Imaging quantum turbulence in ^{3}He-B: Do spectral properties of Andreev reflection reveal properties of turbulence?

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To validate the experimental visualization of turbulence in ^{3}He-B by showing the relation between the vortex-line density and the Andreev reflectance of the vortex tangle in the first simulations of the Andreev reflectance by a realistic 3D vortex tangle,

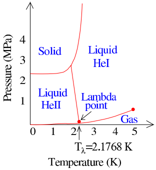
and comparing the results with the first experimental measurements in ^{3}He-B able to probe quantum turbulence on length scales smaller than the inter-vortex separation[1].

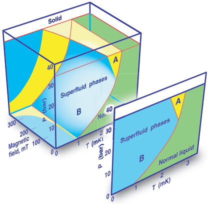
[1] In Prague ^{4}He experiments at temperaturs $1\,\mathrm{K} < T < T_\lambda$, quantum turbulence on the length scales smaller than the intervortex separation was visualized by the Lagrangian dynamics of micrometer-sized solid particles, see e.g. La Mantia and Skrbek, PRB **90**, 014519 (2014).

^{4}He

Phase diagrams

^{3}He-B





Origin of superfluidity

Bose condensation of helium atoms

Bose condensation of Cooper pairs

Transition temperature

$$T_\lambda \approx 2.17\,\mathrm{K}$$

$$T_c \approx 1\,\mathrm{mK}$$

Quantum of circulation

$$\kappa = \frac{2\pi\hbar}{m_4} \approx 0.997 \times 10^{-7}\,\frac{\mathrm{m}^2}{\mathrm{s}}$$

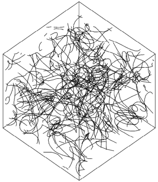
$$\kappa = \frac{2\pi\hbar}{2m_3} \approx 0.662 \times 10^{-7}\,\frac{\mathrm{m}^2}{\mathrm{s}}$$

We will be interested in dynamics of quantized vortex lines and associated flow field at temperatures $\ll T_c$ so that the normal fluid can be neglected.

Each vortex moves in a collective flow field of all other vortices

$\Longrightarrow$ complex dynamics, accompanied by vortex reconnections

$\Longrightarrow$ vortex tangle (quantum turbulence)

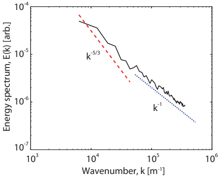


Intencity of quantum turbulence is characterized by the vortex line density, L $[\mathrm{m}^{-2}]$

– total length of vortex lines per unit volume

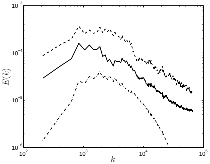
Regimes and spectra of quantum turbulence

Quasiclassical



Large-scale flow

Ultraquantum



No large-scale flow.
The only length scale is the intervortex distance.

Detection and visualization of quantum turbulence

^{4}He: $1\,\mathrm{K} \lesssim T \leq T_\lambda \approx 2.17\,\mathrm{K}$

Second sound

PIV and PTV (Particle Tracking)

He_2^* excimer molecules

^{4}He: $T < 1\,\mathrm{K}$

He_2^* excimer molecules

Electron bubbles

^{3}He-B: N/A for $0.25\,T_c < T < T_c$

^{3}He-B: $T < 0.25\,T_c$

Andreev reflection of quasiparticle thermal excitations

(not yet a truly visualization technique)

Advantages:

1) Non-invasive

2) in combination with numerical simulations, can be regarded as the visualization technique (in particular, allows to measure and interpret fluctuations of the vortex line density)

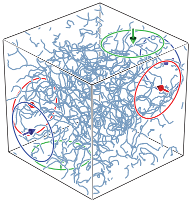
Fisher *et al.* , PRL **63** (1983);
Fisher, Jackson, Sergeev, Tsepelin, PNAS **111** (2014)

Governing equations

$$\mathbf{v}(\mathbf{r},\, t) = -\frac{\kappa}{4\pi}\oint_{\mathcal{L}} \frac{\mathbf{r}-\mathbf{s}}{|\mathbf{r}-\mathbf{s}|^3} \times d\mathbf{s}\,, \qquad \frac{d\mathbf{s}}{dt} = \mathbf{v}(\mathbf{s},\, t)$$

– solved in the cubic box of size 1 mm using the vortex-filament method with periodic boundary conditions. Biot-Savart integrals are evaluated via a tree method.

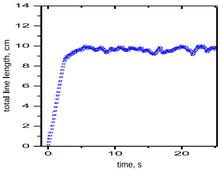
[Baggaley & Barenghi, PRB **84**, 020504 (2011)]



Tangle is generated by vortex loops (radius 200 μm) injected at a frequency $f_i = 10\,\mathrm{Hz}$. To insure isotropy, the loop injection plane is switched at both corners at frequency $f_i = 3.3\,\mathrm{Hz}$.

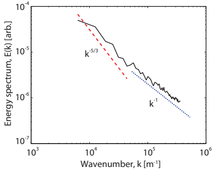
Evolution of the vortex line length

Saturated, time-averaged line density in the statistically steady state: $\langle L\rangle = 9.7 \times 10^7\,\mathrm{m}^{-2}$.



[Baggaley, Tsepelin, Barenghi, Fisher, Pickett, Sergeev, and Suramlishvili, PRL **115**, 015302 (2015)

Note: the simulated tangle is quasiclassical



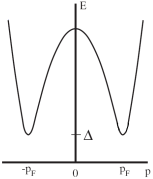
Thermal quasiparticle excitations. Andreev scattering

At $T \ll T_c$ excitation propagate ballistically
(mean free path $\gg$ size of experimental cell)

Quiescent fluid

$$E = \sqrt{\epsilon_p^2 + \Delta^2}, \quad \text{where} \quad \epsilon_p = \frac{p^2}{2m^*} - \epsilon_F$$

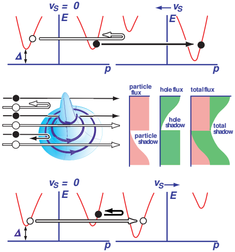
Δ – superfluid energy gap; ϵ_F – Fermi energy; p_F – Fermi momentum



- Quasiparticles: $\epsilon_p > 0$
- ○ Quasiholes: $\epsilon_p < 0$

Flowing ^{3}He-B (e.g. quantized vortex)

$E(\mathbf{p}) \to E(\mathbf{p}) + \mathbf{p} \cdot \mathbf{v}$ (Galilean transformation)



[effective potential barrier: $U_{\text{eff}} = \Delta + \mathbf{p} \cdot \mathbf{v}(\mathbf{r})$]

Fisher *et al.*, PRL **63** (1983);

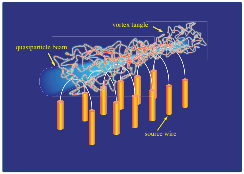
Fisher, Jackson, Sergeev, Tsepelin, PNAS **111** (2014)

Any momentum transfer between excitations and superfluid is minimal

$\Longrightarrow$ the reflected excitations almost exactly retrace the path of incoming quasiparticles

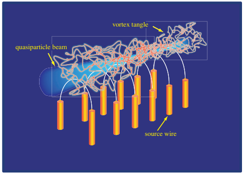
Experimental arrangements (Lancaster)

Sketch of experiment ...

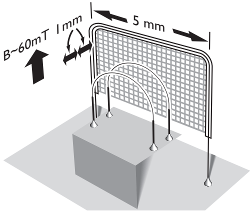


Experimental arrangements (Lancaster)

Sketch of experiment ...



... and its realization



Quasiparticle flux incident in the x-direction on the (y, z)-side of the computational cell:

$$\langle n v_g \rangle^i_{(y,z)} = \int_{\Delta}^{\infty} g(E) f(E) v_g(E)\, dE$$

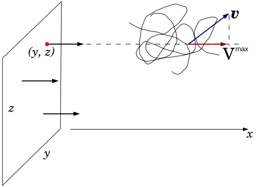
[$g(E)$ – density of states, $f(E)$ – Fermi distribution function]

Quasiparticle flux transmitted through the tangle:

$$\langle n v_g \rangle^t_{(y,z)} = g(p_F) \int_{\Delta + p_F V_x^{\max}}^{\infty} \exp(-E/k_B T)\, dE$$

[$g(p_F)$ – density of momentum states at the Fermi energy]

$V_x^{\max}$ – the highest superfluid velocity encountered along the quasiparticle's trajectory

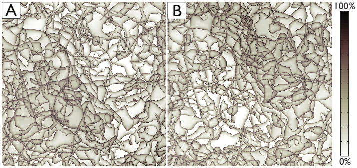


The fraction of quasiparticles Andreev-reflected by a tangle along x-direction at position (y, z):

$$f_{y,z} = 1 - \exp\left(-\frac{p_F V_x^{\max}}{k_B T}\right)$$

A 2D map of Andreev reflection by the tangle

[Baggaley, Tsepelin, Barenghi, Fisher, Pickett, Sergeev, Suramlishvili, PRL **115** 015302 (2015)]



(A) Quasiparticle reflection **(B)** Quasihole reflection

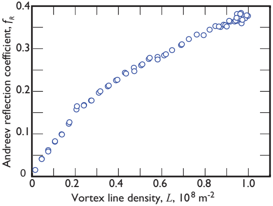
The extended regions of high reflectivity (darker) and low reflectivity (lighter) illustrate the distribution of the large-scale flows

Total Andreev reflection

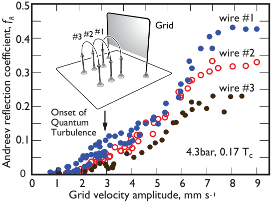
Total reflection $f_R = \langle f_{y,z} \rangle$ (averaged over (y, z)-plane)

The results for quasiparticles and quasiholes are combined to yield the reflection for a full thermal beam

Numerical simulation



Experimental measurements of Andreev reflection from vortices generated by the vibrating grid



- Slower rise of f_R for $L \gtrsim 2 \times 10^7\,\mathrm{m}^{-2}$ is due to the screening effects
- Numerical and experimental data have similar shape, but beware: the onset of turbulence is slightly different in simulations and in the experiment
- Plateau at large velocities/tangle densities: in experiment results from the extra creation of excitations as the grid approaches 1/3 of the Landau critical velocity, in simulation is due to a larger screening

Baggaley, Tsepelin, Barenghi, Fisher, Pickett, Sergeev, Suramlishvili, PRL **115**, 015302 (2015)

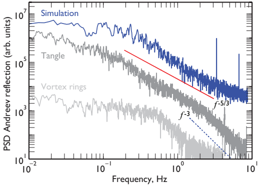
Statistically steady state

Equilibrium, time-averaged values:

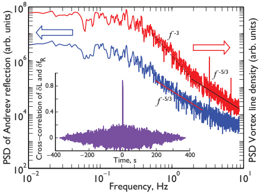
$$\langle L \rangle = 9.7 \times 10^7\,\mathrm{m}^{-2}, \qquad \langle f_R \rangle = 0.37$$

Fluctuations: $\delta L(t) = L(t) - \langle L \rangle$, $\delta f_R(t) = f_R(t) - \langle f_R \rangle$

Power spectral densities (PSD) of the Andreev reflection for numerical simulations and experimental observations



PSD of the simulated Andreev reflection and the simulated line density



- f^{-3} scaling corresponds to the Andreev reflection on length scales *smaller* than the intervortex distance
- *high* f – line density spectrum dominated by *unpolarized*, random vortex lines $\Longrightarrow$ $f^{-5/3}$
- *medium* f – line density spectrum dominated by large scale flows generated by *polarized* vortex lines $\Longrightarrow f^{-3}$
- *cross-over* at frequency $f_\ell \approx \kappa/(2\pi\ell) \approx 1\,\mathrm{Hz}$ corresponding to the intervortex distance
- The fluctuations of the vortex line density and of the Andreev reflection are well correlated. However, their spectral densities behave differently: for large scale flows the line density spectrum scales as f^{-3} while that of Andreev reflection as $f^{-5/3}$; for an unpolarized tangle the scalings are reversed.

Baggaley, Tsepelin, Barenghi, Fisher, Pickett, Sergeev, Suramlishvili, PRL **115**, 015302 (2015)

- The Andreev reflectance of a vortex tangle does indeed reveal the nature of quantum turbulence.

- The $f^{-5/3}$ scaling of the frequency spectrum of the Andreev-retro-reflected signal has been observed earlier in the experiments of the Lancaster ULT group [e.g. Bradley *et al.*, PRL **101**, 065302 (2008)] where it was argued that fluctuations of the reflected signal can be interpreted as fluctuations of the vortex line density.

- The Andreev reflection technique has great potential for elucidating the behavior of pure quantum turbulence.