Experimental and numerical suggestions based on the theoretical models.

Sergey K. Nemirovskii
Institute for Thermophysics,
Novosibirsk, Russia;

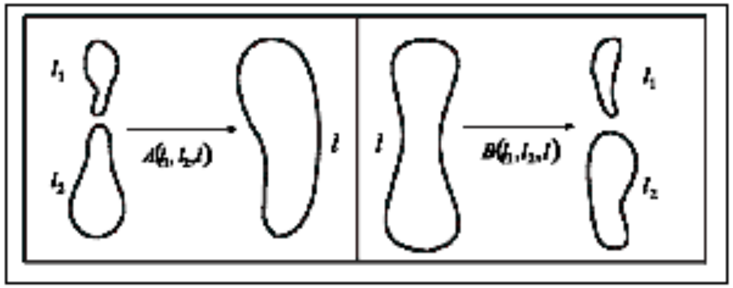
Motivation and goals

- Motivation of this topic for the talk is inspired by the sentence from first announcement of the workshop "We are organizing a workshop to bring together experimentalist and theorists to share ideas about interpretation of currently available experimental measurements, and working out ways for the most effective experiments in future…..".
- Therefore I decided to discuss several theoretical results, which have experimental (or/and numerical) consequences
- Although there is no developed theory of quantum turbulence (QT) so far, rather the isolated modest separate fragments exist, they bear a series predictions which can serve as ideas for experimental and numerical proposals.

Plan of talk

- Kinetics of vortex loops and the theory of quantum turbulence.
- Hydrodynamics of the superfluid turbulence(coarse-grained. Hydrodynamics) and its applications.
- Spectrum of Vortex Line Density (VLD) $\langle \delta L(\omega) \delta L(-\omega) \rangle$.
- Thermal equilibrium state.
- Quasi-classical Turbulence (E as function of L).
- Rotating turbulence.
- 3D Energy spectrum.

QT as a Kinetics of vortex loops



This model describe quantum turbulence as a network of splitting and merging vortex loops. Each of loops has a random walking structure with elementary step ξ_0 connected with the interline space $\delta = \mathcal{L}^{-1/2}$. In this formalism all inner degrees of freedom are ignored. They are immersed in the random walking model. Motivation for this is that due to huge number of reconnections, the inner dynamics doesn't have time to be developed.

Interaction = collisions

Then, the only degree of freedom, which is the length l of the loop. The corresponding problem has the complete analytic solution on the base of the Boltzmann type "kinetic equation" for the distribution function $n(l)$ of number of loops with length l. (Copeland, E. J.; Kibble, T. W. B. & Steer, D. A. *Evolution of a network of cosmic string loops* Phys. Rev. D, 1998,)

Distribution of loops over their sizes

One of the key predictions following from theory concerns the distribution of vortex loops sizes $n(l)$, which is the basis for various applications

$$\boxed{n(l) = \frac{C_{VLD}}{\xi_0^{3/2}} l^{-5/2}.}$$

#

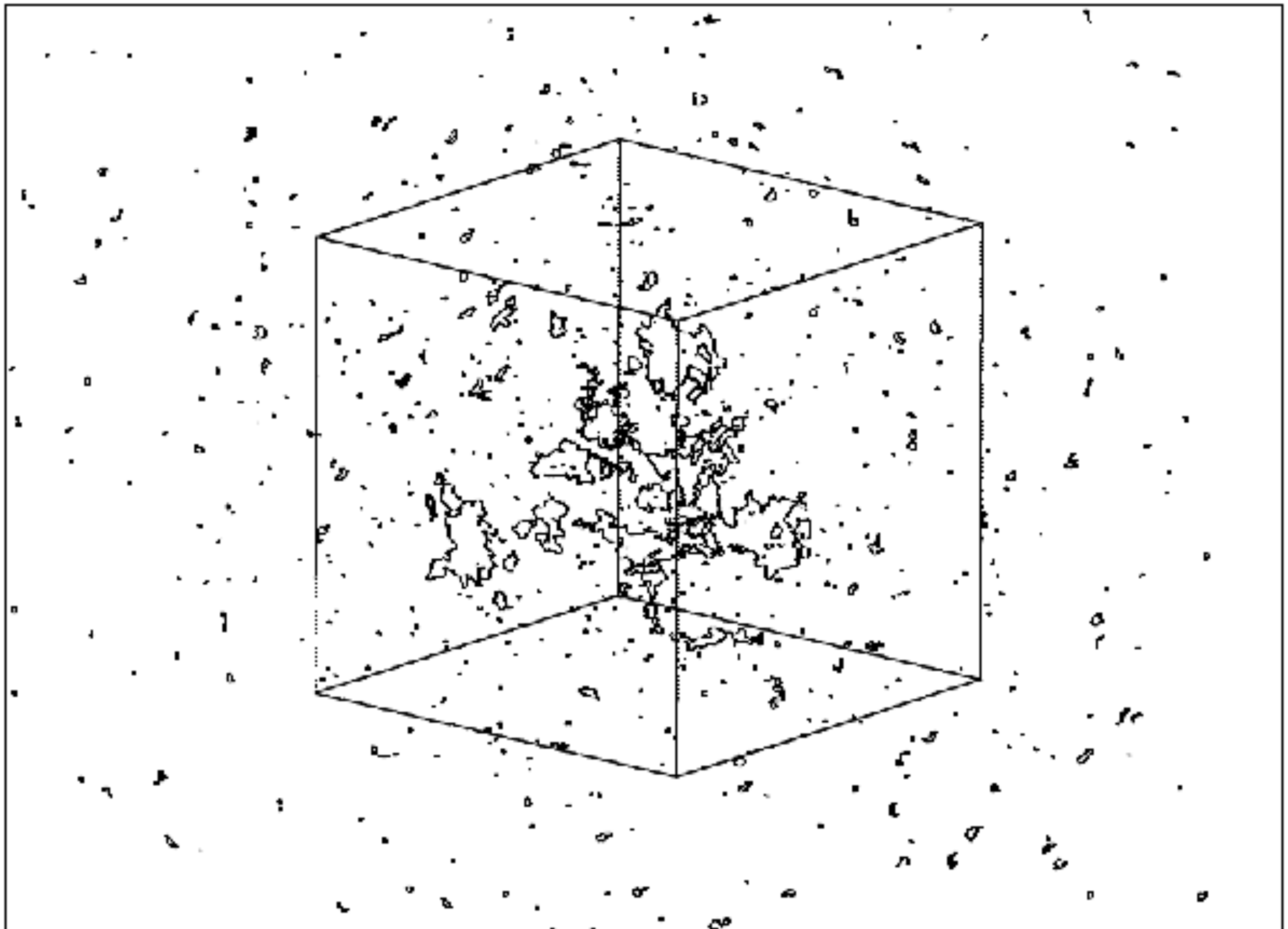
Parameter $C_{VLD} \approx 1.8 \times 10^{-2}$. The law $l^{-5/2}$ was frequently obtained (from thermodynamics) for the distribution of one-dimensional topological defects in other fields of physics (cosmic strings, (Copeland, 1998), or lines of darkness in nonlinear optics (O'Holleran, 2008)

There are numerical results that the distribution of loops in QT obeys a power law $n(l) \propto l^{-a}$, but with a, nearly $1 \div 1.5$ cite: Araki2002, cite: Kondaurova2005.

It is possible that the disagreement between the numerical results and $n(l) \propto l^{-5/2}$ could be associated with the artificial elimination of very small loops in a numerical algorithm, which reduces the number of loops of very small lengths l and makes the distribution more gradual.

The determination of the distribution law $n(l)$ is very important, and is still an open numerical (and expeimental?) problem.

The determination of the size distribution of the loops $n(l)$, constituting the VT from monitoring of the emitted vortex loops $\mathcal{P}(l)$.



Distribution of the emitted vortex loops

Knowing (From the Boltzmann type "kinetic equation") the density of the loops $n(l,x,t)$, the average velocity $V_l(l)$ of the loops, and the free path $\lambda(l)$ (all quantities are l -dependent), we can estimate the spatial flux of loops of size l (see for details cite: Nemirovskii2010), radiated by layer at point x of the VT.

$$\boxed{\mathbf{J}_{rad} = \frac{1}{2} n(l,x,t) V_l(l) \exp(-x/\lambda(l))}$$

(J radiation)

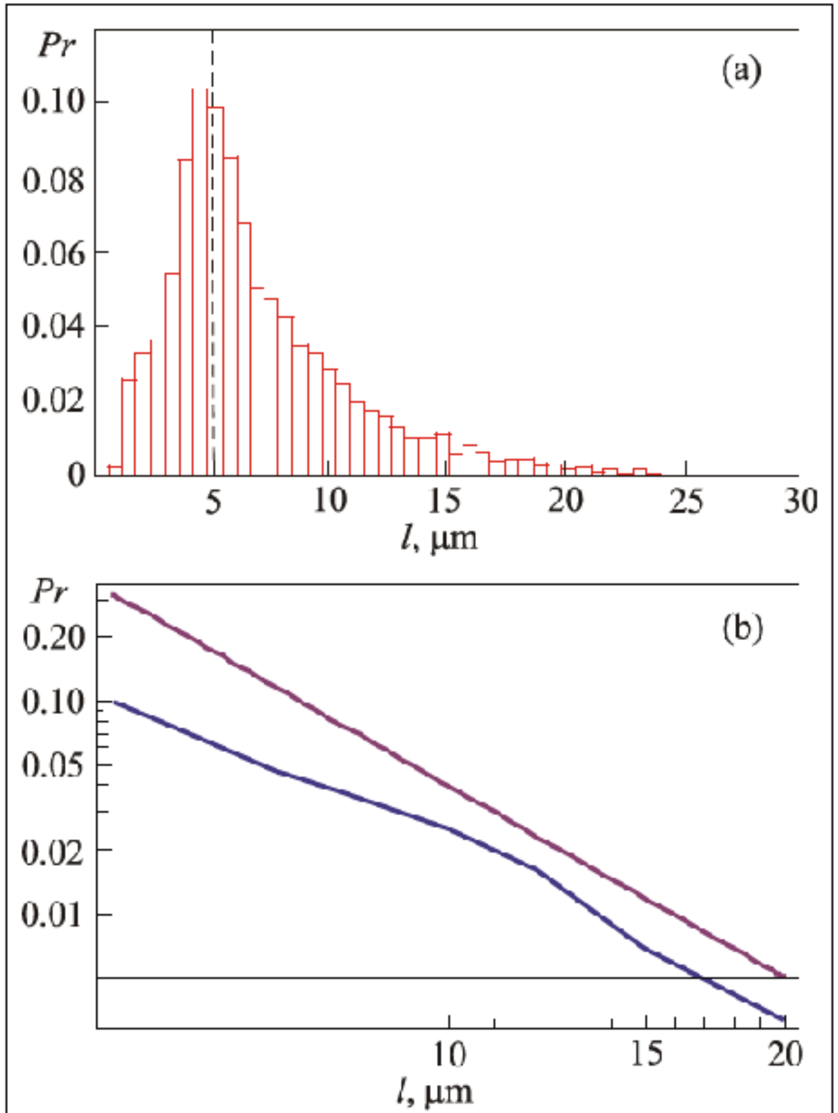
Supposing for simplicity the full uniformity of vortex tangle (inside domain), and integrating out over solid angle $d\theta d\varphi$ and over position of loops dR we obtain the flux of loop through the domain boundary

$$\boxed{\mathbf{J} = \frac{1}{4} \int n(l,x_b,t)\left(\frac{\beta}{\sqrt{l\xi_0}}\right)dl,}$$

(J total)

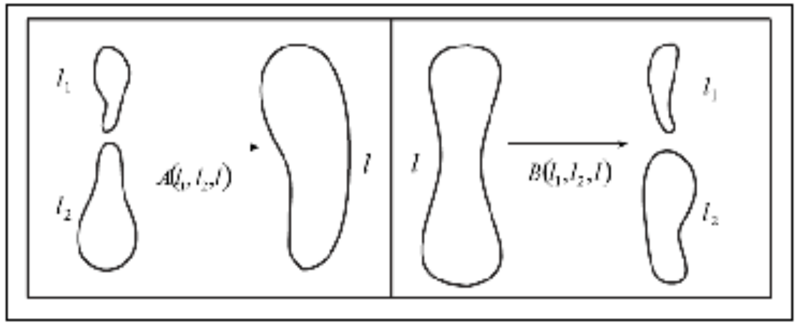
$$\boxed{\mathcal{P}(l) = N(l)/\int N(l)dl \propto l^{-3}}$$

Example of application of stated approach - comparison with work by Nakatsuji, A.; Tsubota, M. & Yano, H., Phys. Rev. B, 2014, 89, 174520



Top. The PDF of the length of emitted vortex loops obtained in cite: Nakatsuji2014. Lower line is the PDF from the upper picture depicted in logarithmic coordinates.The upper curve is the analytical PDF is $\mathrm{Pr}_{an}(l) \approx 50l^{-3}$ obtained from theoretical consideration.

Rate of reconnection



$$A(l_1,l_2,l) = b_m V_l l_1 l_2, \quad B(l_1,l-l_1,l) = b_s \frac{V_l l}{(\xi_0 l_1)^{3/2}}.$$

Quantity $V_l = \kappa/\xi_0$ is (l-independent) characteristic velocity of the approach of elements of the line, b_m and b_s are numerical constants of order of unity.

Reconnection rates for the merging $A(l_1,l_2,l)$ and the splitting $B(l,l_1,l_2)$ of vortex loops are interesting because of their dependence on the sizes l, l_1, l_2.

The full rate of reconnectionsFull rate of reconnections.

$$\dot{N}_{rec} = C_{rec}\kappa\mathcal{L}^{5/2},$$

where C_{rec} one more constant of the order $0.1-0.5$. This result agrees with the recent numerical investigations cite: Tsubota00, cite: Barenghi2004,cite: Kondaurova2014.

The cascade-like fusion and the cascade like breakdown

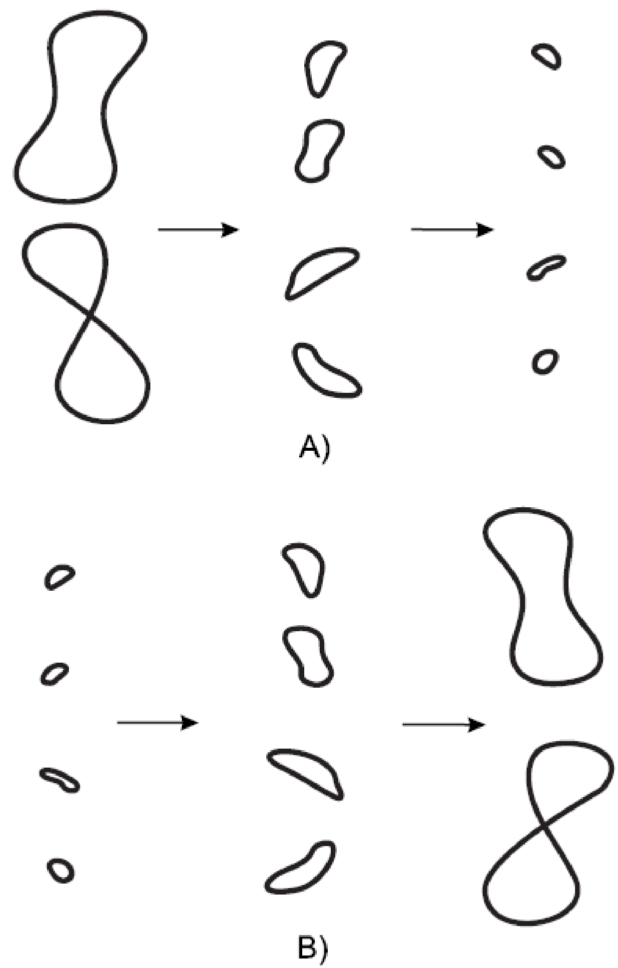


FIG. 5. Pictures illustrating flux of length (or energy, see Ref. 32) in space of the loop sizes. This flux is just redistribution of total length (energy) among the loops of different sizes due to recombination process. (a) Negative flux, or direct cascade appears due to consequent breakdown resulting in formation of smaller and smaller loops. (b). Positive flux or inverse cascade describes consequent fusion of loops leading to formation of larger and larger loops.

Flux of length (energy)

The total length (per unit volume), or the vortex line density $\mathcal{L}(t)$ is defined as follows:

$$\mathcal{L}(t) = \int L(l,t)dl = \int l * n(l,t)dl.$$

(VLD_definition)

Conservation of the vortex line density can be expressed in the form of continuity equation for the length density $L(l,t)$

$$\frac{\partial L(l,t)}{\partial t} + \frac{\partial P(l)}{\partial l} = 0.$$

(continuity equation)

This form of equation states that the rate of change of length is associated with "flux" of length in space of sizes of the loops. Term "flux" here means just the redistribution of the length among the loops of different sizes due to reconnections.

$$P_{net} = P_+ - P_- = |C_F|\kappa\mathcal{L}^2.|$$

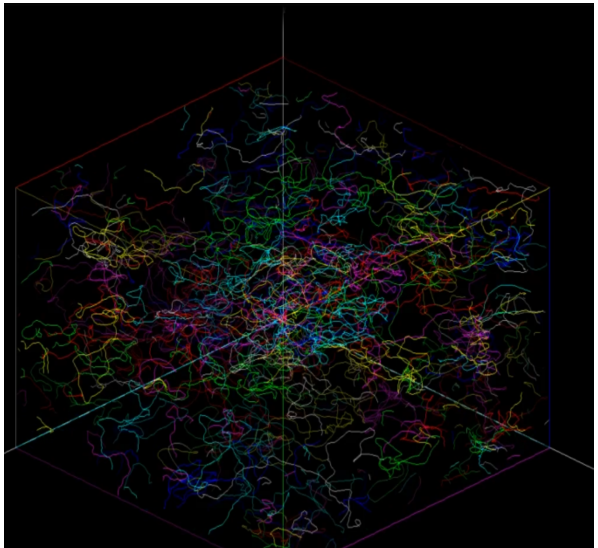
#

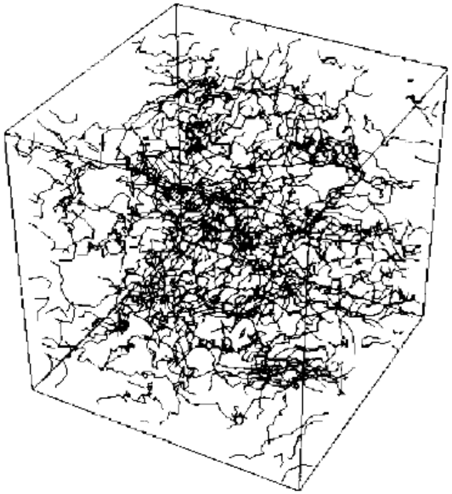
$$C_F \approx (2.22b_m - 3.926c_2^2(T)b_s).$$

(Feynman constant)

The low temperature quantum turbulence $C_F < 0$

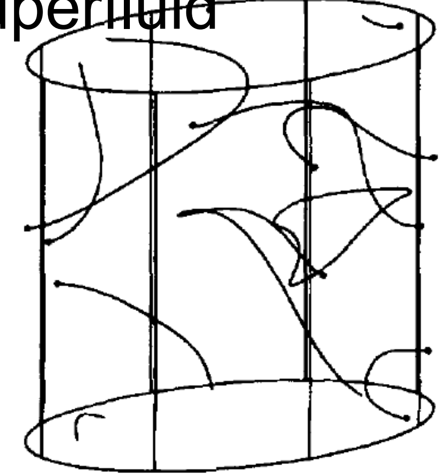
The negative flux appears when the break down of loops prevails and the cascade-like process of generation of smaller and smaller loops forms. There exists a number of mechanisms of disappearance of the vortex energy on very small scales. It can be e.g. the acoustic radiation, nonlinear Kelvin waves, loop escape etc. These dissipative mechanisms balance the grow of the line length due to the mutual friction. As a result, fully developed turbulence with the flux of energy in direction of small scales is formed, what implies highly chaotic picture of the vortex tangle.





The high temperature quantum turbulence $C_F > 0$

The case with inverse flux is less clear. The inverse cascade implies the generation of larger and larger loops. Unlike the previous case of the direct cascade, there is no an apparent mechanism for disappearance of very large loops. The probable scenario is that the parts of large loops are pinned to the walls. Finally, a state with few lines stretching from wall to wall with poor dynamics and rare events is realized, this is a degenerated state of the vortex tangle. Some numerical investigators Schwarz1988, Aarts, report on this situation. This observation can be an alternative explanation for a phenomenon discovered in Helsinki group (Finne2003), who observed transition to superfluid turbulence governed by the temperature.



Hydrodynamics of the superfluid turbulence (HST), or the coarse-grained hydrodynamics and its applications

As it is written in famous book by Donnelly (1991) "Almost all probes of superfluidity are hydrodynamic, and those of us interested in superfluidity, particularly quantum turbulence, have had to learn a good deal of fluid mechanics"

To describe correctly all hydrodynamic phenomena, we need equations of motion of superfluids in presence of the vortex tangle. These equations have to take into account the mutual influence of both velocity field on VT and VT on velocity and temperature field. In other word they decribe the back reaction, i.e. coordinated spacial temporal change of all variables

The hydrodynamics of superfluid turbulence (HST), the theory which unify Vinen equation with the classical Landau two-fluid model

$\boxed{\text{HST} \cup \text{VE, Landau Hydrodynamics}}$

.

The essence of HST

The main aim of formulating the HST is to combine Vinen's equation and the classical two fluid Landau Khalatnikov hydrodynamic equations. (Nemirovskii and Lebedev cite: Nemirovskii1983, Yamada et al. cite: Yamada1989, Guerst cite: Geurst1992)

Vortex line density $\mathcal{L} \to \mathcal{L}(\mathbf{r}, t)$ is considered as field and new variable to $\mathbf{v}_s, \mathbf{v}_n, S, \rho$

$$\delta E_0 = \varepsilon_V \, d\mathcal{L}$$ # (dE of L)

On the ground of conservation laws (Nemirovskii and Lebedev 1983) it is derived

$$\frac{\partial \mathcal{L}}{\partial t} + \nabla \cdot (\mathcal{L} \, \mathbf{v}_{\mathcal{L}}) = \alpha_V \, |\mathbf{v}_{ns}| \, \mathcal{L}^{3/2} - \beta_V \, \mathcal{L}^2$$ # (HST L)

$$\frac{d\rho}{dt} + \nabla \cdot \mathbf{j} = 0 \, ,$$ # (HST rho)

$$\frac{d\mathbf{j}_i}{dt} + \frac{d\Pi_{ik}}{dx_k} = 0 \, ,$$ # (HST j)

$$\frac{dS}{dt} + \nabla \cdot \left[S \, \mathbf{v}_n + S^L \left(v_L - v_n \right) \right] = \frac{1}{T} \left[K \, \mathcal{L} \, v_{ns}^2 + \varepsilon_V \, \beta \, \mathcal{L}^2 \right] ,$$ # (HST S)

$$\rho_s \left\{ \frac{d\mathbf{v}_s}{dt} + (\mathbf{v}_s \cdot \nabla) \, \mathbf{v}_s + \nabla \mu \right\} - b \, \nabla \varepsilon_V - S^L \, b \, \nabla T$$ # (HST vs)

$$= K \, \mathcal{L} \, \mathbf{v}_{ns} + \varepsilon_V \, \alpha_V \, \frac{\mathbf{v}_{ns}}{|\mathbf{v}_{ns}|} \, \mathcal{L}^{3/2} \, .$$

Applications of HST

Interaction of vortices with the second sound

dry friction, action like $\propto \mathbf{V}_{ns}/V_{ns}$

VLD fluctuations

slow decay

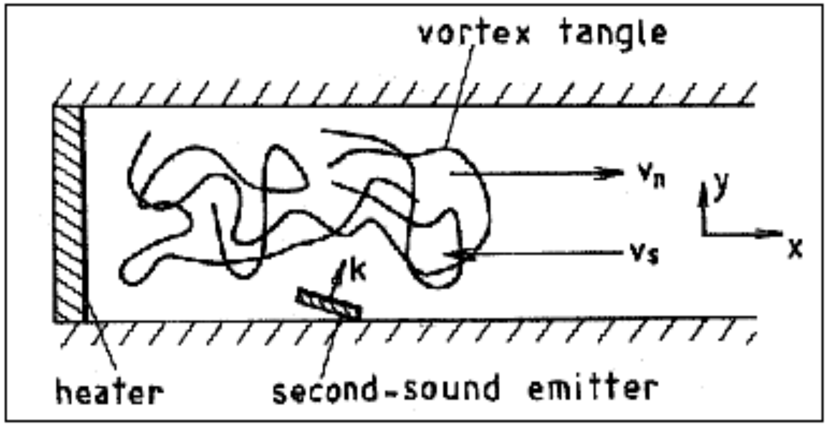
Propagation of intense second sound pulses generating vortex lines and interacting with them

Turbulent fronts and plugs

boiling, cooling with superfluid helium,

more ???

Interaction of vortices with the second sound



The study of ST using acoustical methods, primarily second sound, is one of the most used, widely applied experimental method. The main idea is to take advantage of the extra attenuation of second sound waves resulting from the friction force due to the interaction between the normal component and the VT. The corresponding relation for this damping, which is very much larger than the viscous damping, even for very small VLD, obtained by Vinen cite: Vinen1956 is

$$\boxed{\Gamma_V \ = \ B \ \kappa \ \mathcal{L}_0/6} \qquad \text{\# (attenuation)}$$

dispersion law $\omega(k)$ of a monochromatic second sound wave $\delta T \propto \exp[i(\omega t - k_x x - k_z z)]$, propagating in counterflowing turbulent HeII with a VLD L_0 is (Nemirovskii & Lebedev 1983)

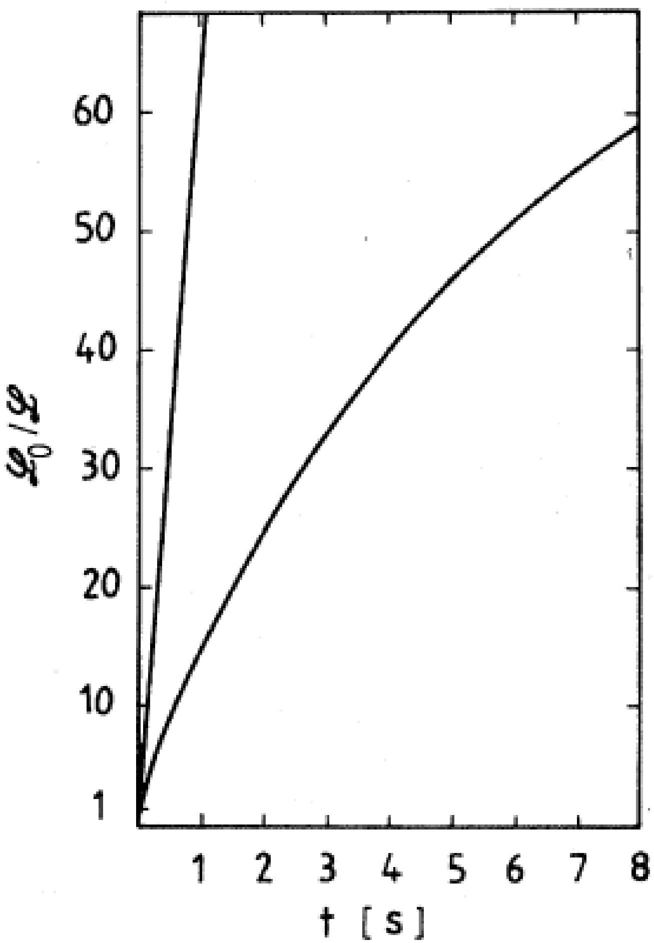
$$\boxed{\omega = u_2\,|\mathbf{k}| + i\left\{\frac{\Gamma_z\,k_z^2}{k^2} + \frac{\Gamma_x\,k_x^2}{k^2} + A_1\,\frac{k_z}{k}\right\} + i\,\frac{A_3\,k_z^2 - A_2\,k\,k_z}{(iu_2 k\tau_V + 1)k^2}\,.}$$

$$\boxed{\Gamma_z = \frac{1}{2}\left(\frac{K\,\rho\,\mathcal{L}_0 I_{\parallel}}{\rho_s\,\rho_n} \pm \frac{\varepsilon_V\,\alpha_V\,\mathcal{L}_0^{3/2}}{\rho_s\,v_s}\right), \qquad \Gamma_x = \frac{K\,\rho\,\mathcal{L}_0 I_{\perp}}{2\rho_s\,\rho_n}\,,}$$

are damping coefficients in the x and z directions, and

$$\boxed{\tau_V = 2\beta_V/(\alpha_V^2 v_{ns}^2)}$$

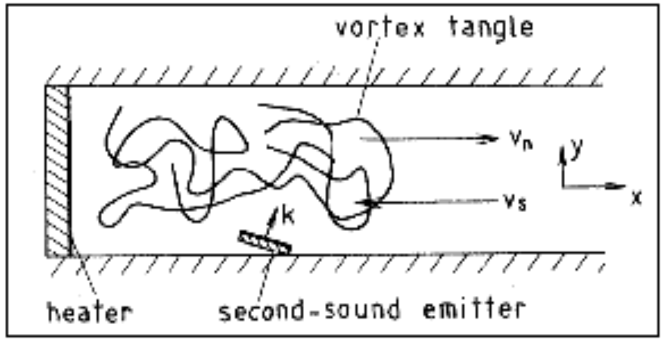
Slow decay of the vortex tangle, HST consideration



After switching off the heater, the field of velocities does not vanish instantaneously. But this, in turn, implies, that the generating (first) term in the Vinen equation is not zero (although it decreases in time), therefore the total decay of the VT should be slower..

The quantity 1/L(t) calculated from a set of HST equations (Kondaurova, 1993). The straight line corresponds to VE.

Slow decay of the vortex tangle, Dresner equation



The HST equations govern both a slow variation of the hydrodynamic variables due to dissipation related to the vortex tangle and fast processes of the first and second sound propagation. Using multi-scale perturbation analysis we show how one can eliminate the fast processes to derive the evolution equation for the slow processes only. We then demonstrate that the long-term evolution of a transient heat load of moderate intensity obeys the nonlinear heat conductivity equation, referred to as the Dresner equation.

$$t_0 = t'; \qquad t_1' = \epsilon t'; \qquad t_2' = \epsilon^2 t', \ \frac{1}{\epsilon} = Sh = \frac{L}{U\tau_d} \qquad \text{\# (t)}$$

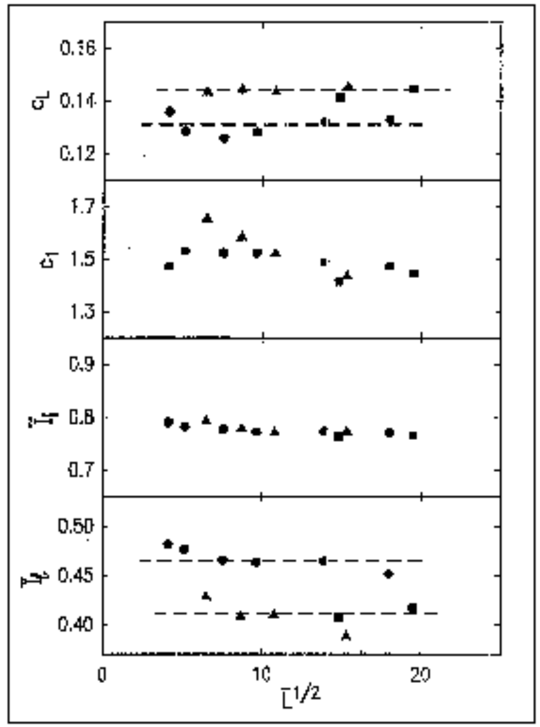
$$\frac{\partial T'}{\partial t_1'} = \epsilon^{-1/3} \frac{\partial}{\partial x'} \left(\frac{\partial T'}{\partial x'} \right)^{1/3} \qquad \text{\# (DR)}$$

Dry friction

Kondaurova, L.; L'vov, V.; Pomyalov, A. & Procaccia, I. Phys. Rev. B, 2014, 89, 014502

> (2) The anisotropy indices $I_\parallel$, $I_\perp$, I_ℓ, and $I_{\ell\perp}$ are practically independent of $V_{\rm ns}$, i.e., $\propto V_{\rm ns}^0$, see Eqs. (4), Fig. 11, and Table IV; $I_\parallel$, $I_\perp \simeq 0.7 \div 0.9$, $I_\ell \simeq 0.5$, $I_{\ell\perp} \approx 0$ (because of the axial symmetry).

Schwarz, K. W. Phys. Rev. B, 1988, 38, 2398-2417



Adachi, H.; Fujiyama, S. & Tsubota, M. Phys. Rev. B, 2010, 81, 104511

> Figure 5(a) shows the anisotropy as a function of v_{ns} and T. The anisotropy is almost independent of v_{ns} and is dependent on T, in agreement with experi-

Dry friction $\mathbf{F} \propto -\mathbf{V}_{ns}/V_{ns}$

Dry friction Sign

$$\rho_s \left\{ \frac{d\mathbf{v}_s}{dt} + (\mathbf{v}_s \cdot \nabla)\mathbf{v}_s + \nabla\mu \right\} - b\,\nabla\varepsilon_V - S^L\, b\,\nabla T$$

$$= K\,\mathcal{L}\,\mathbf{v}_{ns} + \mathrm{Sign}\,\varepsilon_V\,\alpha_V \frac{\mathbf{v}_{ns}}{|\mathbf{v}_{ns}|}\mathcal{L}^{3/2}.$$

#

Nemirovskii and Lebedev, 1983 Sign is +

Based on Feynman's scenario that VT takes the kinetic energy from main flow and returned it in the form of entropy.

Yamada et al.,1989 , Sign is -

Based on the Schwarz's vision that both the growth of VT and its decay is due to mutual friction

Guerst, 1992. Sign is undetermined (coincides with drift of VT wrt the superfluid component)

Based on variationl principle.

Thus, the study of the effects connected to the dry friction term supplies important information:

1. on the macroscopic dynamics of the VT (the explicit form of the Vinen equation)
2. as well as on the microscopic processes describing the stochastic behavior of the whole vortex tangle.

Therefore experimental study of the described effect is extremely important for understanding of counterflowing quantum turbulence

Attempts to observe dry friction effect, Stamm et al. PRB, 1993,

To assess the experimental possibility to study the described effect, consider now the solution of a simple model describing the evolution of a second sound pulse which propagates through such the VT with VLD equal to $\mathcal{L}_0$ taking into account dry friction:

$$\boxed{(dw/dt)_{x-c_{20}t} = -\frac{A_{13}}{2} w \mathcal{L}_0 \mp \frac{A_{14}}{2} \mathcal{L}_0^{3/2}}$$

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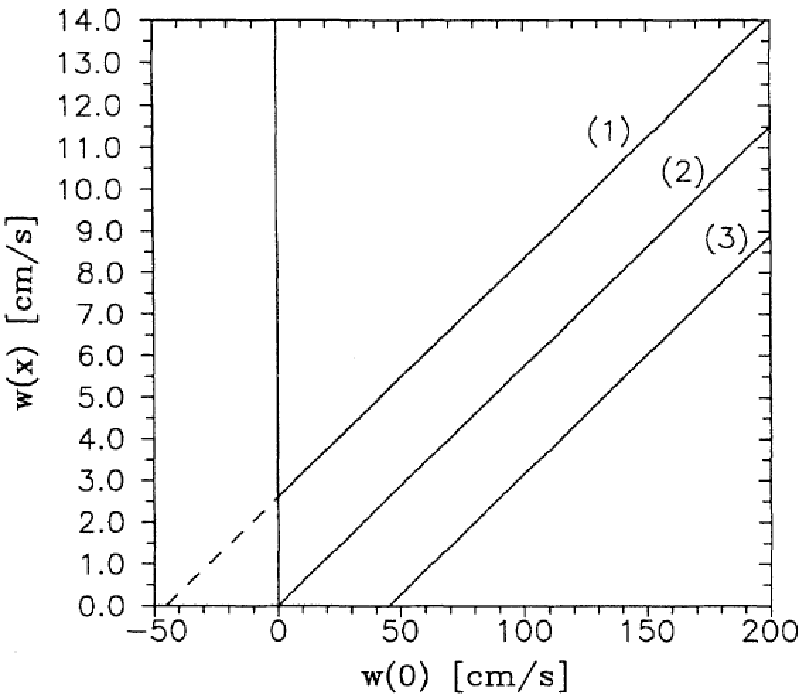
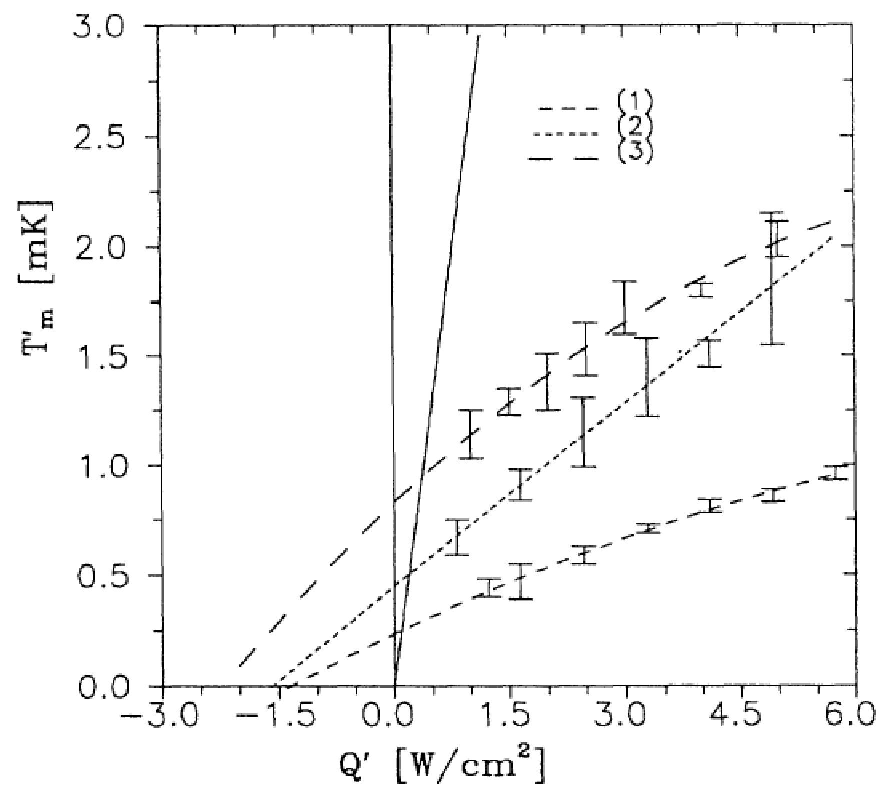




Spectrum Vortex Line Density (VLD) $< \delta\mathcal{L}(\omega)\ \delta\mathcal{L}(-\omega) >$

The important question of the different arrangement of QT, namely whether it is a set of vortex bundles, Vinen disordered state, or mix of these two forms can be experimentally checked from the study of fluctuations of the Vortex Line Density (VLD) $< \delta\mathcal{L}(\omega)\ \delta\mathcal{L}(-\omega) >$ in turbulent flows. The different forms of arrangement of the vortex tangle give various dependencies of these quantities on frequency ω. This conclusion may serve as a basis for the experimental determination of what kind of the turbulence is implemented in different types of generation of QT.

Vinen Equation case

Let us study the reaction of the vortex line density in a fluctuating flow of normal velocity supposing that the dynamics of $\mathcal{L}(t)$ obeys the Vinen equation:

$$\frac{\partial \mathcal{L}}{\partial t} = \alpha_V \left|\mathbf{v}_{ns}\right| \mathcal{L}^{3/2} - \beta_V \mathcal{L}^2. \qquad \text{\# (VE 1)}$$

Linearization of this equation (wrt fluctuation $\delta\mathcal{L}$) leads to

$$\frac{\partial \delta\mathcal{L}}{\partial t} = \alpha_V \frac{\rho}{\rho_s} \mathcal{L}_{0V}^{3/2} \delta \mathbf{v}_n - \beta_V \frac{1}{2} \mathcal{L}_{0V} \delta\mathcal{L}. \qquad \text{\# (viaL0)}$$

Equation (ref: viaL0) shows that the evolution of the fluctuating part of the vortex line density $\delta\mathcal{L}$ bears the relaxation-type character with a characteristic time

$$\tau_V = 2/(\beta_V \mathcal{L}_0), \qquad \text{\# (tau v)}$$

Performing Fourier transform, we obtain the spectrum $\langle \delta\mathcal{L}(f) \delta\mathcal{L}(-f) \rangle$

$$\langle \delta\mathcal{L}(f) \delta\mathcal{L}(-f) \rangle = \frac{4(\alpha_V/\beta_V)^2 \left(\frac{\rho}{\rho_s}\right)^2 \mathcal{L}_{0V} \langle \delta \mathbf{v}_n(f) \delta \mathbf{v}_n(-f) \rangle}{1 + (2\pi f \tau_V)^2}.$$

HVBK case

In terms of HVBK dynamics, the vorticity field $\boldsymbol{\omega}(\mathbf{r},t)$ obeys the following equation (see cite: Khalatnikov1965)

$$\frac{\partial \boldsymbol{\omega}}{\partial t} = \nabla \times [\mathbf{v}_L \times \boldsymbol{\omega}], \quad \mathbf{v}_L = \alpha[\hat{\boldsymbol{\omega}} \times (\mathbf{v}_n - \mathbf{v}_s)]$$

In the stationary case, the coarse- grained superfluid velocity coincides with the normal velocity in the bundle $\mathbf{v}_s = \mathbf{v}_n$, and the steady value $\mathcal{L}_{0b}$ of the vortex line density in the bundle is

$$\mathcal{L}_{0b} = |\nabla \times \mathbf{v}_n|/\kappa$$ # (L 0 b)

Linearization of the HVBK equation (wrt fluctuation $\delta\mathcal{L}$) leads to

$$\frac{\partial(\delta\mathcal{L})}{\partial t} = \alpha\mathcal{L}_{0b}\nabla \times \delta\mathbf{v}_{ns} = \alpha\mathcal{L}_{0b}\nabla \times \delta\mathbf{v}_n - \frac{1}{\tau_b}\delta\mathcal{L},$$

where the relaxation time τ_b of the vortex lines population inside the bundle is:

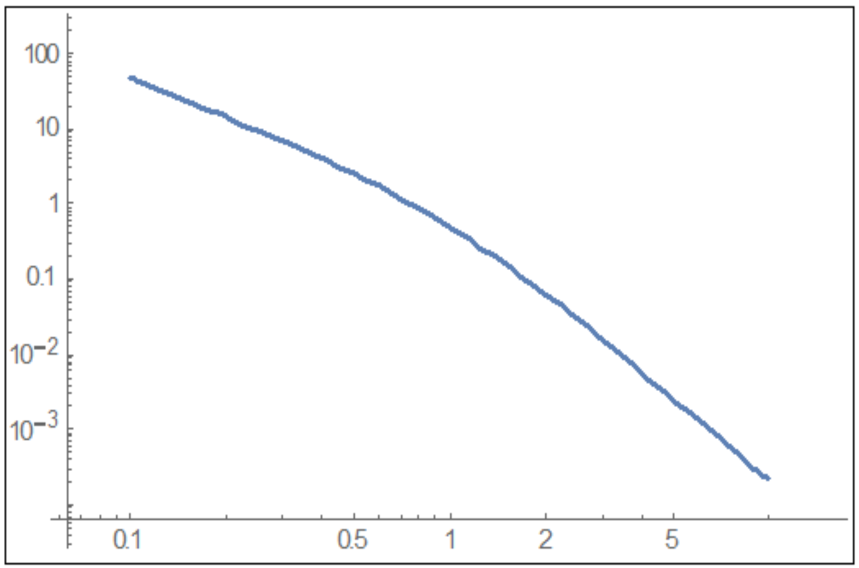
$$\tau_b = \frac{1}{\alpha\mathcal{L}_{0b}\kappa}$$ # (time bundle)

$$< \delta\mathcal{L}(f)\ \delta\mathcal{L}(-f) >= \frac{\alpha^2\mathcal{L}_{0b}^2(\frac{\rho}{\rho_s})^2\tau_b(2\pi f/\bar{v}_n)^2\langle\delta\mathbf{v}_n(f)\delta\mathbf{v}_n(-f)\rangle}{1+(2\pi f\tau_b)^2}.$$

Comparison

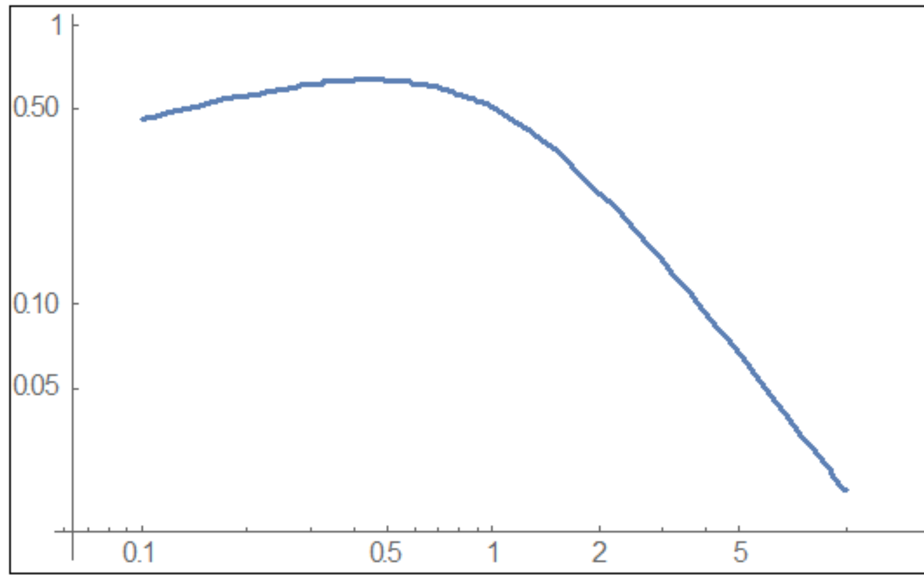
Vinen Equation case

$$\langle\delta\mathcal{L}(f)\delta\mathcal{L}(-f)\rangle = \frac{4(\alpha_V/\beta_V)^2\ (\frac{\rho}{\rho_s})^2\mathcal{L}_{0V}\langle\delta\mathbf{v}_n(f)\delta\mathbf{v}_n(-f)\rangle}{1+(2\pi f\tau_V)^2}$$



HVBK case

$$<\delta\mathcal{L}(f)\ \delta\mathcal{L}(-f)> = \frac{\alpha^2\mathcal{L}_{0b}^2(\frac{\rho}{\rho_s})^2\tau_b^2k^2\langle\delta\mathbf{v}_n(f)\delta\mathbf{v}_n(-f)\rangle}{1+(2\pi f\tau_b)^2}.$$



These results may serve as a basis for the experimental determination of what kind of the turbulence is implemented in different types of generation.

Energy grows as $\mathcal{L}^{4/3}$

In the disordered, Vinen turbulence $E \propto \mathcal{L}$. In the quasi-classical case the energy per unit mass E, contained in the inertial interval can be evaluated from spectrum

$$E_K(k) = C\,\varepsilon^{2/3} k^{-5/3}, \qquad \text{\# (eq-Kolmogorov)}$$

In vortex bundles conception the intervortex distance δ and the flux ε. are not independent quantities, they relate to each other. To show this, let us evaluate the vorticity at the scale δ (see paper by L'vov, Nazarenko & Rudenko 2007). This can be done with the use of Equation (ref: eq-Kolmogorov), and by observing that the energy per unit mass (in the k space) is related to the squared vorticity as $E(k) = \langle \mathbf{v_k v_{-k}} \rangle = \langle \boldsymbol{\omega}_\mathbf{k} \boldsymbol{\omega}_\mathbf{-k} \rangle / k^2$. Then

$$\langle \omega^2 \rangle \sim \int^{1/\delta} k^2 E(k) dk \sim \varepsilon^{2/3} \delta^{-4/3}. \qquad \text{\# (omega at crossover)}$$

Taking the value of vorticity from Eq. $\omega = \kappa \mathcal{L}$ they obtained that $\varepsilon \sim \kappa^3 \delta^{-4}$. To find how the energy scales with the VLD $\mathcal{L}$, let us use the estimate for the energy $E \sim C\varepsilon^{2/3} D^{2/3}$,written above, and the expression for the dissipation rate via an intervortex space $\varepsilon \sim \kappa^3 \delta^{-4}$. The comparison leads to:

$$E \sim C D^{2/3} \kappa^2 \mathcal{L}^{4/3}. \qquad \text{\# (E(L) CT)}$$

Thus, energy as a function of the VLD $\mathcal{L}$ grows faster than for the disordered state (where $E \propto \mathcal{L}$), and slower than for the solid-body rotating case (where $E \propto \mathcal{L}^2$). This observation would be of interest for the experimental study of the quasi-classical behavior of QT.

Langevin and Fokker -Planck statement of the problem. Chaotic vortices in the thermal equilibrium

- The Langevin approach is a powerful method allowing the treatment of various types of statistical dynamics, ranging from thermal equilibrium to a fully non-equilibrium turbulent state. These options are achieved by properly choosing a random (stirring) force correlator.
- Although our workshop is devoted to quantum turbulence, I would like to attract attention to another case of chaotic vortices - namely vortex filaments in thermal equilibrium state. Particular importance of such a formulation gives the fact that it has a complete analytic solution. This allows us to understand the mechanisms of emergence and development of various structures in the chaotic set of vortex filaments, such as, for example, vortex bundles or anisotropy

Langevin and Fokker-Planck equation

In case of nonzero counterflowing velocity $\mathbf{v}_{ns} = \mathbf{v_n} - \mathbf{v_s}$ equation of motion (in Langevin statement) of vortex line element reads

$$\boxed{\dot{\mathbf{s}} = \mathbf{B}(\xi) + \mathbf{v}_s + \alpha \mathbf{s}'(\xi) \times (\mathbf{v}_{ns} - \mathbf{B}(\xi)) + \alpha' \mathbf{s}'(\xi) \times \mathbf{s}'(\xi) \times (\mathbf{v}_{ns} - \mathbf{B}(\xi)) + \mathbf{f}(\xi_0, t).}$$

The corresponding Fokker-Planck equation probability distribution functional (PDF) $\mathcal{P}(\{\mathbf{s}(\xi)\}, t) = \langle \delta(\mathbf{s}(\xi) - \mathbf{s}(\xi, t)) \rangle$ is

$$\boxed{\frac{\partial \mathcal{P}}{\partial t} + \int d\xi \frac{\delta}{\delta \mathbf{s}(\xi)} \left\{ [\mathbf{B}(\xi) + \mathbf{v}_s + \alpha \mathbf{s}'(\xi) \times (\mathbf{v}_n - \mathbf{v}_s - \mathbf{B}(\xi))] + F \frac{\delta}{\delta \mathbf{s}(\xi)} \right\} \mathcal{P} = 0}$$

We assert that this dynamics also has equilibrium solution and our goal now is to find the analogue of Gibbs distribution in this case.

$$\boxed{\mathcal{P}(\{\mathbf{s}(\xi)\}, t) = \mathcal{N} \exp(-\frac{\tilde{H}\{\mathbf{s}(\xi)\}}{k_B T}).}$$

Here the Hamiltonian $\tilde{H}\{\mathbf{s}(\xi)\}$ is

$$\boxed{\tilde{H} = E\{\mathbf{s}(\xi)\} - \mathbf{P_L}(\mathbf{v_n} - \mathbf{v_s}),}$$

where energy $E\{\mathbf{s}(\xi)\}$ and $\mathbf{P}_L$ the so called Lamb Impulse are defined as (see, e.g. cite: Batchelor1967)

$$\boxed{E\{\mathbf{s}(\xi)\} = \frac{\rho_s \tilde{\kappa}^2}{8\pi} \int_{\Gamma} \int_{\Gamma'} \frac{\mathbf{s}'(\xi)\mathbf{s}'(\xi)}{|\mathbf{s}(\xi) - \mathbf{s}(\xi')|} d\xi d\xi', \qquad \mathbf{P}_L = \frac{\rho_s \kappa}{2} \int \mathbf{s}(\xi) \times \mathbf{s}'(\xi)\, d\xi.}$$

The partition function

The partition function (below $\beta = 1/k_B T$) is

$$Z = \int D\mathbf{s}(\xi) \exp\left[-\beta \left(\frac{\rho_s \kappa^2}{8\pi} \oint\oint d\xi_1 d\xi_2 \frac{\mathbf{s}'(\xi_1)\cdot\mathbf{s}'(\xi_2)}{|\mathbf{s}(\xi_1)-\mathbf{s}(\xi_2)|} d\xi_1 d\xi_2 - \frac{\rho_s \kappa}{2} \int \mathbf{s}(\xi)\times\mathbf{s}'(\xi)\, d\xi \right) \right]$$

The similar quantity was studied in series of work on the role of the string-like objects (dislocations, vortices etc.) in the phase transition physics(see for details cite: Kleinert1990,cite: Edwards1979, cite: Copeland1991, cite: Chorin1994).

In spite of its formidable form, it can be evaluated, therefore the problem of chaotic vortex filament has the complete analytical solution.

That allows to follow all details of the chaotic vortex dynamics.

Rotating turbulence

- Rotating turbulence We have described a number of effects that arise in the combined flow when rotation and quantum turbulence coexist (see subsection (<ref>rotating</ref>)). This topic has not been very much studied. As for numerical simulations, there is only result when the axial counterflow superimposed on the rotating vortex lattice (see <cite>Tsubota2003,Tsubota2004</cite>). To our knowledge the "opposite" case of the rotation of the pre-prepared QT has not been studied numerically. It is also of interest to explore the theoretical prediction on nonlinear period shift and the extra dissipation predicted by Eqs. (<ref>period shift</ref>)-(<ref>exta dissipation</ref>).

Instead of Summary

- So we made several proposals for experimental studies on dynamics of quantum turbulence based on theoretical models.
- It is understood that is not always the predicted theoretical predictions come true.
- I understand as well that not all will rush to perform the suggested proposal.
- Thus, this exposition can be regarded merely as a certain style of presentation of the theoretical results

Thank You !