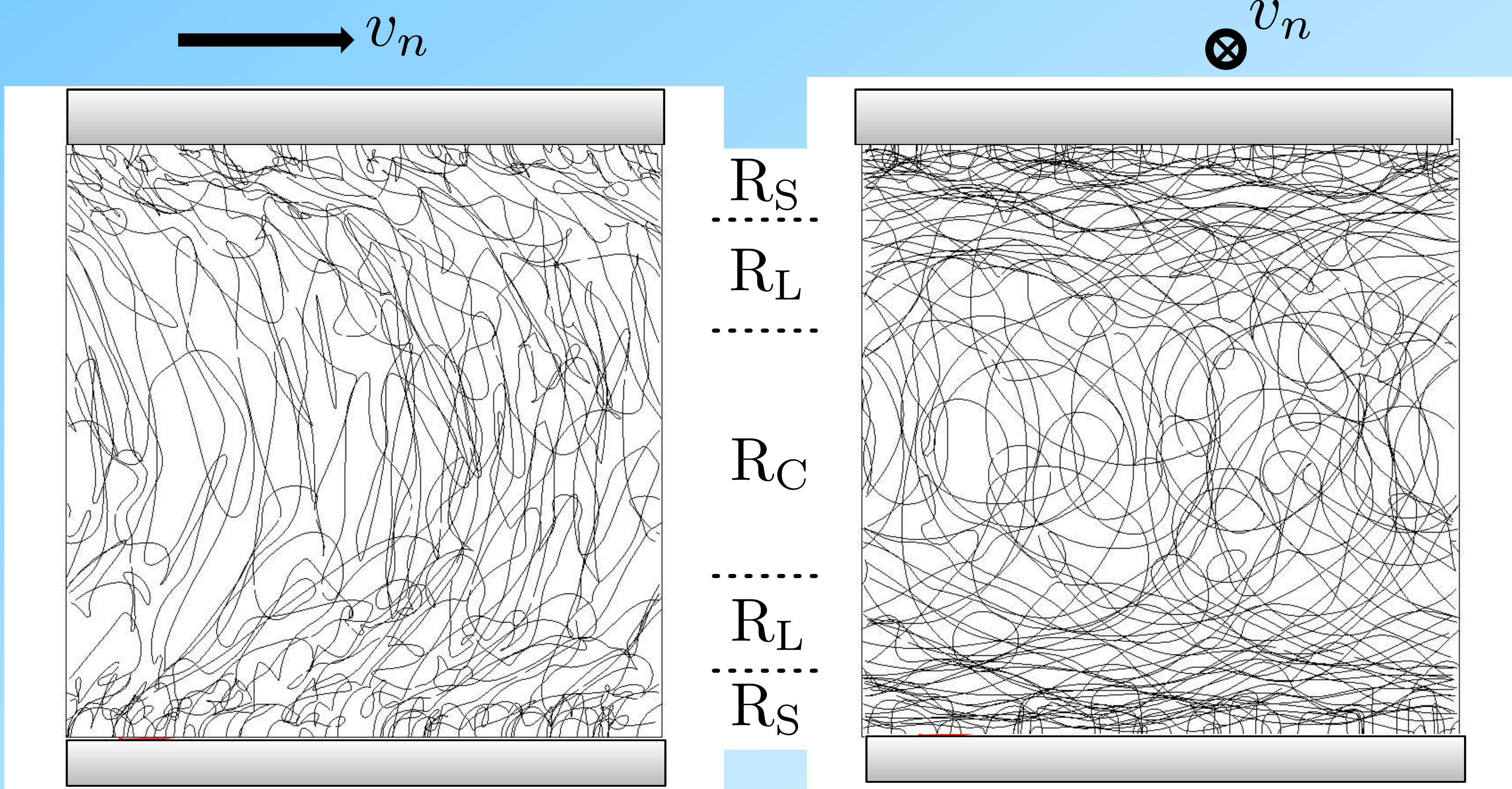
Momentum is transferred toward the walls.

Every behavior of quantum turbulence comes from the dynamics and the configuration of quantized vortices, even the log-law.

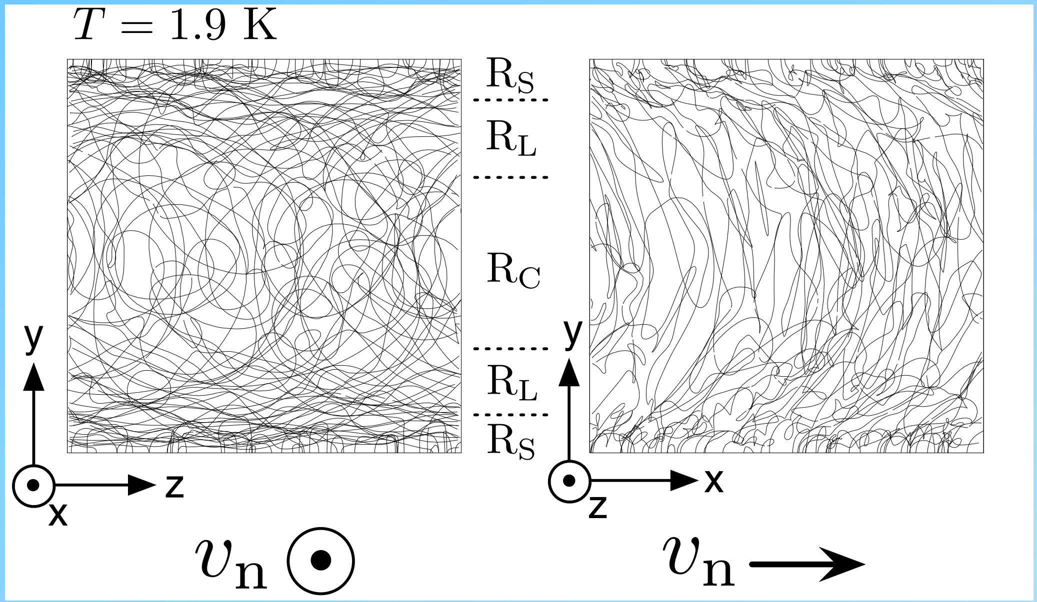
R_L: the log-law region

T=1.9K

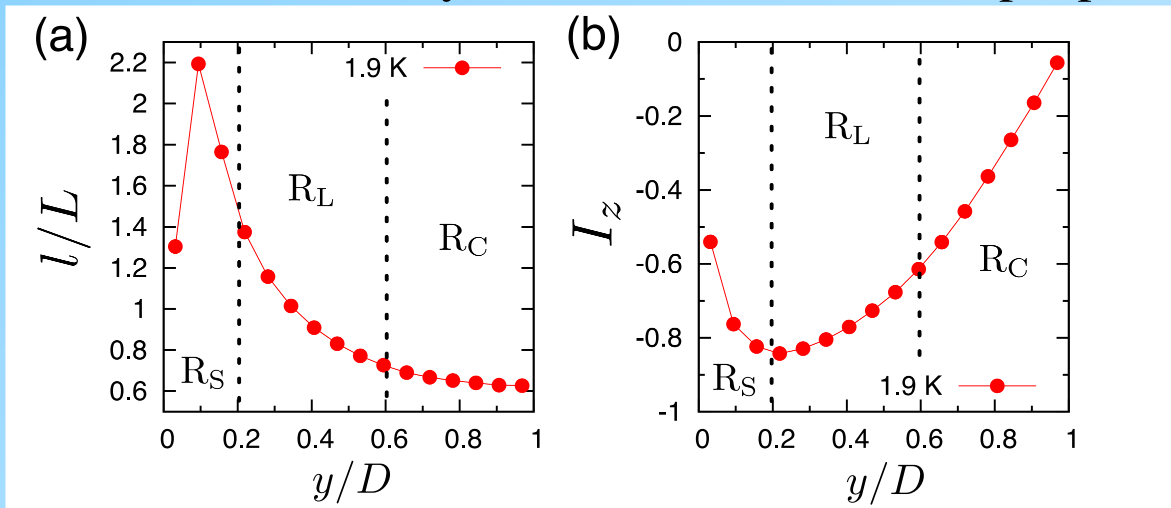
$\overline{v_n}$ =0.9cm/s



Configuration of vortices



l: local vortex density $I_Z \propto \boldsymbol{s}' \cdot \hat{\boldsymbol{z}}$: Anisotropic parameter



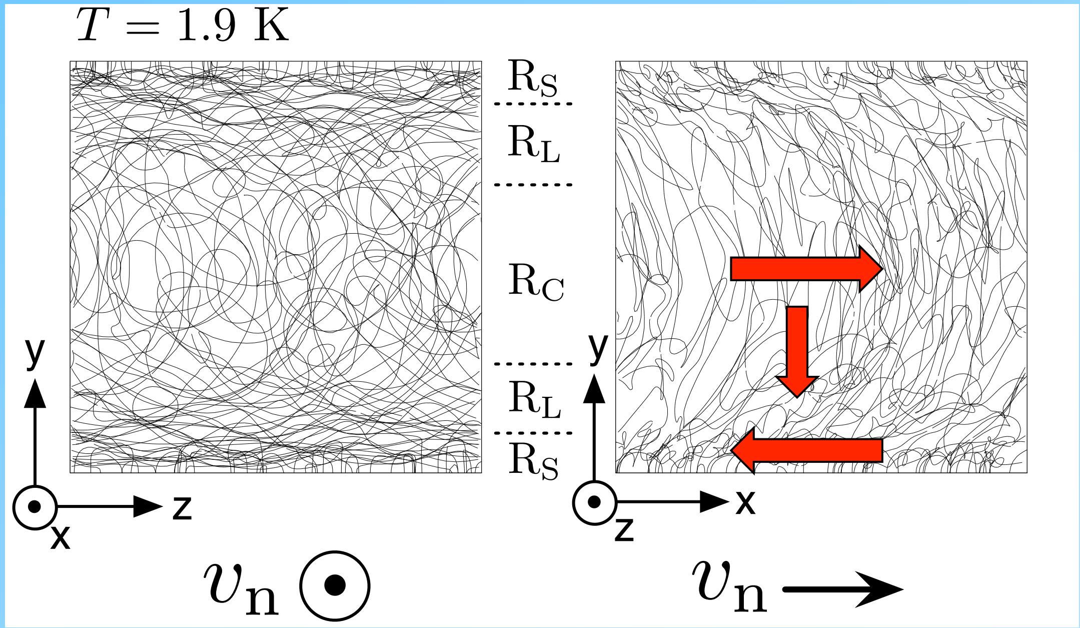
Three regions

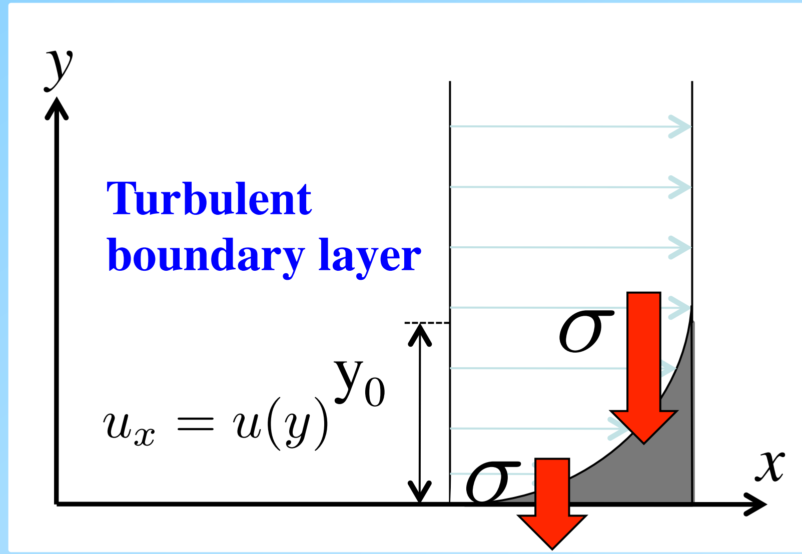
R_C: Central low density region

R_L: Log-law region, vortices are parallel to z

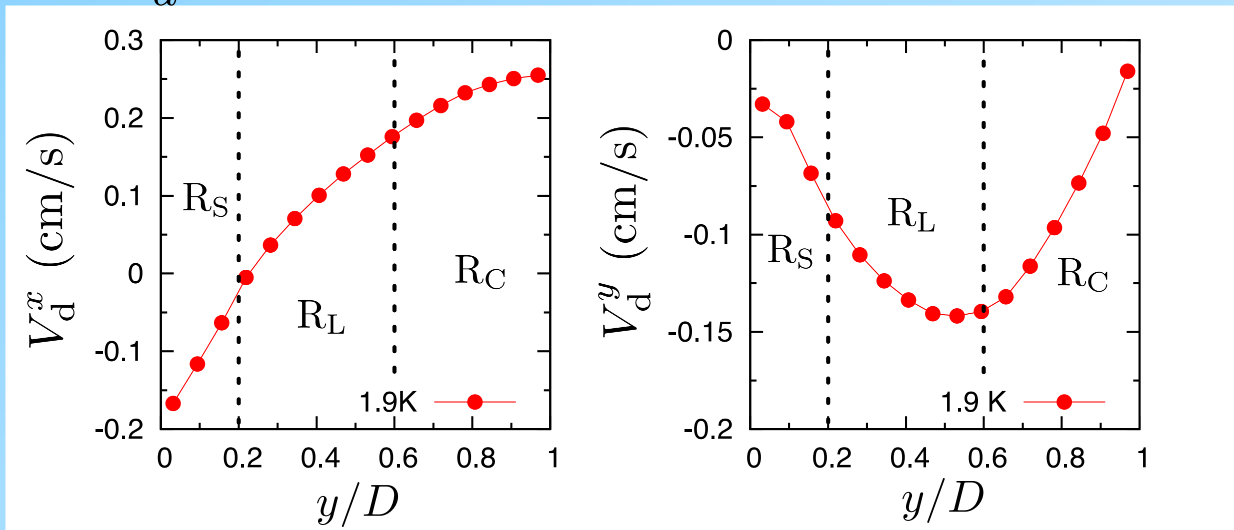
R_S: High-density region near the wall, consisting of small curved loops

Dynamics of vortices



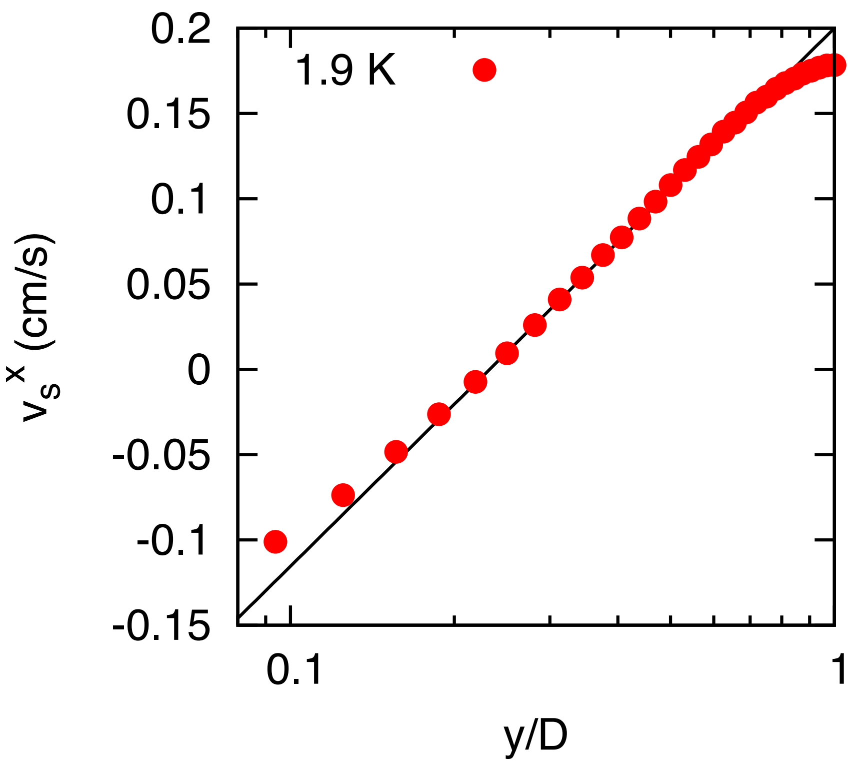


$V_d^i \propto \dot{\boldsymbol{s}} \cdot \hat{\boldsymbol{r}}_i$: Drift velocity



The vortices move towards the walls.
→ Transfer of momentum

How about the Karman constant?



$$v_s^x = \frac{v_q^*}{\kappa_q}\left[\log\left(\frac{y}{D}\right) + c\right]$$

T (K)	v_0 (s/cm)	$v_{\mathrm{q}}^*/\kappa_{\mathrm{q}}$ (s/cm)	c –
1.9	0.184	0.141	1.46
1.6	0.079	0.070	1.40
1.3	0.025	0.028	1.14

We know v_q^*/κ_q from the fitting. Since we have no theory for v_q^*, however, we cannot obtain the Karman constant κ_q.

Summary

1. We review the simulation of VFM in mogeneous counterflow.

2. The recent visualization experiments open the door of the new era.

3. We discussed the two topics in inhomogeneous case.

3-1. Inhomogeneous turbulence in a square channel

S. Yui, M. Tsubota, Phys. Rev. B91, 184504(2015)

3-2. Log-law in turbulent boundary layer

S. Yui, K. Fujimoto, M. Tsubota, arXiv:1508.01347