Workshop on Interpretation of measurements in 4He superfluid turbulence
Saclay, France, Sept 14-18, 2015

The decay of quantum turbulence energy in superfluid helium-4

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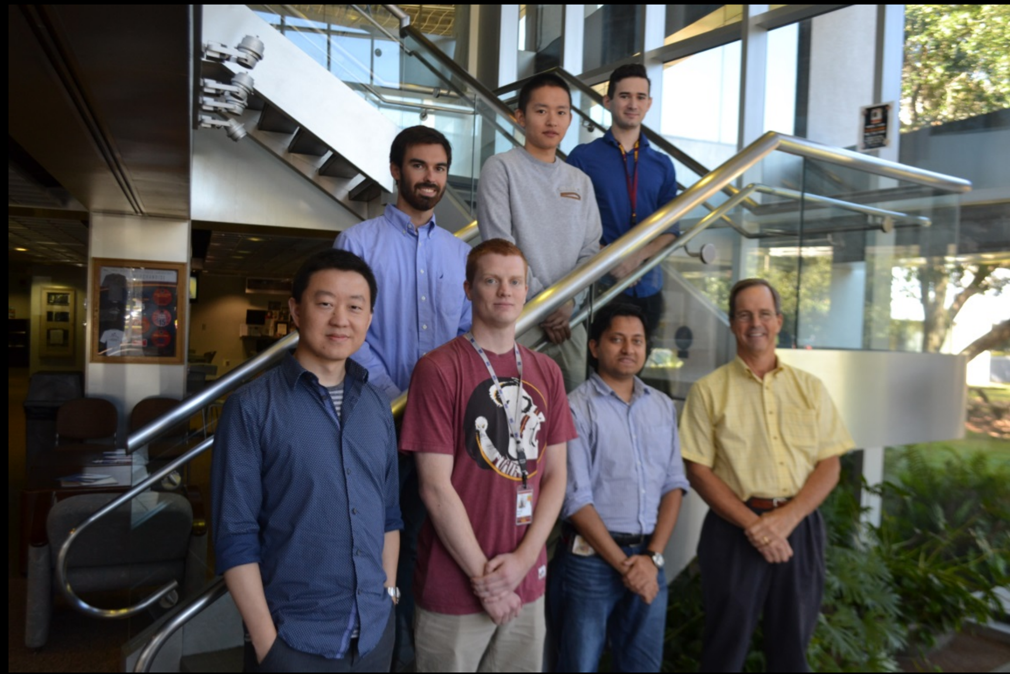


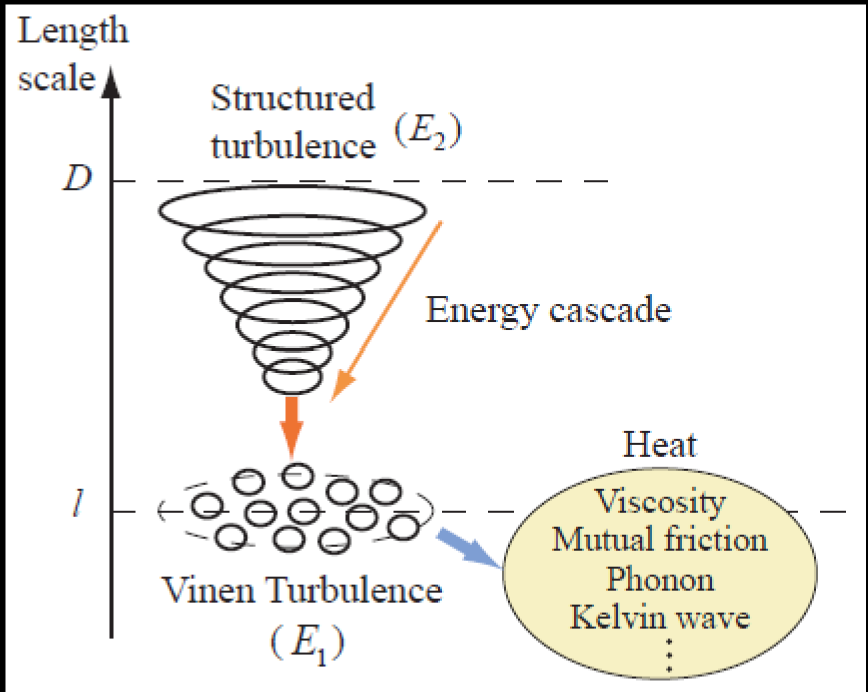
National High Magnetic Field Laboratory

Outline

- Decay of quantum turbulence energy
  - Turbulence energy of a vortex tangle and its decay
  - Some outstanding questions
- Experimental methods
  - 2nd sound attenuation
  - He_2^* molecular tracer-line tracking
- Study on decaying thermal counterflow
  - Existing knowledge and open questions
  - The decay of vortex density and turbulence energy
  - Determination of the effective viscosity
- Future measurements

Decay of quantum turbulence energy

- Turbulence energy of a vortex tangle and its decay



- The total turbulence energy can be divided into two parts:

$$E = E_1 + E_2$$

E_1 is the energy associated with flows at intervortex spacing scale

$$E_1 \approx \frac{1}{4\pi}\frac{\rho_s}{\rho}\ln\left(\frac{l}{a}\right)\cdot\kappa^2 L \sim \kappa^2 L$$

E_2 accounts for the energy of large scale flows due to vortex polarization.

$$E_2 = \begin{cases} 0 & \text{Vinen turbulence (no polarization)} \\ \propto L^2 & \text{Solid body rotation (complete polarization)} \\ E_2(L, ?) & \text{General large-scale turbulence} \end{cases}$$

How to correlate the turbulence energy and vortex density?

- For classical turbulence: $$\frac{dE}{dt} = -\nu < \omega^2 >$$
- As an analogy, for quantum turbulence: $$\frac{dE}{dt} = -\nu'(\kappa L)^2$$

Consequence of the energy decay rate equation:

$$\frac{dE_1}{dt} + \frac{dE_2}{dt} = -\nu'(\kappa L)^2 \Rightarrow \frac{1}{4\pi}\frac{\rho_s}{\rho}\ln\left(\frac{l}{a}\right)\cdot\kappa^2\frac{dL}{dt} + \frac{dE_2}{dt} = -\nu'(\kappa L)^2$$

$$\Rightarrow \quad \boxed{\kappa^2\frac{dL}{dt} + \frac{dE_2}{dt} = -\nu'(\kappa L)^2}$$

The above equation describes the decay behavior of vortex density in general turbulence.

Some special cases:

(1) For Vinen turbulence: $E_2 = 0 \Rightarrow \kappa^2 \frac{dL}{dt} = -\nu'(\kappa L)^2$

This is the same as the Vinen's equation: $\frac{dL}{dt} = -\frac{\chi_2 \kappa}{2\pi} L^2 \Rightarrow \frac{\nu'}{\kappa} = \frac{\chi_2}{2\pi} \cdot \frac{\rho_s}{4\pi\rho} \ln\left(\frac{l}{a}\right)$

→ Vinen decay: $L^{-1}(t) = L_0^{-1} + \left(\frac{\chi_2 \kappa}{2\pi}\right) \cdot t$

(2) For grid turbulence: $\frac{dE_1}{dt} \sim \kappa^2 \frac{dL}{dt} \ll \frac{dE_2}{dt} \sim -\frac{\Delta U_0^3}{2D}(1+t/2\tau)^{-3}$

→ Quasiclassical decay: $L(t) \sim (1+t/2\tau)^{-3/2}$

In general, one needs to know dE_2/dt in order to evaluate the decay behavior of the vortex density.

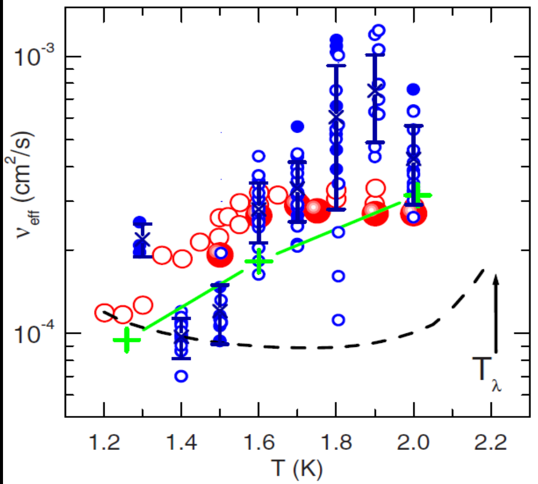
- Outstanding questions

(1) Can we test the energy decay rate equation experimentally?

$$\frac{dE}{dt} = \kappa^2 \frac{dL}{dt} + \frac{dE_2}{dt} = -\nu'(\kappa L)^2$$

(2) Can we determine of the effective viscosity reliably?

$$L(t) = \frac{(3C)^{3/2} D}{2\pi\kappa\nu^{1/2}} (t - t_0)^{-3/2}$$



T.V. Chagovets, A.V. Gordeev, and L. Skrbek, PRE 76, 027301 (2007)

(3) Is the effective viscosity the same for different forms of QT?

We need to measure both L and E_2 as a function of the decay time:

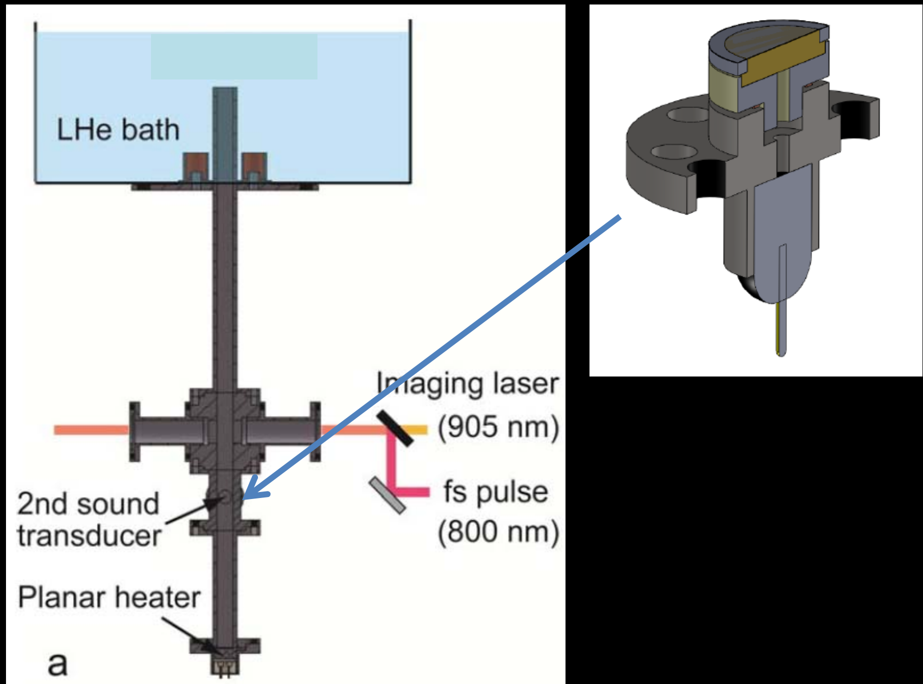
L : 2nd sound attenuation

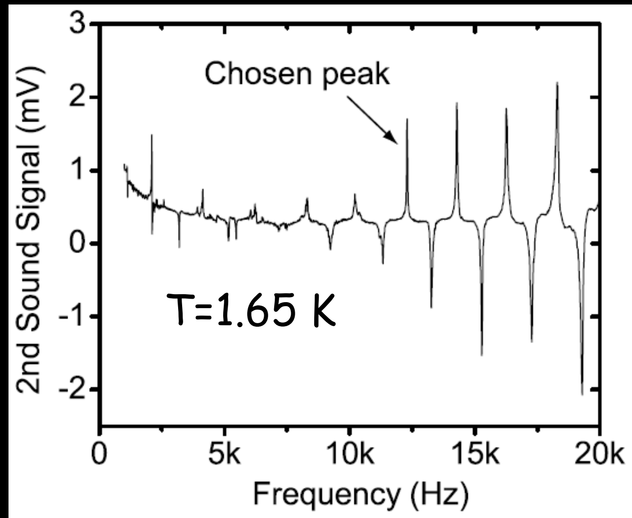
$E_2 = \frac{3}{2}\Delta U^2$: Flow visualization

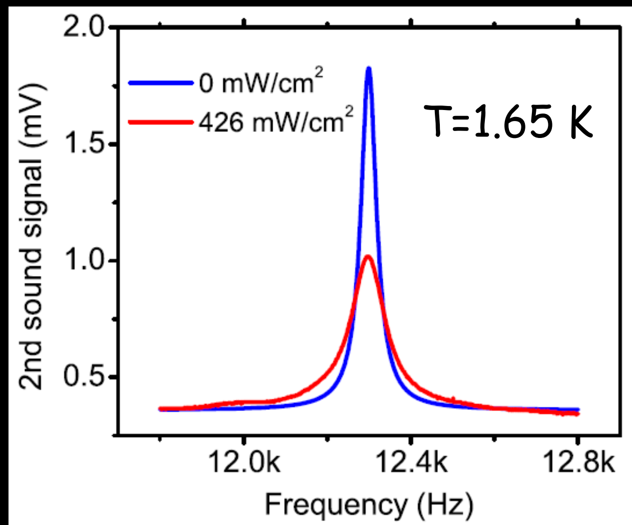
(We choose to study decaying thermal counterflow)

Experimental methods

- Measuring vortex line density: 2nd sound attenuation







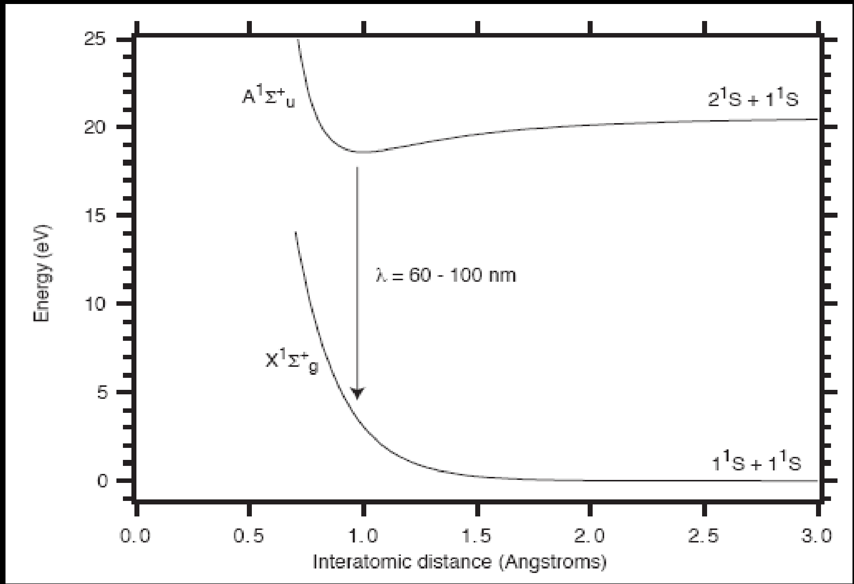
Attenuation of 2nd sound by vortices allows the determination of the mean vortex line density:

$$L = \frac{6\pi\Delta f_0}{B\kappa}\left(\frac{\bar{A}_0}{\bar{A}} - 1\right)$$

(expression is taken from Chagovets, et al., PRE 76, 027301 (2007))

Experimental methods

- Flow visualization



➢ Metastable He_2^* molecules as tracers.

They can be easily created. No need of foreign particle injection:

$e^- + He^+ + He \rightarrow He^* + He \rightarrow He^*_2$

singlet state $A^1\Sigma_u^+$ lifetime: ~10 ns

triplet state $a^3\Sigma_u^+$ lifetime: **~13s**

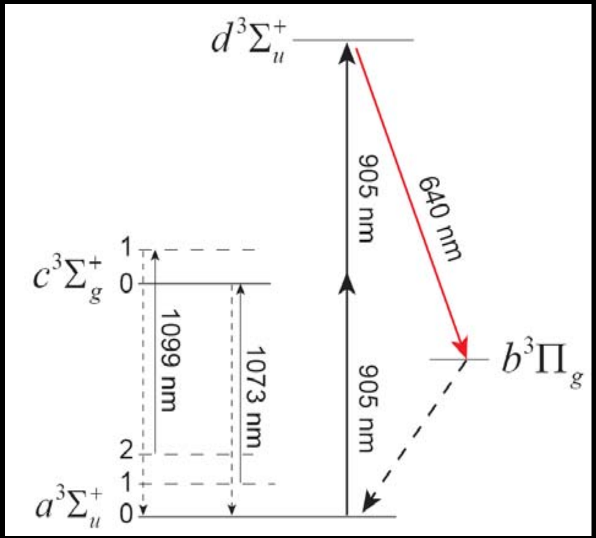
➢ He_2^* molecules form little bubbles in LHe and dense helium gas (R~6Å)

➔ Above 1K : molecules trace the normal-fluid component only.

➔ Below 0.6 K : molecule bubbles should bind to vortex lines

(D. Zmeev, et al, Phys. Rev. Lett., 110, 175303 (2013))

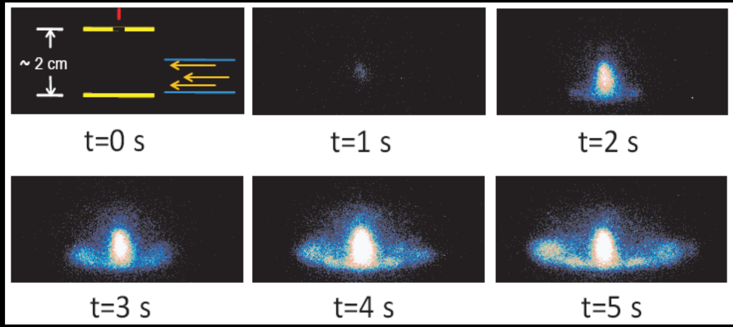
Imaging He_2^* molecular tracers via laser-induced fluorescence:



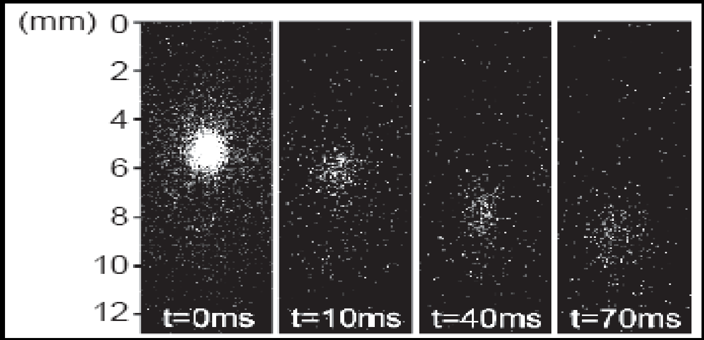
W.G. Rellergert *et al.*, Phys. Rev. Lett, 100 (2008).

For molecules in the triplet ground state a(0):

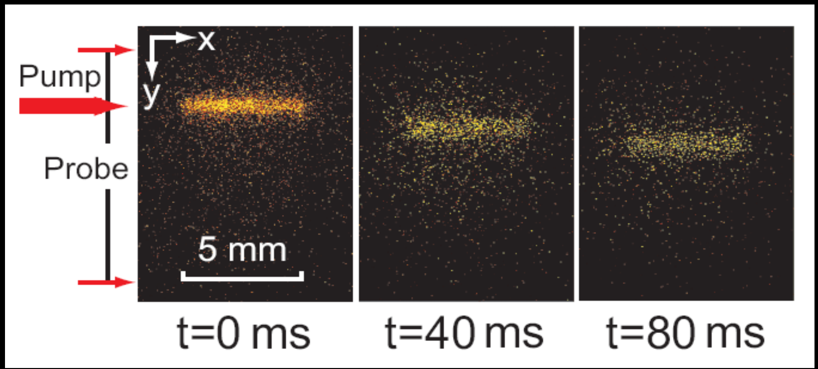
- A 905 nm pulsed laser is used to drive a cycling transition.
- Fluorescent light emitted at 640 nm.



Guo, et al., J. Low Temp. Phys., 158, 346 (2009)



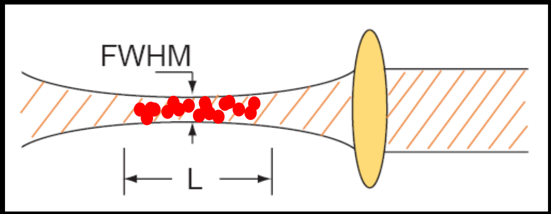
Guo, *et al.*, Phys. Rev. Lett., 102, 235301 (2009)



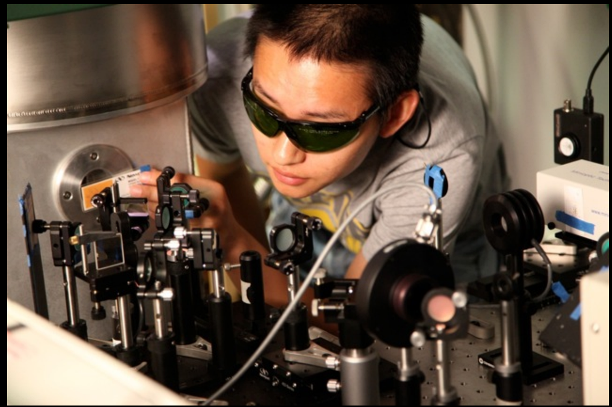
Guo, et al., Phys. Rev. Lett. 105, 045301 (2010).

New technique: He_2^* molecular tracer-line tracking

Femtosecond laser field ionization in helium:



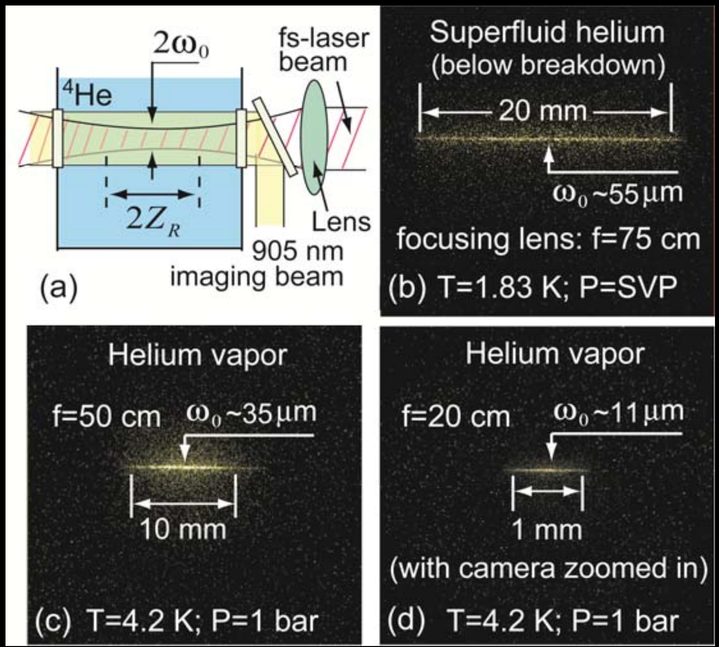
$I \geq 10^{13}$ W/cm^2



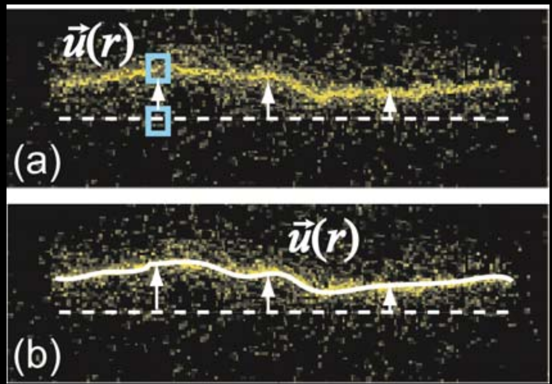
Pulse length: 35 fs

Pulse energy: up to 4 mJ

Rep rate: up to 5 kHz



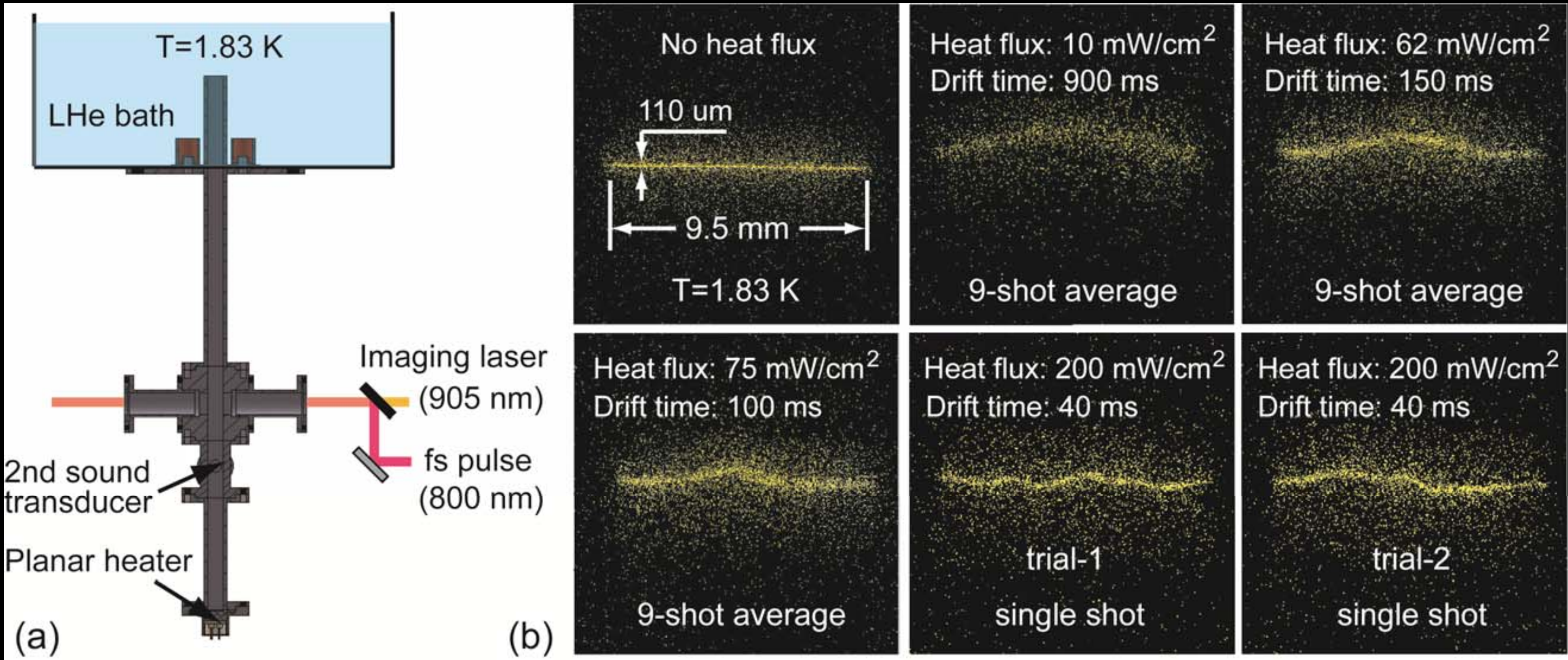
J. Gao, et al., Rev. Sci. Instrum. 86, 093904 (2015)



W. Guo, *et al.*, PNAS, 111, 4653 (2014)

- Thin tracer lines can be produced and tracked, allowing high precision flow field measurement.
- This technique is applicable to He-I and gaseous helium.

Application to steady-state thermal counterflow:



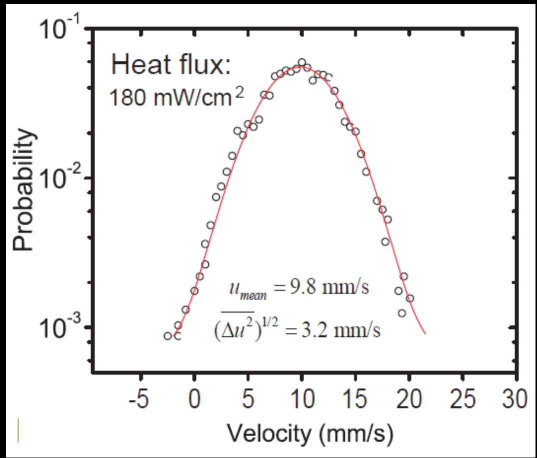
A. Marakov, G. Jian, et al., Phys. Rev. B 91, 094503 (2015).

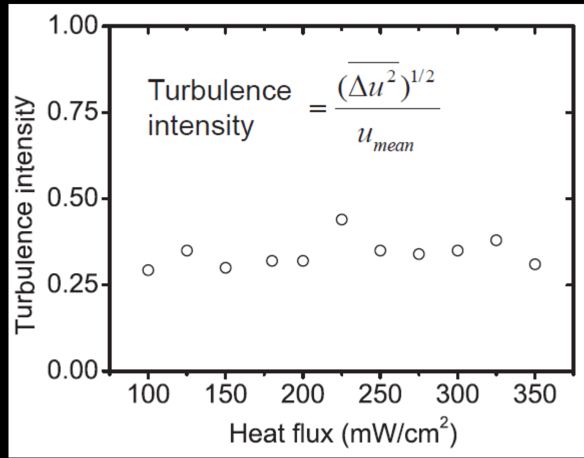
Major observations:

1) Three distinct velocity profiles of the normal fluid were observed:

Parabolic laminar profile → Tail-flatten laminar profile → Distorted turbulent profile

2) The velocity PDF in turbulent normal fluid is found to be a Gaussian. The turbulence intensity is measured to be about 35%.



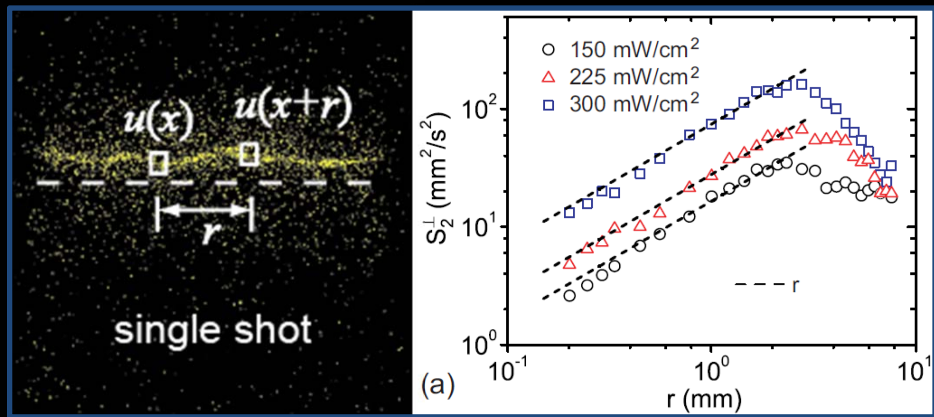


3) The 2nd order transverse structure function is calculated

$$S_2^{\perp}(R,r) = \overline{\left\langle \left(u(R+r) - u(R)\right)^2 \right\rangle}$$

$$S_2(r) \propto r^n \Leftrightarrow E(k) \propto k^{-(n+1)}$$

$$\Rightarrow \tilde{E}(k) \propto k^{-2}$$



Study on decaying thermal counterflow

- Existing knowledge and open questions

According to Vinen's equation:

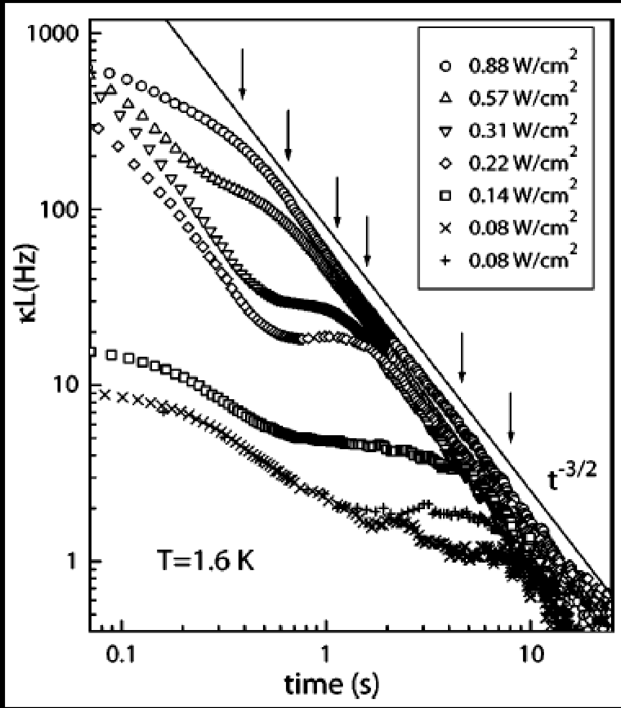
$$\frac{dL}{dt} = \frac{\rho_n B \chi_1}{2\rho} u_{ns} L^{3/2} - \frac{\kappa \chi_2}{2\pi} L^2 = 0$$

1) In steady state counterflow:

$$L_0 = \gamma^2 u_{ns}^2$$

2) In decaying counterflow:

$$L(t) = L_0 \left(1 + \frac{\chi_2 \kappa L_0}{2\pi} t\right)^{-1}$$



L. Skrbek, A.V. Gordeev, and F. Soukup, PRB 67, 047302 (2003)

However, Skrbek et al discovered that at long decay time:

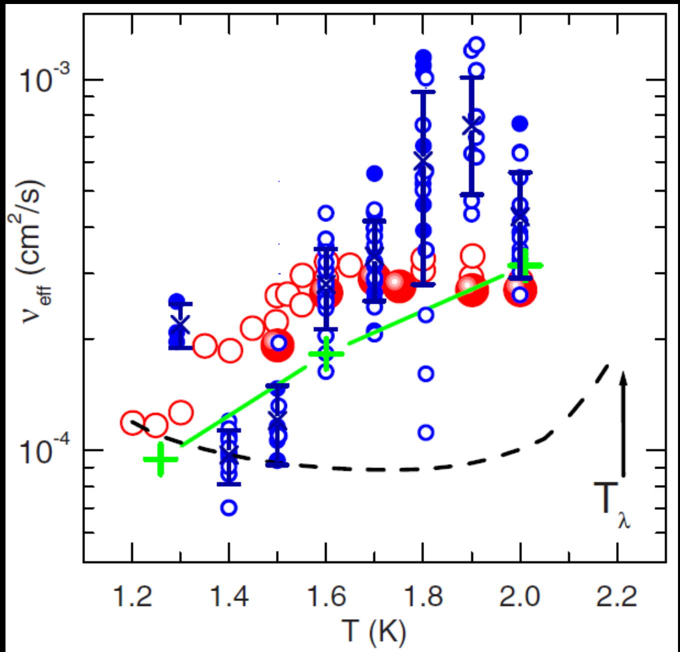
$$L(t) \propto t^{-3/2}$$

- The two fluids are coupled in the absence of the heat source (in about 10 ms). (Justified, see Vinen, PRB 61, 1410 (2000))
- The turbulence at long decay time has a classical Kolmogorov energy spectrum with an energy-containing eddy size D.

With the assumed Kolmogorov turbulence energy form:

$$\frac{dE}{dt} = -\nu\omega^2 = -\nu'(\kappa L)^2$$

$$\Rightarrow \quad L(t) = \frac{(3C)^{3/2} D}{2\pi\kappa\nu^{1/2}}(t-t_0)^{-3/2}$$



T.V. Chagovets, A.V. Gordeev, and L. Skrbek, PRE 76, 027301 (2007)

Questions:

1) Why does the vortex density decay differently from Vinen's equation?

2) Is there indeed a Kolmogorov spectrum at long decay time?

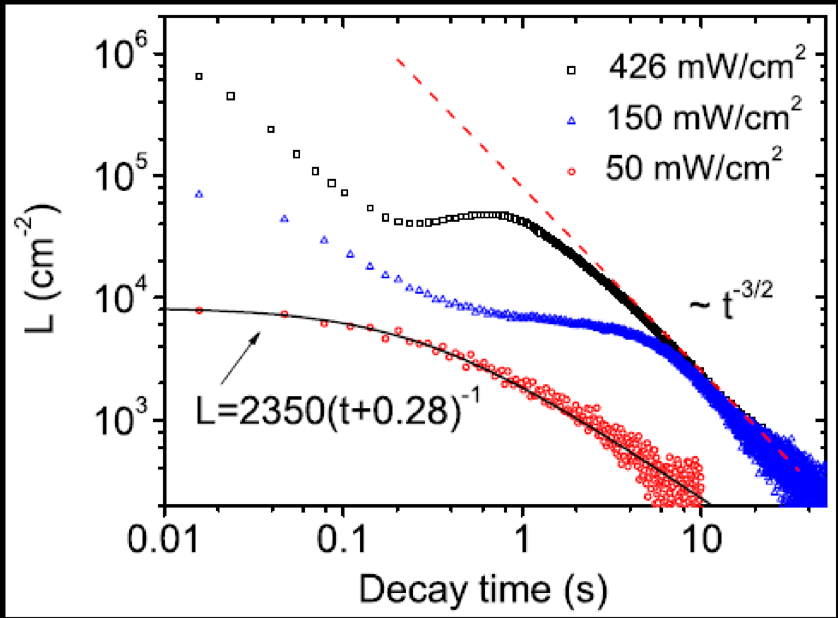
3) Can we test the energy decay rate equation?

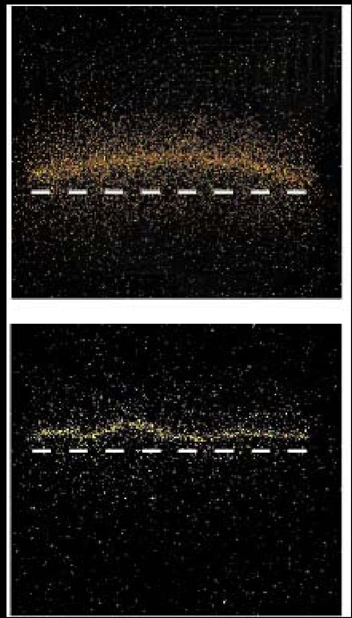
4) Is the reported ν' reliable?

We can now answer them all with the combination of 2nd sound and flow visualization !

Results in decaying counterflow

- The decay of vortex line density (1.65 K)





Laminar regime:

$Q <\sim 60\ mW/cm^2$

Turbulent regime:

$Q >\sim 60\ mW/cm^2$

Observations:

(1) L~ t^(-3/2) at long decay times is seen when the initial heat flux is large.

(2) We see pure Vinen decay when the initial heat flux is low. (new)

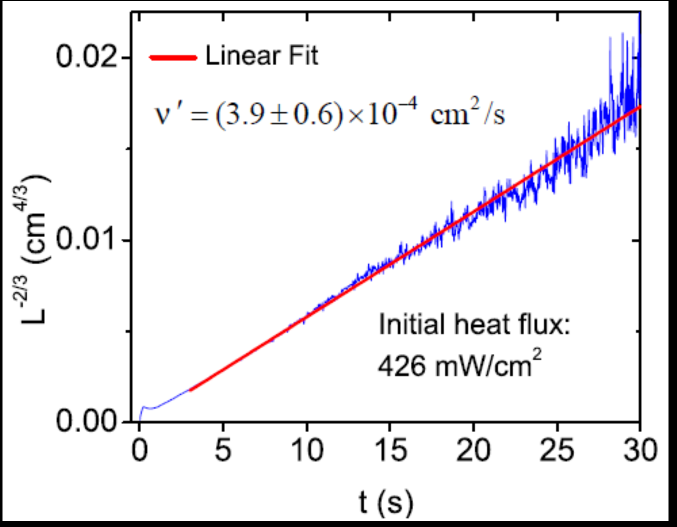
(3) Visualization in the normal fluid shows that the t^(-3/2) behavior appears only when the normal fluid is turbulent in steady state. (new)

(4) From the vortex density data, we can determine the effective viscosity for the decay of both large scale turbulence and Vinen turbulence:

1) fit the t^(-3/2) part of the vortex curve at long decay time:

$$L(t) = \frac{(3C)^{3/2} D}{2\pi\kappa \nu_{eff}^{1/2}} (t - t_0)^{-3/2}$$

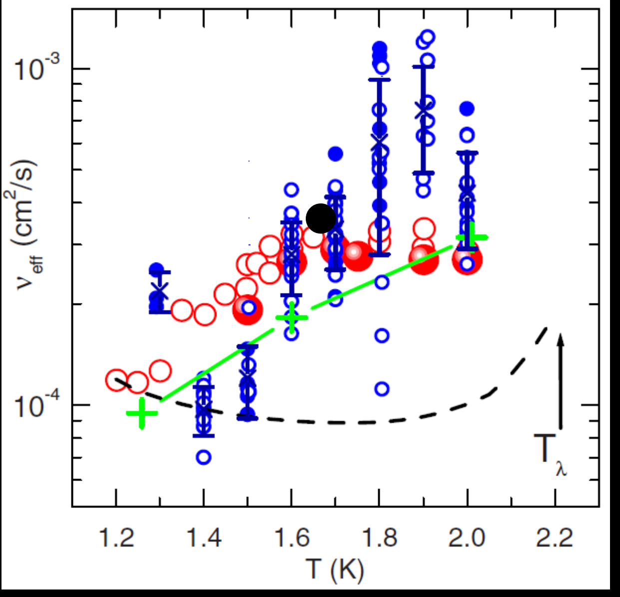
$$\nu' = (3.9 \pm 0.6) \times 10^{-4} \ \text{cm}^2/\text{s}$$



2) fit the Vinen decay data at small heat fluxes:

$$L^{-1}(t) = L_0^{-1} + \left(\frac{\chi_2 \kappa}{2\pi}\right) \cdot t$$

$$\nu' = (3.9 \pm 0.3) \times 10^{-4} \ \text{cm}^2/\text{s}$$

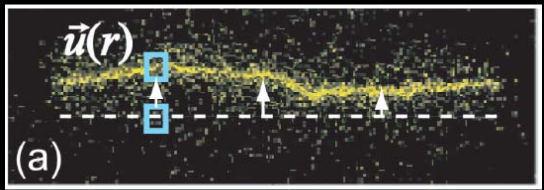


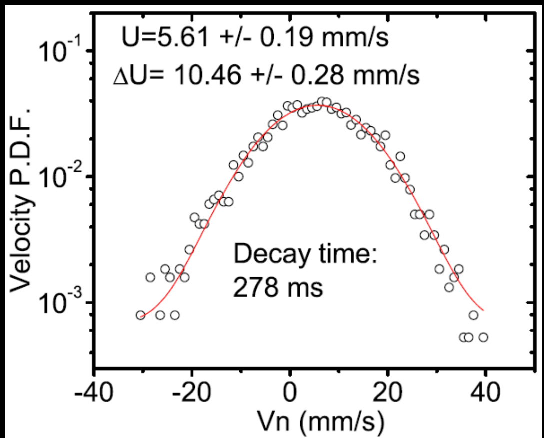
T.V. Chagovets, A.V. Gordeev, and L. Skrbek, PRE 76, 027301 (2007)

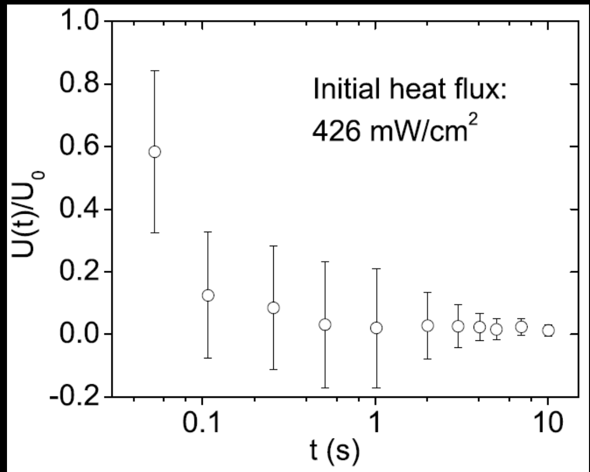
- The decay of turbulence energy (1.65 K)

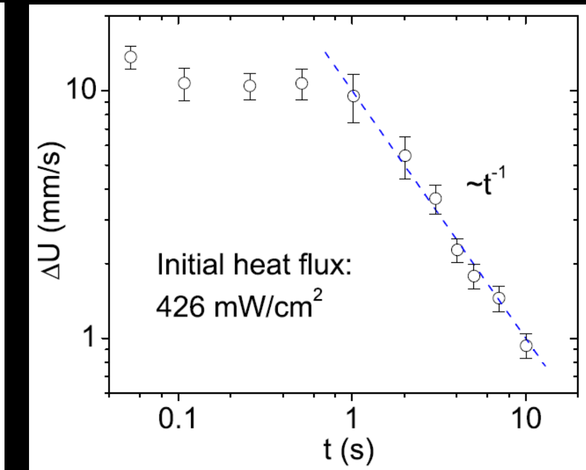
Note: the normal fluid and superfluid become coupled in a few tens of ms after the decay starts

→ $U_n = U_s$ at scales greater than vortex spacing !







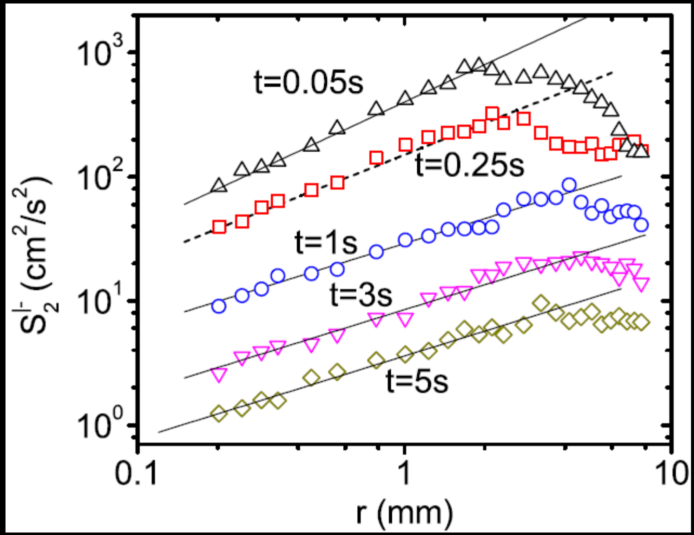


(1) By tracking the tracer lines, we can determine the normal fluid velocity and analyze its PDF.

(2) We can determine the mean velocity and turbulent velocity fluctuation from the PDF.

- Mean velocity decays to zero quickly (< 0.1 s).
- Velocity fluctuation decays as t^(-1) after ~ 1 s.

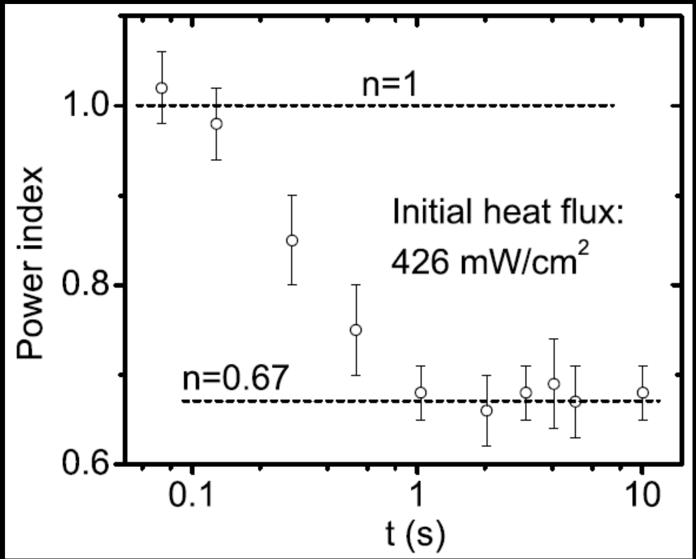
(3) The 2nd order transverse structure function shows spectrum evolution



$$S_2^{\perp}(R,r) = \overline{\left\langle \left(u(R+r) - u(R)\right)^2 \right\rangle}$$

which contains information about the turbulence energy spectrum:

$$S_2(r) \propto r^n \Longleftrightarrow \tilde{E}(k) \propto k^{-(n+1)}$$



Observations:

- At short decay times: n=1

$$\Rightarrow \tilde{E}(k) \propto k^{-2}$$

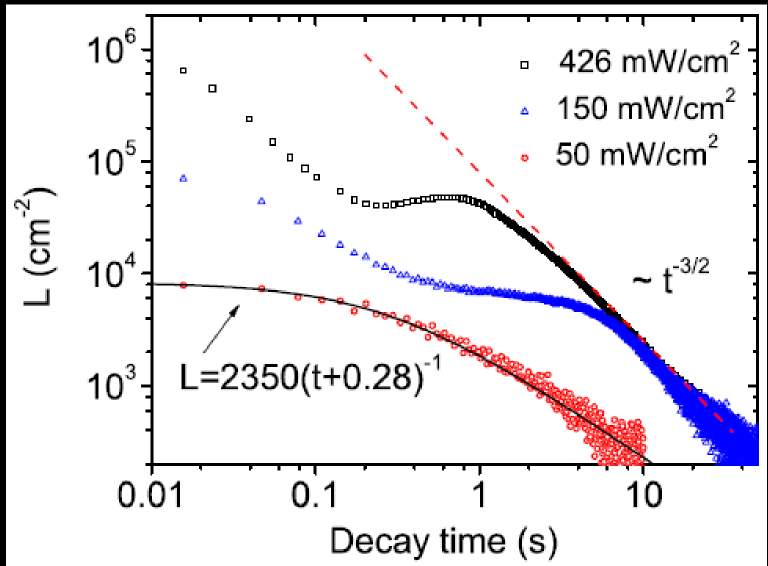
- At long decay times: n=2/3

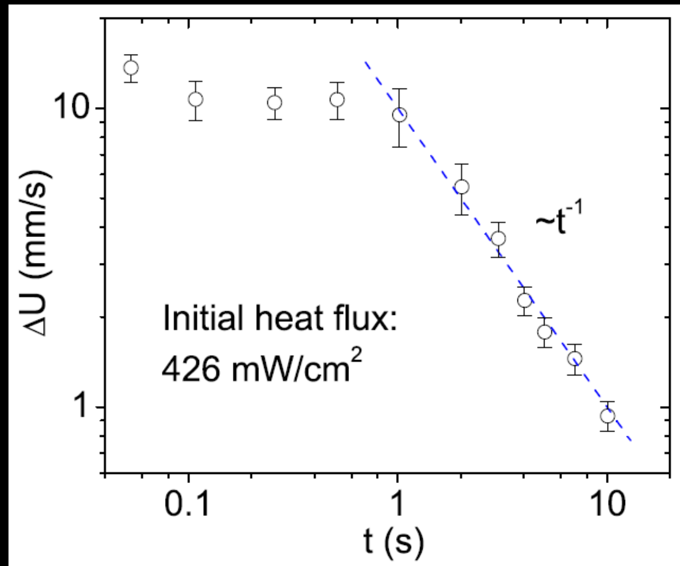
$$\Rightarrow \tilde{E}(k) \propto k^{-5/3}$$

(Confirms Kolmogorov energy spectrum !)

- Testing the turbulent energy decay rate expression:

$$\frac{dE}{dt} \simeq \kappa^2 \frac{dL}{dt} + \frac{dE_2}{dt} = -\nu'(\kappa L)^2 \Leftrightarrow E(t) \simeq \kappa^2 L(t) + E_2(t) = \nu' \cdot \int_t^\infty \kappa^2 L^2(t')dt'$$

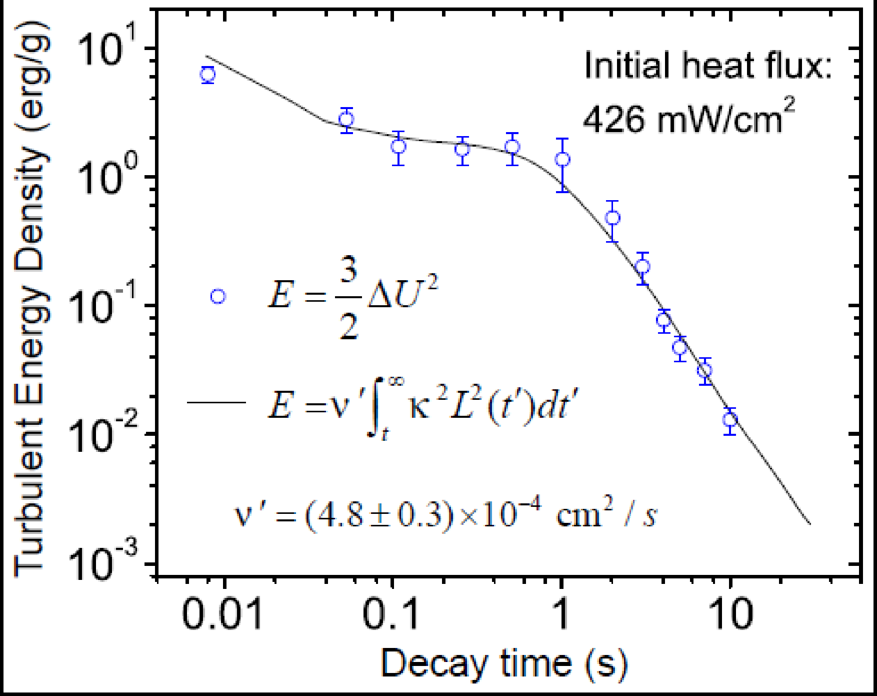




One can easily see that at t > 0.1 s :

$\kappa^2 L(t)$ is only a few percent of $E_2(t) = \frac{3}{2}(\Delta U)^2$

Let's compare $\frac{3}{2}(\Delta U)^2$ and $\nu' \cdot \int_t^\infty \kappa^2 L^2(t')dt'$



J. Gao, W.F. Vinen, and W. Guo, in preparation (2015).

(1) The profiles of $\frac{3}{2}(\Delta U)^2$ and $\nu' \cdot \int_t^\infty \kappa^2 L^2(t')dt'$ agree quite well.

➔ Energy decay rate equation is tested.

(2) The effective viscosity ν' as a scaling factor can be determined by requiring best fit of the two curves at long decay times.

$$\nu' = (4.8 \pm 0.3)\times 10^{-4}\ \text{cm}^2/\text{s}$$

Summary of effective viscosity (1.65 K) :

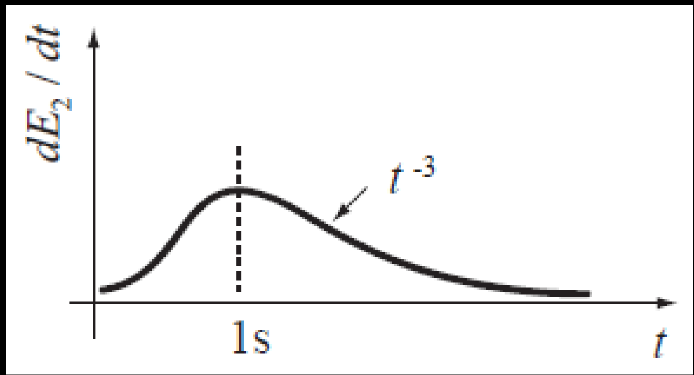
	Vinen decay	Vortex density fit	Energy decay fit
ν' $(10^{-4}\text{cm}^2/\text{s})$	3.9 ± 0.3	3.9 ± 0.6	4.8 ± 0.3

- Some additional comments :

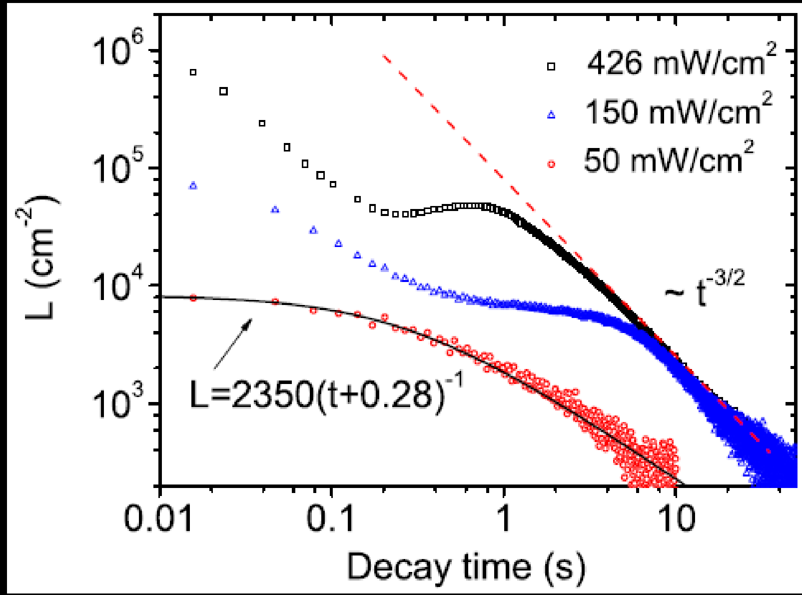
(1) vortex density decay

$$\frac{dE}{dt} \simeq \kappa^2 \frac{dL}{dt} + \frac{dE_2}{dt} = -\nu'(\kappa L)^2$$

dE_2 / dt is nearly zero initially due to its k^(-2) spectrum. As the spectrum evolves, this term becomes dominant:



The "bump" structure in the vortex density decay is caused by the spectrum evolution of the large scale turbulence.



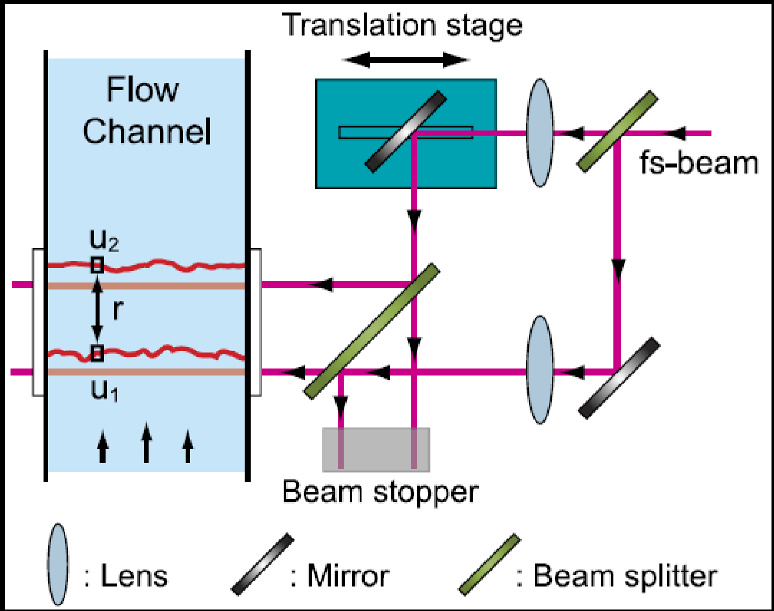
(2) Our tracer-line tracking technique currently has a resolution of about 100 um. It is comparable to the vortex spacing:

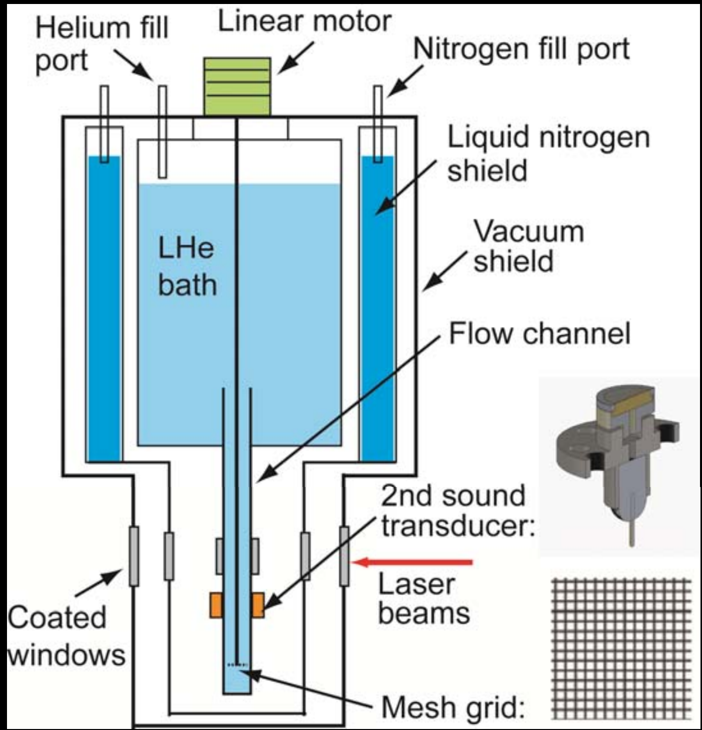
L=10^5: line spacing ~ 30 um

L=10^4: line spacing ~ 100 um

Future measurements

(1) We plan to create two tracer lines for measuring the longitudinal structure functions.





$$S_n^{\parallel}(x,r) = \overline{\left\langle \left(u(x+r) - u(x) \right) \cdot \hat{r} \right\rangle^n}$$

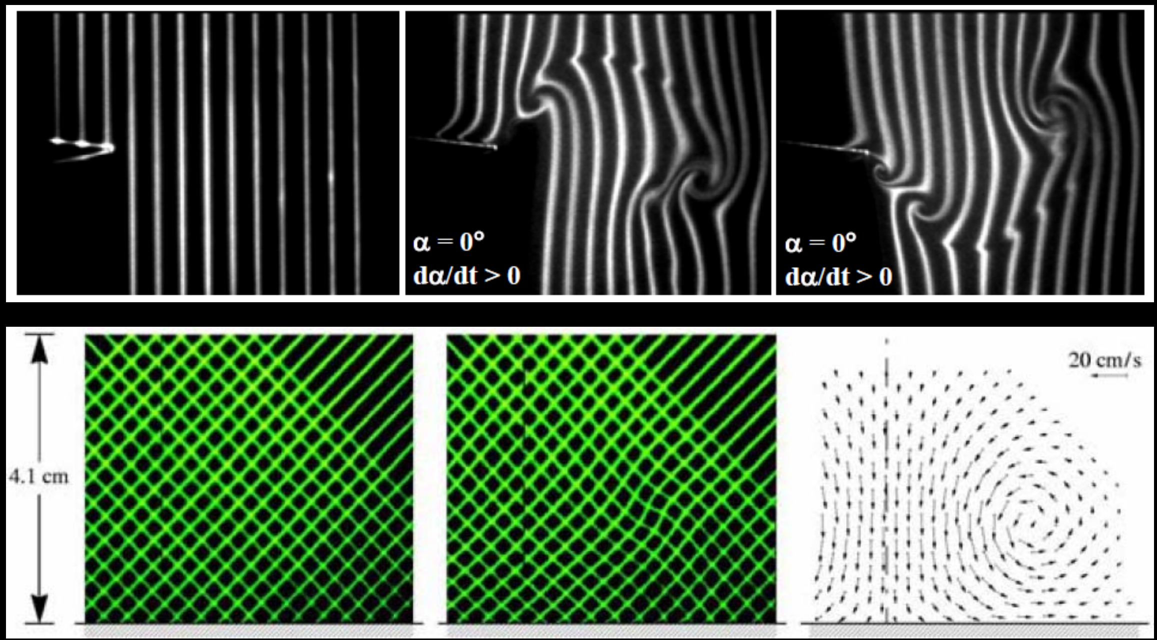
We will apply it to the study of towed-grid generated turbulence. For turbulence with a Kolmogorov spectrum:

$$S_n^{\parallel}(r) \propto r^{n/3}$$

We will examine the deviation to study intermittency in grid QT.

In the future, we hope to design optical systems to create grids and cross tracer-line patterns:

State-of-art MTV experiments in classical fluids



M.M. Koochesfahni and D.G. Nocera, "Molecular tagging velocimetry maps fluid flows", Laser Focus World 37 (6), 103 (2001).

High precision measurements of all velocity components can be made. More complex flow parameters such as the vorticity field can be probed.

Summary

1) The energy decay rate expression is directly tested for the first time.

2) In decaying counterflow, vortex decay is affected by large scale turbulence in steady state.

3) We confirm a Kolmogorov spectrum at long decay time.

4) Previously reported effective viscosity of QT is reliable.

END

Questions?