Superfluid turbulence near the intervortex scale

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Collaborators:

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SHREK

SUPERFLUIDE À HAUT REYNOLDS EN ECOULEMENT DE VON KÁRMÁN

ENSL, CEA/SBT, CEA/SPEC, I. NEEL, LEGI, Luth ET SYSTÈMES COMPLEXES

SHREK Collaboration

CEA/SBT: Rousset, Girard, Diribarne, …
NEEL: Roche, Gibert, Hebral,Rusaouen
LEGI: Baudet, Bourgoin,….
ENS Lyon: Chila, Chevillard, Salort
CEA/SPEC: Dubrulle, Daviaud, Gallet, Moukharski,Braslau
Luth: Lehner

SHREK – FeliSia campaign: SuperFluid Helium Turbulence near intervortex scale

SHREK Collaboration + SN, Golov, Lvov

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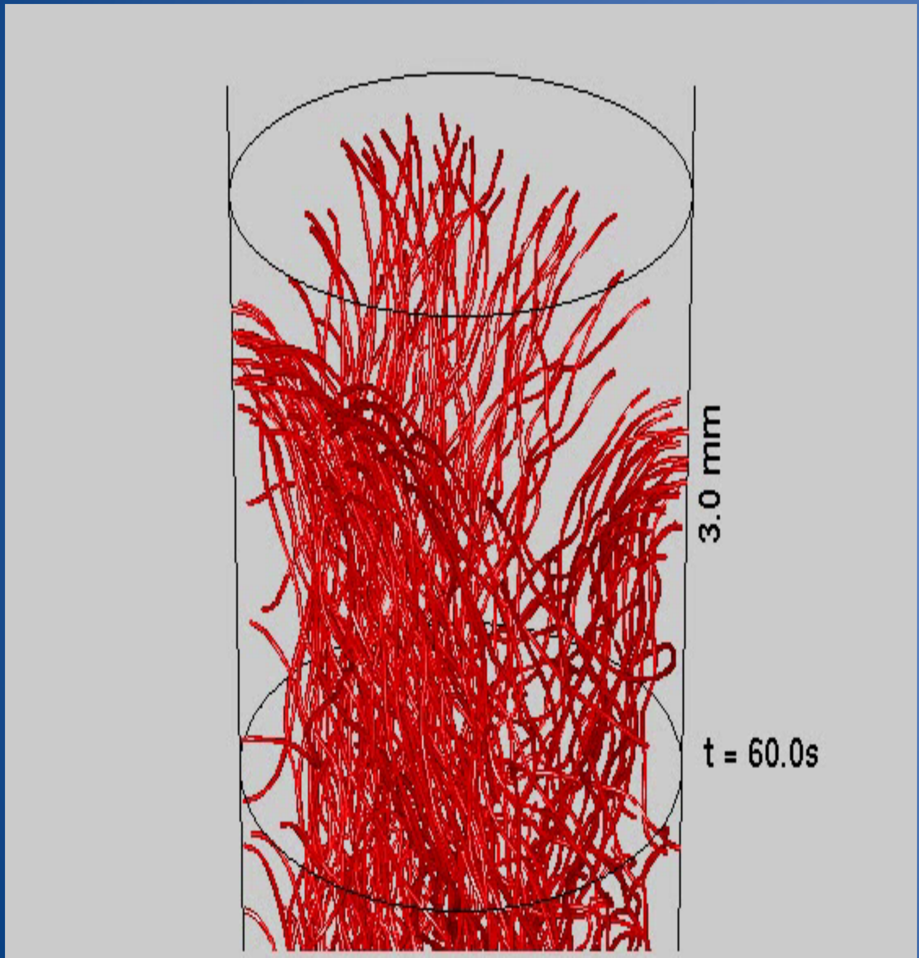
Superfluid Turbulence at T=0



Vortex tangle described by Biot-Savart equation

$$\frac{d\mathbf{r}}{dt} = \frac{\kappa}{4\pi} \int \frac{d\mathbf{s} \times (\mathbf{r} - \mathbf{s})}{|\mathbf{r} - \mathbf{s}|^3}$$

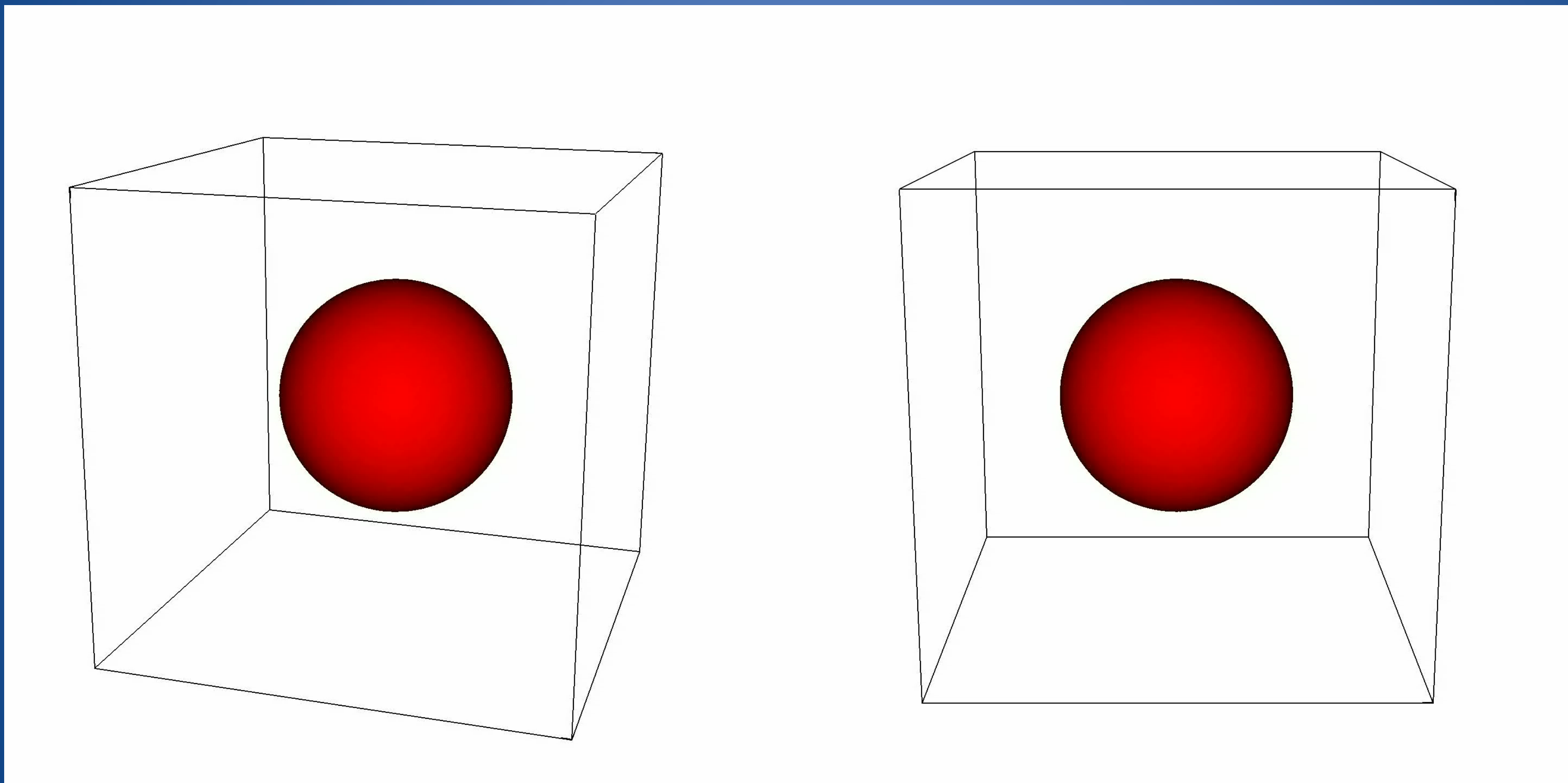
Circulation quantum $\kappa = h/m$.

Rotating turbulence: Hänninen et al (2007)

Numerical method of Schwarz (1985)

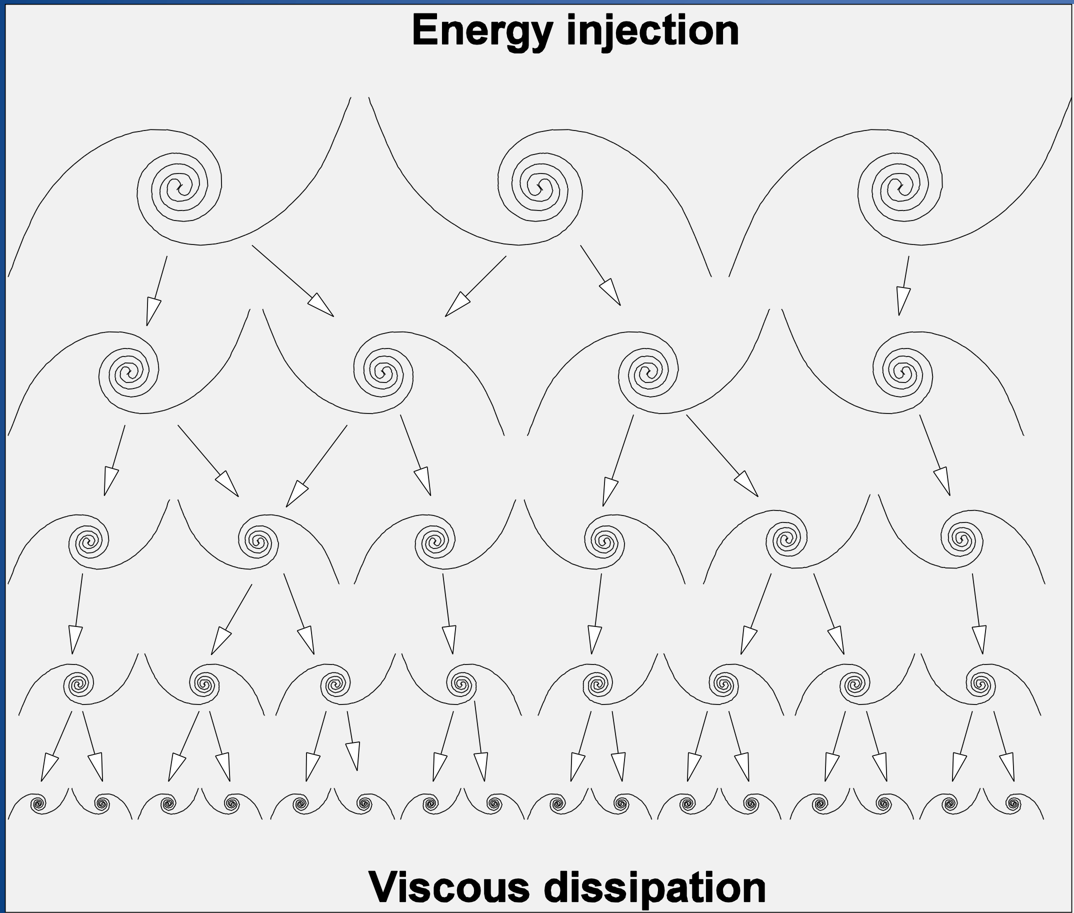
Superfluid turbulence

Gross-Pitaevskii Equation $$i\frac{\partial\psi}{\partial t}+\nabla^2\psi+|\psi|^2\psi=0$$



Formation and decay of vortex tangle in GP model (Proment et al, 2015)

Classical turbulence at scales > ℓ



Richardson cascade

Kolmogorov spectrum

$E(k) = C\,\varepsilon^{2/3}\,k^{-5/3}$

- Same picture for ST at scales larger than inter-vortex separation ℓ ?
- There is no viscosity in ST. What happens when cascade reaches scale ℓ ?

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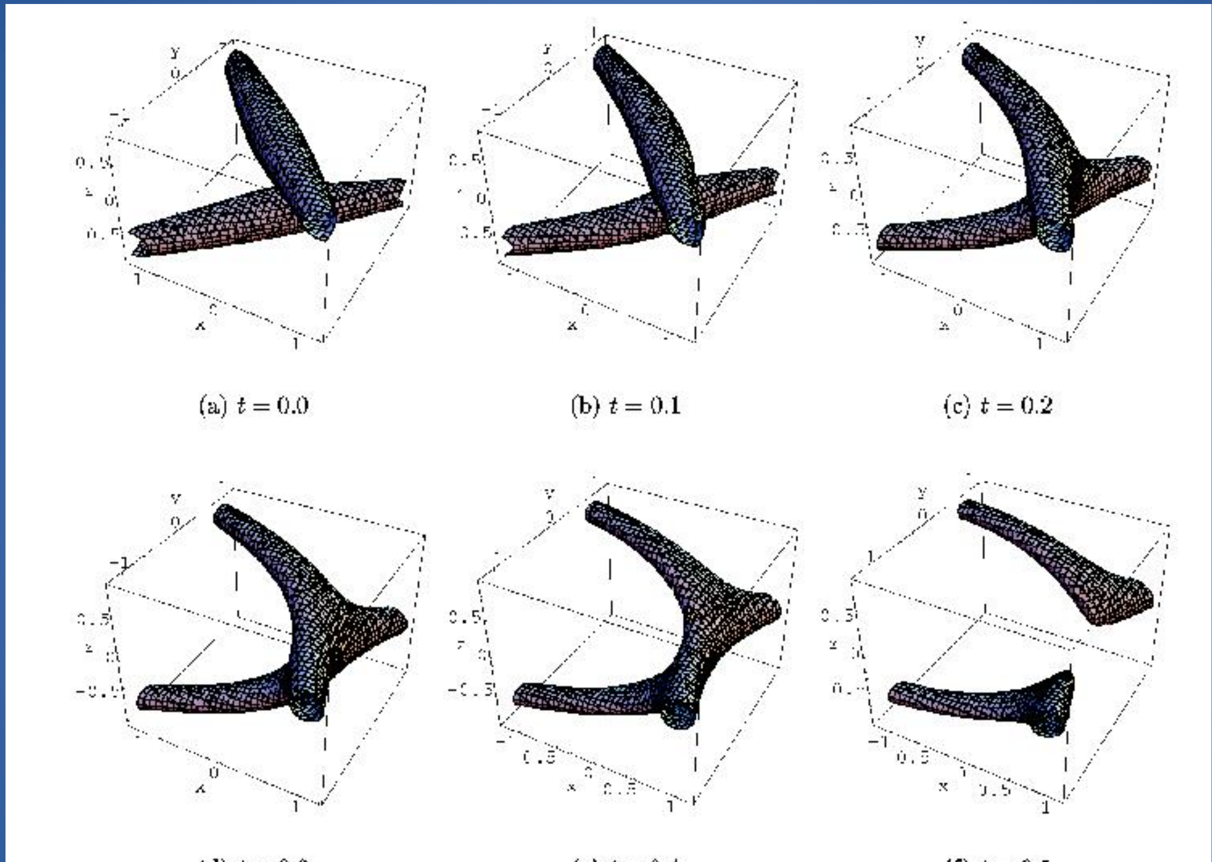
At scale ℓ : reconnections.

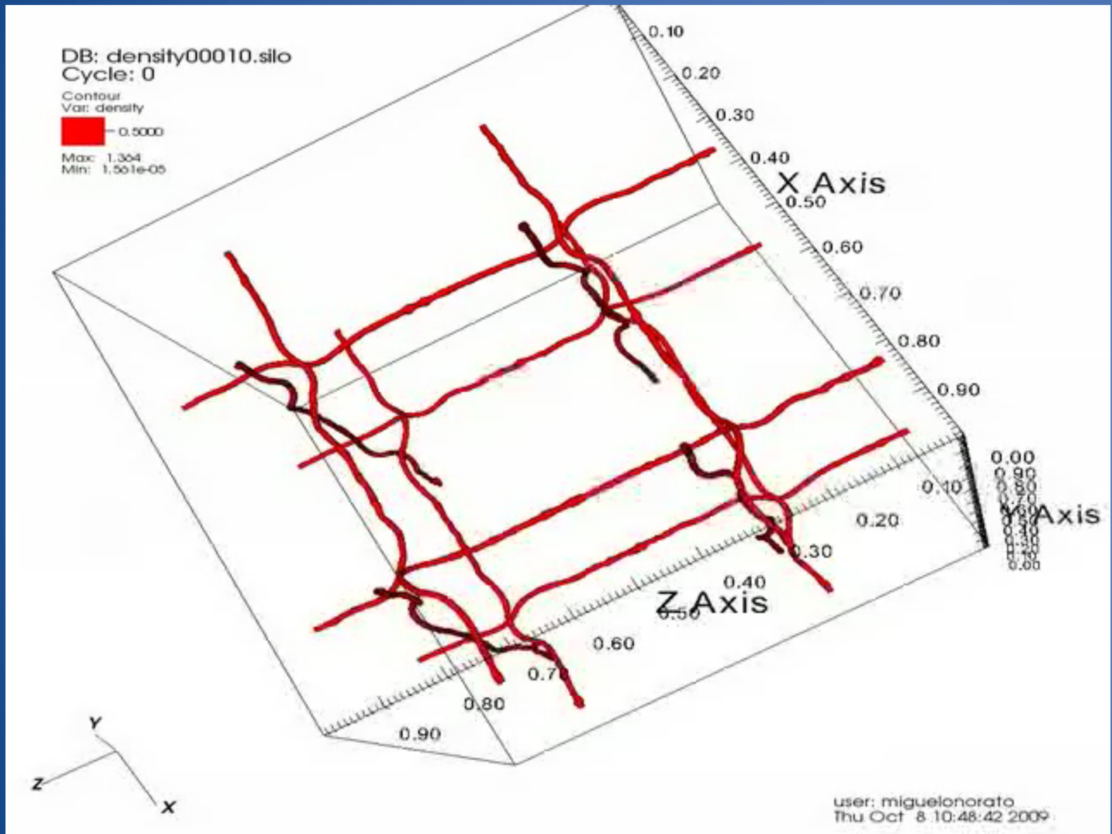


(a) $t = 0.0$ (b) $t = 0.1$ (c) $t = 0.2$

(d) $t = 0.3$ (e) $t = 0.4$ (f) $t = 0.5$

Numerics of GP: Koplik & Levine (1993).
Analytics of GP: SN & West (2003).
Helium experiment: Lathrope et al (2008).

Below ℓ : Kelvin Waves



GPE simulation, Proment et al (2010)

- Waves on vortex lines
- Nonlinear wave-wave interactions transfer energy to smaller scales < ℓ.

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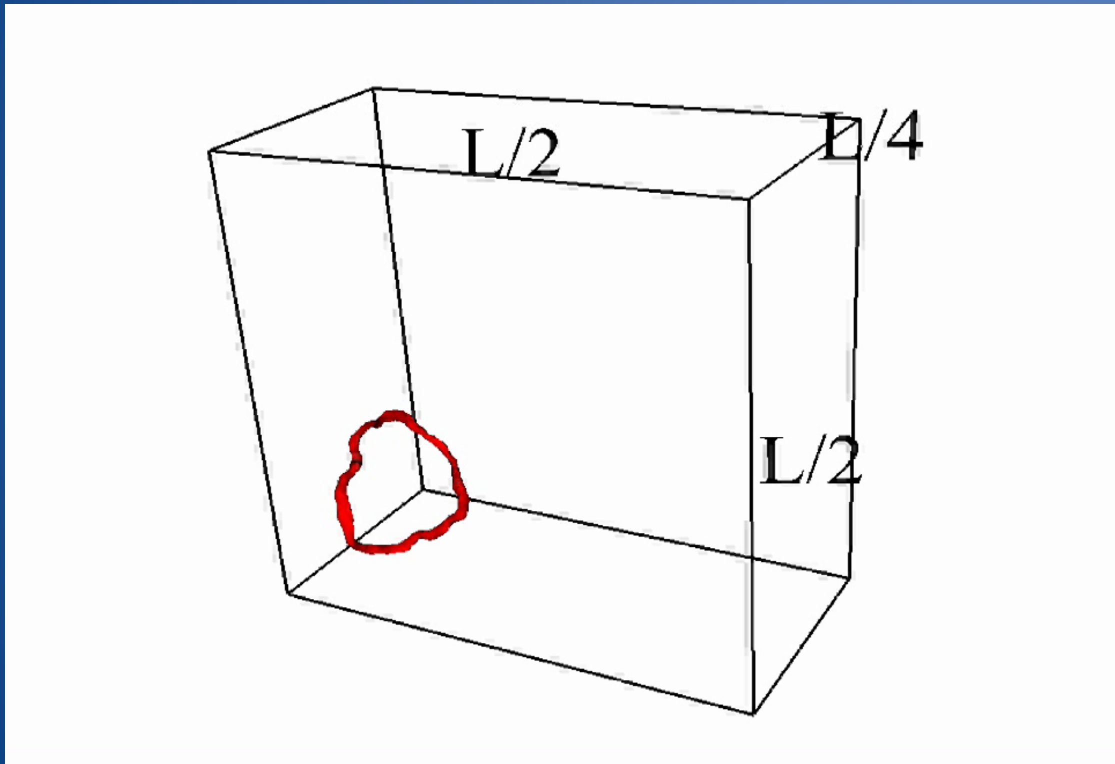
Below ℓ : Kelvin Waves



GPE simulation, Proment et al (2010)

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Kelvin Wave Turbulence

$$\frac{d\mathbf{r}}{dt} = \frac{\kappa}{4\pi}\int \frac{d\mathbf{s}\times(\mathbf{r}-\mathbf{s})}{|\mathbf{r}-\mathbf{s}|^3}$$ – Biot-Savart

⬇ **k-space**

$$\frac{da_k}{dt} = \frac{\delta H}{\delta a_k};$$

$$H = \int \omega_k a_k^* a_k d\vec{k} + \int d\vec{k}_1 \ldots d\vec{k}_6 \delta_{1,2,3}^{4,5,6} W_{1,2,3}^{4,5,6} a_1 a_2 a_3 a_4^* a_5^* a_6^*$$

Hamiltonian description of weakly nonlinear KW's: Kozik & Svistunov (2004);

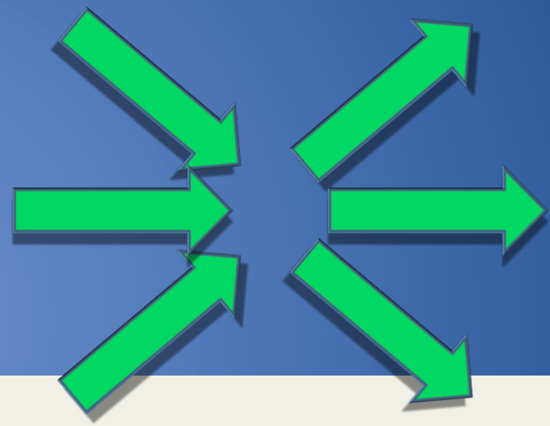
Correct expression for W: Laurie, Lvov, SN, Rudenko (2009)

Statistical description of KW turbulence

Wave Turbulence approach: Peierls (1929), Litvak (1959), Zakharov (1965) …

Wave spectrum:

$$\langle a(\vec{k},t)\, a^*(\vec{k}',t)\rangle = n_k(t)\,\delta(\vec{k}-\vec{k}')$$



$$\frac{dn_k}{dt} = \int d\vec{k}_1 \ldots d\vec{k}_5\, |W^{4,5,6}_{k,1,3}|^2\, \delta^{4,5,6}_{k,1,3}\, \delta(\omega_k + \omega_1 + \omega_2 - \omega_3 - \omega_4 - \omega_5)$$
$$\times n_k n_1 n_2 n_3 n_4 n_5 \left(n_1^{-1} + n_2^{-1} + n_3^{-1} - n_4^{-1} - n_5^{-1} - n_6^{-1}\right)$$

– Kinetic Equation for interacting KW sextets: Kozik & Svistunov (2004)

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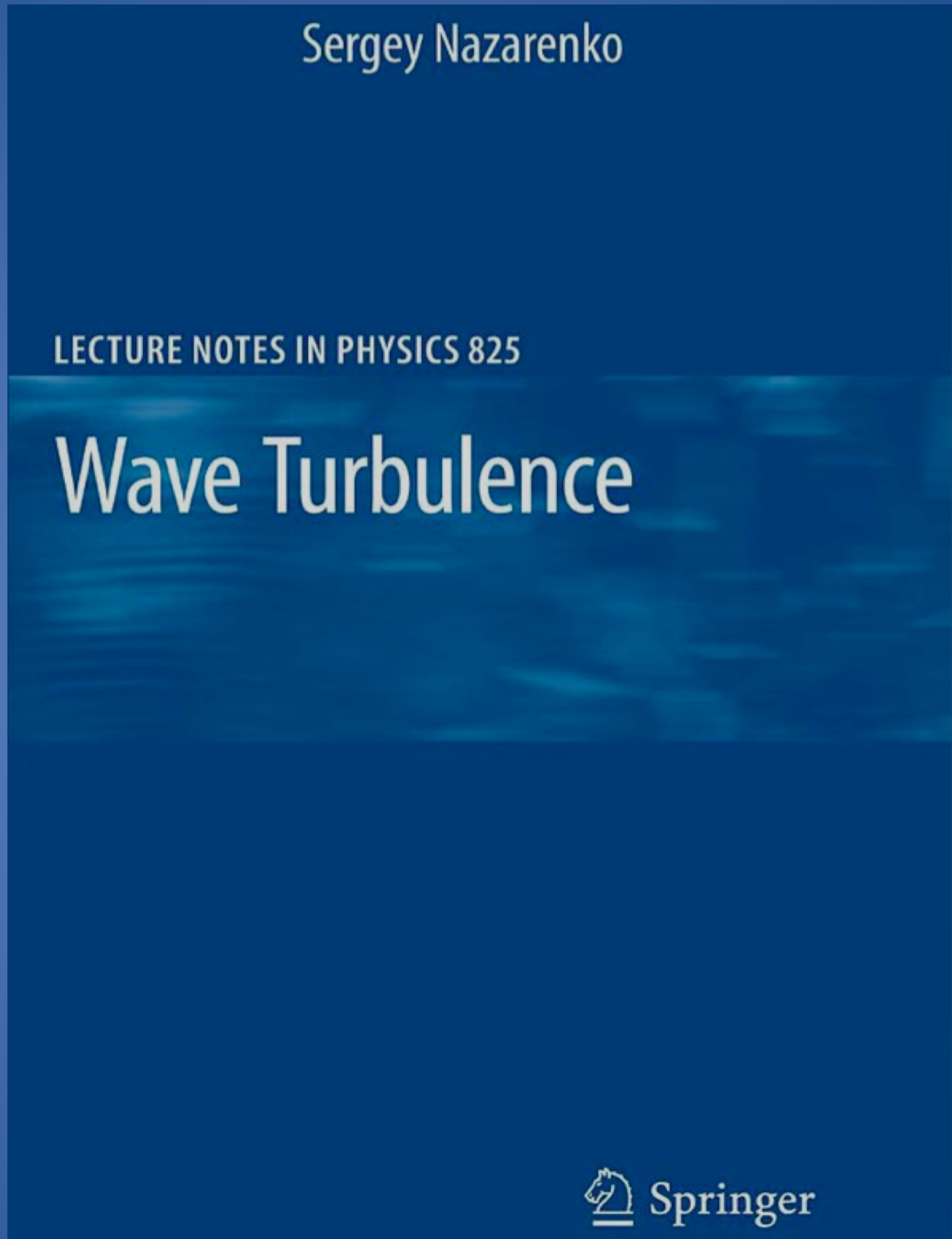

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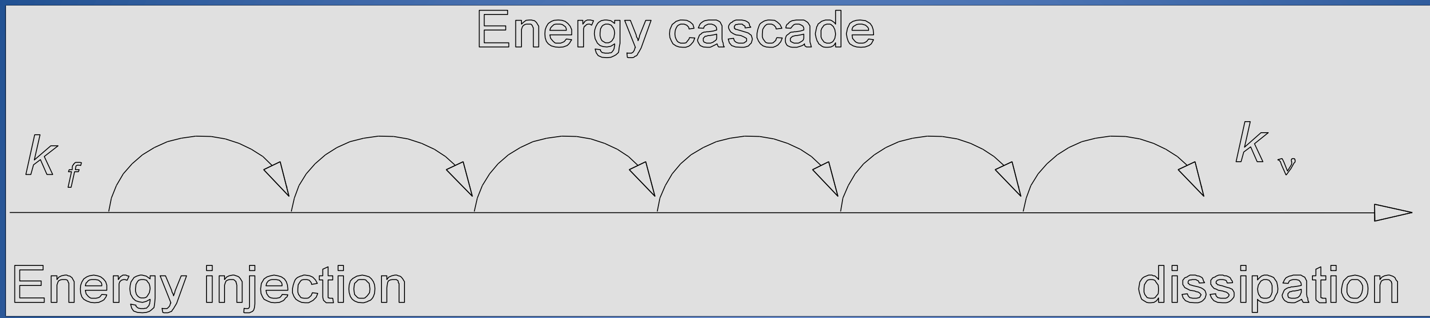
Wave energy cascade



Reconnections & crossover

Phonon radiation

Kolmogorov-Zakharov spectrum; Zakharov (1965).

For KW's – Kozik-Svistunov spectrum (2004):

$$E_k = C_{kw}\Lambda\left(\kappa^7\epsilon/\ell^8\right)^{1/5} k^{-7/5}; \qquad \Lambda = \ln\frac{\ell}{a}$$

Matching the classical and quantum ranges

$$E_k = C_{hd} \epsilon^{2/3} k^{-5/3} \qquad at\ scales > \ell,$$

$$E_k = C_{kw} \Lambda \left(\kappa^7 \epsilon / \ell^8 \right)^{1/5} k^{-7/5} \qquad at\ scales < \ell.$$

•Effective viscosity measured by turbulence decay rate. Stalp *et al* (2000).

• Assume that K41 extends down to ℓ. Vicinity of ℓ contains most vorticity (hence vortex line density).

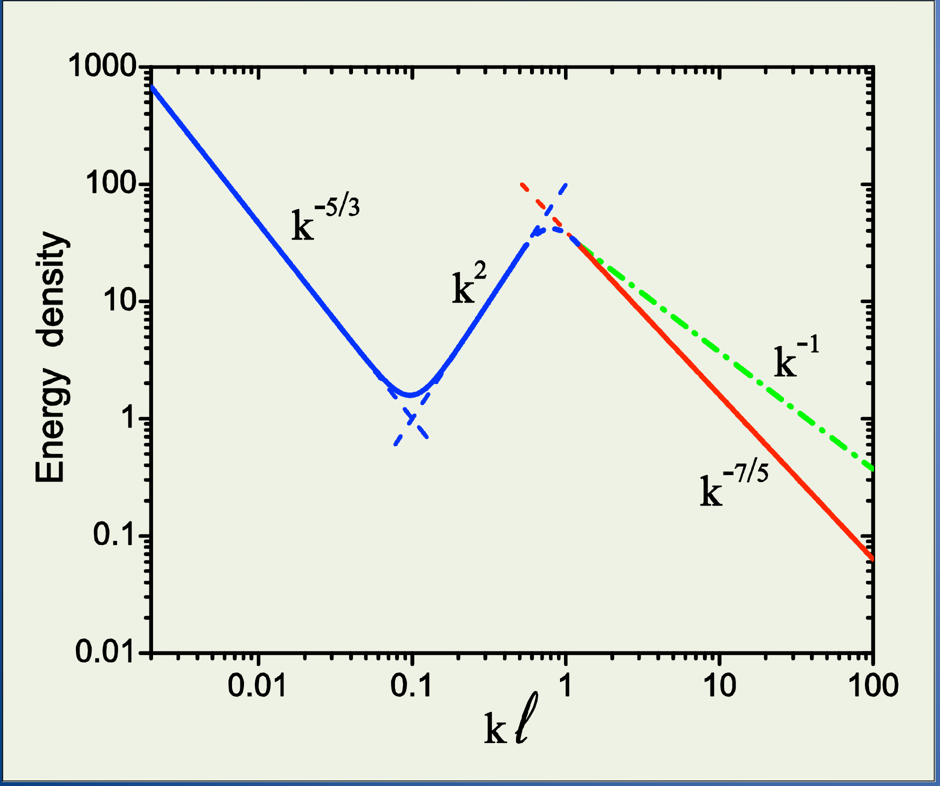
•Turbulence decays like classical with effective viscosity:

$$\nu' \sim \frac{\epsilon \ell^4}{\kappa^2} \sim \kappa$$

Bottleneck crossover at ℓ

$$E_k = C_{hd}\epsilon^{2/3}k^{-5/3} \qquad \Leftarrow \quad \neq at\ \ell \quad \Rightarrow$$

$$E_k = C_{kw}\Lambda\left(\kappa^7\epsilon/\ell^8\right)^{1/5} k^{-7/5}$$



- Kolmogorov and KW spectrum cannot be joined continuously at $1/\ell$.
- Wave turbulence is less efficient than strong hydro turbulence and cannot cope with Kolmogorov cascade → *bottleneck*. Lvov et al (2007).
- "Warm cascade" solution. Connaughton & SN (2004):

$$E_k = k^2[2.2\,\epsilon k^{-11/2} + T^{3/2}]^{2/3}$$

$$\nu' = \Lambda^{-5}\kappa$$ Reduced effective viscosity

Effective viscosity in experiment

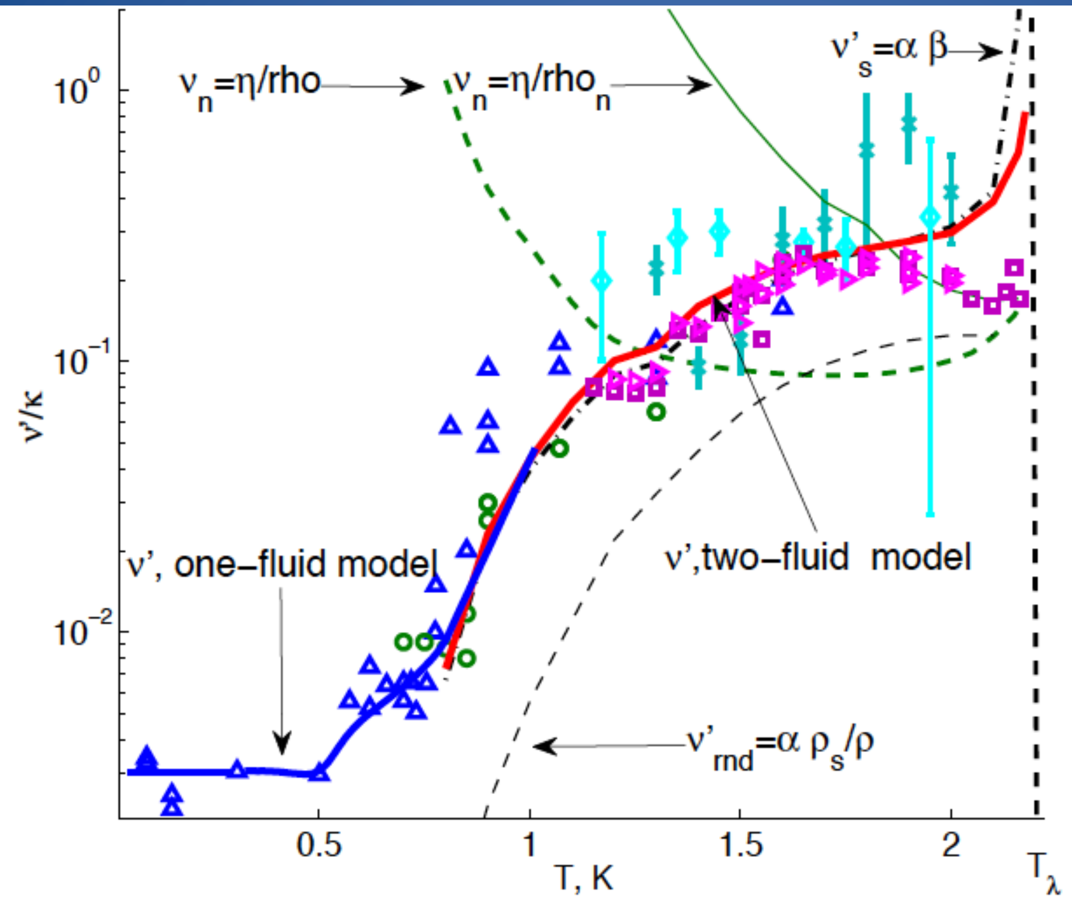


FIG. 5: Color online. Comparison of the experimental, numerical and analytical results for the temperature dependence of the effective kinematic viscosities: Blue triangles – Manchester spin-down experiments[19]; Green empty circles – Manchester ion-jet experiments[20]; sea-green diamonds with error-bars – Prague counterflow experiments[15]; cian crosses with error-bars – Prague decay in grid co-flow experiments[16]; Magenta empty squares – Oregon towed grid experiments[26];Pink right triangles–Oregon towered grid experiments[27]. Solid green line – experimental results[66] for the normal-fluid kinematic viscosity $\nu_n = \mu/\rho_n$ (normalized by the normal-fluid density); Dashed green line – He-II kinematic viscosity $\nu \equiv \mu/\rho$, (normalized by the total density) – see also Tab. I. Thin black dash line – effective viscosity for the random vortex tangle ν'_{rnd}, estimated by Eq. (31); Thick dot-dashed black line – the Vinen-Niemela estimate[10] of the effective superfluid viscosity, ν'_s, given by Eq. (32). Blue solid line – $\nu'(T)$ at $T < 1.1$ K from numerical solution of Eqs. (51b) for the one-fluid differential model of gradual eddy-wave crossover; Red solid line – $\nu'(T)$ at $T > 0.9$ K from numerical simulations in Sec. III B of gradually damped two-fluid HVBK Eqs. (1) in the Sabra shell-model approximation (33).

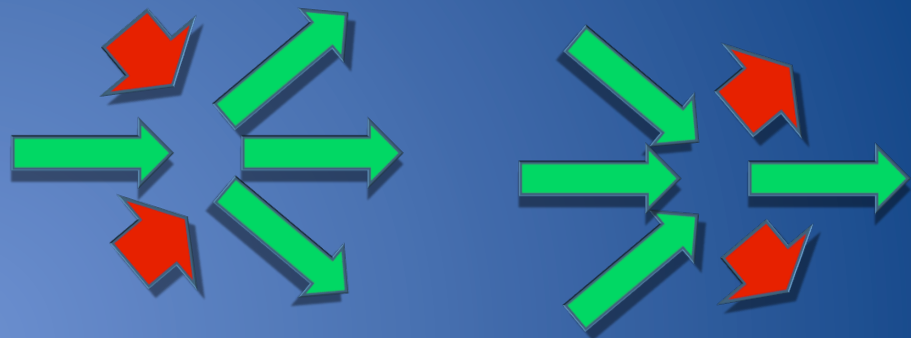
Nonlocality of KS. New 4-wave theory

KZ spectra *only valid if local* (i.e. RHS integral in KE converges).

KS spectrum is nonlocal: Laurie, Lvov, SN & Rudenko (2009).

Two k's in sextet is << other four k's .

→ Effective 4-wave interaction on vortex line with large-scale (random) curvature:



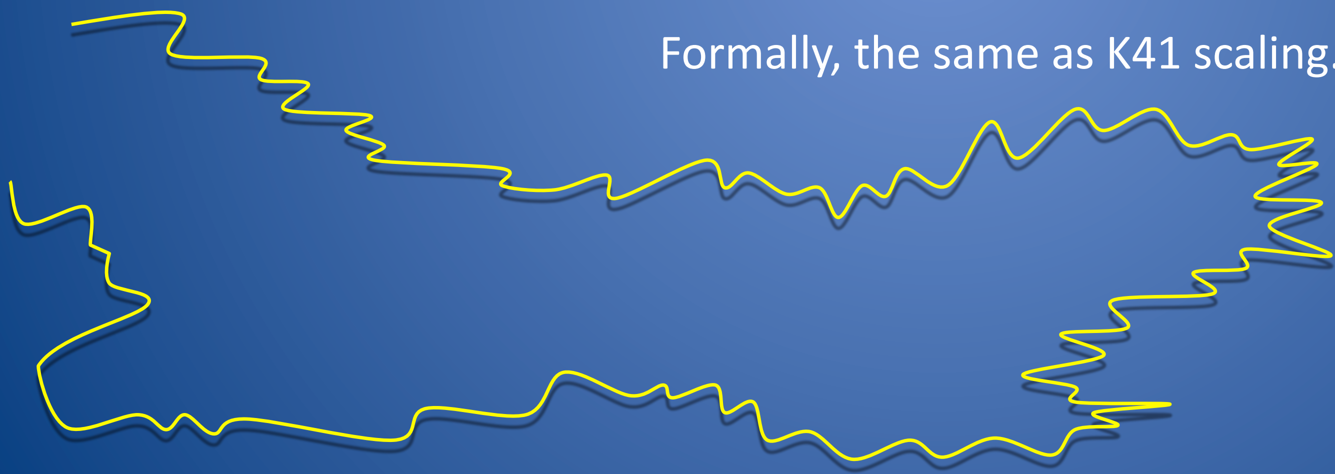
New Spectrum of weak KW turbulence

Local spectrum of the 4-wave theory. Lvov & Nazarenko (2009):

$$E_k = C\,\frac{\Lambda \kappa\, \epsilon^{1/3}}{\Psi^{2/3}}\, k^{-5/3},$$

where Ψ is mean large-scale angle.

Formally, the same as K41 scaling.



Interacting wave quartets on a randomly curved vortex line

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Bio-Savart simulations of KW turbulence

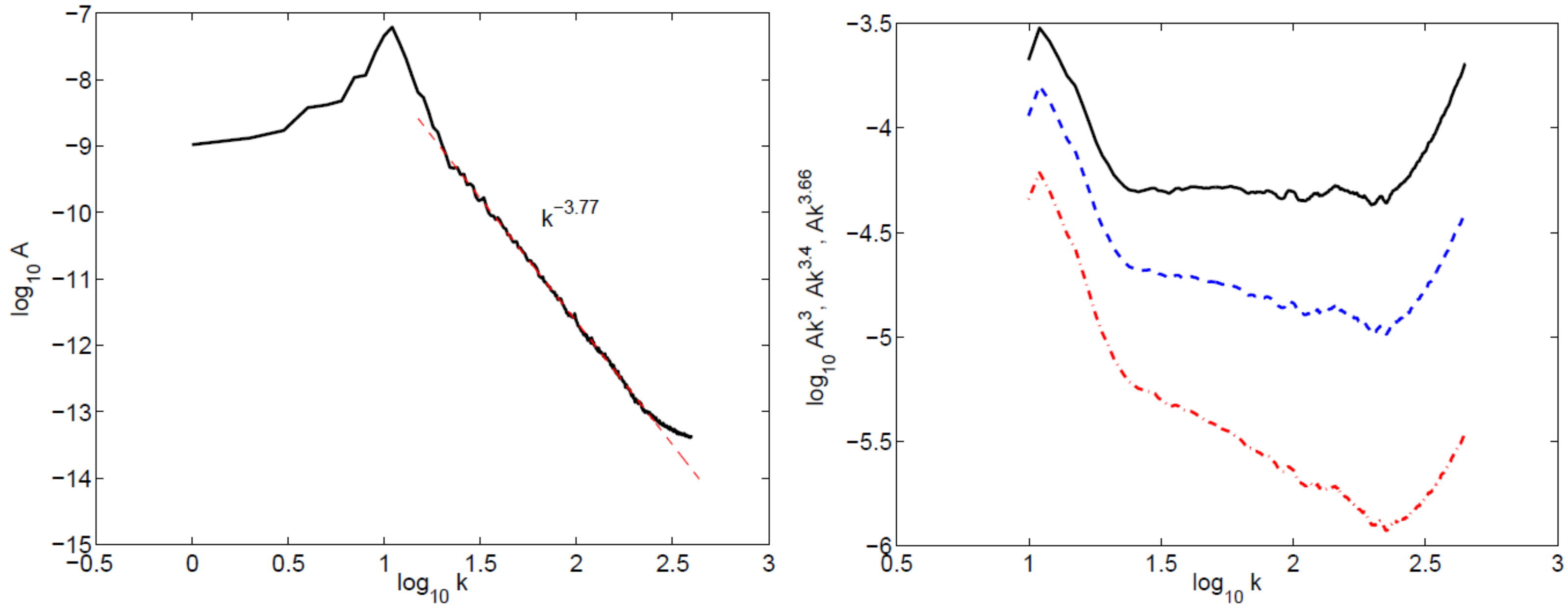

FIG. 2: (left) Amplitude spectrum A (arbitrary units) vs wavenumber k (cm^{-1}). The dashed line is the computed line of best fit corresponding to $A \sim k^{-3/7}$. (right) The corresponding compensated spectra: $A(k)k^{3.66}$ (solid line), $A(k)k^{3.4}$ (dashed line), $A(k)k^3$ (dot-dashed line). The amplitude of the forcing $\mathcal{F} = 0.1$ cm/s.

A. Baggaley and J. Laurie, 2012

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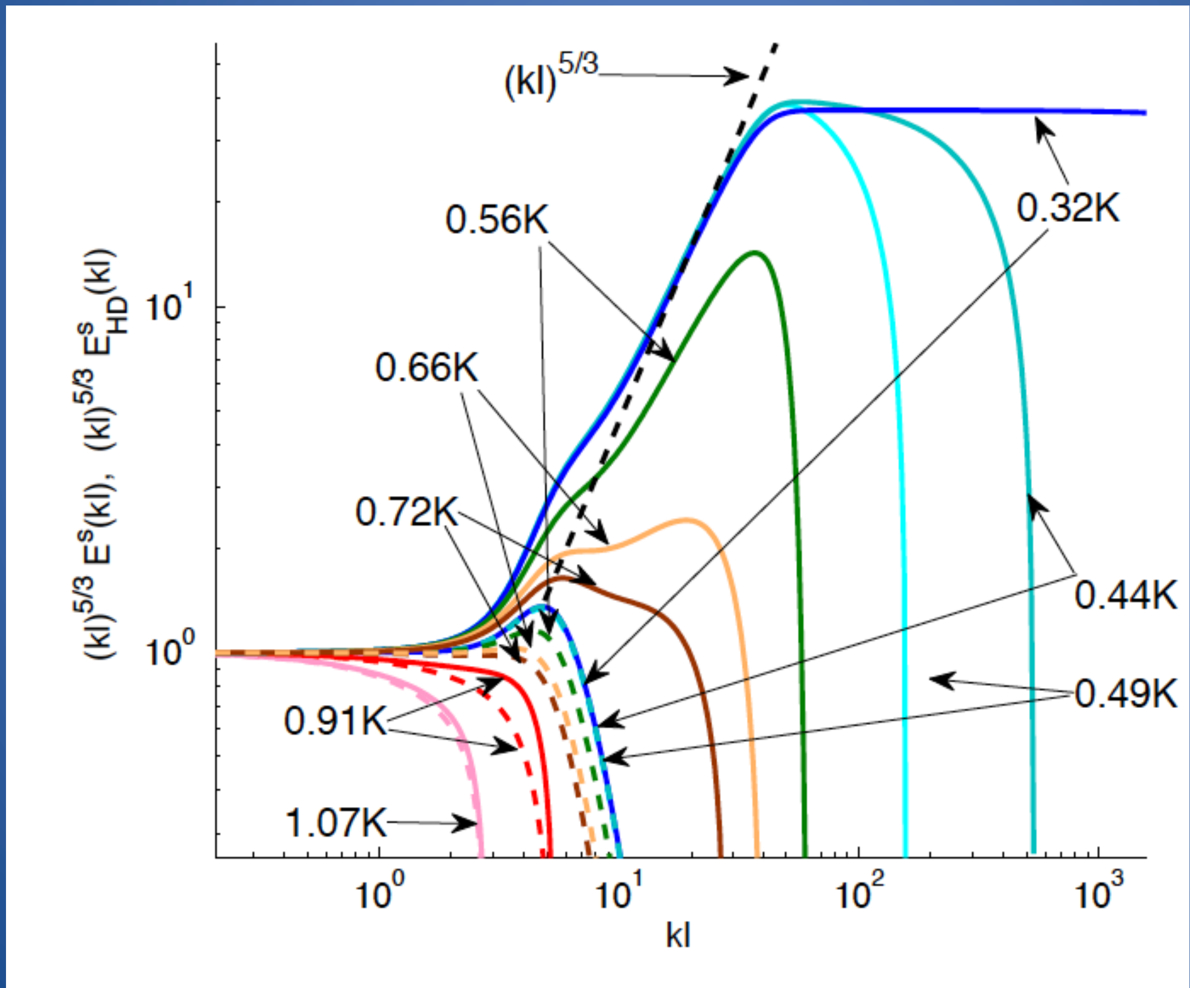
Corrections and extensions of the bottleneck description

- Nonlocality of KW does not change the bottleneck prediction
- KW and HD motions coexist – gradual transition with milder bottleneck (Lvov, Nazarenko, Rudenko, 2008). 1-fluid model, blending function. Equi-partition of the KW energy.
- Finite-T suppression of KW and bottleneck. 1-fluid model with an effective viscosity term.
- Polarization enhancement of the bottleneck effect via suppression of reconnections (e.g. in presence of external rotation).

Finite-T suppression of KW and bottleneck. 1-fluid model with an effective viscosity term (Boue et al 2015)



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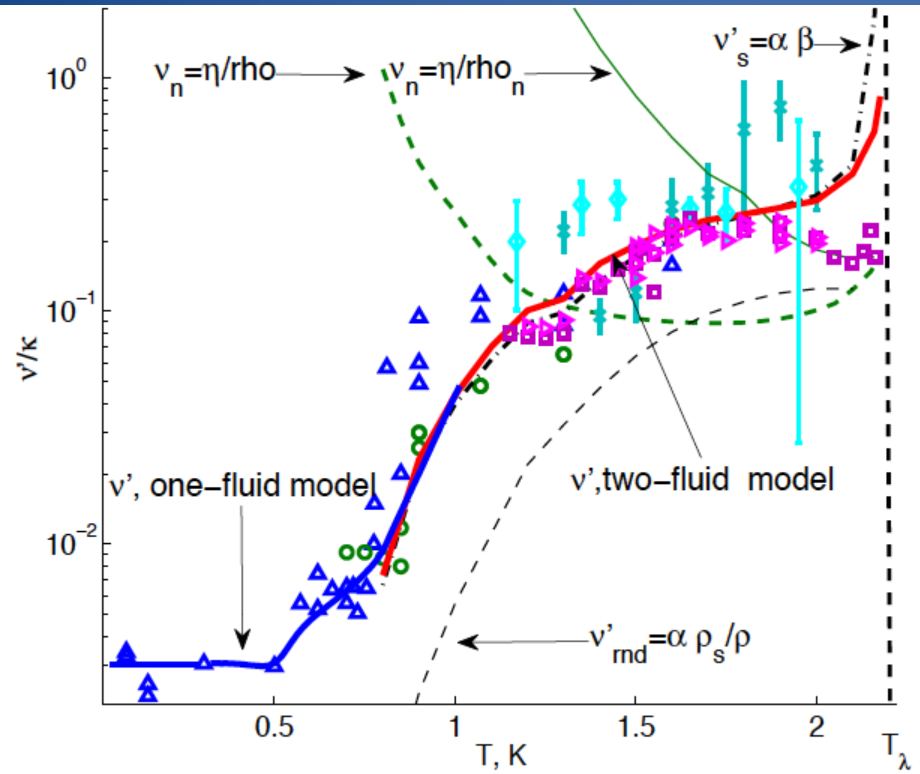
1-fluid and 2-fluid models (Boue et al 2015)



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Reconnections are the main mechanism for dissipation via mutual friction.
Normal and superfluid components get unlocked during fast reconnection dynamics. Thus Vinen-Niemela model (with still normal component) gives good description at high T.

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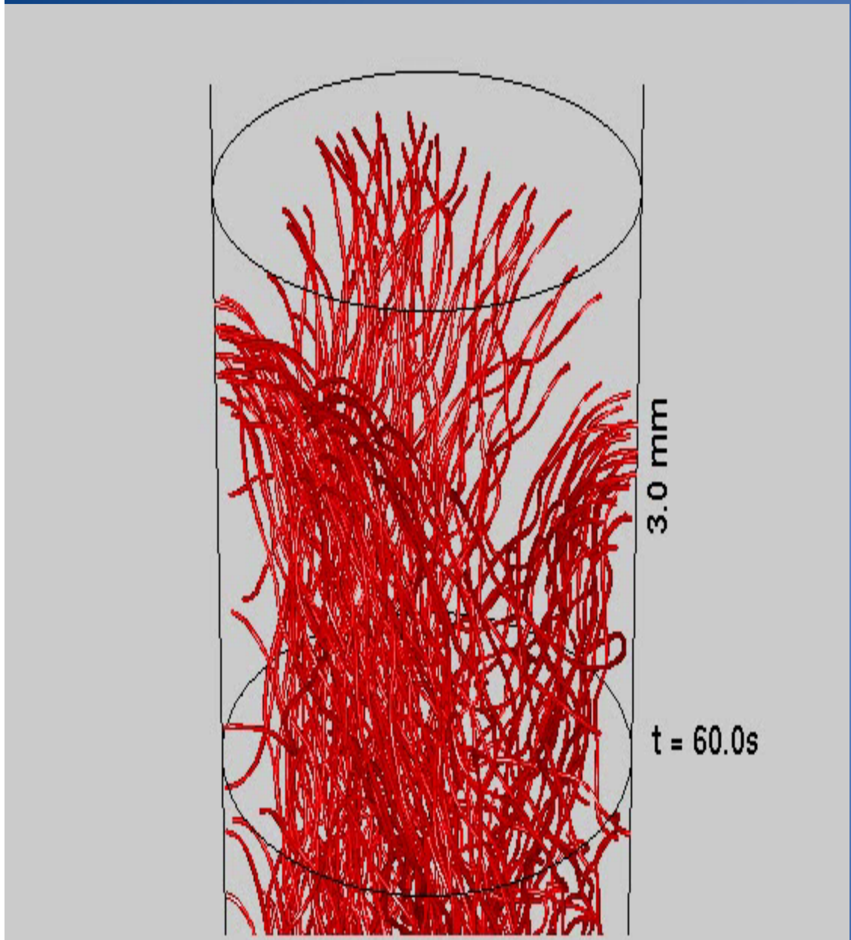
Polarization enhancement of the bottleneck



- Polarization enhancement of the bottleneck effect via suppression of reconnections (e.g. in presence of external rotation).
- Poor man's model: view polarized tangle as a direct sum of an unpolarized and fully polarized components:

$$\frac{\nu'}{\kappa} \simeq (1-P)\,\alpha^{1/3}\left(\frac{\Lambda}{4\pi}\right)^{4/3}$$

- This effect is possibly seen in SHREK, which can access spectra (not just the net vortex line decay).
- December SHREK mission dedicated to study of crossover scales – FeliSia campaigh.

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Summary

• ST is a unique system where gradual transition from the classical to the quantum physics is taking place along the cascade. ST is a rich system: vortices, polarised tangles, bundles, reconnections, waves.

•Asymptotically exact theory is available for small-scale ST for T=0.

•The bottleneck classical-quantum crossover leads to reduction of ν' .

•The gradual crossover theory predicts existence of a range where the spectrum is wave dominated but the flux is eddy dominated.

•Finite T kills KWs and the bottleneck. Polarisation enhances the bottleneck.

•Theory has many (reasonable) assumptions. Accurate numerics needed for turbulence with a wide scale range including those > and <than ℓ.

•Experiment is planned at CEA Grenoble SHREK facility to probe the cross-over region.

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