



Universitas Carolina  
Charles University in Prague



## Recent Results on Quantum Turbulence in $4\text{He}$ Obtained Using Visualization, and Oscillating Objects in Prague

**L. Skrbek**, Faculty of Mathematics and Physics, Charles University in Prague



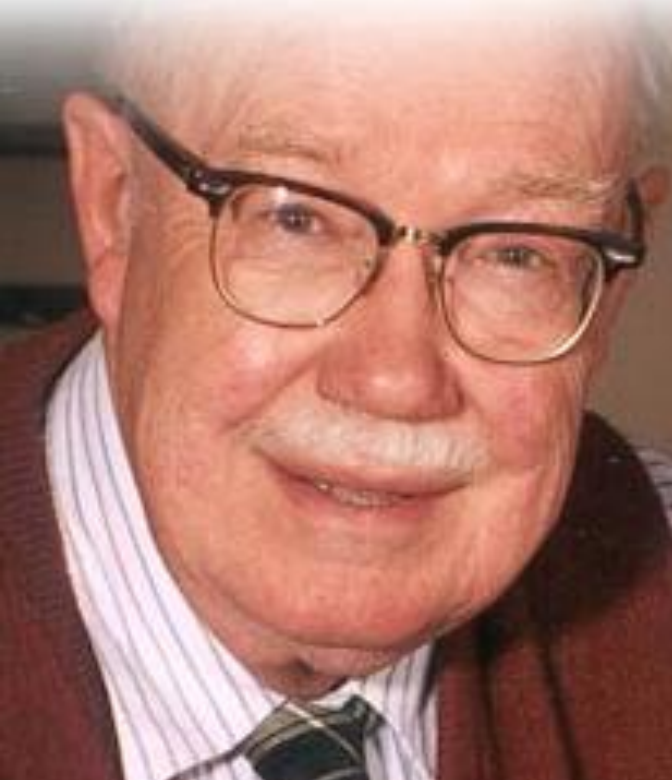
Prague group:

**M. La Mantia, M. Jackson, D. Duda, E. Varga S. Babuin, M. Rotter, LS, D. Schmoranzer**



Saclay, September  
14-18, 2015

Russell J. Donnelly , Professor of Physics  
University of Oregon



\* April 16, 1930  
† June 13, 2015



# 1958 - Periodic Boundary Layer Experiments in Liquid Helium

ANNALS OF PHYSICS: **3**, 320-345 (1958)

## Periodic Boundary Layer Experiments in Liquid Helium\*

R. J. DONNELLY

*Institute for the Study of Metals and Department of Physics, University of Chicago,  
Chicago, Illinois*

AND

A. C. HOLLIS HALLETT

*Department of Physics, University of Toronto, Toronto, Ontario*

A hydrodynamical study of oscillation experiments in helium I and helium II has been made using the concept of the boundary layer. Experimental results show that the hydrodynamical flow of helium II changes with amplitude of oscillation. At low amplitudes the simple two-fluid model is applicable; at higher amplitudes the normal and superfluid components are in common motion and at the highest amplitudes the flow is turbulent. A table of values of the kinematic viscosity of helium II for the condition when the two fluids move together has been given. The principle of dynamical similarity has been discussed and a dimensionless similarity parameter is given which governs the onset of turbulence in liquid helium. In helium II a new similarity parameter is required at lower amplitudes. An empirical expression is given for this parameter. Some new theoretical questions have been raised.

2015 – Repeat using modern technology, such as imaging and data processing and interpret using current knowledge

# Recent Prague results on visualization of cryogenic helium flows

**Marco La Mantia, Daniel Duda, Patrick Švančara, Miloš Rotter, LS**

**Numerics: Emil Varga**

**Coworkers: Carlo Barenghi, Yuri Sergeev**



## Investigated flows:

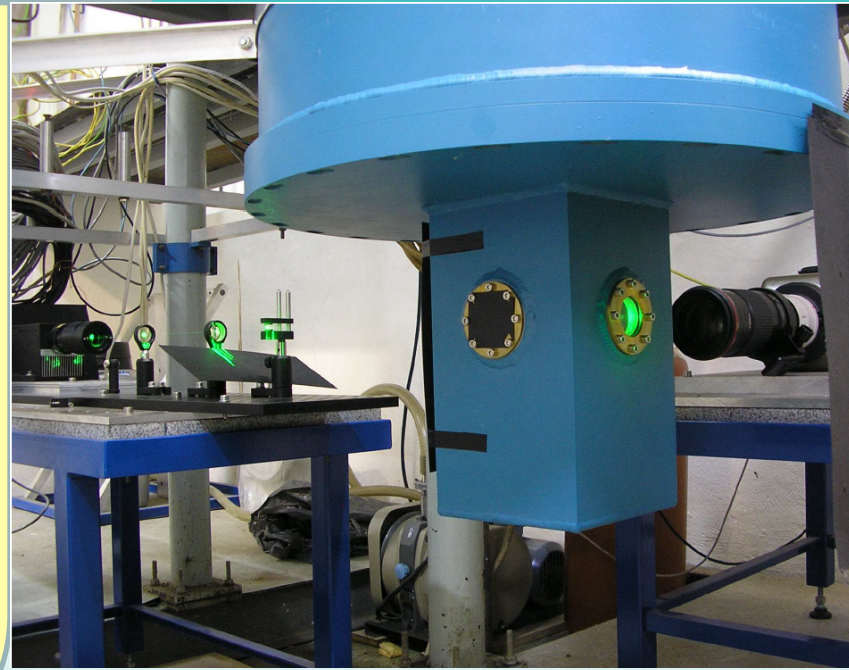
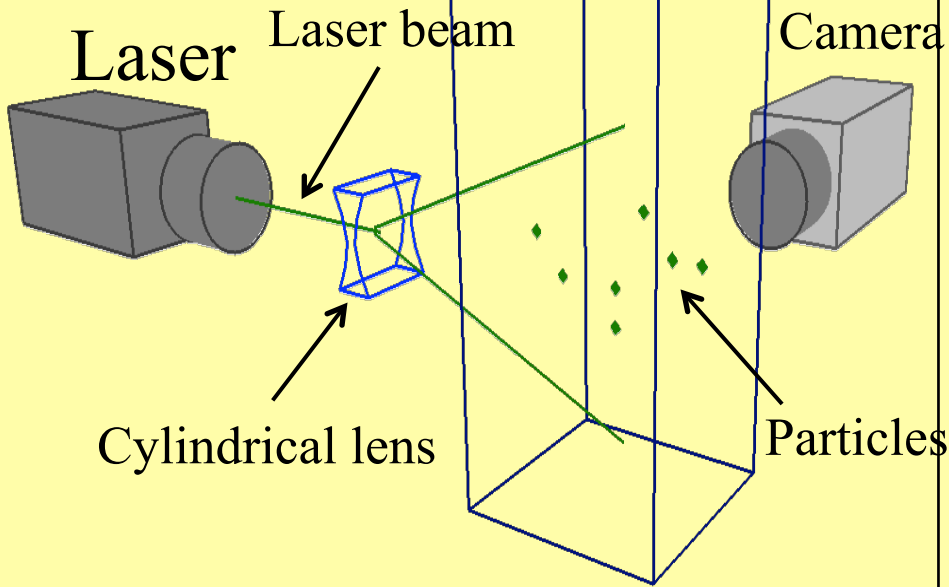
**Thermal counterflow**

**Counterflow past a cylinder**

**Flows of normal and superfluid 4He due to oscillating objects**

- M. La Mantia, T. V. Chagovets, M. Rotter and LS *Testing the performance of a cryogenic visualization system on thermal counterflow by using hydrogen and deuterium solid tracers.* **Rev. Sci. Instrum. 83, 055109 (2012)**
  - M. La Mantia, D. Duda, M. Rotter and LS, *Lagrangian accelerations of particles in superfluid turbulence.* **JFM 717, R9 (2013)**
  - M. La Mantia and L. Skrbek, *Quantum, or classical turbulence?* **EPL 105, 46002 (2014)**
  - D. Duda, M. La Mantia, M. Rotter and LS, *On the visualization of thermal counterflow of He II past a circular cylinder.* **JLTP 175, 331-338 (2014)**
  - W. Guo, M. La Mantia, D. P. Lathrop and S. W. Van Sciver, *Visualization of two-fluid flows of superfluid helium-4.* **PNAS 111, 4653-4658 (2014)**
  - M. La Mantia and LS, *Quantum turbulence visualized by particle dynamics.* **Phys. Rev. B 90, 014519 (2014)**
  - D. Duda, P. Švančara, M. La Mantia, M. Rotter and LS, *Visualization of viscous and quantum flows of liquid 4He due to an oscillating cylinder of rectangular cross section.* **Phys. Rev. B, in print**
- E. Varga, C. F. Barenghi, Y. A. Sergeev, LS *Backreaction of Tracer Particles on Vortex Tangle in Helium II Counterflow***  
**Submitted to JLTP –Proc. of QFS 2015**

# Prague Visualization Laboratory



- Custom-built **low-loss cryostat** with five sets of windows that **minimise heat input into the helium bath**, enabling horizontal as well as vertical optical access
- **Continuous wave solid state laser**, **fast digital camera** and relevant **hardware and software** to implement the **PIV** and **PTV** techniques for cryogenic flows analysis



**Micron-sized  
hydrogen/deuterium  
tracers**

## Experimental conditions

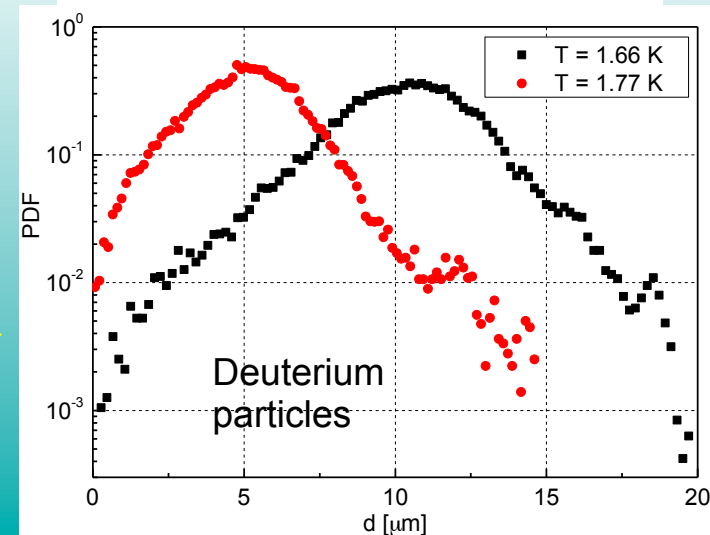
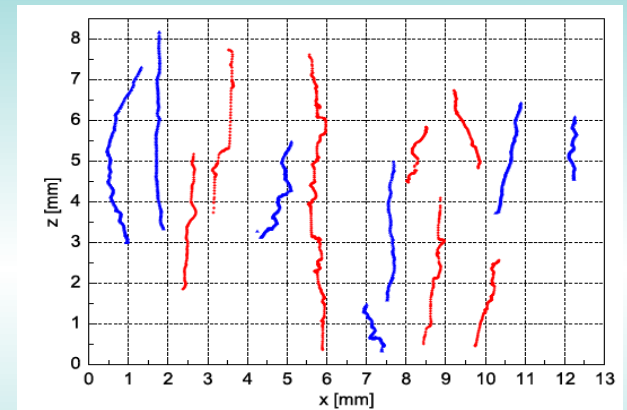
Thermal counterflow of superfluid  $^4\text{He}$   
in a square channel  
of 25 mm sides and 100 mm long  
experimentally studied by visualization

Temperature:  $1.3 \text{ K} < T < 2.1 \text{ K}$

Applied heat flux:  $50 \text{ W/m}^2 < q < 500 \text{ W/m}^2$

Counterflow velocity  $v_{ns}$  up to 10 mm/s

SF  
↓  
↑  
NF

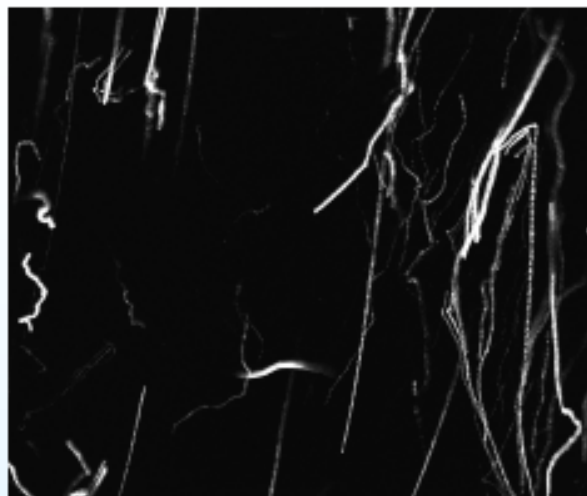


The particles' radii are calculated by assuming that the particles are spherical and that the buoyancy force is balanced by the Stokes drag,

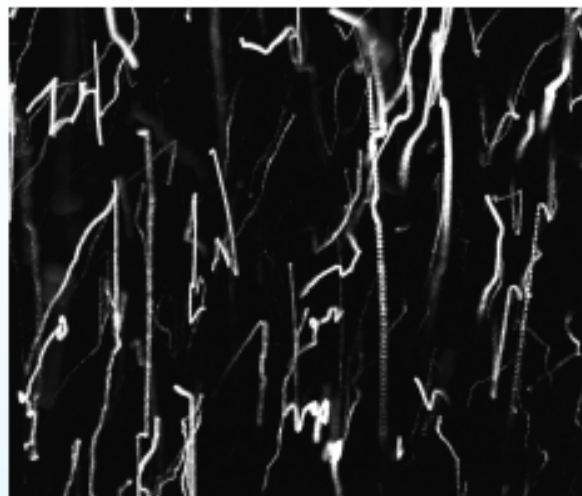
$$R_p = \sqrt{\frac{9 \mu v_1}{2 g (\rho - \rho_p)}}$$

# Results

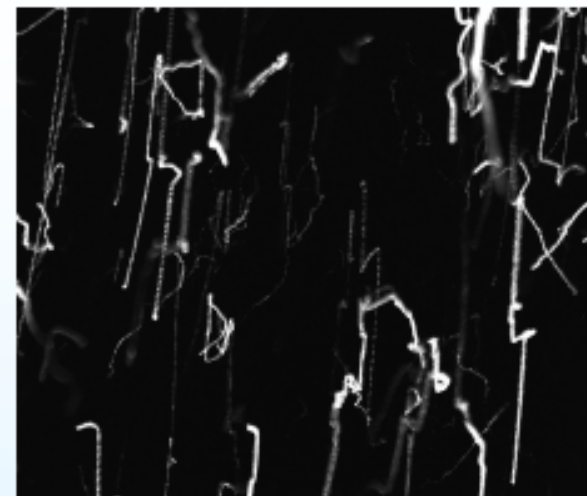
$T = 1.95 \text{ K}$



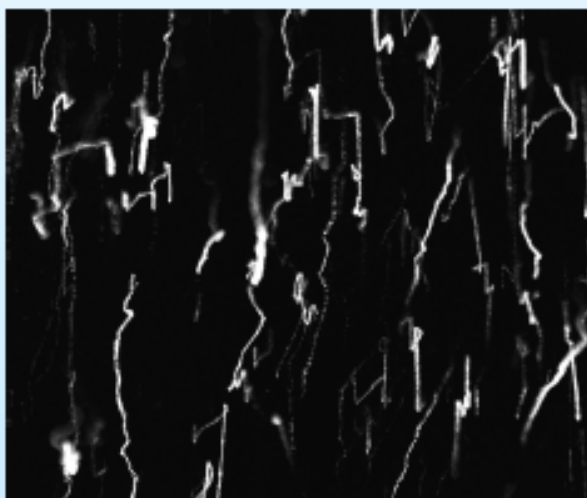
$q = 97 \text{ W/m}^2$ ;  $v_{ns} = 0.80 \text{ mm/s}$



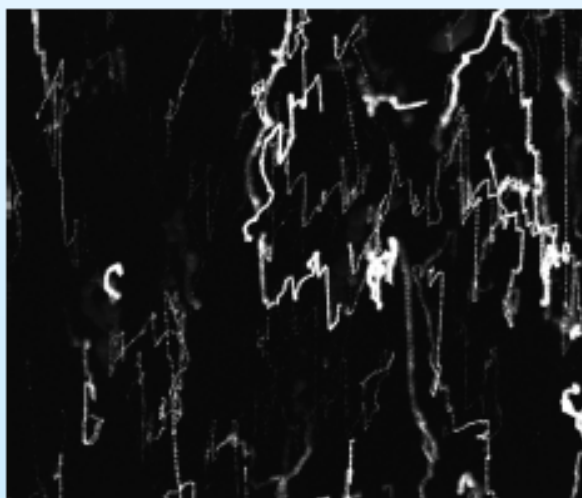
$q = 121 \text{ W/m}^2$ ;  $v_{ns} = 1.00 \text{ mm/s}$



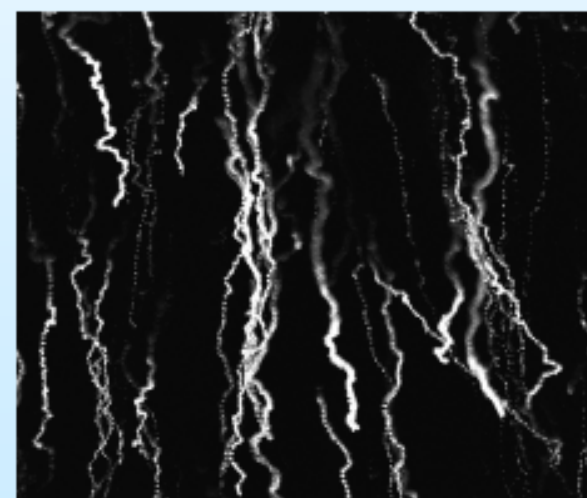
$q = 146 \text{ W/m}^2$ ;  $v_{ns} = 1.21 \text{ mm/s}$



$q = 193 \text{ W/m}^2$ ;  $v_{ns} = 1.60 \text{ mm/s}$



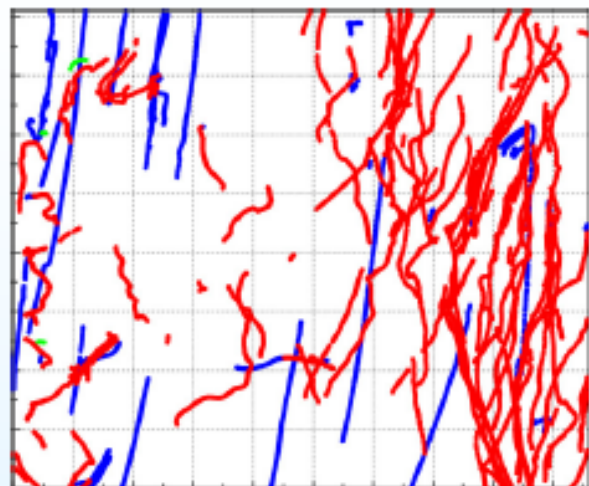
$q = 242 \text{ W/m}^2$ ;  $v_{ns} = 2.01 \text{ mm/s}$



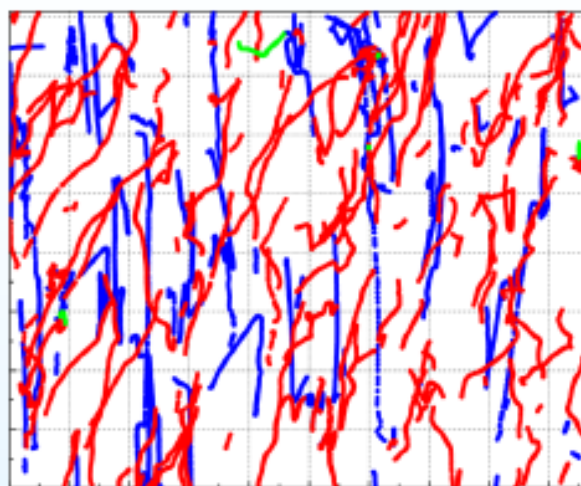
$q = 484 \text{ W/m}^2$ ;  $v_{ns} = 4.01 \text{ mm/s}$

$T = 1.95 \text{ K}$

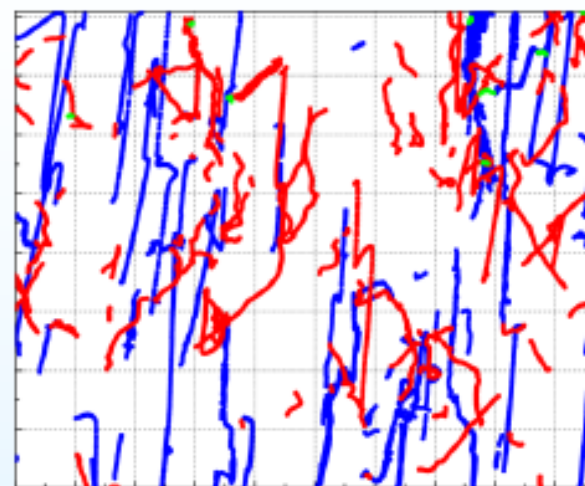
# Results



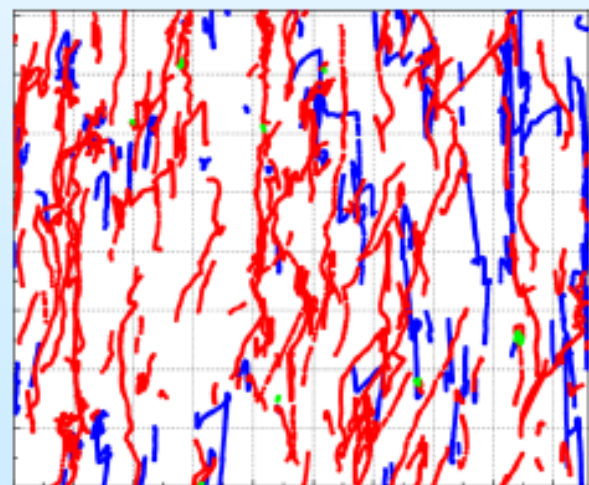
$q = 97 \text{ W/m}^2$ ;  $v_{ns} = 0.80 \text{ mm/s}$



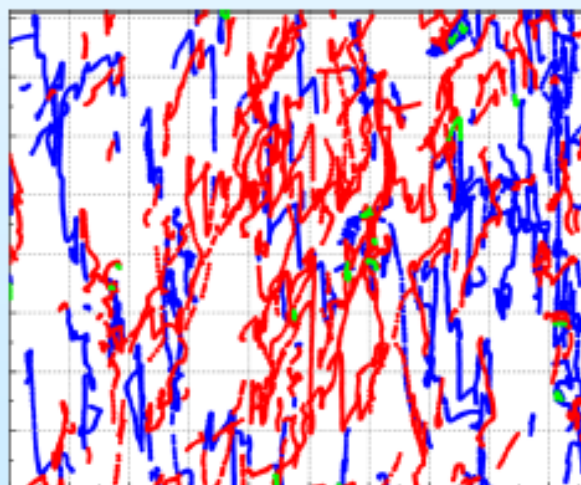
$q = 121 \text{ W/m}^2$ ;  $v_{ns} = 1.00 \text{ mm/s}$



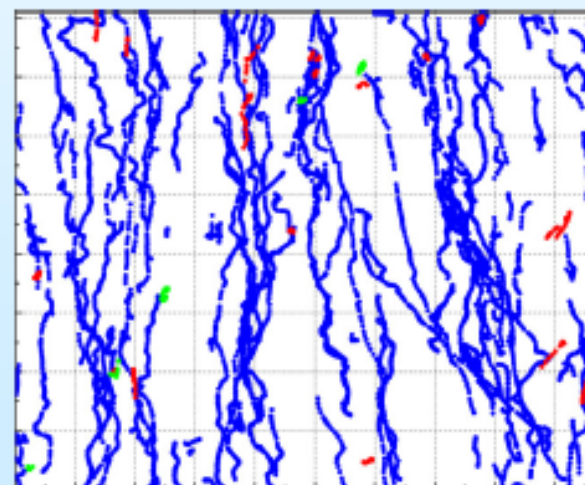
$q = 146 \text{ W/m}^2$ ;  $v_{ns} = 1.21 \text{ mm/s}$



$q = 193 \text{ W/m}^2$ ;  $v_{ns} = 1.60 \text{ mm/s}$



$q = 242 \text{ W/m}^2$ ;  $v_{ns} = 2.01 \text{ mm/s}$



$q = 484 \text{ W/m}^2$ ;  $v_{ns} = 4.01 \text{ mm/s}$



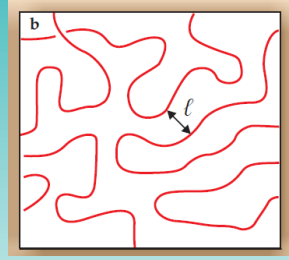
# Characteristic length scales in 4He counterflow turbulence

$$v_s = \frac{\kappa}{2\pi r}$$



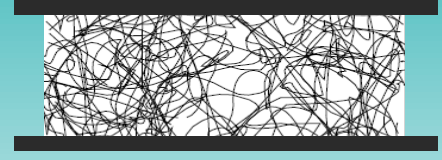
$$\xi \approx 10^{-8} \text{ cm}$$

**Vortex core size**



$$l = 1/\sqrt{L} \approx 100 \mu\text{m} = 10^{-2} \text{ cm}$$

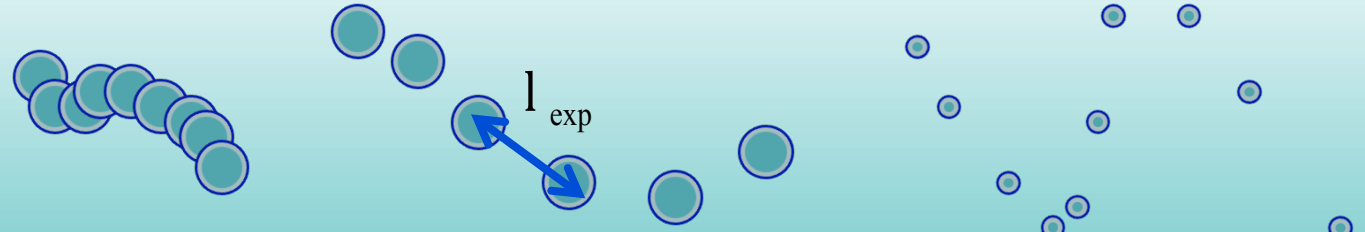
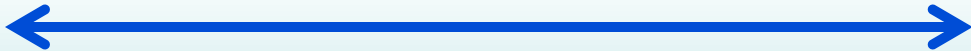
**Mean intervortex distance**



$$D \approx 1 \text{ cm}$$

**Outer scale**

Scales experimentally accessible by particle tracking



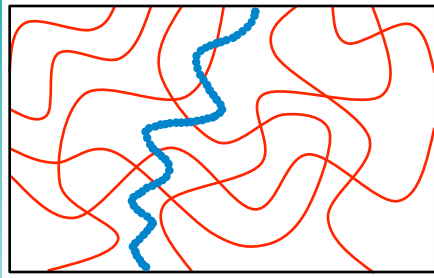
$$l_{\text{exp}} \approx l/10$$

$$l_{\text{exp}} \approx 10 l$$

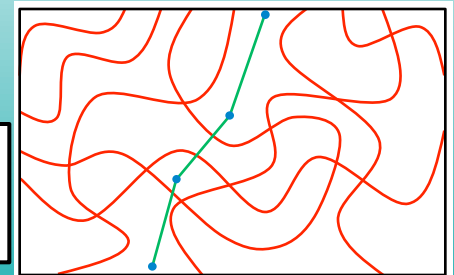
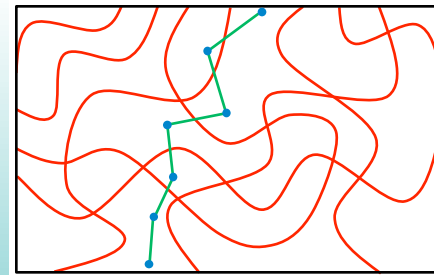
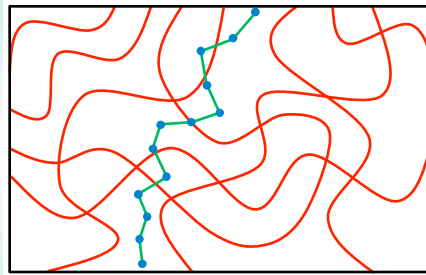
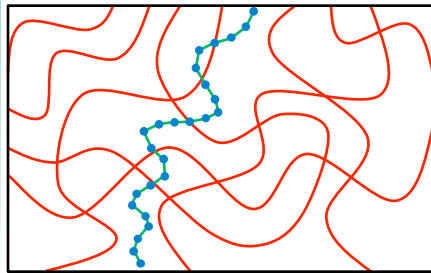
Vinen (ultraquantum) QT → crossover →  
???

Kolmogorov (quasiclassical) QT

# Data processing



Minimum time between frames  
(maximum frame rate)  
Minimum distance between particle positions



Maximum time between frames  
(minimum frame rate)  
Maximum distance between particle positions

time between frames decreases  
(frame rate increases)  
distance between particle positions  
decreases

time between frames increases  
(frame rate decreases)  
distance between particle positions increases

## Quantum signature: velocity and acceleration PDFs

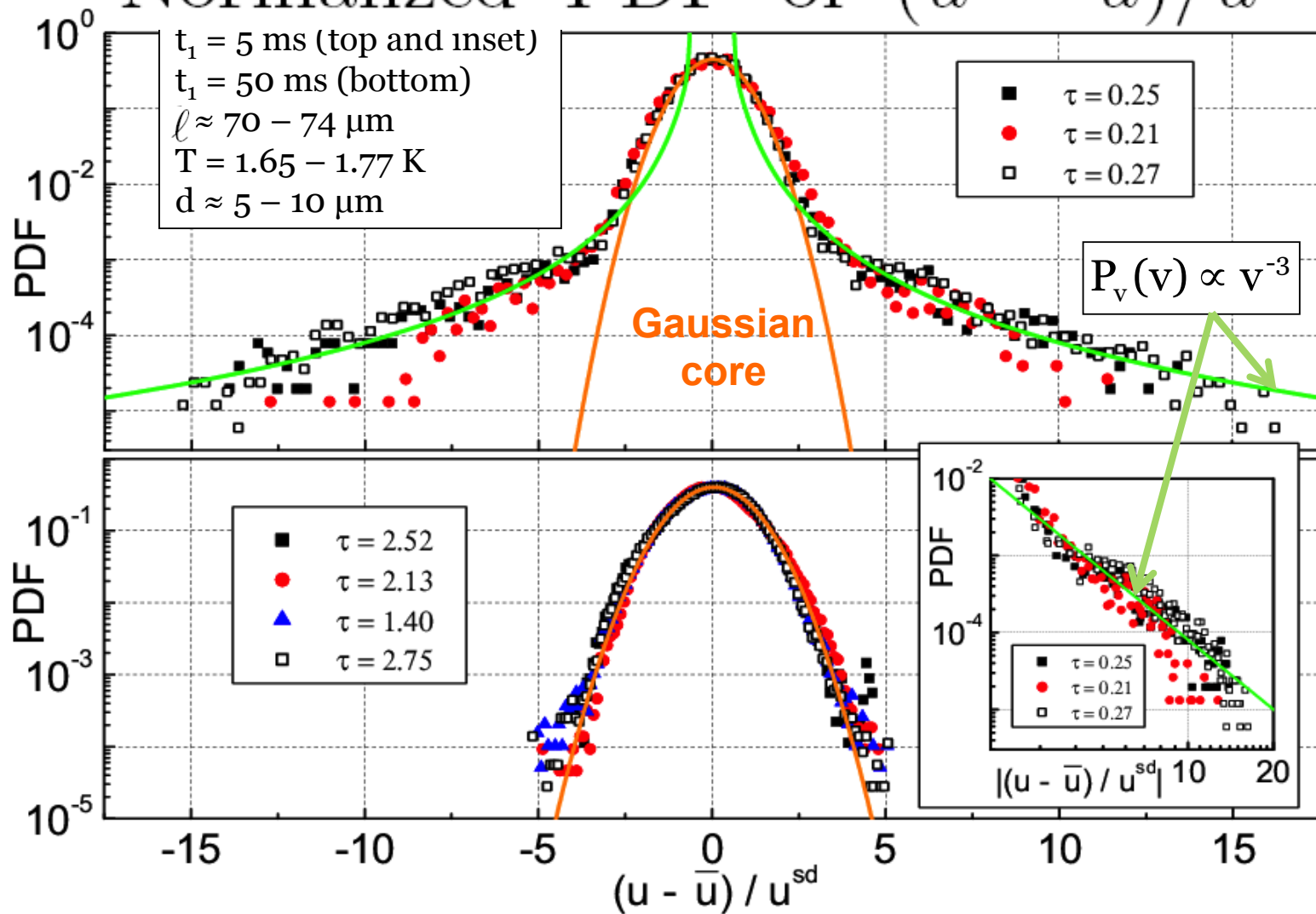
The probability of observing a velocity  $V$  in the proximity of a quantized vortex can be written as

$$P(V) \propto \int \delta(V - v_s) r dr = \int \delta\left(V - \frac{\kappa}{2\pi r}\right) r dr \propto V^{-3}$$

Similarly, the probability of observing an acceleration  $A$  in the proximity of a quantized vortex is obtained as

$$P(A) \propto \int \delta(A - a_s) r dr = \int \delta\left(A - \frac{v_s^2}{r}\right) r dr \propto A^{-5/3}$$

# Normalized PDF of $(u - \bar{u})/u^{sd}$



## Quantum, or classical turbulence?

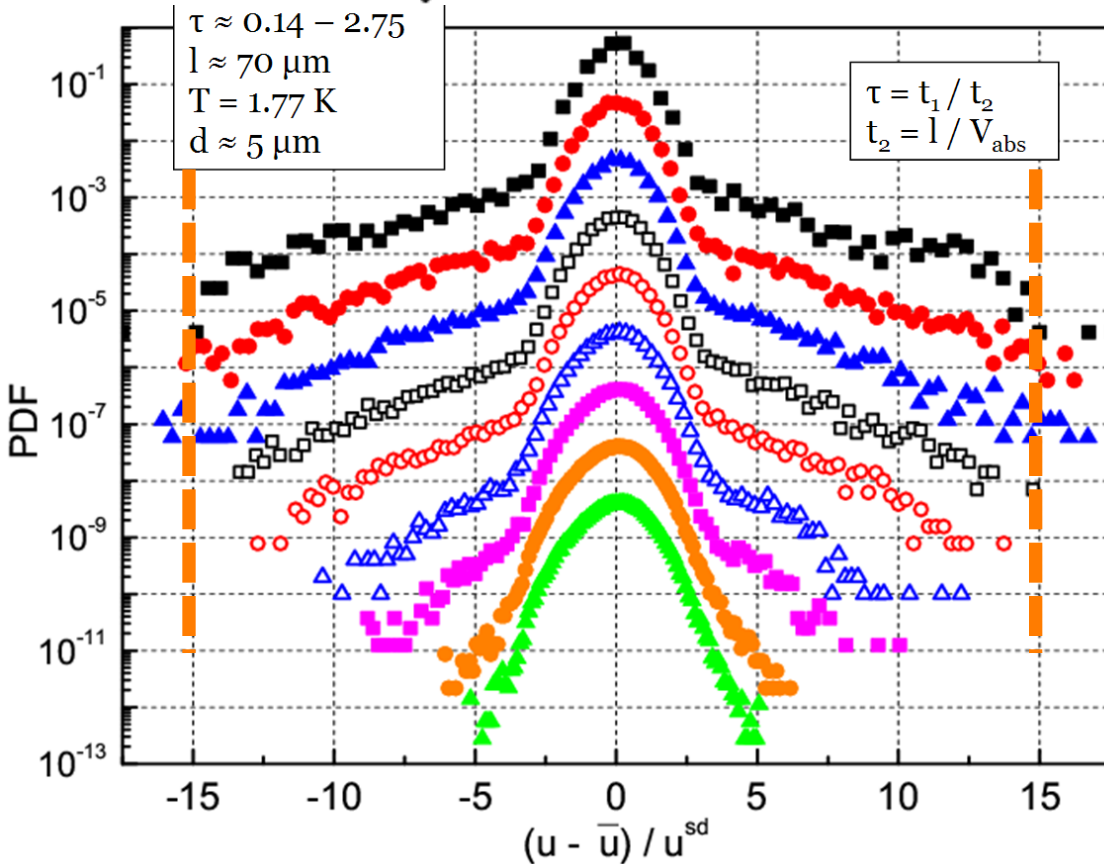
M. LA MANTIA<sup>(a)</sup> and L. SKRBK

EPL, 105 (2014) 46002

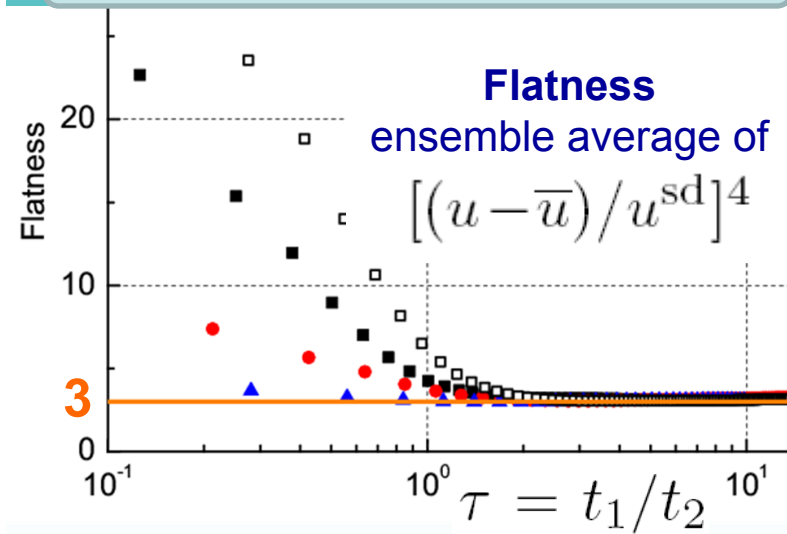
doi: 10.1209/0295-5075/105/46002

Faculty of Mathematics and Physics, Charles University - Ke Karlovu 3, 121 16 Prague, Czech Republic

# Crossover from Quantum to Classical Behavior



## Quantitative confirmation



The flatness at  $\tau < 1$  ( $l_{\text{exp}} < l$ ) depends on the frame rate used to collect the images due to the detection of less events of large magnitude

The width of the tails of the velocity PDF (a quantum effect) due to the finite size of the particles. The upper limit results from the assumption that the particle is subject to the tension of a quantized vortex line, which is balanced by the viscous Stokes drag, leading to a maximum velocity

$$v_{\text{max}} = \frac{\rho_s \kappa^2}{6\pi^2 d \eta} \ln \left( \frac{d}{\xi} \right)$$

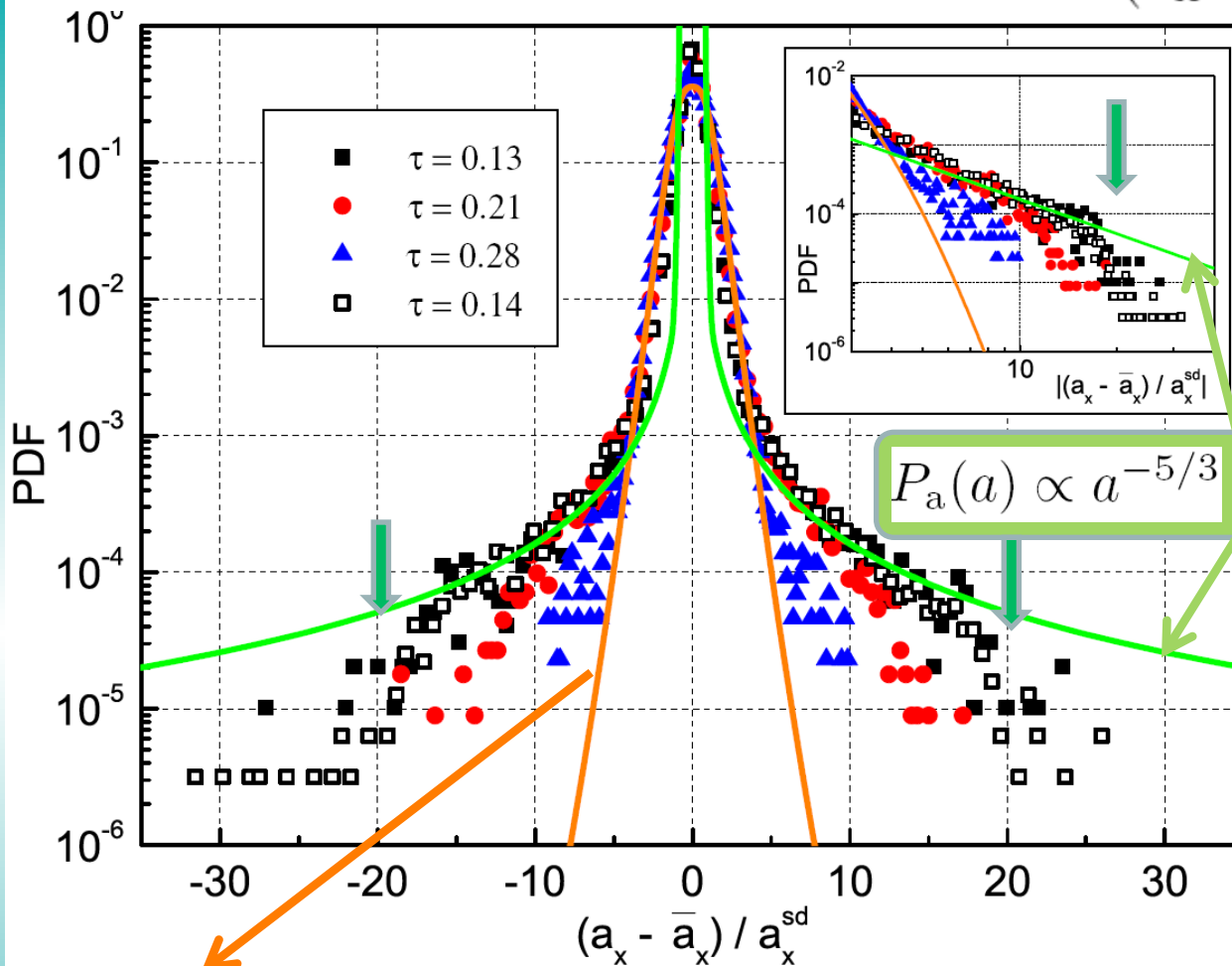


$$v_{\text{max}} \approx 30 \text{ mm/s}$$



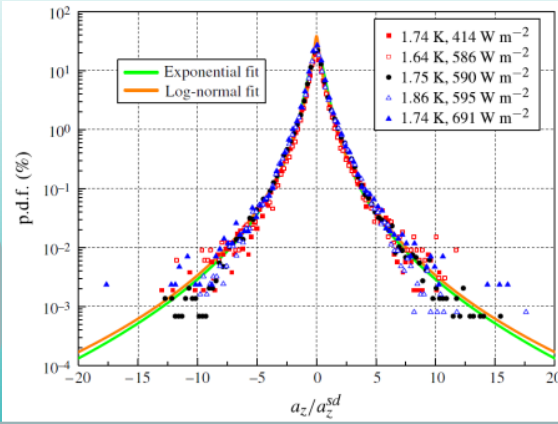
$$\text{corresponds to about } 15u^{\text{sd}}$$

# PDF of the non-dimensional acceleration $(a_x - \bar{a}_x)/a_x^{sd}$



Prediction for the tails:  
 about  $20|a_x^{sd}|$   
 corresponds to the acceleration of a few micrometer-sized particle touching a vortex core (note that the particle size  $d$  has a large influence on the acceleration magnitude as  $a \propto d^{-3}$ )

log-normal distribution of the acceleration magnitude of classical turbulent flows: Mordant, Crawford, Bodenschatz, PRL **93**, 214501 (2004); Qureshi et al PRL **99**, 184502 (2007); Eur. Phys. J. B **66**, 531 (2008)



$$\text{PDF} = \frac{\exp(3s^2/2)}{4\sqrt{3}} \left[ 1 - \text{erf} \left( \frac{\ln|a/\sqrt{3}| + 2s^2}{\sqrt{2}s} \right) \right]$$

$s$  is a fitting parameter

$s = 1$  – fluid particles,  $S < 1$  – inertial particles

$$s = 0.4$$

For  $l_{\text{exp}}; l$  the classical-like log-normal PDF shape is recovered La Mantia, Duda, Rotter, and LS, J. Fluid Mech. **717**, R9 (2013).

# Summary: velocity and acceleration PDFs

Quantum turbulence, generated in thermal counterflow of superfluid  $^4\text{He}$ , has been probed **experimentally** at length scales straddling about two orders of magnitude across the **mean distance  $l$  between quantized vortices**.

- Quantum signature is apparent from the velocity and acceleration distributions, at length scales **smaller than  $l$**  and from the temperature and velocity dependence of the acceleration magnitude, at length scales **of the order of  $l$**
- Classical-like signature is found, from the velocity and acceleration distributions, at length scales **larger than  $l$**
- Indirect evidence of **macroscopic, classical-like vortical structures** is obtained at length scales **larger than  $l$  and smaller than the integral length scale**

# Results: added mass

inviscid fluid

$$\rho_p \frac{du_p}{dt} = \rho_f \frac{Du_f}{Dt} + C\rho_f \left( \frac{Du_f}{Dt} - \frac{du_p}{dt} \right) \Rightarrow \frac{du_p}{dt} = \frac{1+C}{\rho_p/\rho_f + C} \frac{Du_f}{Dt} = K_p \frac{Du_f}{Dt}$$

- i) if  $\rho_p = \rho_f$ ,  $K_p = 1$ , i.e., the particle has the same acceleration as the fluid
- ii) if  $\rho_p > \rho_f$ ,  $K_p < 1$ , i.e., the particle accelerates less than the fluid
- iii) if  $\rho_p \gg \rho_f$ ,  $K_p \ll 1$ , i.e., the particle accelerates much less than the fluid
- iv) if  $\rho_p < \rho_f$ ,  $K_p > 1$ , i.e., the particle accelerates more than the fluid
- v) if  $\rho_p \ll \rho_f$ ,  $K_p \approx (1+C)/C$ , e.g., negative ion in He II

solid hydrogen,  $\rho_H \approx 80 \text{ kg m}^{-3}$     deuterium,  $\rho_D \approx 200 \text{ kg m}^{-3}$

helium     $\rho_f = \rho_{\text{He II}} \approx 146 \text{ kg m}^{-3}$

$$K_H \approx 1.36$$

$$K_D \approx 0.80$$

$$K_H / K_D \approx 1.70 \text{ (sphere)}$$

$$K_H / K_D \approx 1.5 - 2 \text{ (prolate spheroid)}$$

Prediction: hydrogen particles in He II are expected to accelerate roughly 1.5 to 2 times more rapidly than deuterium particles

**This is in agreement with experimental observation**

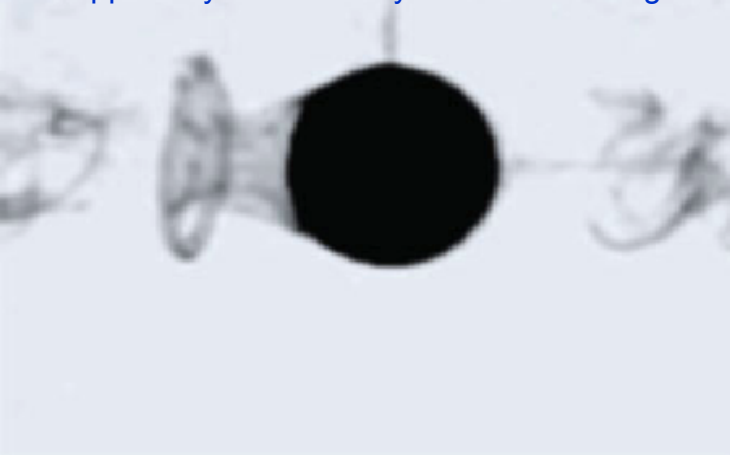
•M. La Mantia and LS *Dynamics of particles visualizing quantum turbulence*  
Phys. Rev. **B 90**, 014519 (2014)



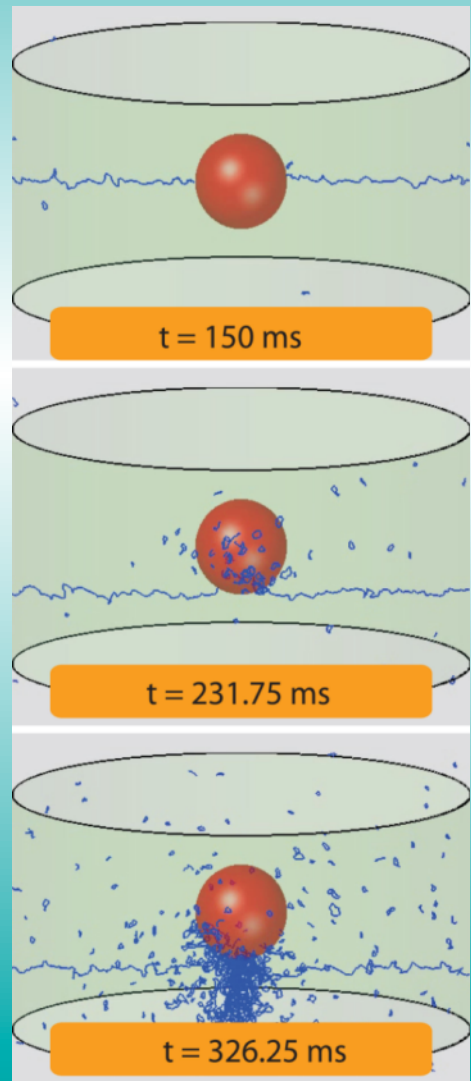
# Large vortex structures exist in classical viscous flows



Oscillating sphere in water;  
adapted from an original photograph  
supplied by R J Donnelly and R Hershberger



Oscillating sphere in He II,  
simulation adapted from original work  
by Hanninen R, Tsubota M, Vinen WF  
(2007) Phys Rev B 75:064502.



## Do large vortex structures exist in (various) quantum flows ???

### Experiments

- M. Murakami, M. Hanada, and T. Yamazaki, Jpn. J. Appl. Phys. Suppl.26-3, 107 (1987).
- G. Stamm, F. Bielert, W. Fiszdon, and J. Piechna, Physica B 193, 188 (1994).

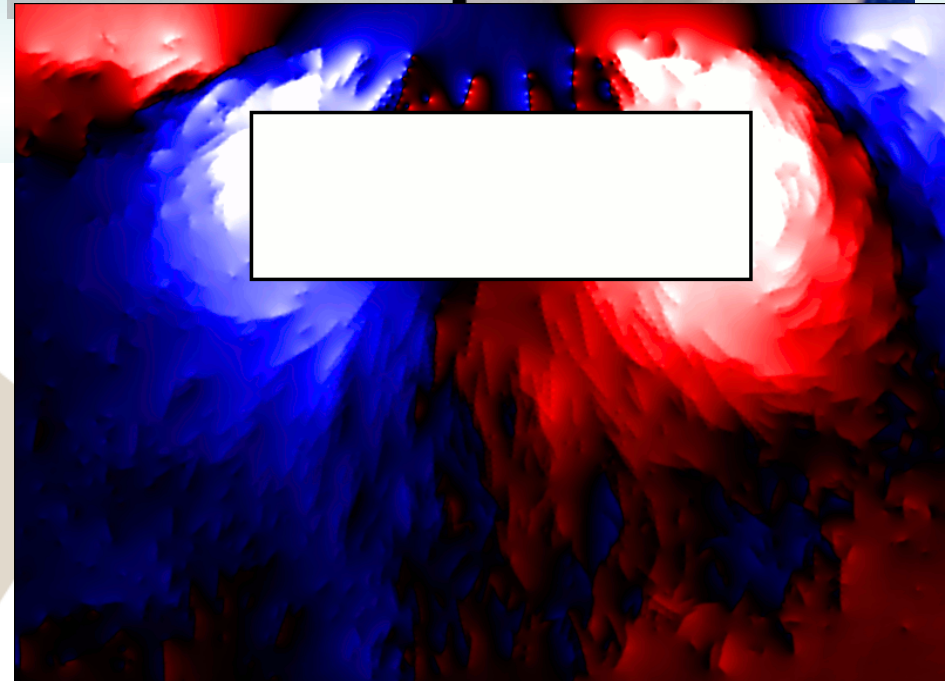
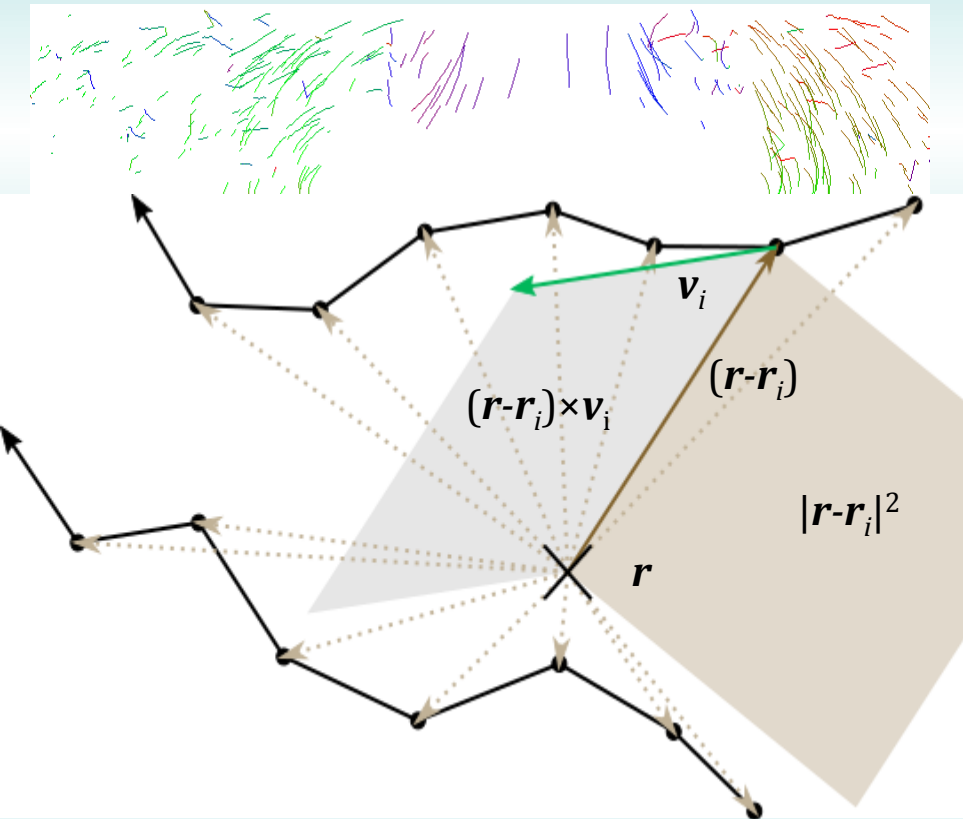
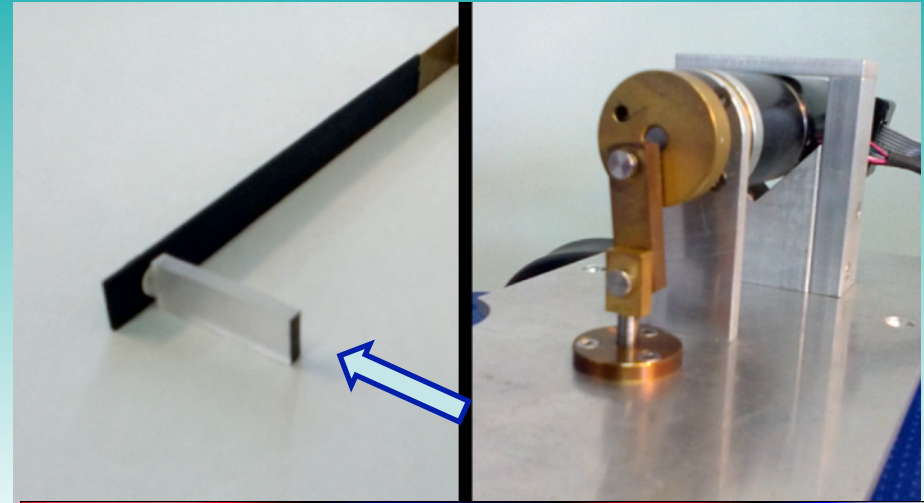
# Oscillating cylinder of rectangular cross-section 3 x 10 mm

Oscillations:

frequency 0.5 Hz, amplitude 5 mm

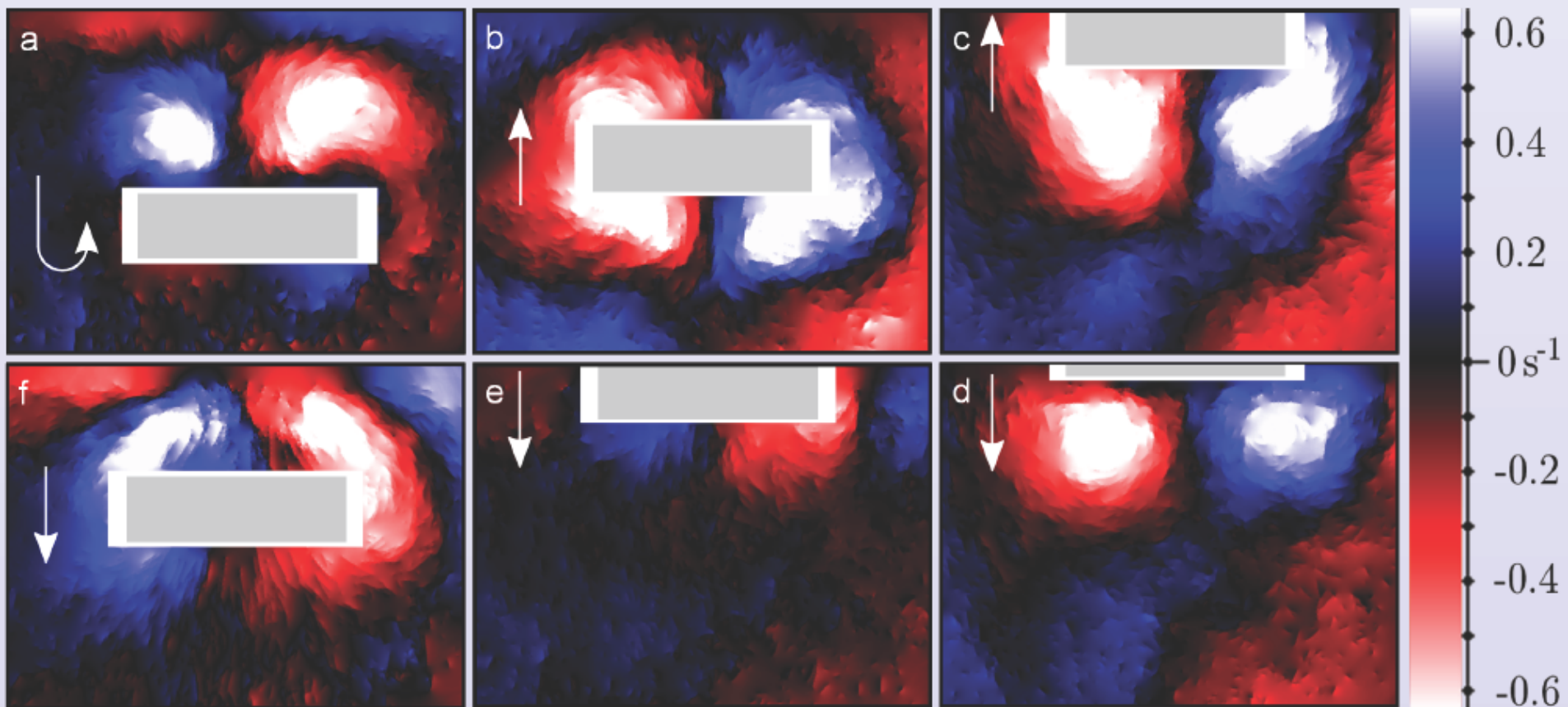
20 s entire video,  
camera frequency 100 Hz (exposition time 5 ms),  
phase averaged,  
trajectories of min 5 points shown,  
laser power 1.05 W

**T=1.24 K**

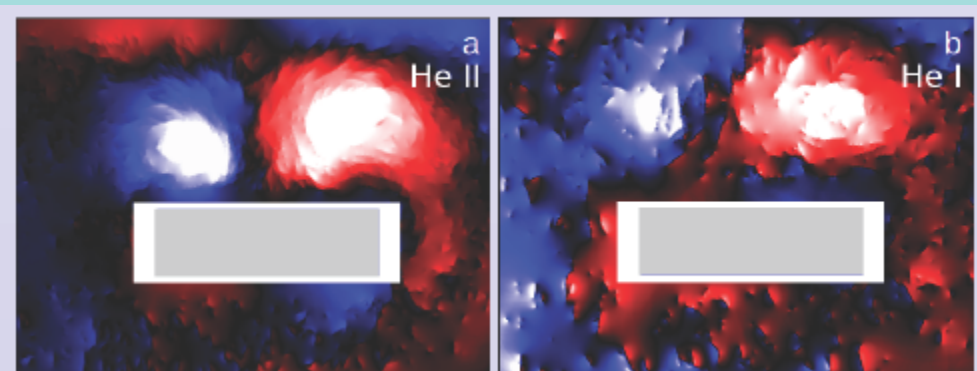


$R_M$  – radius of region of interest  
 $\Phi$  – phase window half-width

During the cycle, two pairs of vortices of opposite direction are shed



$T = 1.24 \text{ K}$ ,  $f = 0.5 \text{ Hz}$ ,  $a = 5 \text{ mm}$



a: He II,  $T = 1.24 \text{ K}$ ,  $a = 5 \text{ mm}$ ,  $f = 0.5 \text{ Hz}$

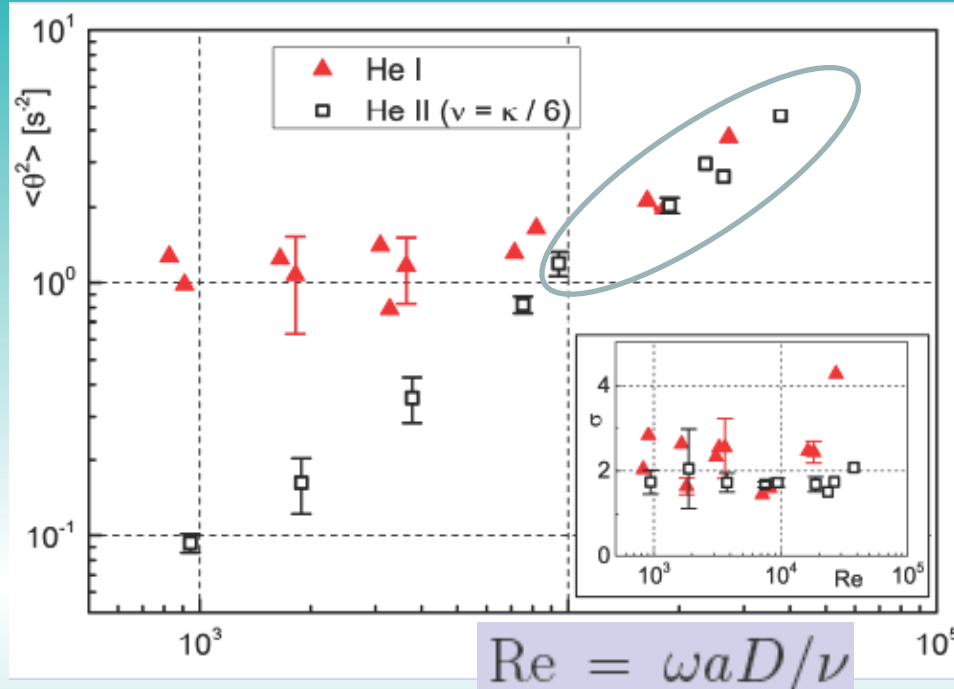
b: He I,  $T = 2.18 \text{ K}$ ,  $a = 5 \text{ mm}$ ,  $f = 0.5 \text{ Hz}$

**Vortices appear similar in He I and He II**

$\langle \theta^2 \rangle$ : ensemble average of the  $\theta^2$  parameter

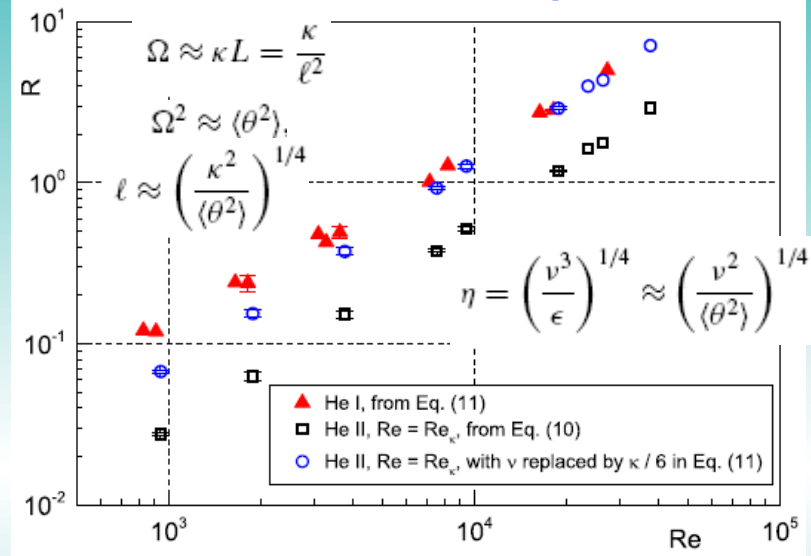
**Kinematic viscosity:**

for He II,  $\nu = \kappa/6$ .



$\omega$  and  $a$  angular frequency and amplitude  $D = 10$  mm

**Ratio  $R$  between the probed length scale and the flow scale plotted vs  $Re$**



Visualization of viscous and quantum flows of liquid 4He due to an oscillating cylinder of rectangular cross section  $D$ . Duda, P. Švančara, M. La Mantia, M. Rotter, LS PRB, in print

At large enough length scales (larger than Kolmogorov dissipation length and quantum length scale - average distance between quantized vortices) He I and II behave similarly.

Similarly to thermal counterflow, both viscous and quantum features can be observed in mechanically driven flows of He II, depending on the length scales at which the quantum flow is probed

**Warning:** particles can significantly affect both average tangle properties and also the statistics of particle motion; current PTV experiments, however, use safe, low numbers of particles.

**Numerical study Backreaction of Tracer Particles on Vortex Tangle in Helium II Counterflow**

By E. Varga, C. F. Barenghi Y. A. Sergeev, LS

# Recent Prague results on $4\text{He}$ quantum flows due to oscillating objects

David Schmoranzer, Martin Jackson, Tamara Skokankova, LS



Coworkers: Joe Vinen, Viktor Tsepelin, Andrew Woods, Oleg Kolosov, Javier Luzuriaga, Eddy Collin



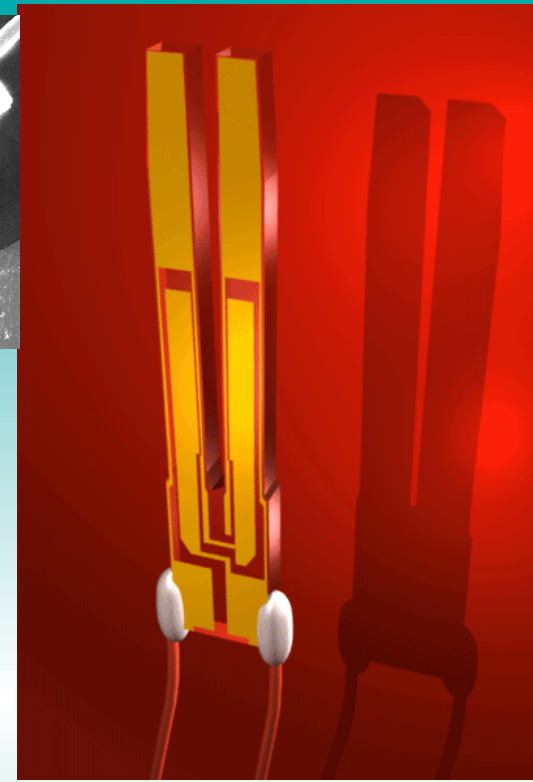
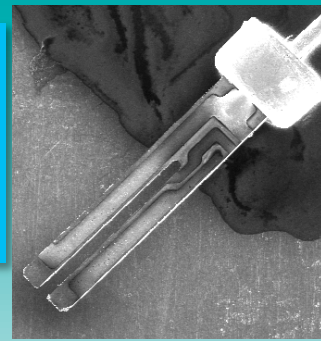
Quantum flows and sound emission due to :

- Quartz tuning forks
- Microwires
- Double paddle oscillators
- Oscillating disc

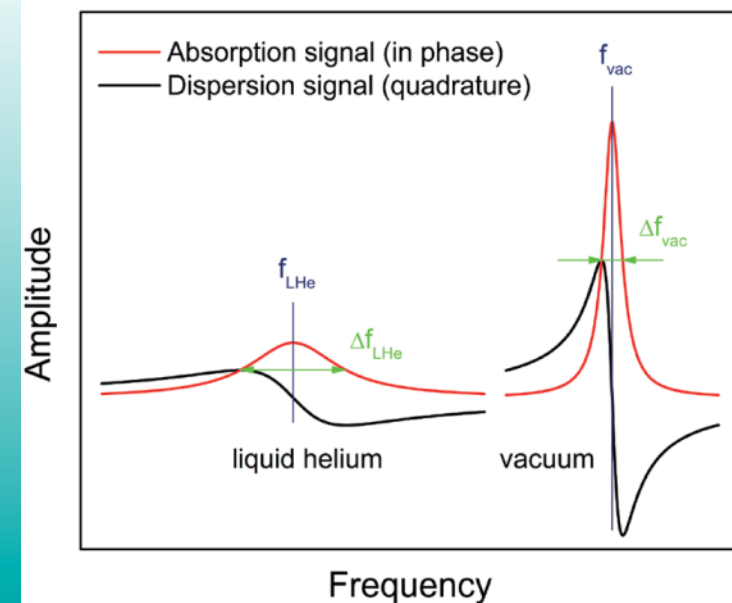
D. Schmoranzer, M.J. Jackson, LS, O. Kolosov, V. Tsepelin, A.J. Woods *Measurements of Vortex Line Density Generated by a Quartz Tuning Fork in Superfluid  $4\text{He}$ , submitted to JLTP*

M.J. Jackson, D. Schmoranzer, LS, and A.J. Woods *Turbulent Transition in Superfluid  $4\text{He}$  due to Quartz Tuning Forks*  
**In preparation**

# Quartz Tuning Forks



- Used as Time Standards
- Piezoelectric Oscillator - Oscillates with applied voltage
- Oscillation induces a current  $\propto$  Velocity
- Damping governed by excitations
  - Thermometry
- Can create and detect turbulence



# Classical Oscillatory Flow - High-Frequency Limit

Introduce the dimensionless quantities

$$u = Uu', \quad \nabla = 1/l \nabla', \quad t = 1/\omega t', \quad p = \rho U^2 l^2 p',$$



to yield the dimensionless Navier-Stokes Equation

$$l\omega/U \partial u' / \partial t' + u' \cdot \nabla' u' + \nabla' p' = \nu/U l \Delta' u'.$$

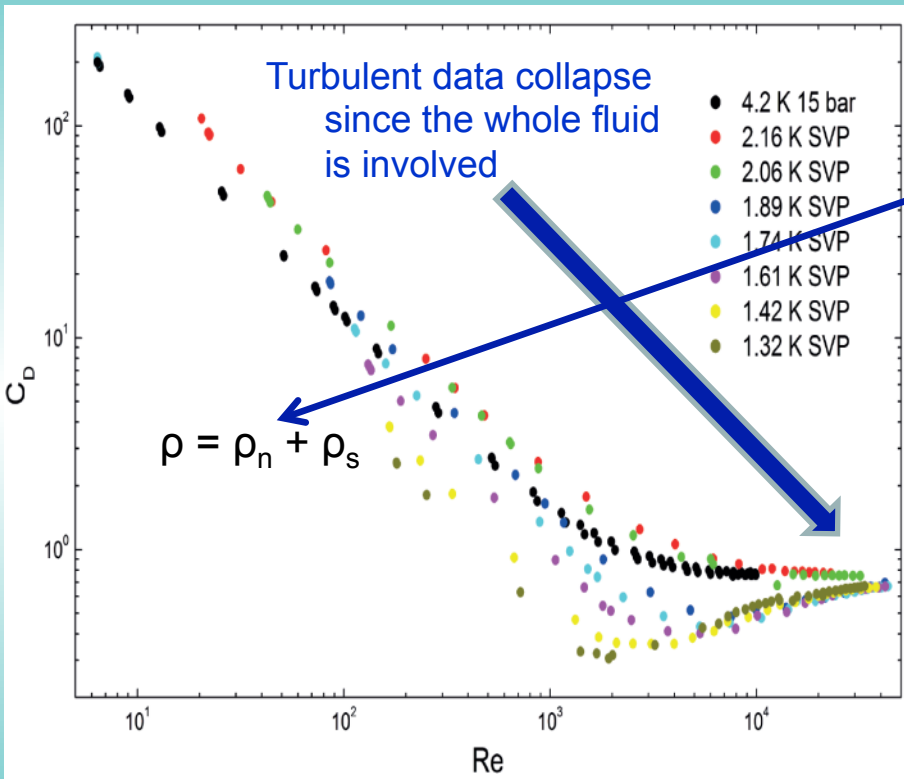
$2\pi / Kc$

$1/Re$

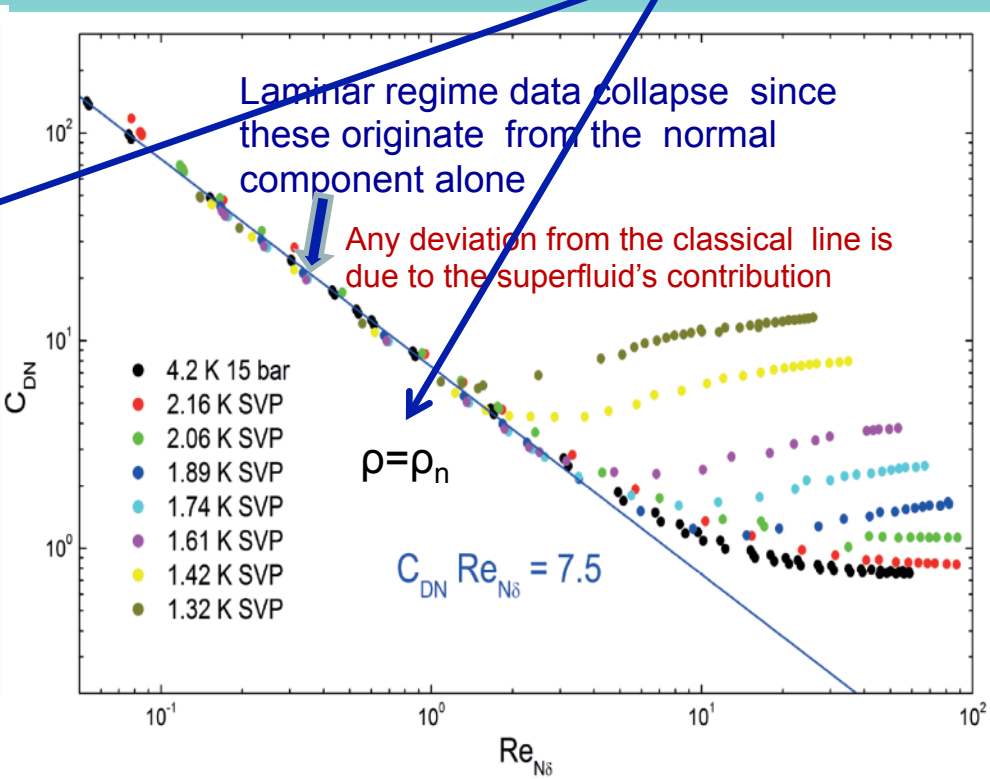
- At high frequencies, velocity changes occur on the scale of the viscous penetration depth  $\delta = \sqrt{2\nu/\omega}$ .
- Thus  $Kc/\pi = Re = U\delta/\nu$
- Therefore, one parameter is sufficient to characterise such flow, provided  $\delta \ll D$  (We neglect surface roughness).

# Measure damping force ( $F$ ) against velocity ( $U$ ) and convert to **drag coefficient** and Reynolds number

$$C_{Dx} = \frac{2F}{\rho_x A U^2}$$



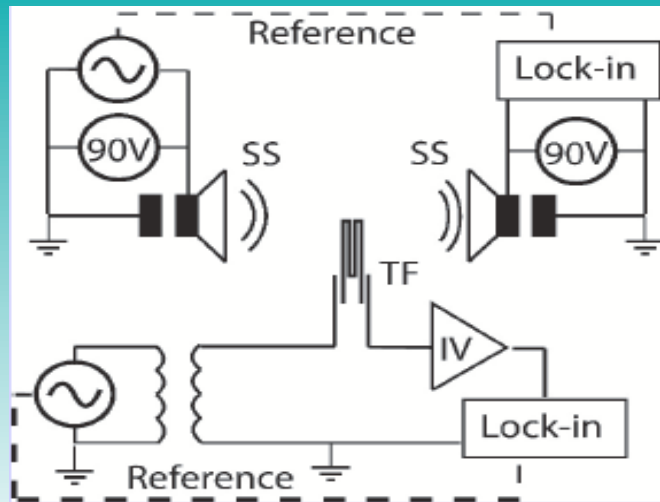
based on the size of the fork, dynamic viscosity and total fluid density of He



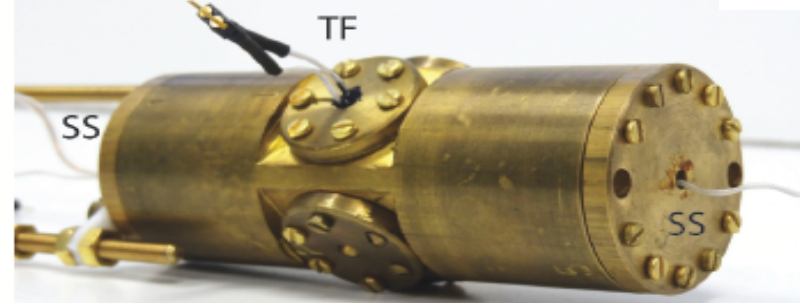
based not on the size of the fork, but on the viscous penetration depth in the normal fluid



# Does the oscillating tuning fork produce quantized vortices ?

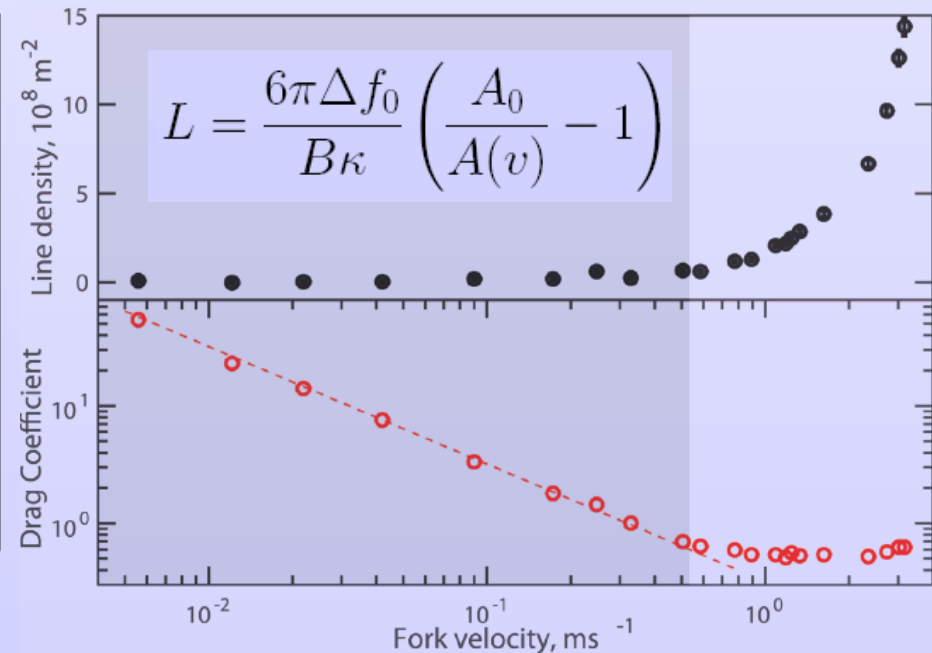
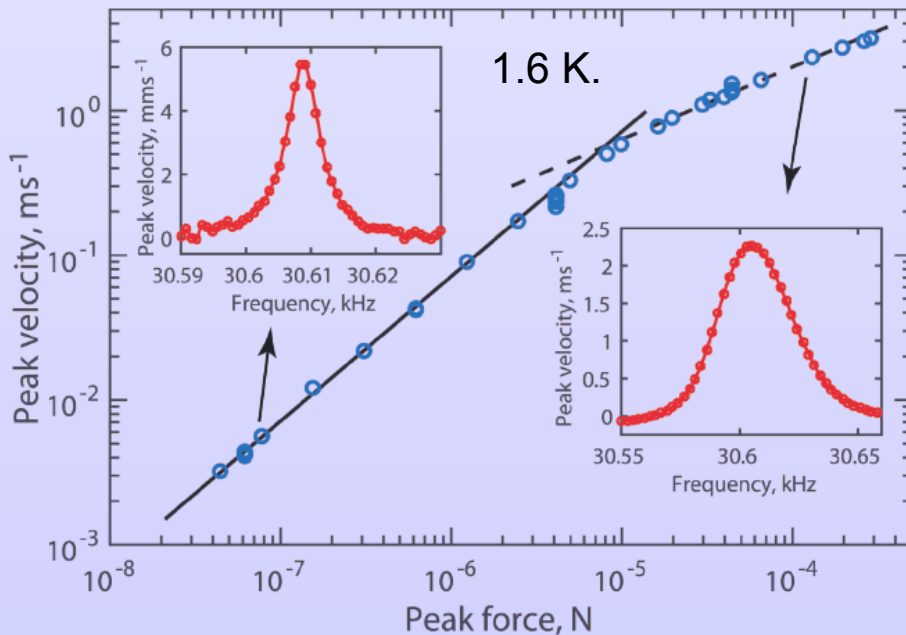


cylindrical second sound resonator



Measurements of Vortex Line Density Generated by a Quartz Tuning Fork in Superfluid 4He

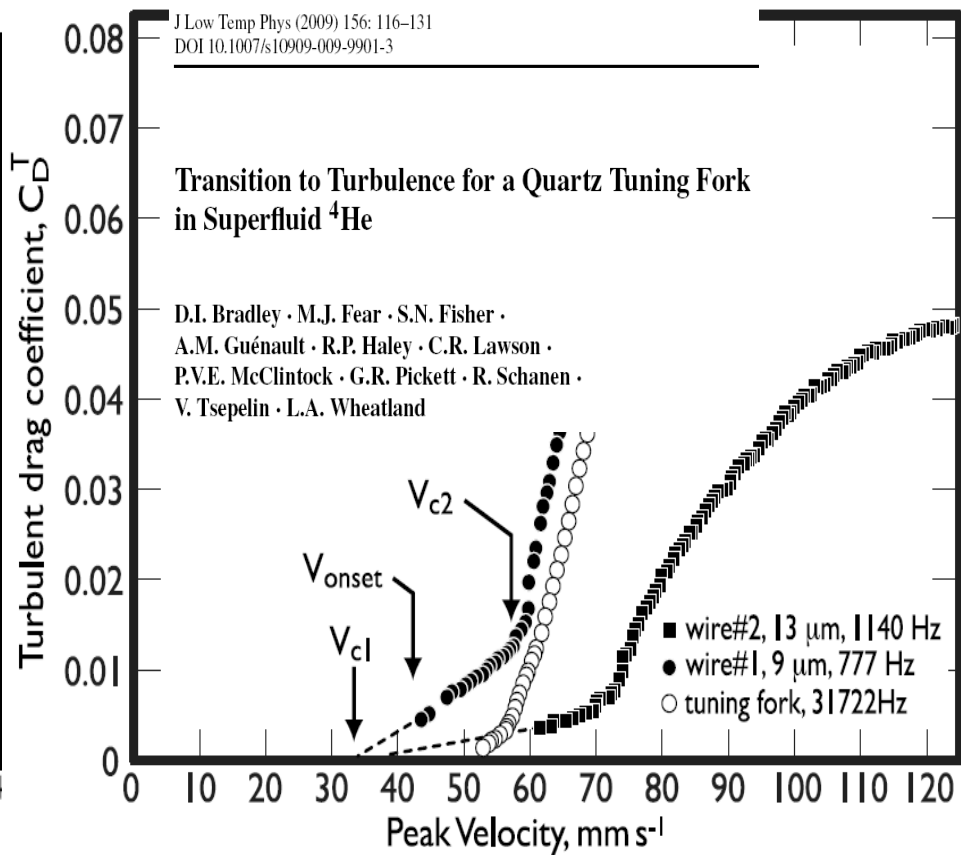
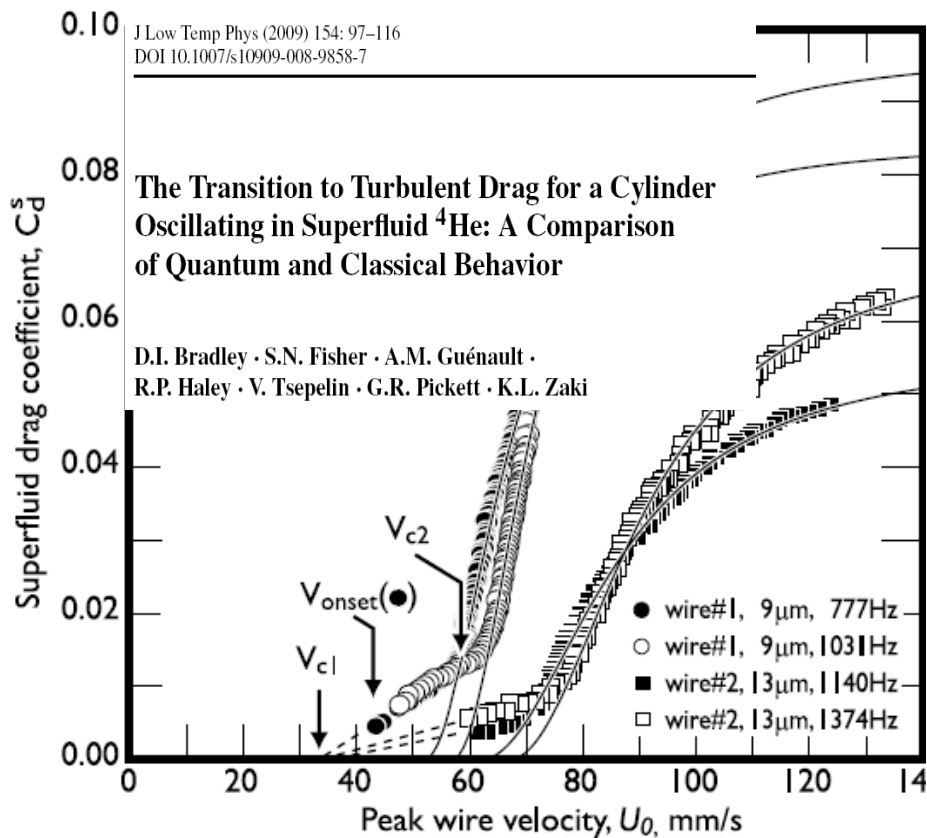
D. Schmoranzer, M.J. Jackson, LS, O. Kolosov, V. Tsepelin, A.J. Woods



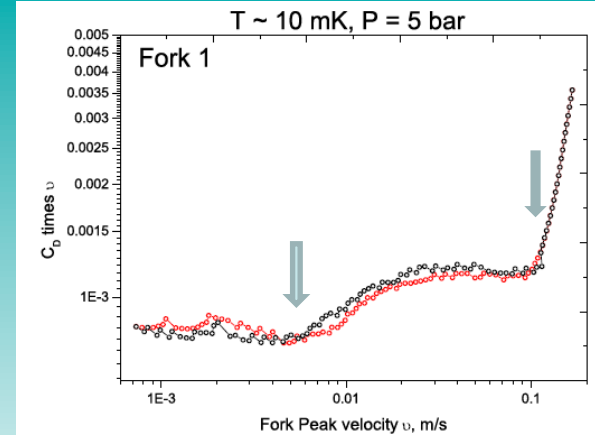
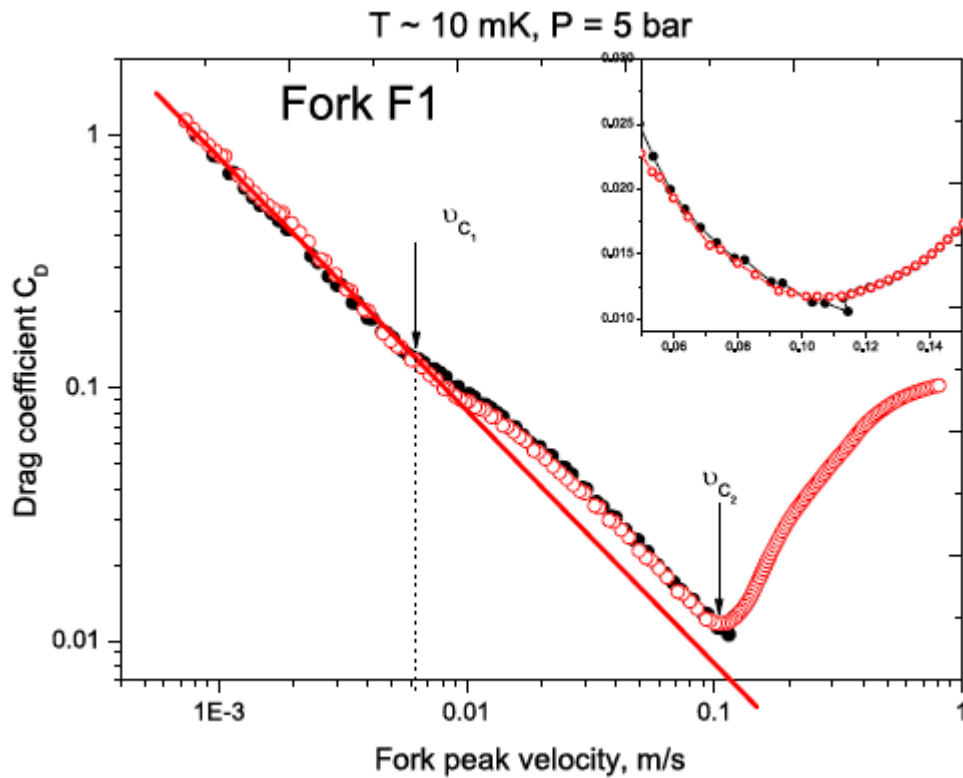
Directly tested by second sound attenuation:

Production of quantized vortices is directly related to the onset of excess damping

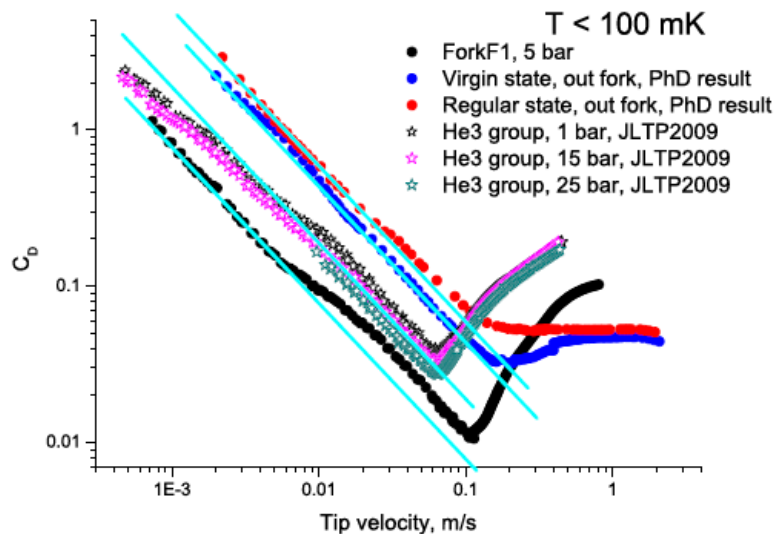
# Two critical velocities?!?



The superfluid drag coefficient as a function of peak velocity for the two wire resonators in superfluid  $^4\text{He}$  and comparison with the standard 32 kHz quartz tuning fork



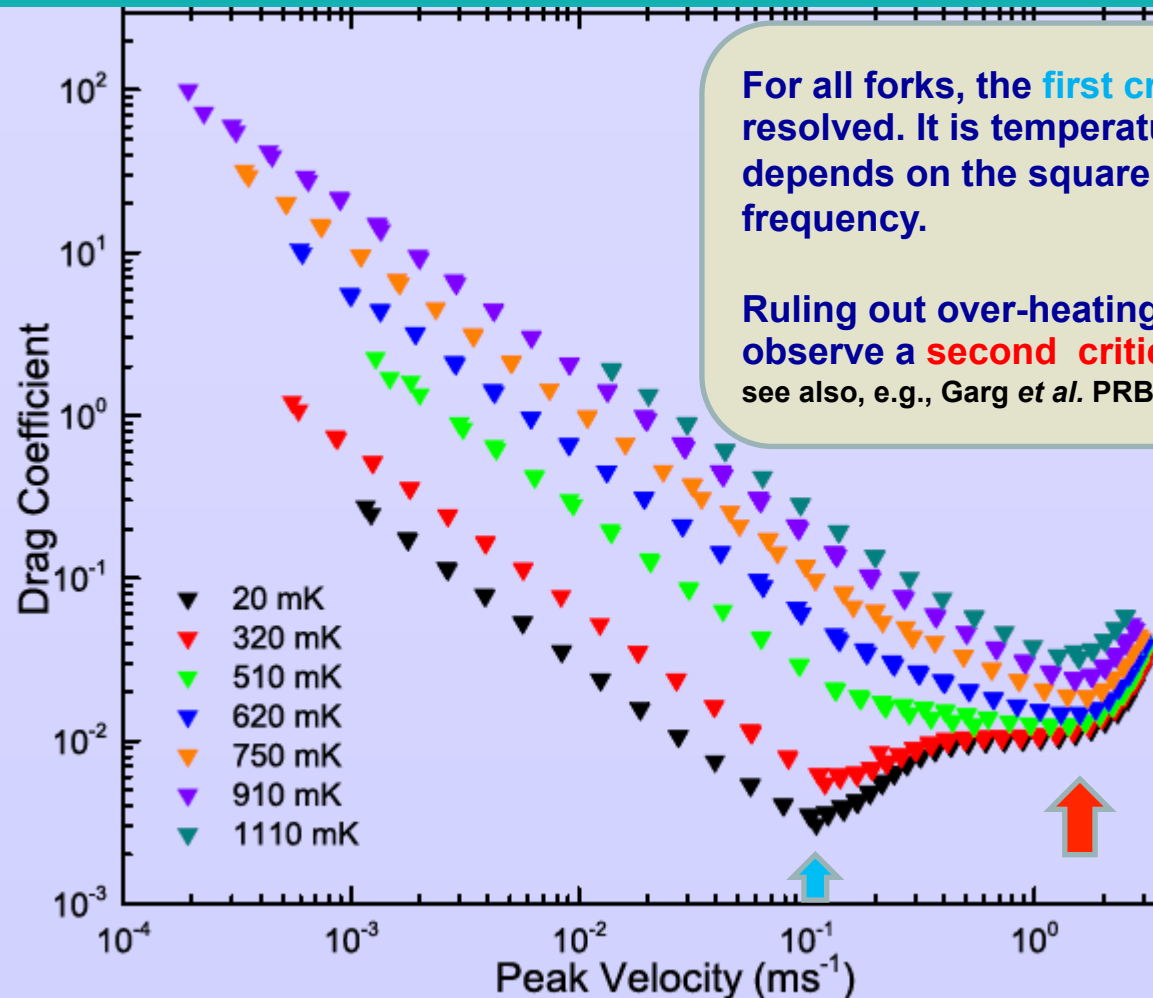
D. Garg, V. B. Efimov, M. Giltrow, P. V. E. McClintock LS and W. F. Vinen: **On Flow of Isotopically Pure  $^4\text{He}$  due to Vibrating Quartz Forks in the  $T \rightarrow 0$  Limit**  
 PHYSICAL REVIEW B **85**, 144518 (2012)



• **Two critical velocities are clearly observed**

• **The behaviour in the turbulent drag regime is not always reproducible, at low  $T$  the drag coefficient does not approach the classical value of order unity and is poorly understood**

# Evidence for two critical velocities 6 kHz fork, first overtone at 40 kHz



For all forks, the **first critical velocity** is clearly resolved. It is temperature independent, but it depends on the square root of the oscillation frequency.

Ruling out over-heating and cavitation, we also observe a **second critical velocity** of order 1 m/s. - see also, e.g., Garg *et al.* PRB 85 144518 (2012)

## POSSIBLE SCENARIO OF THE TRANSITION

Laminar flow of the normal component, potential flow of the superfluid

1st instability (in the superfluid component) - nucleation of quantized vortices leads to coupling of the components through mutual friction force

Quasi-laminar flow of the coupled components

Further instabilities trigger classical-like transition to turbulence

## Quantum turbulence generated by oscillating structures

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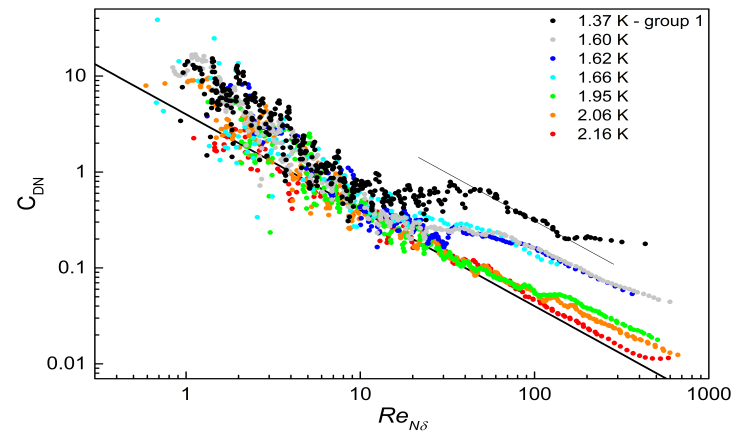
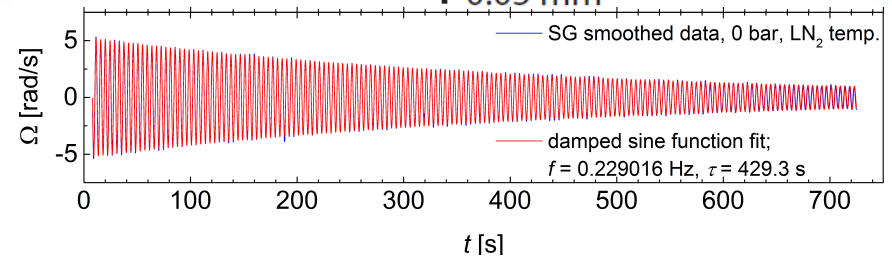
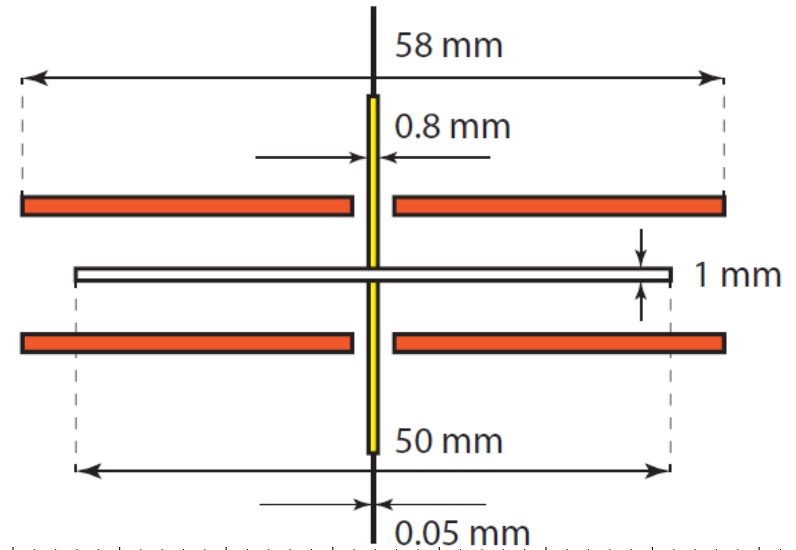
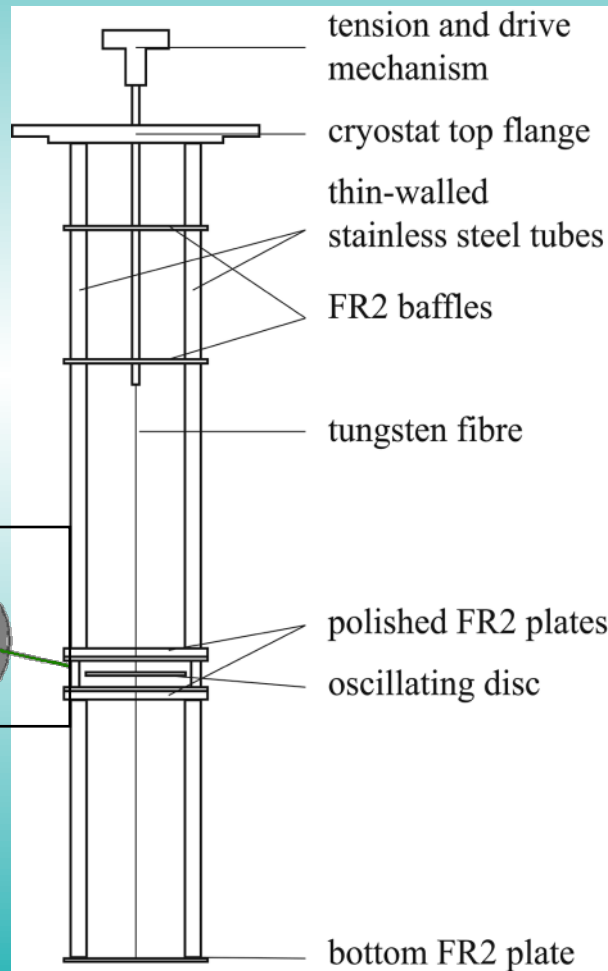
Edited by Katepalli R. Sreenivasan, New York University, New York, NY, and approved December 12, 2013 (received for review July 22, 2013)

The paper summarizes important aspects of quantum turbulence that have been studied successfully with oscillating structures. It describes why some aspects are proving hard to interpret, and it outlines the need for new types of experiment and new developments in theoretical and computational work.



# Torsionally oscillating disc experiment revisited in Prague

R.J. Donnelly, A.C. Hollis Hallett:  
*Annals of Physics* 3 (1958) 320



# Summary

- All forms of  $4\text{He}$  - cryogenic helium gas, normal liquid He I and superfluid He II - serve as outstanding working fluids for **cryogenic fluid dynamics** and **quantum turbulence**

- Extremely high  $Re$  and  $Ra$  flows can be studied under controlled laboratory conditions
- Quantum turbulence** has been investigated over 50 years -a lot is known about it, but it is still only partly understood
- In the zero temperature limit QT represents the simplest prototype of turbulence
- At finite temperature, in the wo-fluid regime, QT is more complex than classical turbulence, combining classical turbulence in the normal fluid with the dynamics of the vortex tangle in the superfluid, coupled by the mutual friction force

**Does  $4\text{He}$ , together with other quantum fluids, hold the key to unlocking the underlying physics of fluid turbulence?**

Plenty of interesting physics to play with...

