Interpretation of ... SuperFluid Turbulence in Paris:

Interpretation of Prague decay measurements as Coexistence and interplay of quantum and classical turbulence

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based on two just submitted to arXiv:1504.09XX papers:

1. S. Babuin[†], V.S. L'vov[*] , A. Pomyalov[*], L. Skrbek[†] and E. Vargag[†], Coexistence and interplay of quantum and classical turbulence of superfluid ^{4}He.
2. D. Khomenko[*], V. S. Lvov[*] , A. Pomyalov[*], and I. Procaccia[*] Counterflow decoupling in superfluid turbulence.

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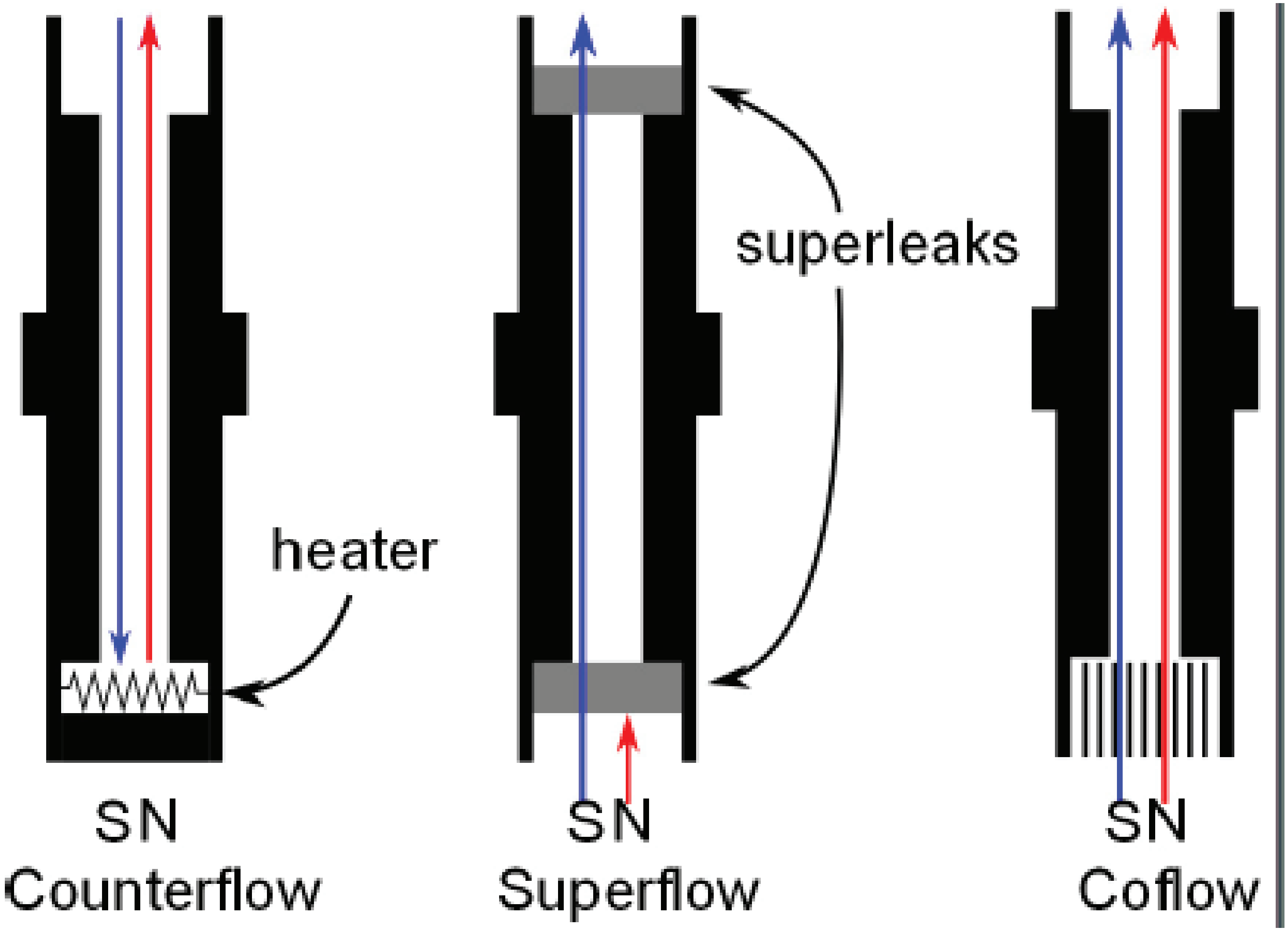
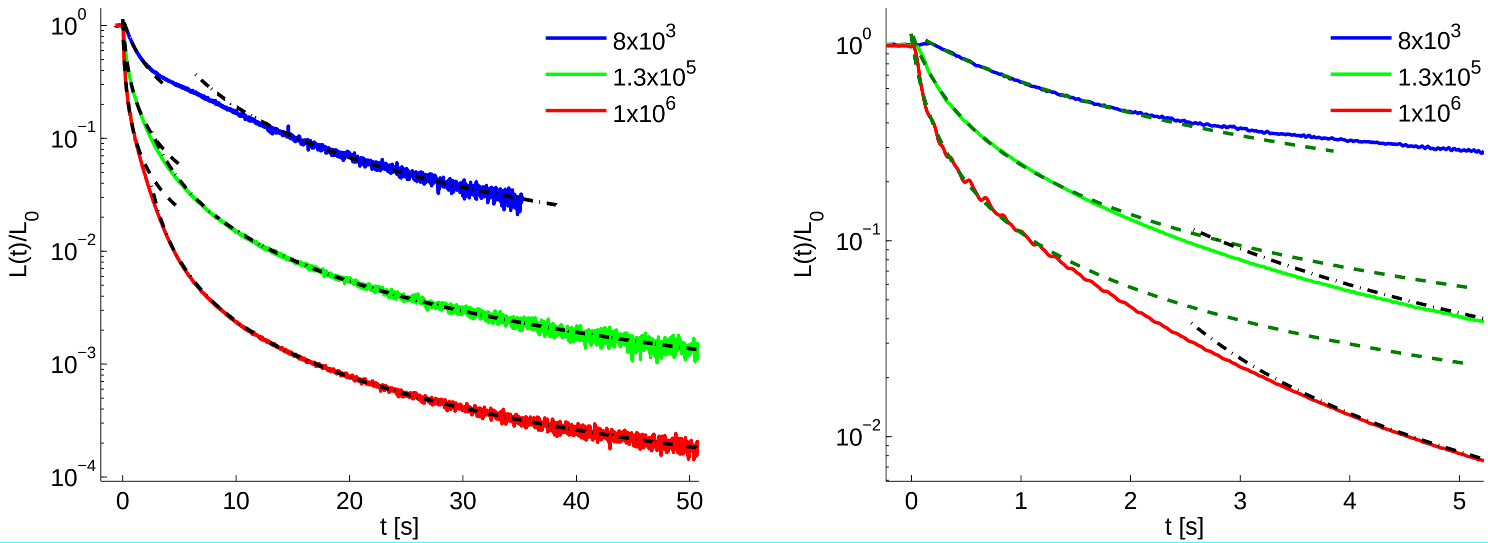


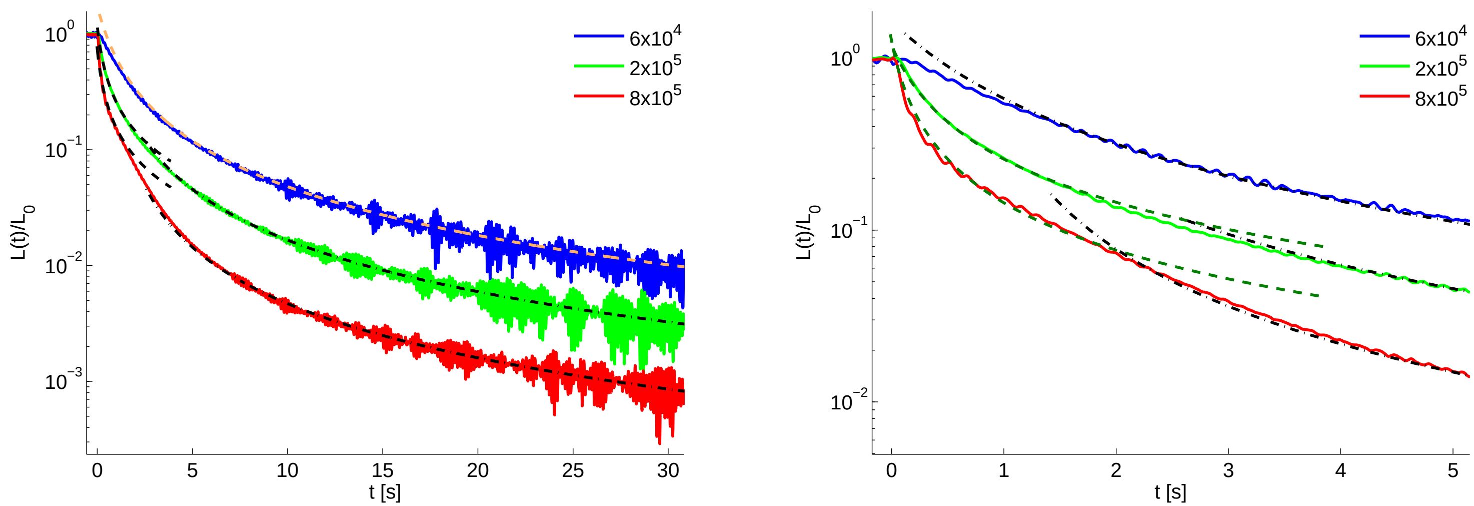
Prague flow channels for the study of counter-flow, super- and co-flow. S and N stand for super-fluid and normal components. Counter-flow is produced thermally by a heater. Super-flow and co-flow are driven mechanically by a bellows.

Prague data for the VLD decay $\mathcal{L}(t)/\mathcal{L}(0)$ in the co-flow, $D = 7\,$mm,

$T = 1.35\,$K



$T = 1.45\,$K.



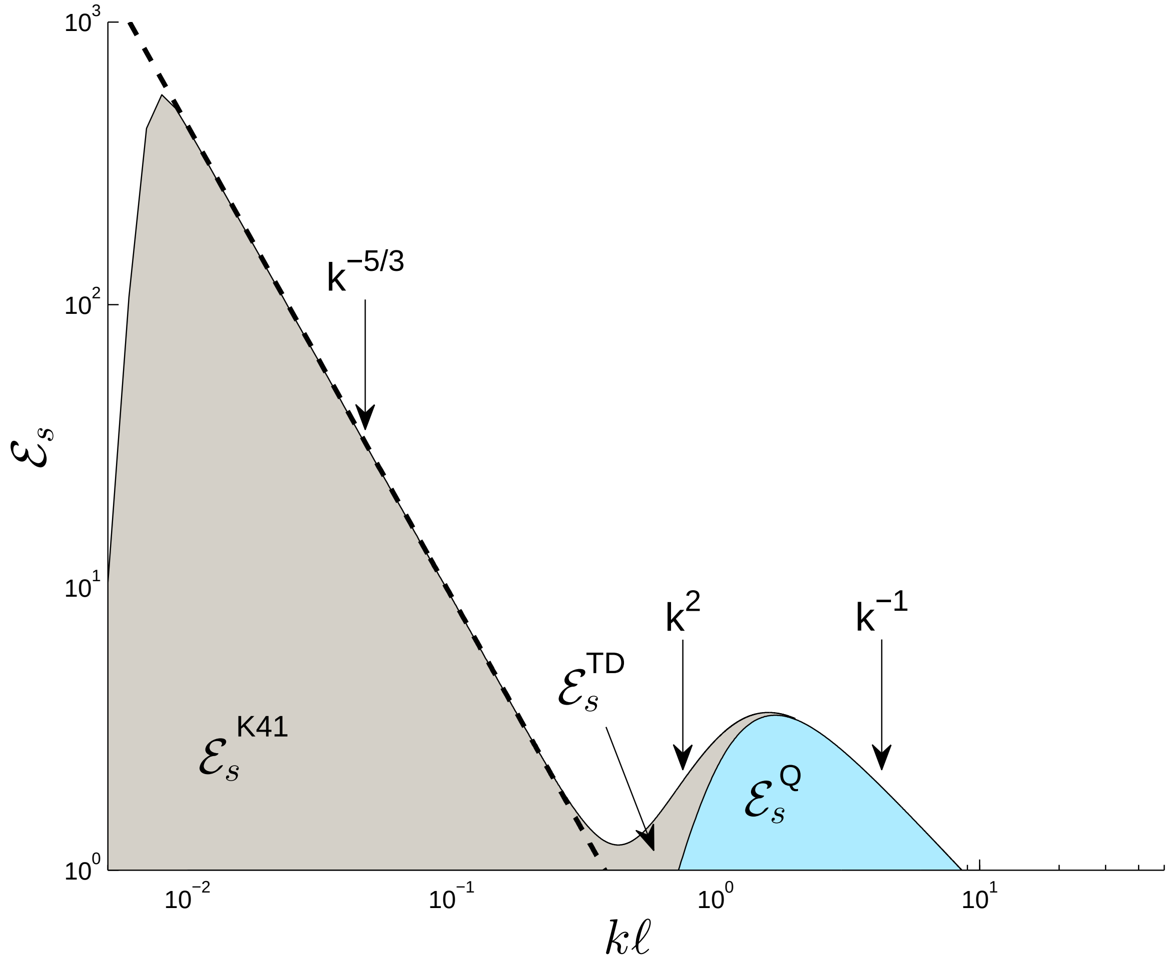
Quantum t^{-1}-fits – green dashed lines, classical $t^{-3/2}$-fits – black dash-dotted lines.

Coexistence of the classical (grey) and quantum (cyan) turbulence in co-flow

Classical energy spectrum consists of cascade part $\mathcal{E}_{\rm s}^{\rm K41}(k) \propto k^{-5/3}$

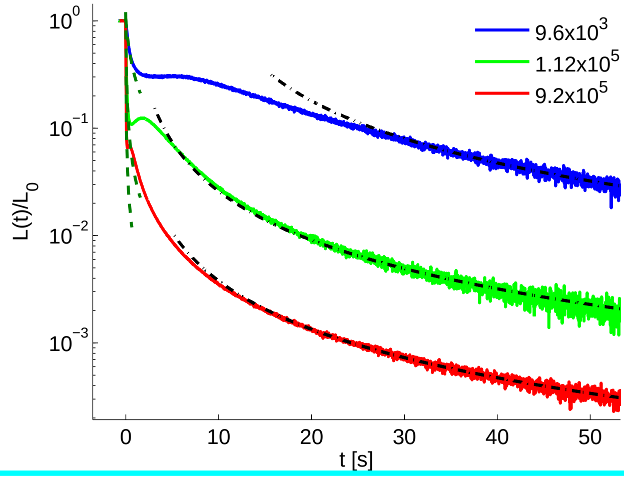
and thermodynamic equilibrium part $\mathcal{E}_{\rm s}^{\rm TD}(k) \propto k^{2}$

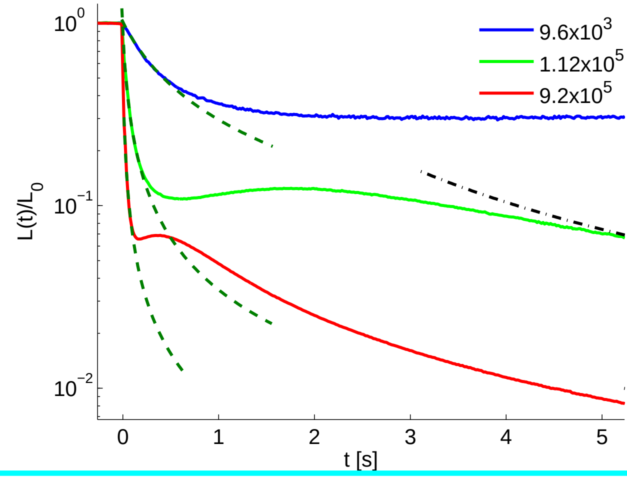
Quantum energy spectrum of random tangle has $1/k$ large k-asymptotics



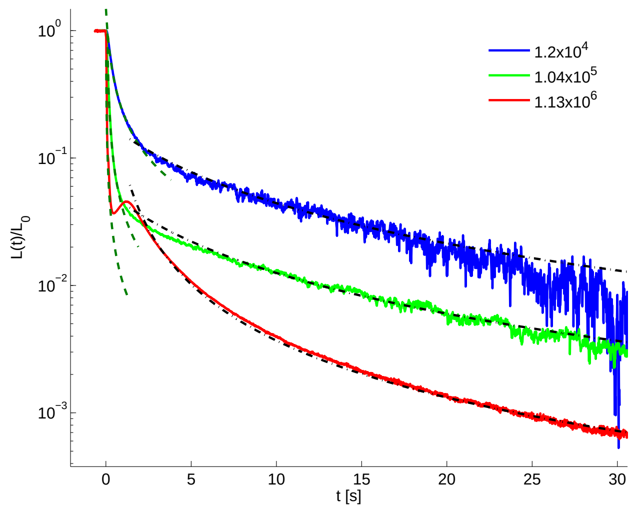
Prague data for the VLD decay $\mathcal{L}(t)/\mathcal{L}(0)$, $T = 1.45\,\mathrm{K}$ $D = 10\,\mathrm{mm}$,

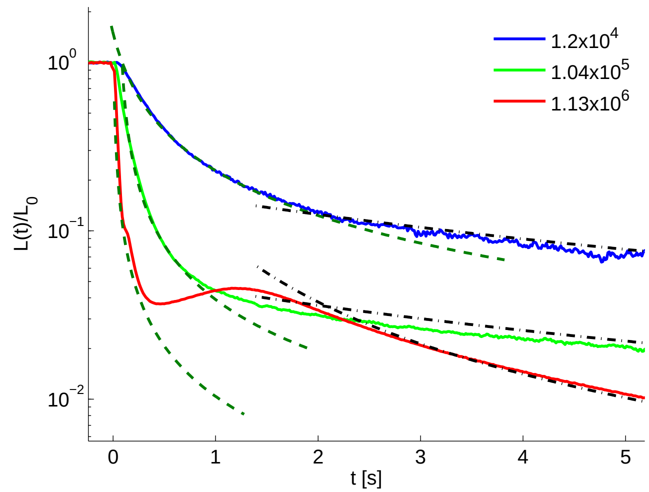
Counter-flow: Quantum t^{-1}-fits – green dashed lines, classical $t^{-3/2}$-fits – black dash-dotted lines.





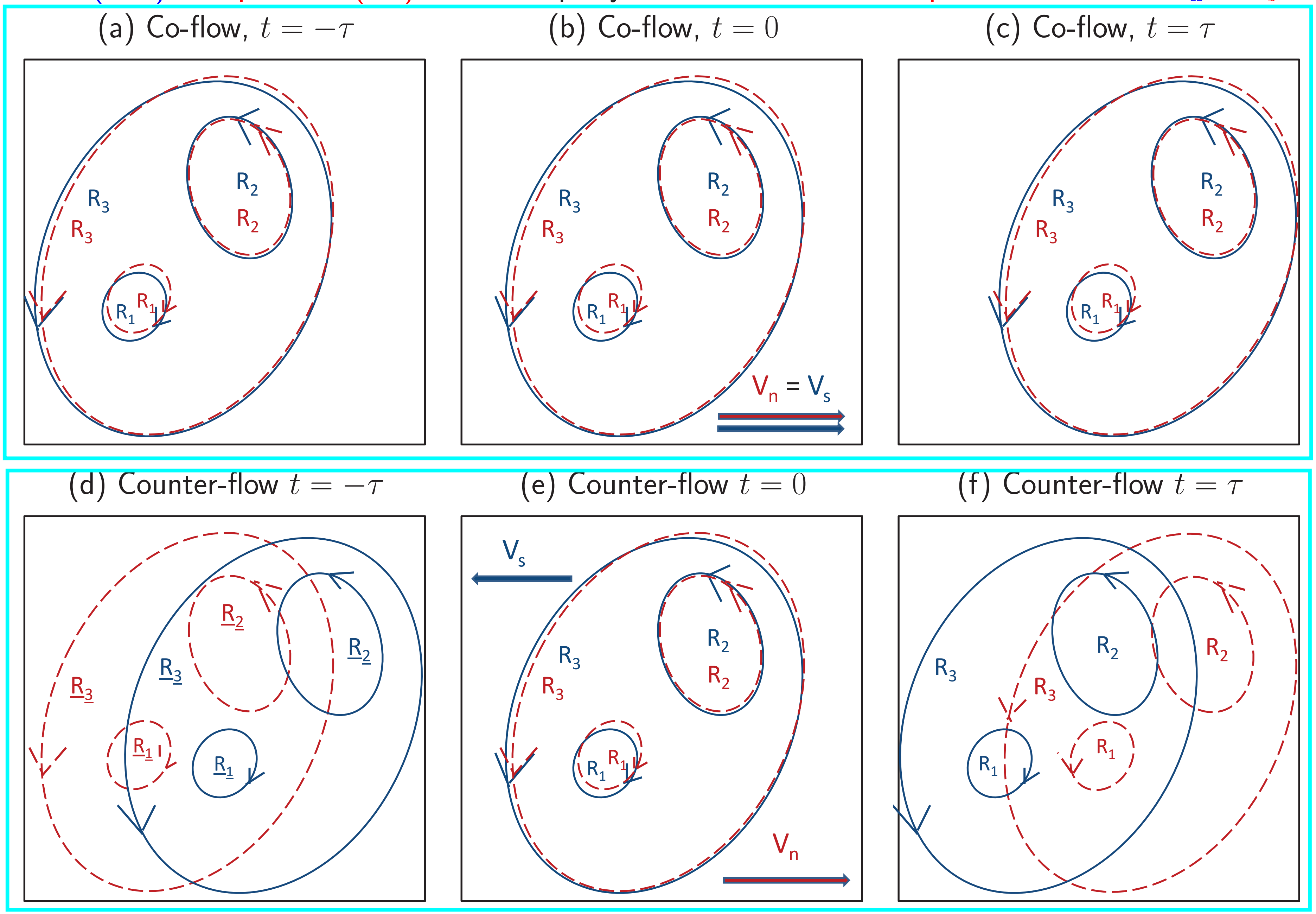
Superflow demonstrates behavior $\Downarrow\Downarrow$ very similar to that of counter-flow $\Uparrow\Uparrow$





A way to understand "bump" is to assume delay in the classical-energy supply of quantum tangle

Normal- (blue) & super-fluid (red) eddies swept by the mean normal- & super-fluid velocities $U_{\rm n}$ & $U_{\rm s}$



Time $\tau \simeq R_2/U_{\rm ns}$ is of the order of overlapping time of the middle-scale R_2-eddies.

Stationary energy spectra of counter- and pure super-flow turbulence

as a consequence of mutual-friction suppression due to

Counterflow decoupling of the normal- and super-fluid velocities

– Interaction (overlapping) time of scale R-eddies: $\boldsymbol{\tau}_{\mathbf{int}} = R/U_{\rm ns}$ ($\boldsymbol{U}_{\mathbf{ns}}$ – Counter-flow velocity)

– Mutual friction coupling time:

kind of the HVBK eqs:

$$\frac{\partial \boldsymbol{u}_{\rm s}}{\partial t} + \ldots \simeq \alpha\kappa\mathcal{L}\left[\boldsymbol{U}_{\rm ns} + \boldsymbol{u}_{\rm n}(\boldsymbol{r},t) - \boldsymbol{u}_{\rm n}(\boldsymbol{r},t)\right], \tag{1}$$

$$\frac{\partial \boldsymbol{u}_{\rm n}}{\partial t} + \ldots \simeq -\alpha\frac{\rho_{\rm s}}{\rho_{\rm n}}\kappa\mathcal{L}\left[\boldsymbol{U}_{\rm ns} + \boldsymbol{u}_{\rm n}(\boldsymbol{r},t) - \boldsymbol{u}_{\rm n}(\boldsymbol{r},t)\right], \tag{2}$$

$$\boldsymbol{u}_{\rm ns} \equiv \boldsymbol{u}_{\rm n} - \boldsymbol{u}_{\rm n}\,,\ \boldsymbol{\Omega}_{\mathbf{mf}} \equiv \frac{\alpha\rho}{\rho_{\rm s}}\kappa\mathcal{L}\,, \qquad \frac{\partial \boldsymbol{u}_{\rm ns}}{\partial t} + \ldots \simeq -\Omega_{\rm mf}\left[\boldsymbol{U}_{\rm ns} + \boldsymbol{u}_{\rm ns}(\boldsymbol{r},t)\right]. \tag{3}$$

$$\tag{4}$$

– Counterflow decoupling parameter $\boldsymbol{\zeta(R) = 1/\tau_{\mathbf{int}}\Omega_{\mathbf{mf}}} \quad \Rightarrow \boldsymbol{\zeta(k) = \frac{k U_{\mathbf{ns}}}{\Omega_{\mathbf{mk}}}}$

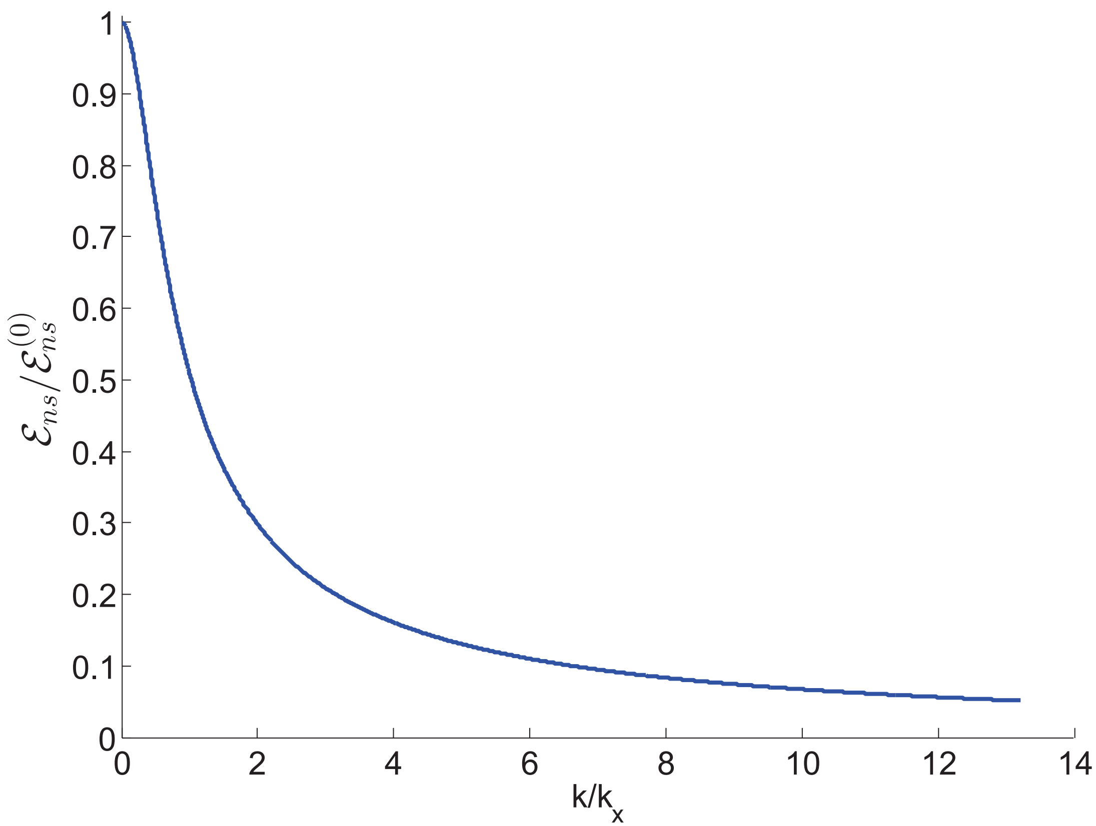
Analytical theory of the coupling-decoupling processes, developed in Ref.[2] results in the equation for the dimensionless decoupling function $\mathcal{D}(k)$, which depends on k via decoupling parameter $\zeta(k)$:

$$\boldsymbol{\mathcal{D}(k) = D[\zeta(k)] \equiv \frac{E_{\mathbf{ns}}(k, U_{\mathbf{ns}})}{E_{\mathbf{ns}}(k, 0)} = \frac{\mathbf{arctan}[\zeta(k)]}{\zeta(k)}}\,. \tag{5}$$

Here $E_{\rm ns}(k, U_{\rm ns}) = \langle \boldsymbol{u}_{\rm s}(\boldsymbol{k}) \cdot \boldsymbol{u}_{\rm n}(\boldsymbol{k})\rangle$ is cross-correlation function of the normal- and superfluid velocities in Fourier $\boldsymbol{k}$-representation.

$$D(\zeta) = 1 - \frac{\zeta^2}{3}\,,\ \text{for } \zeta \ll 1\,, \qquad D(\zeta_\times) = \frac{1}{2}\,,\ \text{for } \zeta_\times \approx 2\,, \quad D(\zeta) = \frac{\pi}{2\,\zeta}\,,\ \text{for } \zeta \gg 1\,. \tag{6}$$

k-dependence of the decoupling function $\boldsymbol{D(k)}$



$\zeta_{\max} = \zeta(k_{\max})$ at the highest value of k, $k_{\max} \simeq \pi/\ell$.
With $\ell \simeq 1/\sqrt{\mathcal{L}} \simeq 1/(\gamma_{\mathcal{L}} U_{\rm ns})$ this gives a simple $U_{\rm ns}$-independent estimate of $\zeta_{\max}$:

$$\boldsymbol{\zeta_{\max} \simeq \frac{\pi}{\alpha_{\rm ns}\,\kappa\,\gamma_{\mathcal{L}}} \sim 40\,,}$$

Estimate:

$$\boldsymbol{D_{\min} = D(\zeta_{\max}) \simeq 0.04\,,}$$
$$\boldsymbol{\frac{k_{\max}}{k_{\times}} \simeq 20\,,}$$

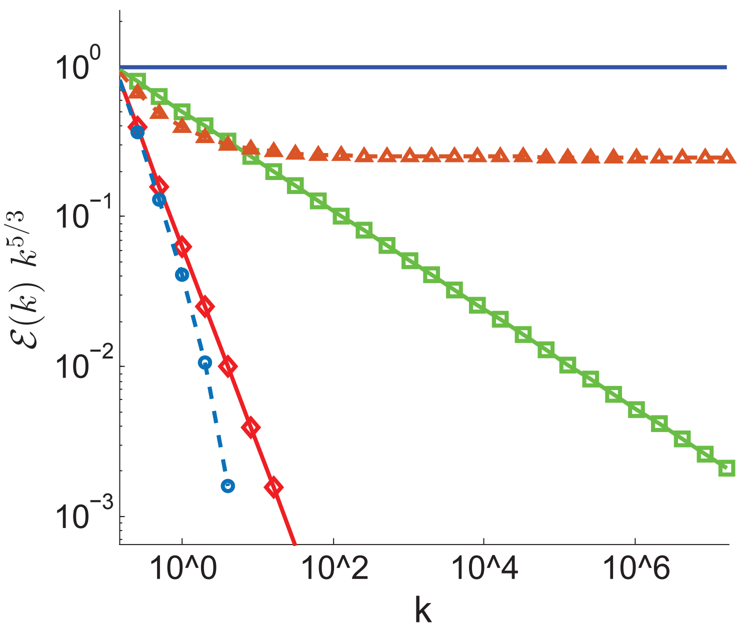
for $T = 1.45\,$K.

$$\frac{\partial \mathcal{E}_{\rm s}(k,t)}{2\,\partial t}+\mathcal{NL}_{\rm s} \;=\; \alpha\kappa\mathcal{L}\big[\mathcal{E}_{\rm ns}(k,t)-\mathcal{E}_{\rm s}(k,t)\big]\simeq -\alpha\kappa\mathcal{L}\mathcal{E}_{\rm s}(k,t)\big[1-D(k)\big]\,, \tag{7a}$$

$$\frac{\partial \mathcal{E}_{\rm n}(k,t)}{2\,\partial t}+\mathcal{NL}_{\rm n} \;=\; \frac{\alpha\kappa\mathcal{L}\rho}{\rho_{\rm s}}\big[\mathcal{E}_{\rm ns}(k,t)-\mathcal{E}_{\rm n}(k,t)\big]\simeq -\frac{\alpha\kappa\mathcal{L}\rho}{\rho_{\rm s}}\big[\mathcal{E}_{\rm n}(k,t)\big[1-D(k)\big]\,. \tag{7b}$$

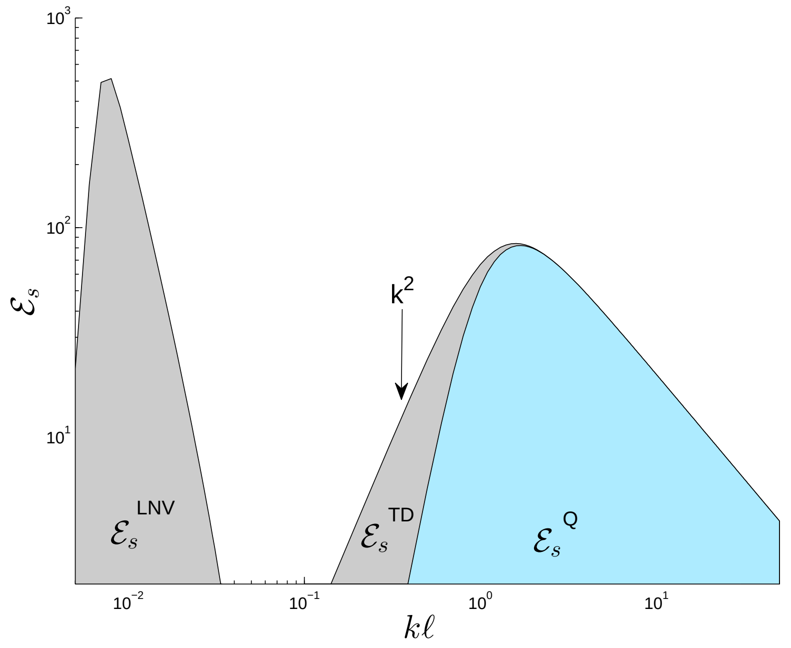
Here $\mathcal{NL}_{\rm s,n}$ are nonlinear terms which we do not specified at this stage of the research. For $k \gg k_\times$, $D(k) \ll 1$ and situation become similar to the equation for $\mathcal{E}_{\rm s}$ for the superfluid turbulence in ^{3}He, where mutual friction drastically suppresses the energy spectrum $\mathcal{E}_{\rm s}(k)$. Instead of classical Kolmogorov spectrum $\mathcal{E}(k) \propto k^{-5/3}$ Lvov, Nazarenko and Volovik (LNV) (JETP Letters, 2004) found the spectra

$$\boldsymbol{\mathcal{E}_{\rm s}(k) \propto \frac{1}{k^{5/3}\Big[\frac{1}{k^{2/3}} \pm \frac{1}{k_{\rm cr}^{2/3}}\Big]^2}}\,, \quad \text{critical: } \propto k^{-3},\ k_{\rm cr}\to\infty, \text{ subcrititical (with } -\text{), supercritical } (+).$$

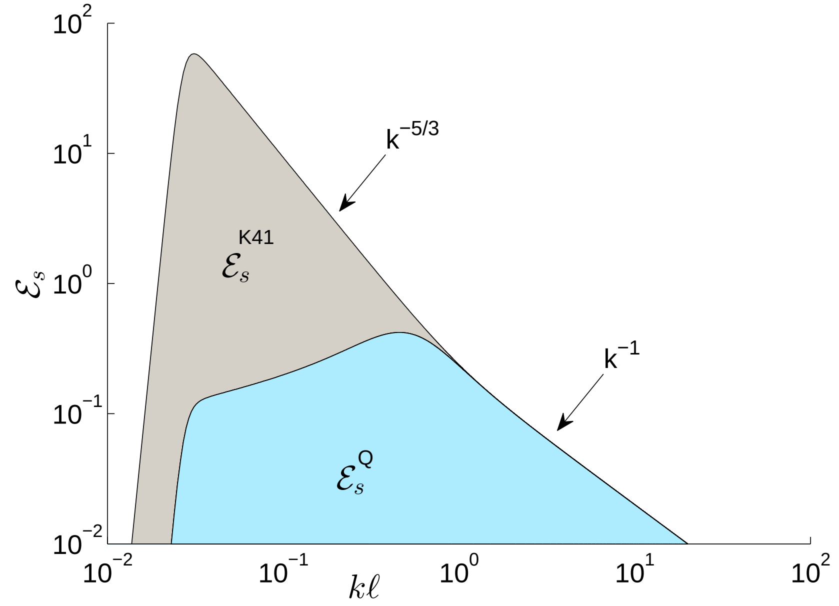


Sketch of the superfluid turbulent energy spectra

Counter- & super-flow stationary spectra



Late time asymptotics

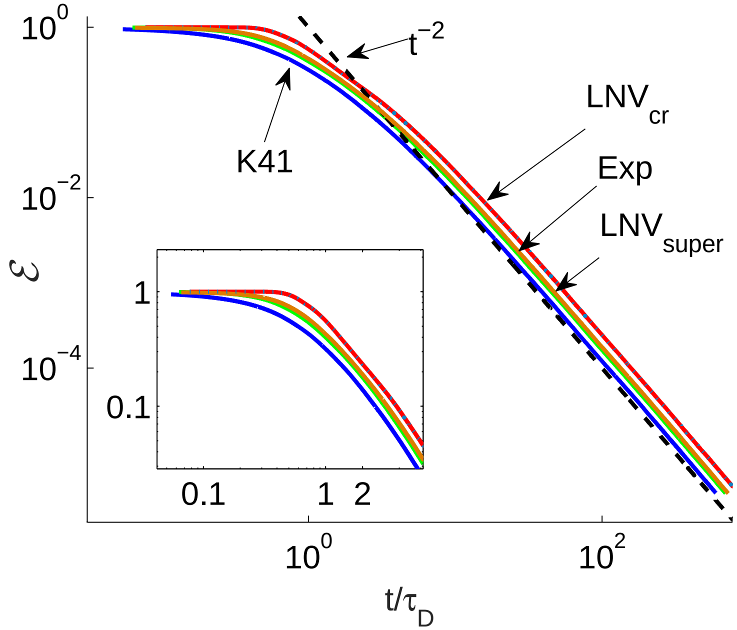


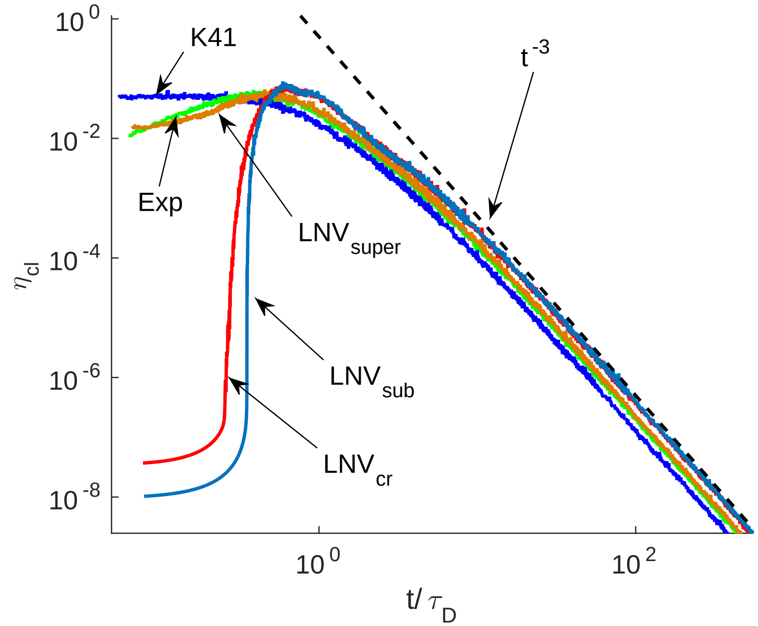
After switching off the counter- or super-flow the stationary energy spectrum of super-/counter-flow (left panel) evolve to the spectrum, shown in right panel, switching on the energy flux toward quantum vortex tangle after some delay.

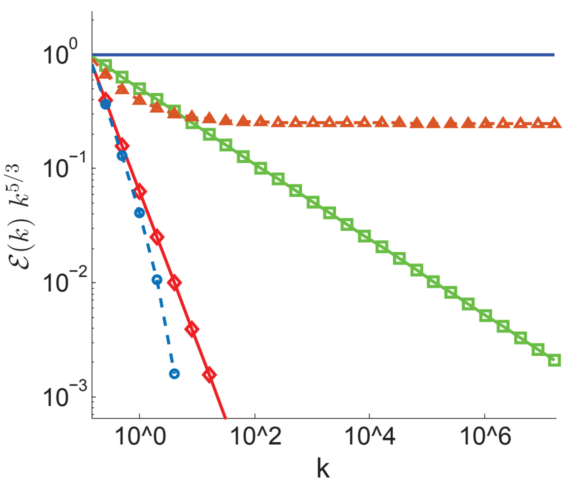
Numerical simulations of the classical turbulence decay by Sabra-shell model

After ensemble averaging over 10^4 realizations we got time dependence of

Left: total energy (with t^{-2} asymptotics) Right: Energy flux toward large k (with t^{-3} asymptotics)

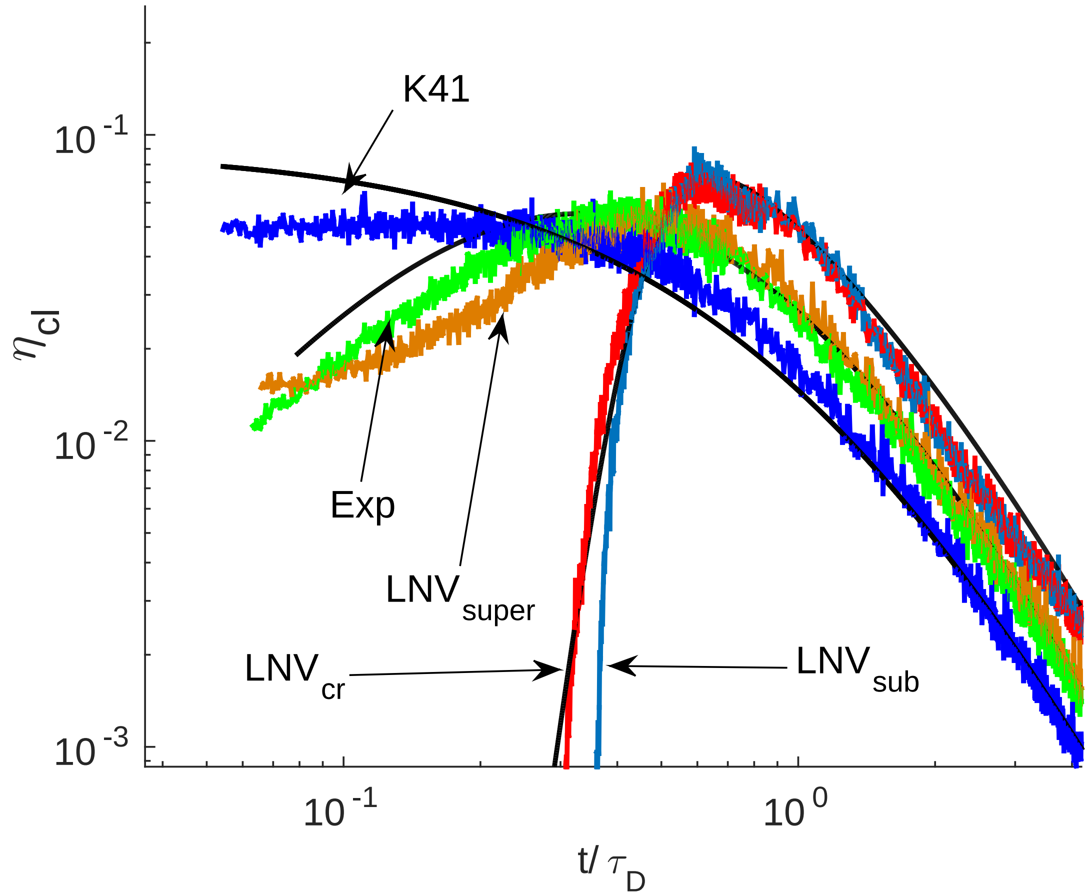






For K41 initial condition (IC) – solid blue lines there are no delay in the energy dissipation. For LNV critical (red line) and sub-critical (blue dashed line) there is clear delay with sharp switching on, while for weakly localized IC ("experimental" k^{-2} spectrum (green) and LNV super-critical IC orange dashed lines) there is smooth switching on.

Classical source function $\eta_{\rm cl}(t)$ and delay function $F_{\rm del}(t)$



Fit of decay from K41 IC by classical source function is given by the classical source function

$$\eta_{\rm cl}(t) = \frac{D^2}{\kappa^2(t-\tau_2)^3}$$

Introduce delay function

$$F_{\rm del}(t,\tau_{\rm del}) = f_n^2(t,\tau_{\rm del})\,,$$

$$f_n(t,\tau_{\rm del}) = \frac{t^n}{(t^n+\tau_{\rm del}^n)}\,,$$

and delayed source function:

$$\eta_{\rm cl,del}(t) = F_{\rm del}(t,\tau_{\rm del})\eta_{\rm cl}(t+\tau_{\rm del})\,.$$

Fit of the decays from k^{-2} and LNV supercritical spectra is given by the delayed source

$$\eta_{\rm cl,del}(t) = \frac{t^n}{(t^n+\tau_{\rm del}^n)}\frac{D^2}{\kappa^2(t+\tau_{\rm del}-\tau_2)^3}\,,$$

with $n=1$, while fit for the well localized (LNV critical and subcritical) IC with $n=6$.

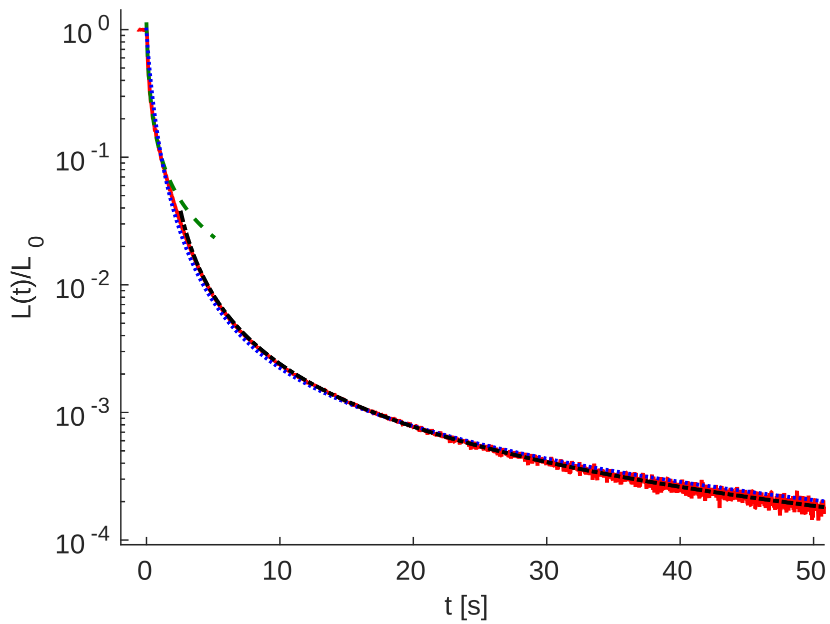
Basic and improved models of VLD decay

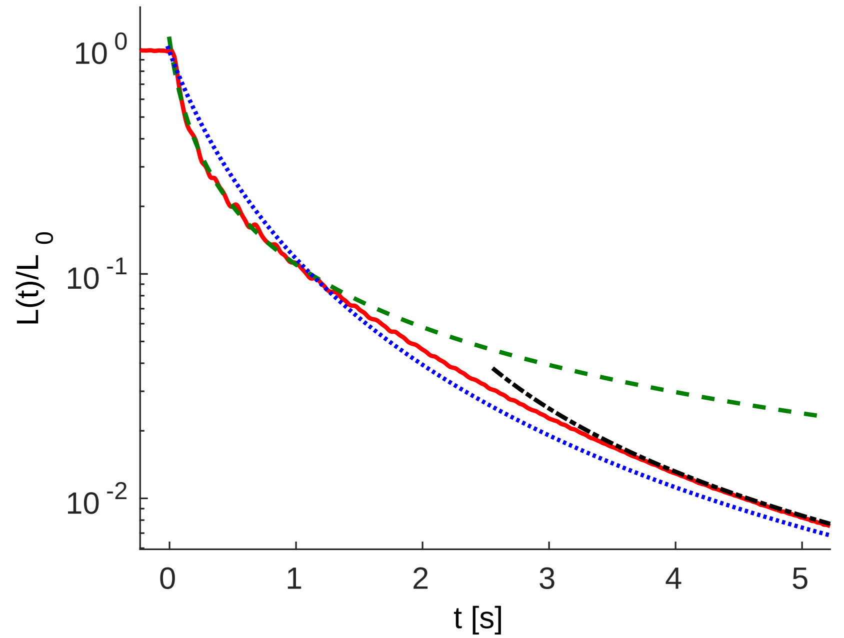
- Basic model vs co-flow decay experiment

$$\frac{d\mathcal{L}}{dt} = \frac{2\,(d_2 D)^2}{\kappa^2 (t-\tau_2)^3} - \frac{\alpha\,\kappa}{d_1}\mathcal{L}^2\,, \Rightarrow\ \mathcal{L}_1(t) = \frac{b_1 \mathcal{L}_0 |\tau_1|}{t-\tau_1}\,,\quad \mathcal{L}_2(t) = \frac{b_2 \mathcal{L}_0 |\tau_2|^{3/2}}{(t-\tau_2)^{3/2}}\,. \tag{8}$$

Here $\mathcal{L}_0 = \mathcal{L}(t=0)$, d_2, b_1 and b_2 are dimensionless phenomenological parameters. τ_1 and τ_2 – virtual origin times for the quantum $\mathcal{L}_1$ and classical $\mathcal{L}_2$ asymptotics. Approximate solution:

$$\boldsymbol{\mathcal{L}_{\mathrm{bas}}(t) = \mathcal{L}_2(t) \coth\left[\frac{\mathcal{L}_2(0)}{\mathcal{L}_1(t)}\right]}\,. \tag{9}$$





Red solid lines, Prague co-flow delay data with $T = 1.35$ K, $\mathcal{L}_0 \approx 10^6\,\mathrm{cm}^{-2}$.

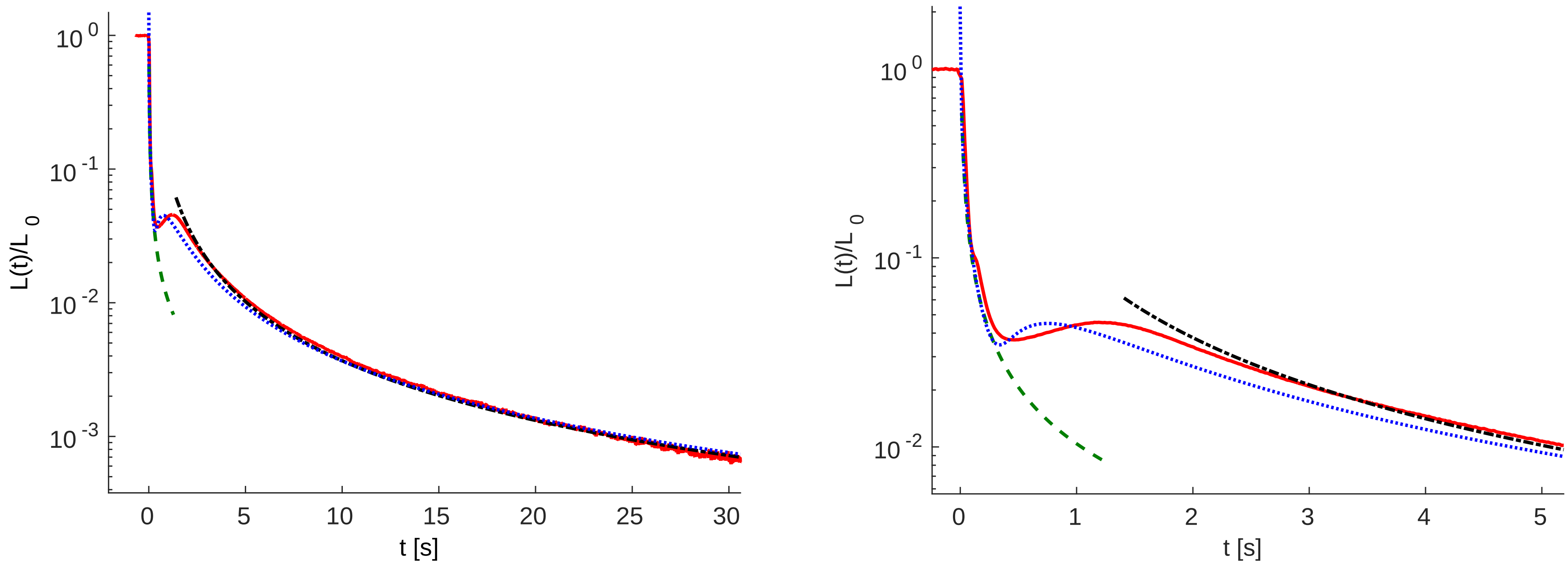
Blue dotted lines – basic model $\mathcal{L}_{\mathrm{bas}}(t)$ prediction with $\tau_1 = 0.5$ s, $\tau_2 = 1$ s and $b_2 = 0.075$.

- Improved model vs super-flow decay experiment

$$\frac{d\mathcal{L}}{dt} = F_{\rm del}(t,\tau_{\rm del})\frac{2\,(d_2 D)^2}{\kappa^2(t+\tau_{\rm del}-\tau_2)^3} - \frac{\alpha\,\kappa}{d_1}\,. \tag{10}$$

Approximate solution:

$$\boldsymbol{\mathcal{L}_{\rm im}(t) = f_n(t,\tau_{\rm del})\mathcal{L}_2(t+\tau_{\rm del})\coth\left[\frac{f_n(t,\tau_{\rm del})\mathcal{L}_2(0)}{\mathcal{L}_1(t)}\right]}\,. \tag{11}$$



Red solid lines, Prague super-flow delay data with $T = 1.45\,$K, $\mathcal{L}_0 \approx 10^6\,$cm^{-2}.

Blue dotted lines – improved model $\mathcal{L}_{\rm im}(t)$ with $n=2$, $\tau_1 = 0.015\,$s, $\tau_2 = 0.1\,$s and $b_2 = 0.22$.

1 Summary and perspectives

Based on good agreement between the experimental observations and the analytical predictions we conclude that

the basic and improved models adequately reflect the underlying physical processes responsible for the decay of superfluid ^{4}He turbulence, including

– partial decoupling of the normal- and super-fluid velocities in super- and counter-flowing turbulence;
– resulting suppression of energy spectra in these flows;
– time-delay in the energy flux in the energy flux from classical to quantum parts of super-fluid turbulence.

Nevertheless much more experimental, numerical and analytical work is required to formulate really advanced model of decaying superfluid turbulence, which will account in details for the interplay of coexisting classical and quantum forms of superfluid turbulent energy. This requires, for example,

– inclusion into the model the real stationary energy spectrum of counter- and super-flowing ^{4}He and its evolution after the switching off the flow,
– accounting for the energy flux to the quantum tangle from the classical thermal bath, and
– analysis of the affect of spatial inhomogeneity on the turbulence decay and its time evolution.