Small-Scale Energy Transfer Mechanisms in Quantum Turbulence

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Interpretation of Measurements in Superfluid Turbulence
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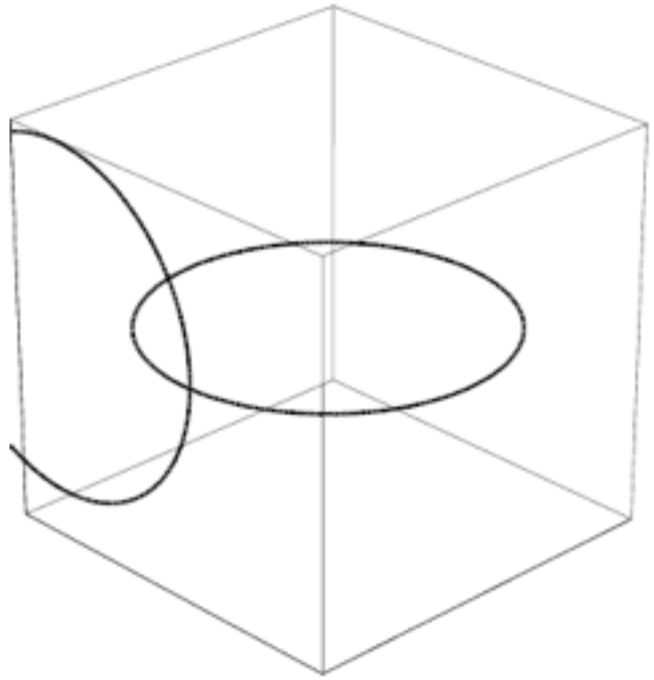
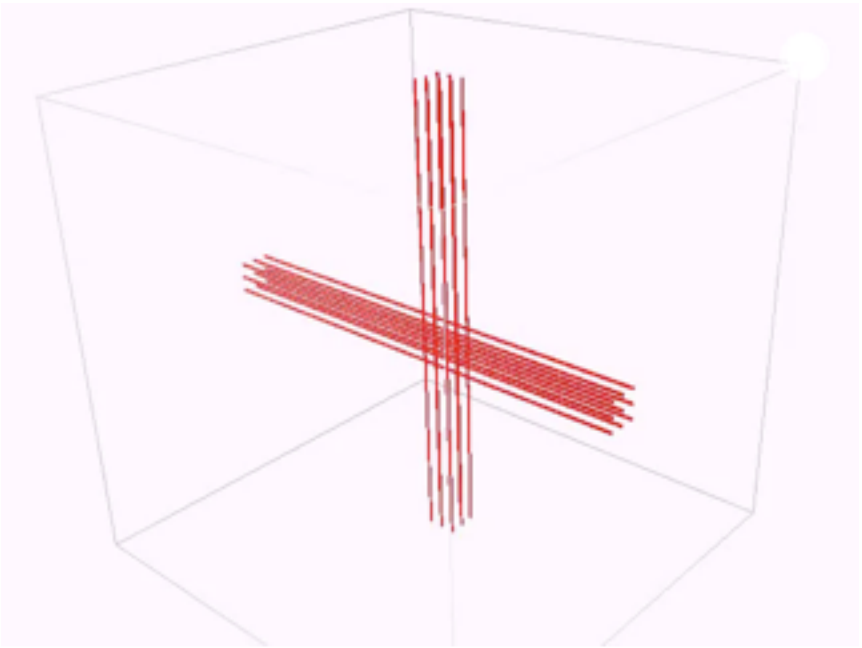
Motivation





Courtesy of Andrew Baggaley and Sultan Alamri

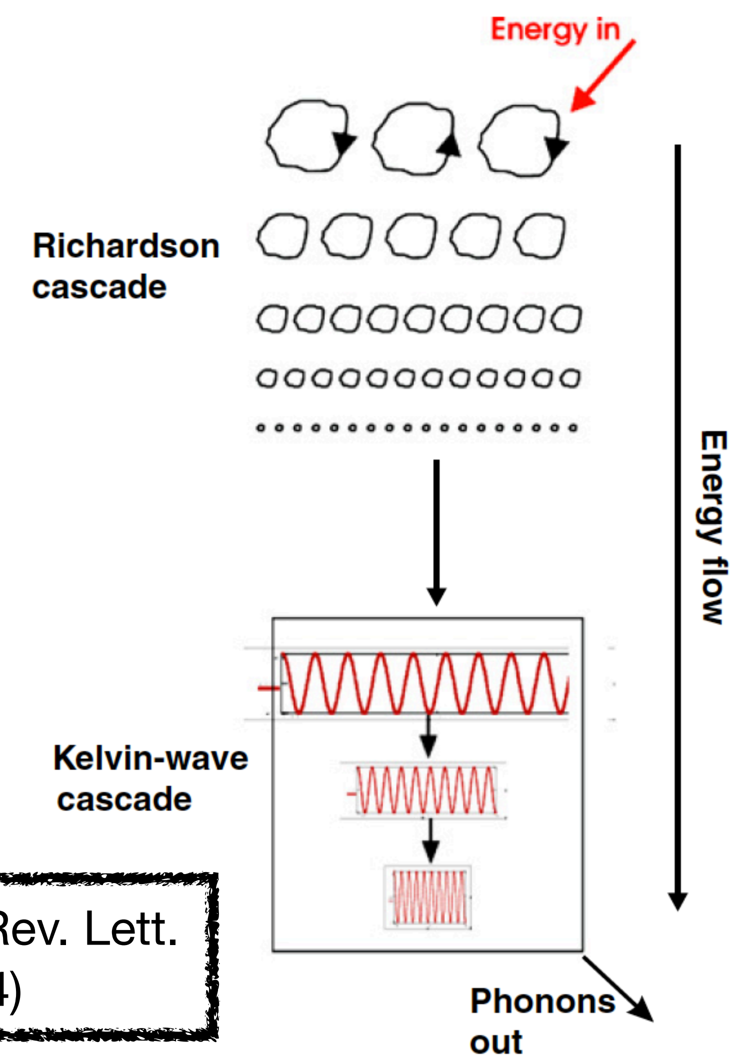
Mechanisms of energy transport

1. Generation of Kelvin waves
   - The Kelvin wave cascade
2. Sound emission
   - Direct excitation of phonons
3. Vortex ring creation
   - Transfer of energy and momentum to other areas of the tangle or dissipation at boundaries

Popular consensus for small scale QT

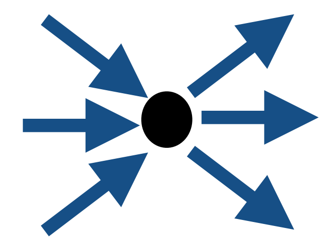
The Kelvin wave cascade at zero temperature

- Vortex reconnections transfer large-scale energy to Kelvin waves at superfluid cross-over region
- Weakly nonlinear Kelvin wave interactions transfer energy to even smaller scales



Wave turbulence description

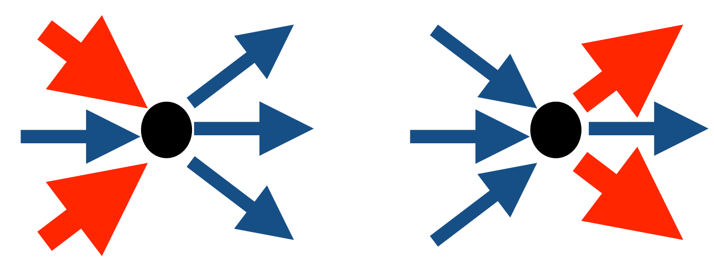
- Characterized by six-wave resonant interactions



$$E_k = C_{KS}\,\Lambda\,\kappa^{7/5}\,\epsilon^{1/5}\,k^{-7/5}$$

Kozik, Svistunov, Phys. Rev. Lett. **92**, 035301, (2004)

Nonlocality and effective 4-wave description



$$E_k = C_{LN}\,\Lambda\,\kappa^{7/5}\,\epsilon^{1/3}\,\Psi^{-2/3}\,k^{-5/3}$$

JL *et al.* Phys. Rev. B, **81**, 104526, (2010)
L'vov, Nazarenko, Low Temp. Phys. **36**, 785, (2010)

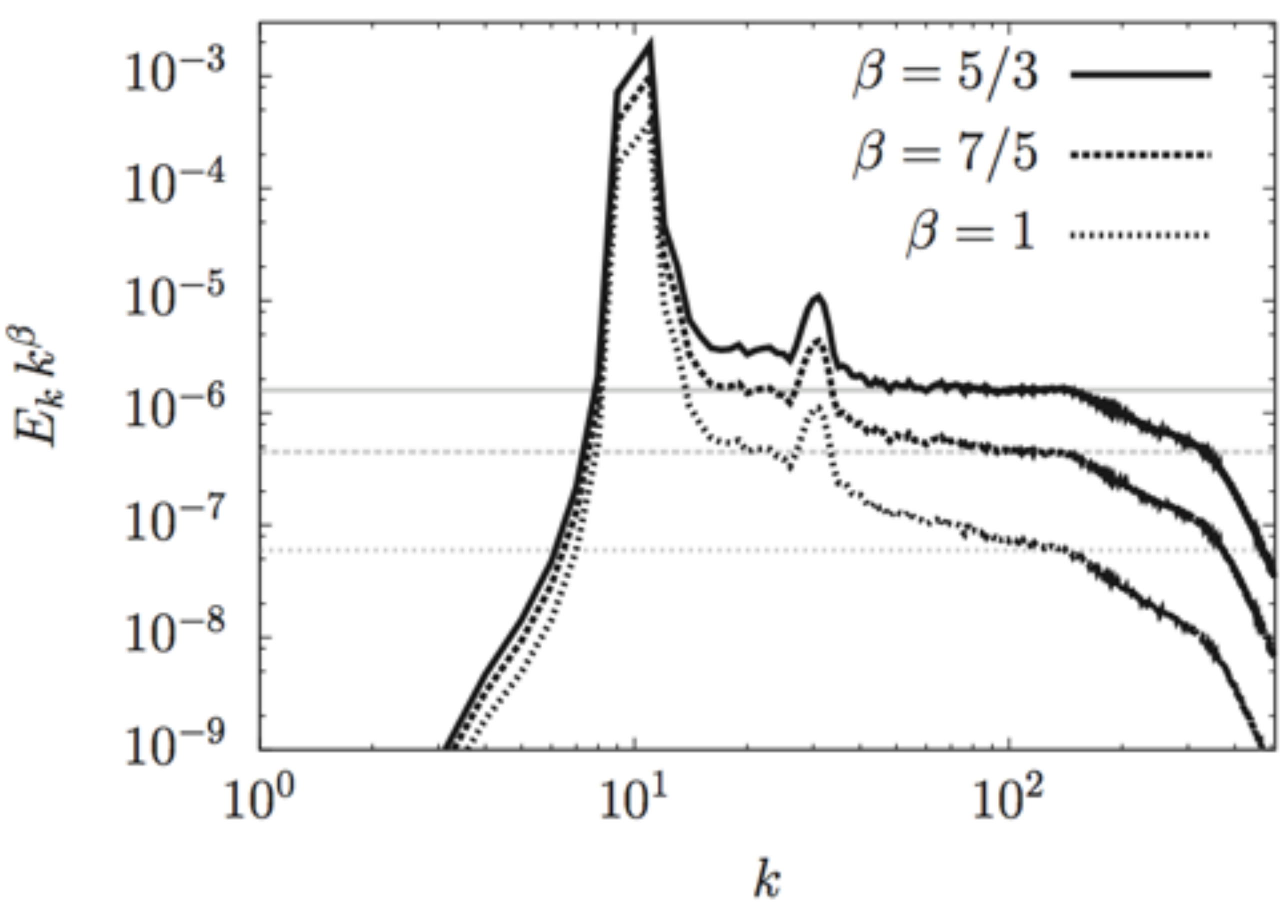
Evidence for Effective 4-Wave Interactions

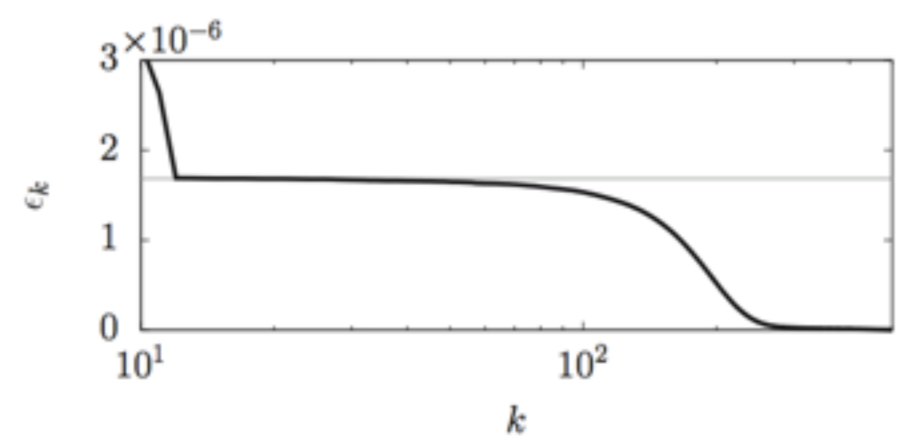
Computation of spectrum prefactor

- Power law exponents are relatively close, making numerical verification difficult
- Prefactors should provide an easier indicator
- 4-wave collision integral simple enough to be numerically solved

$$C_{LN} = (128\pi)^{1/3} \left(\frac{dI(x)}{dx}\bigg|_{x=11/3} \right)^{-1/3} = 0.304$$

Boué *et al.* Phys. Rev. B, **84**, 064516, (2011)





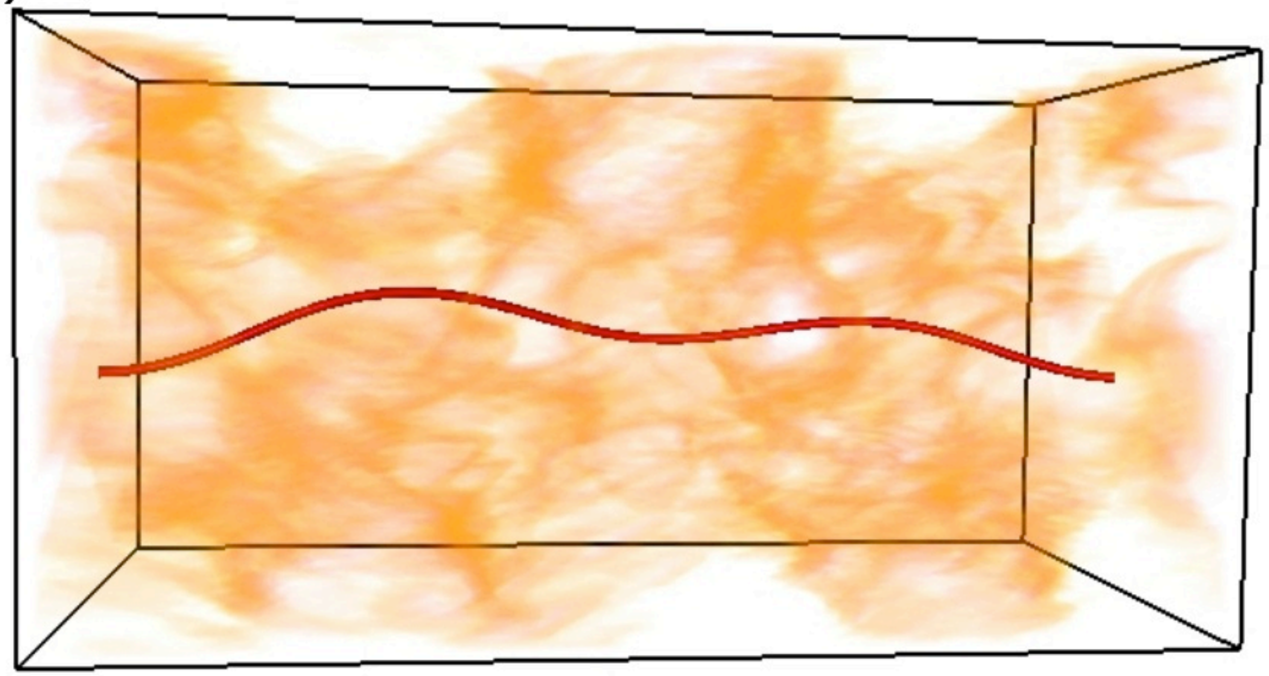
$$C_{LN}^{num} = 0.318$$

$$C_{KS}^{num} = 0.0087$$

Baggaley, JL, Phys. Rev. B, **94**, 025301, (2014)

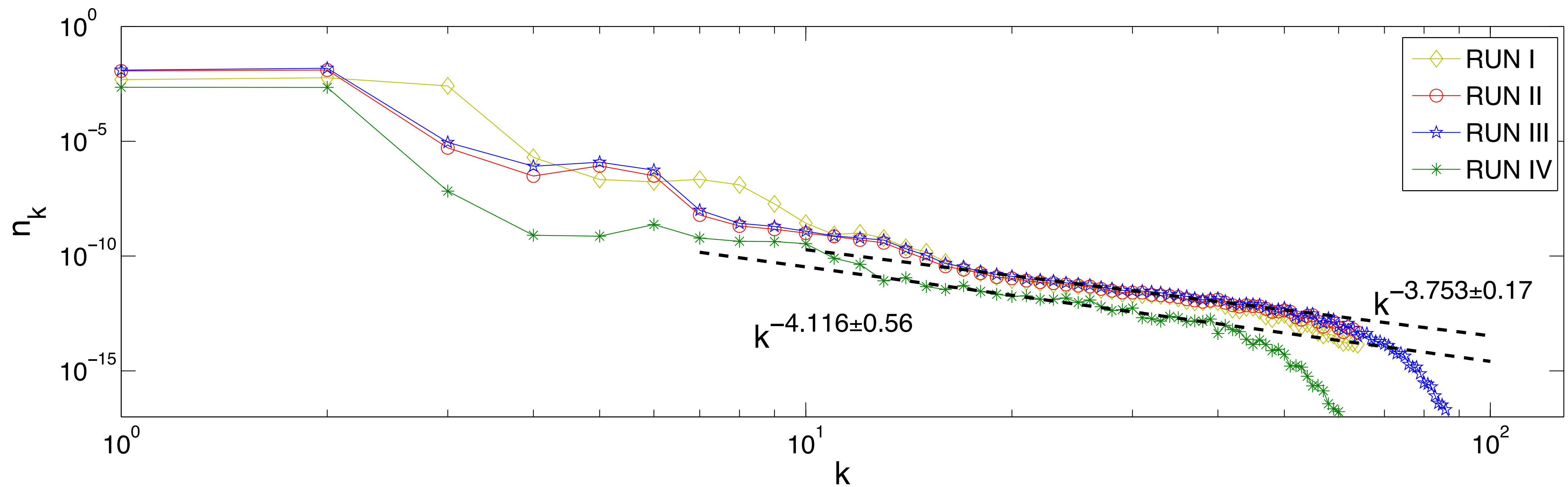
Kelvin Wave Cascade in Gross-Pitaevskii

a)

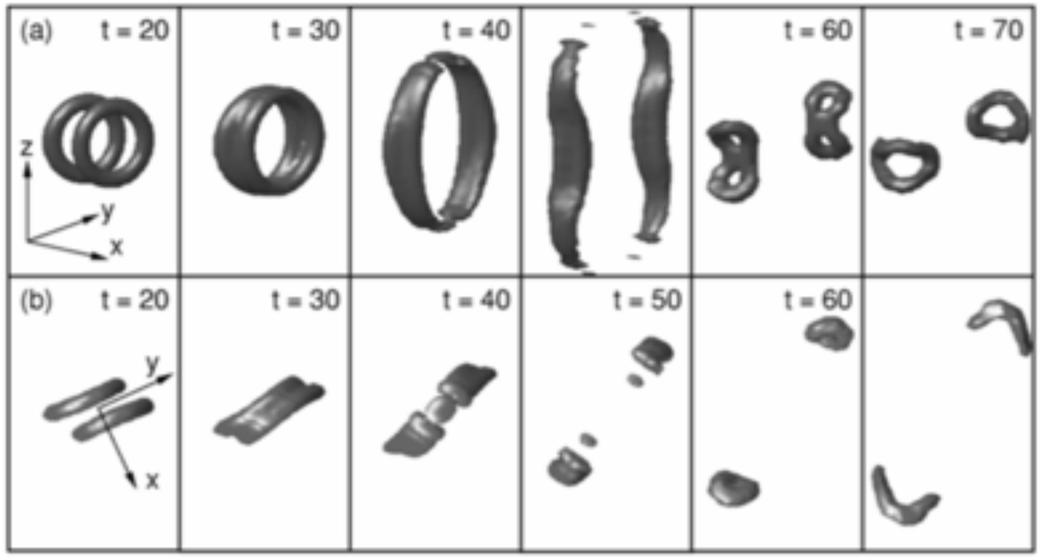


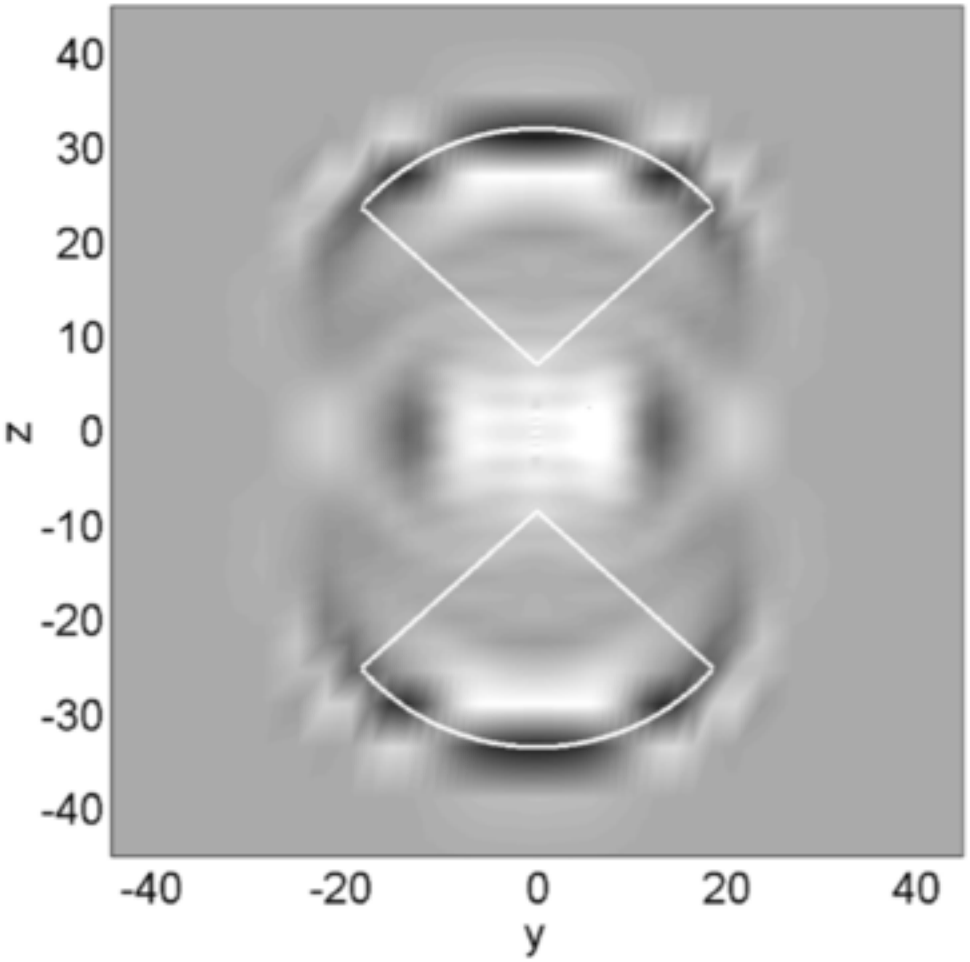
Krstulović, Phys. Rev. E, **86**, 055301, (2012)

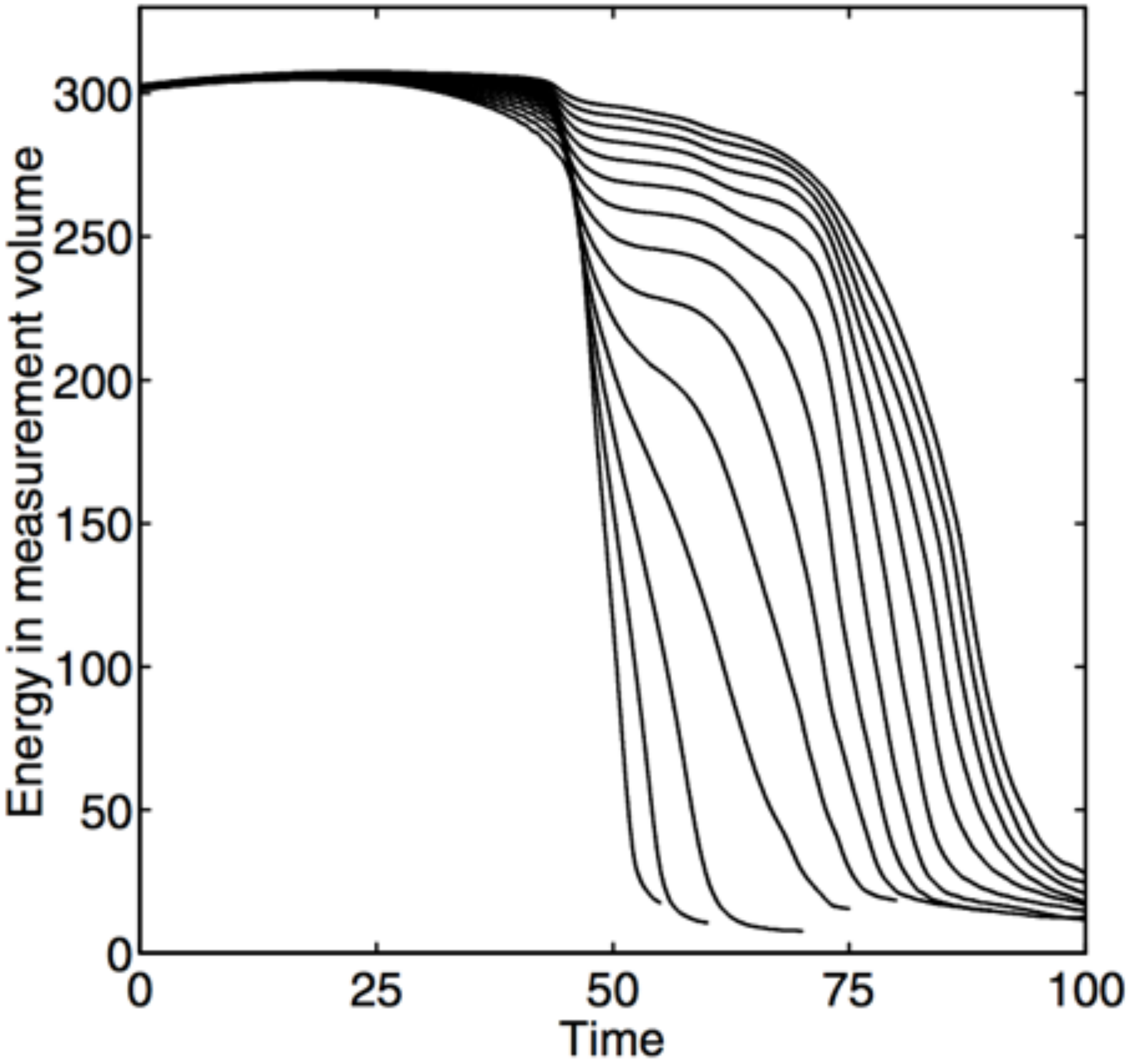
$$i\dot{\Psi} = -\nabla^2\Psi + \Psi\,|\Psi|^2$$



Sound Emission in Gross-Pitaevskii





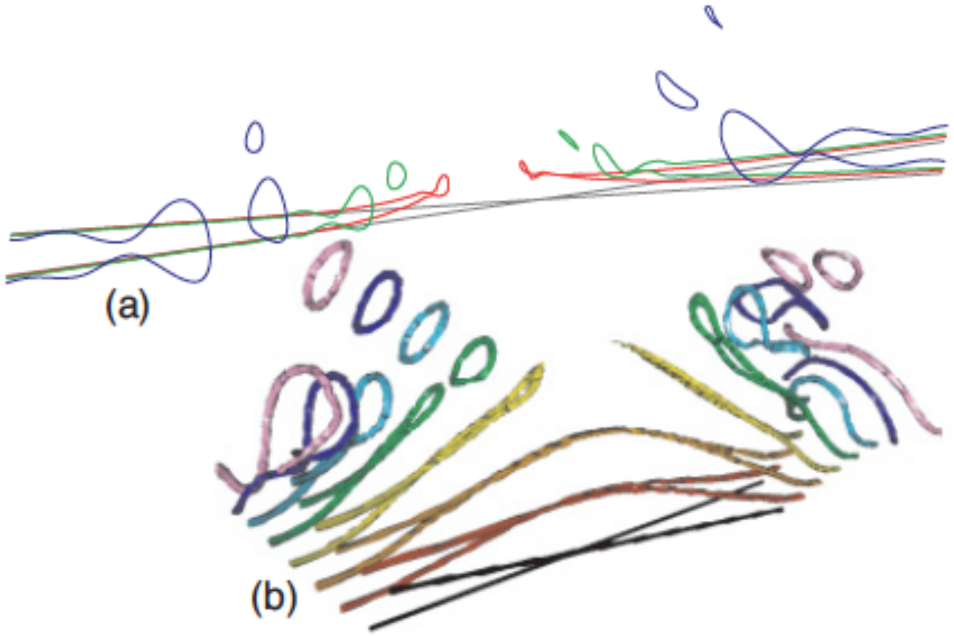


Leadbeater *et al.* Phys. Rev. Lett. **86**, 1410, (2001)

Is There a Role for Vortex Ring Emission?

Vortex ring cascade at large angles

- A vortex reconnection of two (almost) anti-parallel vortices lead to a series of self-reconnections and the emission of multiple vortex rings
- Critical angle for ring generation in the Biot-Savart model is $\theta_c \simeq 0.942\pi$

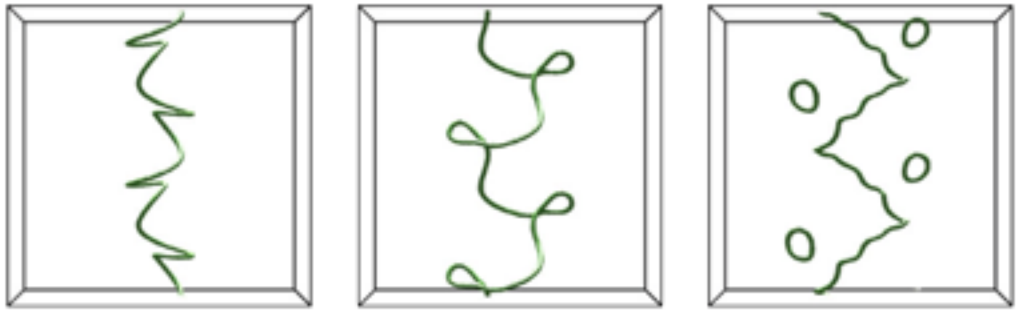


Kursa *et al.* Phys. Rev. B, **83**, 014515, (2011)
Kerr, Phys. Rev. Lett. **106**, 224501, (2011)

Modulational instability and self-reconnection

- Strongly nonlinear Kelvin waves can lead to modulational instability and self reconnections

Salman. Phys. Rev. Lett. **111**, 165301, (2013)



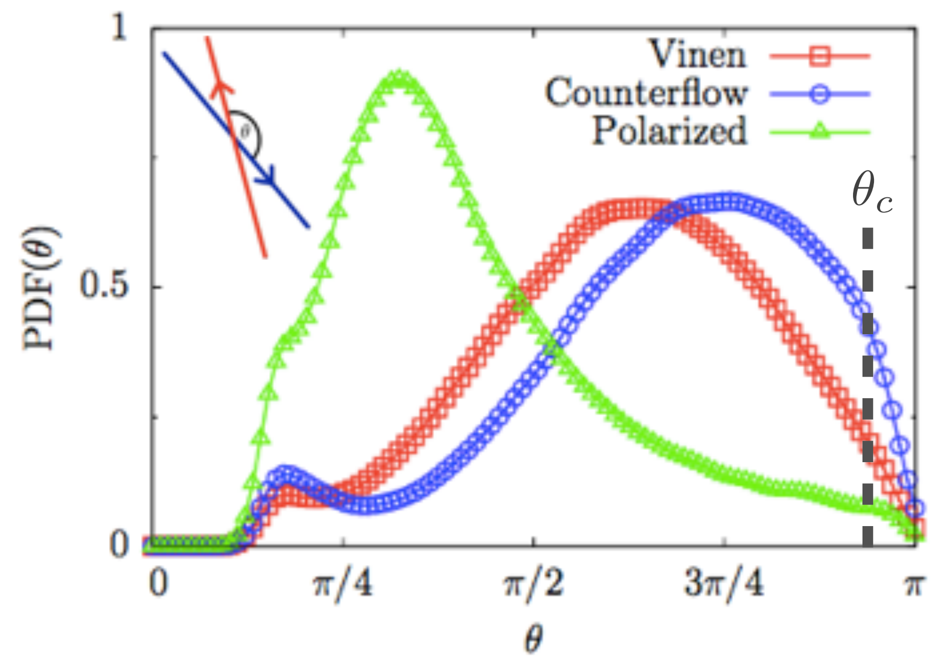
Can we quantify these processes in quantum turbulence?

How Common is Vortex Ring Emission?

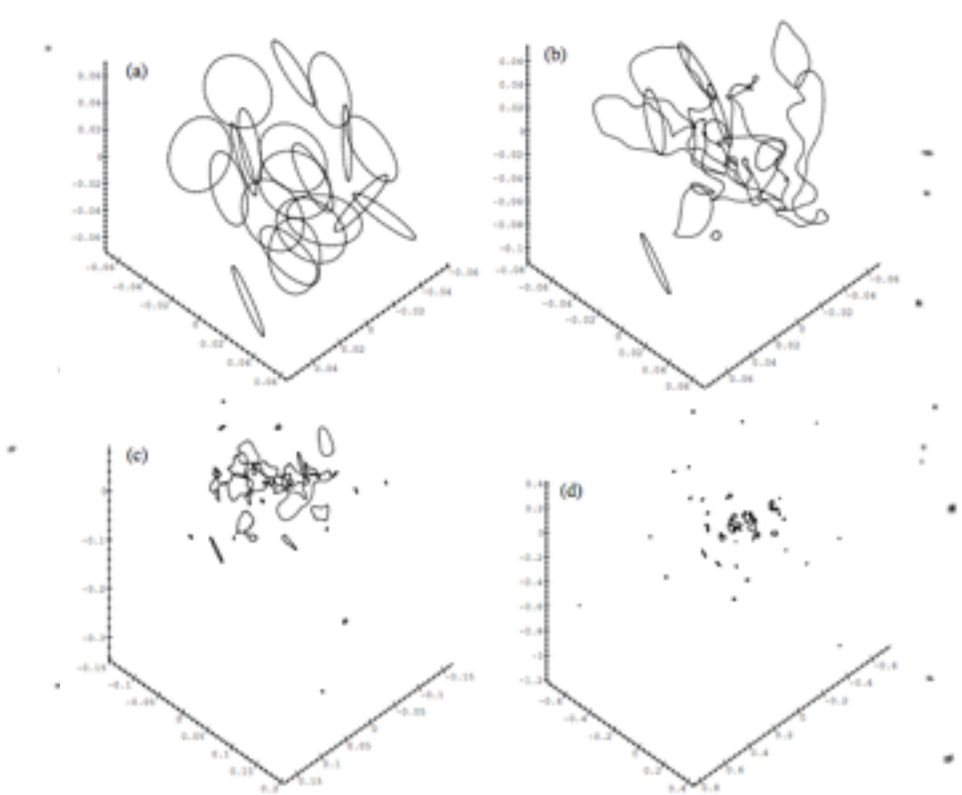
Reconnection angles in QT tangles

- Suppression of large angle reconnection in polarized tangles
- Majority of reconnection will not lead to ring cascade

4% (Counterflow), 2% (Vinen), 1% (Polarized)



Clear observation of ring emission in localized tangles



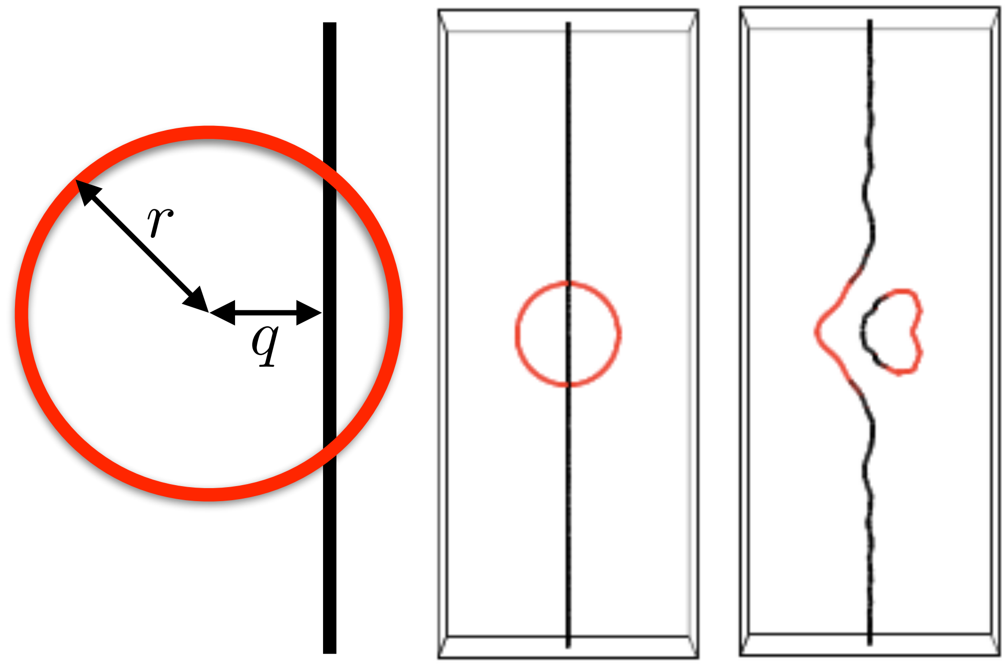
- Can we identify types of ring emission in quantum turbulence?
- Kursa *et al.* (2011) made estimates based on ring cascade showing this could be the main dissipation mechanism for sparse low temperature tangles
- What about larger vortex rings that undergo reconnections?

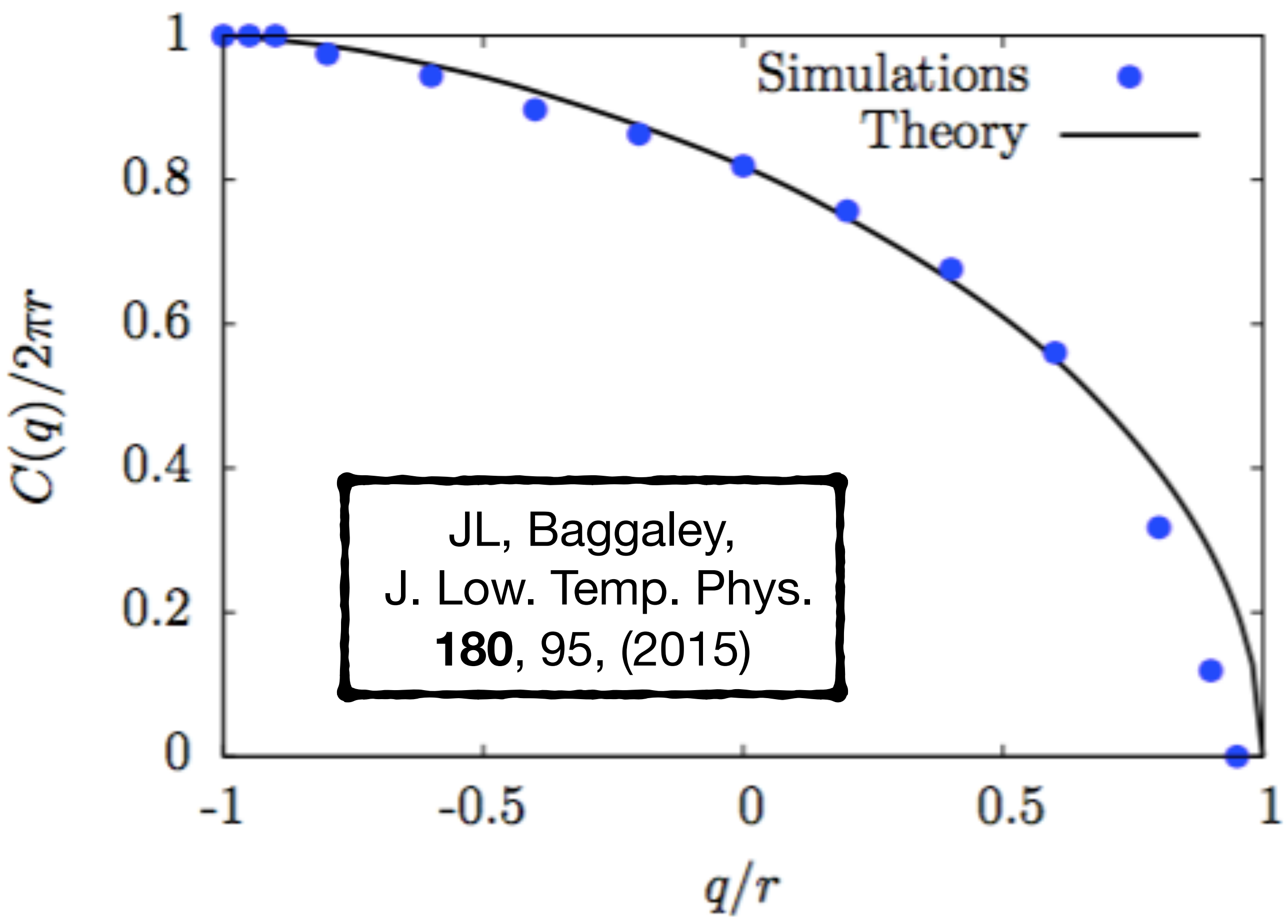
Ring-Line Reconnections

Ideal vortex ring-line reconnection

Post-reconnection circumference

$$C(q) = 2\left(r^2 - q^2\right)^{1/2} + 2r\arccos\left(\frac{q}{r}\right)$$





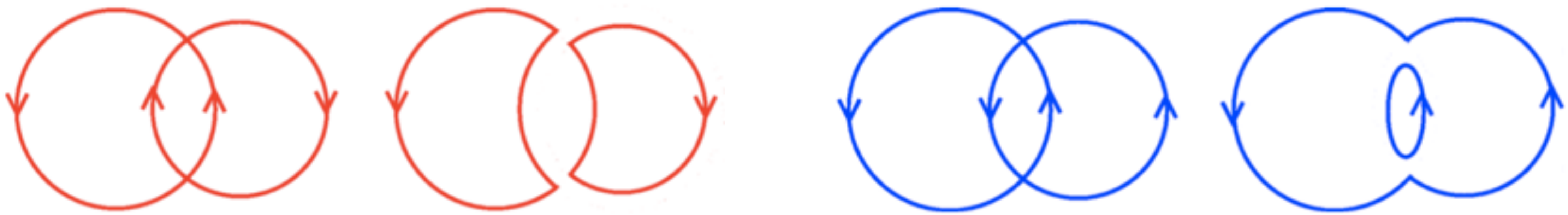
- Reconnection leads to robust outgoing vortex ring and stable vortex line
- Numerical data suggests almost ideal circumference after reconnection
- Assuming that the impact factor is uniformly distributed we can compute the mean post reconnection ring radius

$$\langle r_{post}\rangle = \frac{3r}{4}$$

Mean post-reconnection radius

Ring-Ring Reconnections

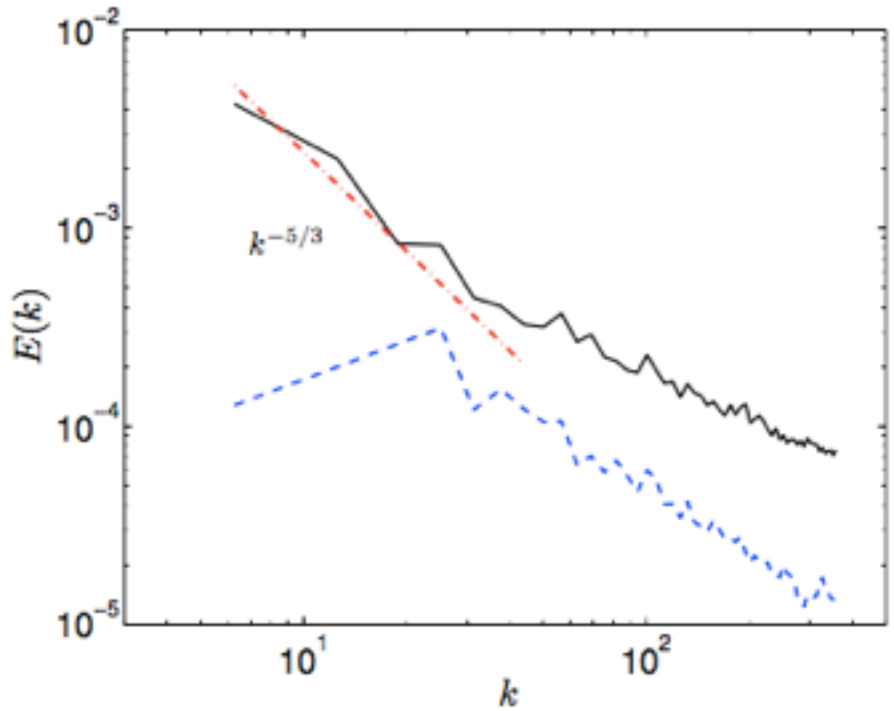
Naïve ring-ring reconnections



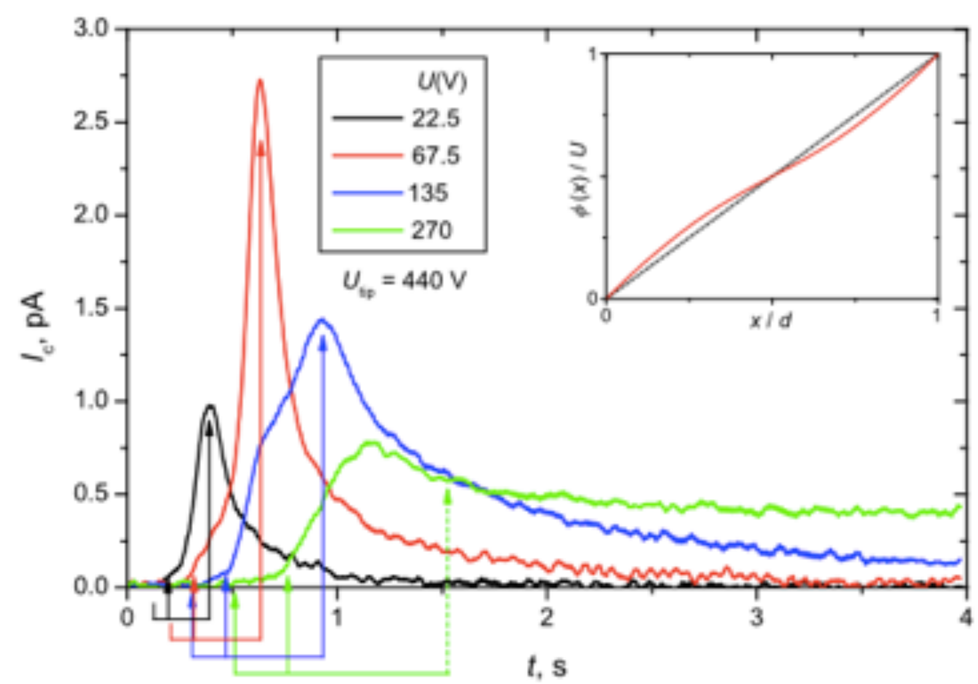
- Isotropic tangles would have an equal distribution of these reconnections

Unidirectional vortex ring collisions

- Restricting the system to favour one type of reconnection can affect turbulence



Baggaley *et al.* Phys. Rev. E, **89**, 013002, (2014)



Walmsley *et al.* Phys. Rev. Lett. **113**, 125302, (2014)

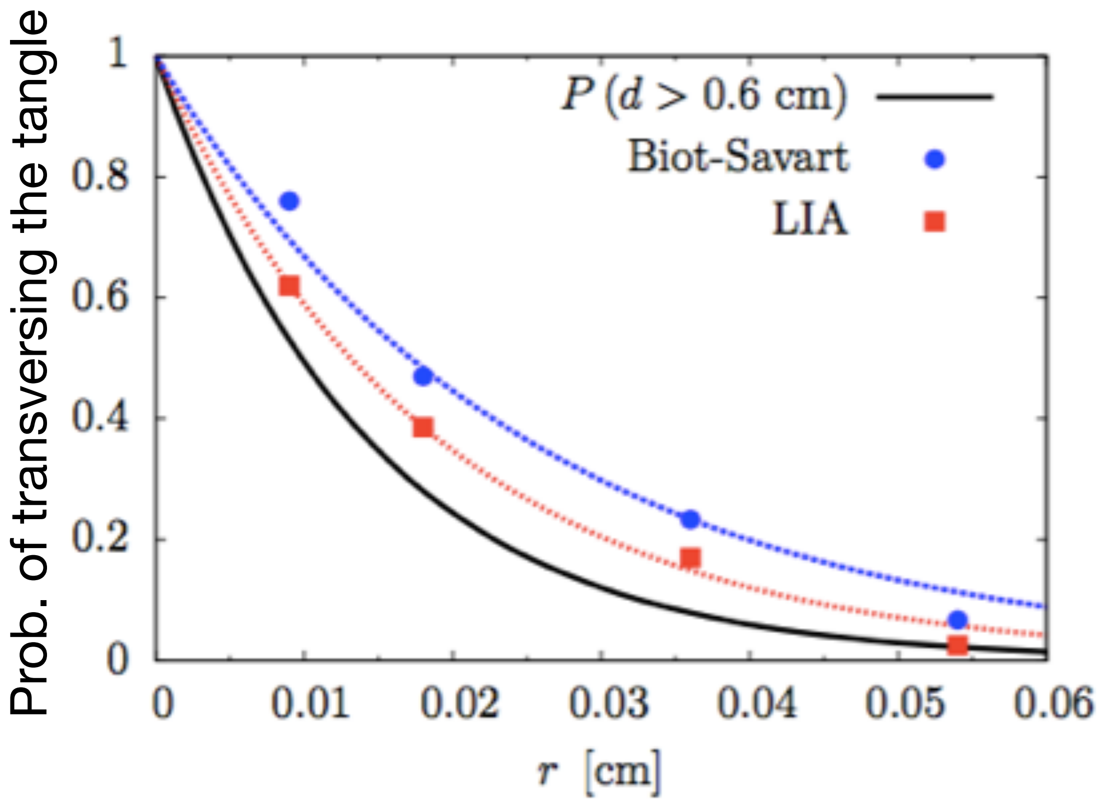
Ring Propagation Through a Tangle

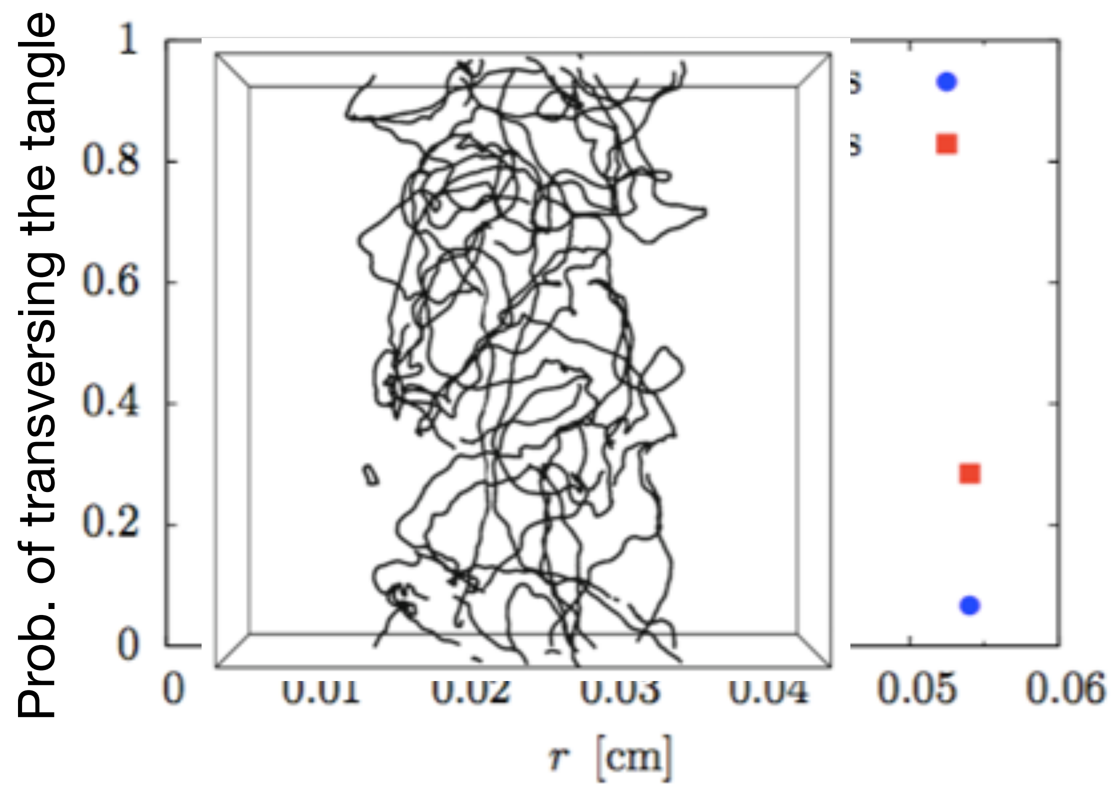
What is the survival of a vortex ring in a vortex tangle?

- Typical mean free path estimate is $\langle d \rangle = 1/2rL$
- Assuming exponential distribution, a naïve estimate gives $P(d > W) = \exp\left(-2rLW\right)$

Firing a vortex ring through a tangle

$W = 0.6 \text{ cm} \quad \ell = 0.13 \text{ cm}$





- Our numerical data implies that the usual mean free path estimate underestimates the distance for reconnection by 75% (Biot-Savart)
- Upon reconnection inside a tangle there is still a significant chance of the ring surviving

JL, Baggaley,
J. Low. Temp. Phys.
180, 95, (2015)

Energy Transport by Vortex Rings

Scale of ring generation in self-reconnections

- Can estimate ring radius generated through self-reconnections $r \sim \ell / \left[\ln(\ell/a)\right]^{1/2}$

A naïve estimate of ring energy transport

Manchester experiment

$D = 4.5 \text{ cm}$ $\quad L \simeq 10^3 \text{ cm}^{-2}$ $\quad \ell \simeq 3.2 \times 10^{-2} \text{ cm}$

Our numerical simulations

Mean free path estimate $\simeq 1/1.14 rL$ $\quad$ Ring survival $> 93\%$

- For a randomly generated ring, assuming ideal ring-line reconnections can transverse a typical tangle in with only 4 reconnections on average
- Consequently, up to 65% of the original ring energy can reach the boundaries
- For higher VLDs $L \simeq 10^6 \text{ cm}^{-2}$ this drops to only 5%

Conclusions

Vortex rings seem to be robust structures that are capable of transporting energy across a quantum vortex tangle even when undergoing reconnections

Important open problems

- What happens at boundaries? Ring dissipation? Reflection?
- Can we further quantify vortex ring generation in quantum tangles?
- Complex reconnection topologies (angled rings, bundles, …)