Turbulence and irreversibility in space plasmas

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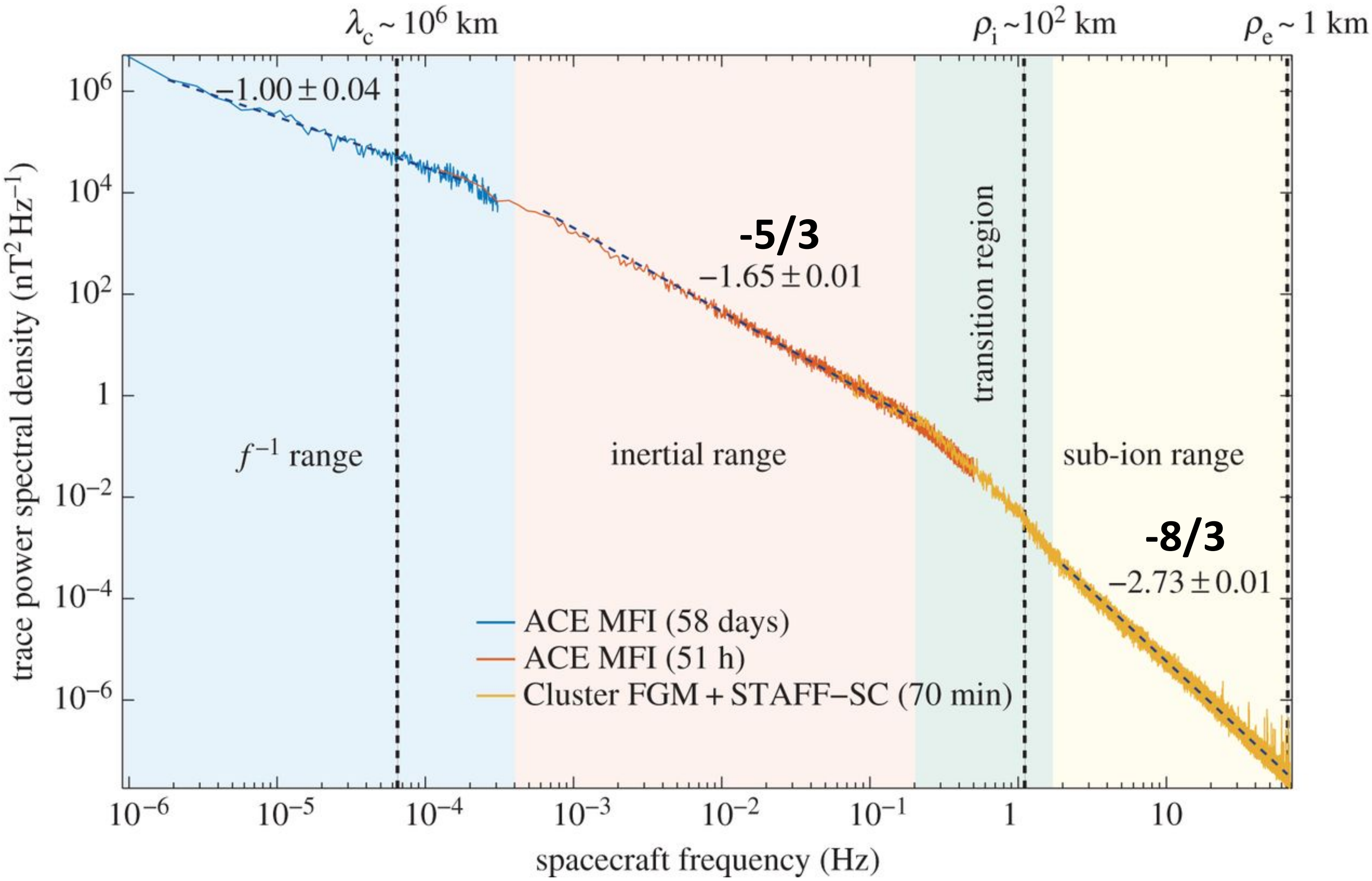
Solar wind (wave) turbulence

at 1 AU



[Kiyani+, PTRSA, 2015]

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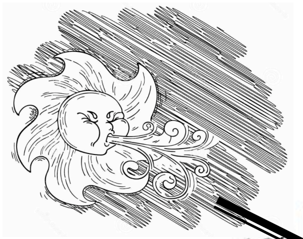
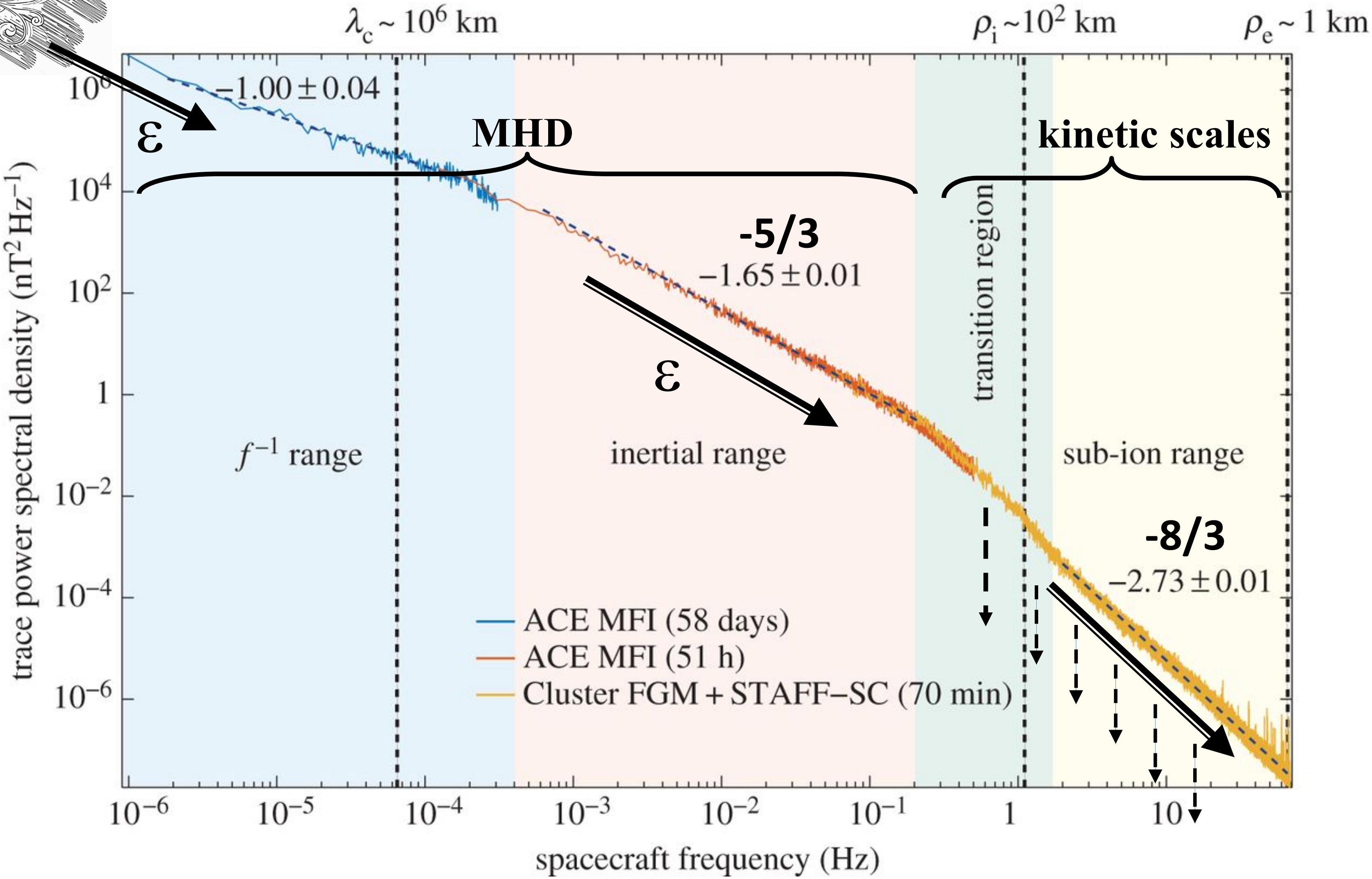




[Kiyani+, PTRSA, 2015]

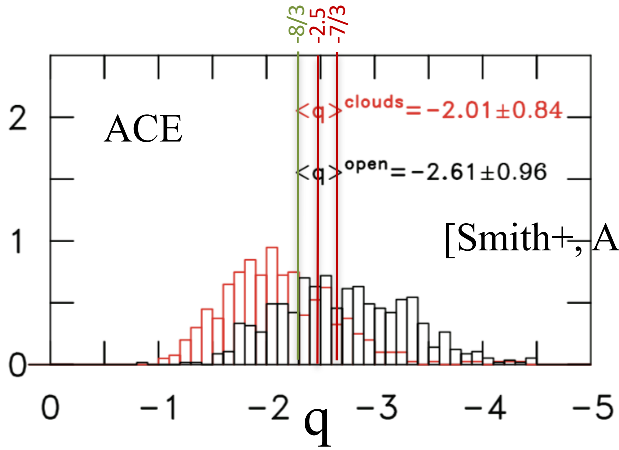
Discussion about

1) Turbulence *versus* reversibility at kinetic scales: -8/3

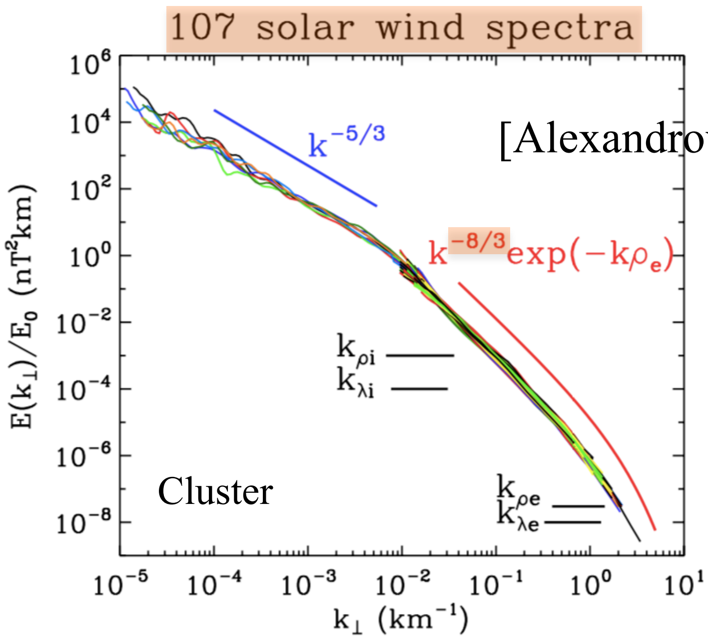
2) Exact laws and anomalous heating: ε

1) SW turbulence at kinetic scales

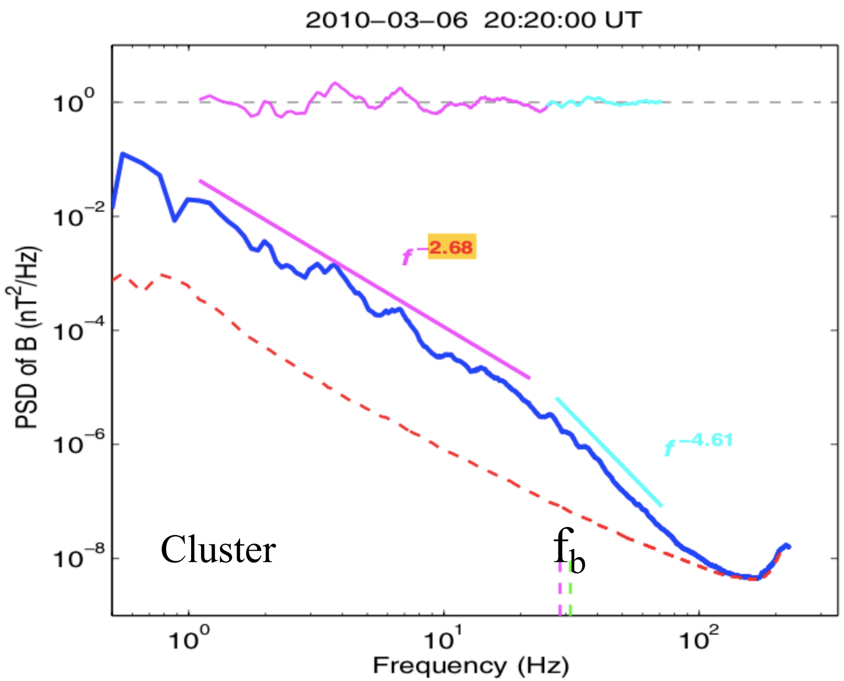
[see also: Behannon, RGSP, 1978; Denskat+, JGZG, 1983; Leamon+, JGR, 1998]

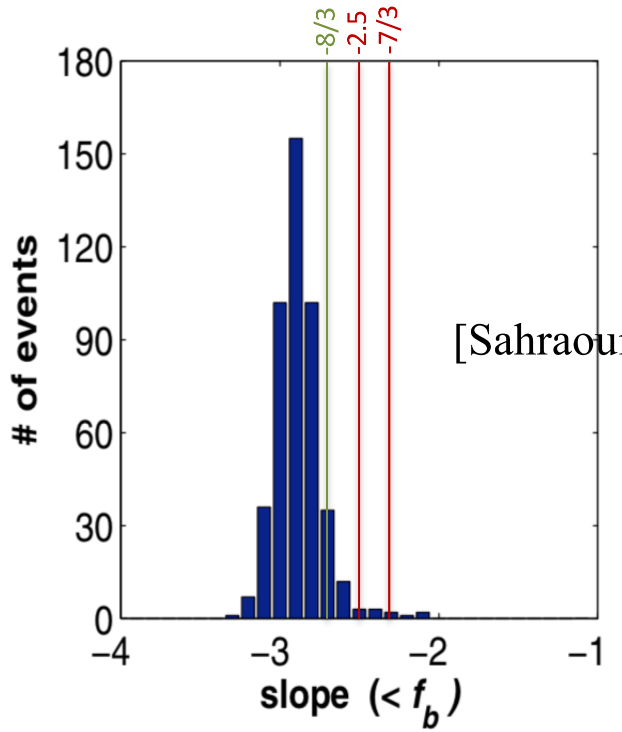


[Smith+, ApJ, 2006]



[Alexandrova+, ApJ, 2012]



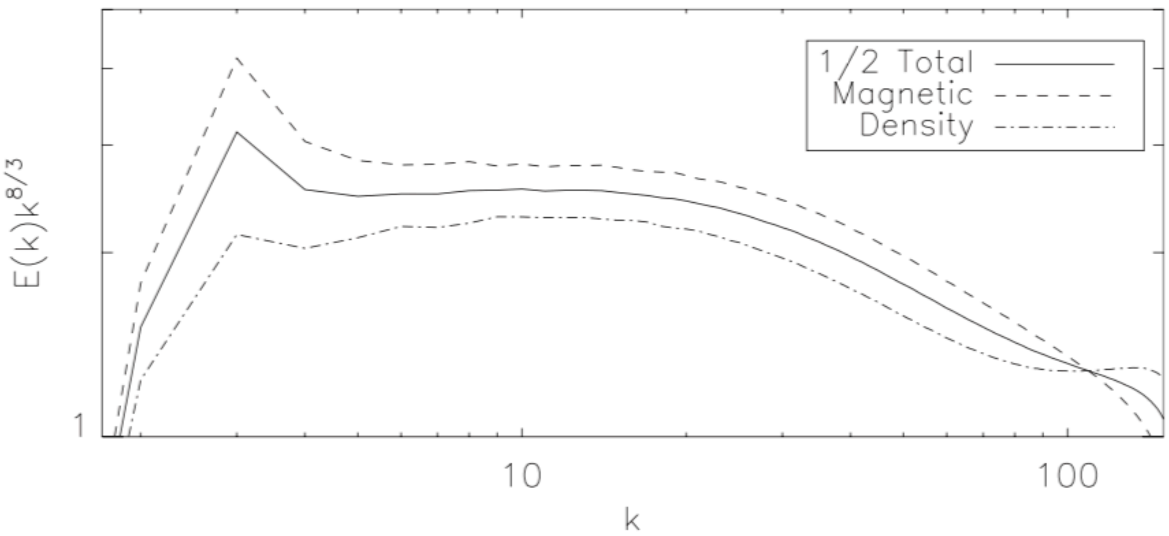


[Sahraoui+, ApJ, 2013]

Phenomenological/analytical predictions

Theoretical predictions:

- Strong (wave) turbulence → **-7/3** (for k or $k_\perp$)
  [Biskamp+, PRL, 1999]
- Weak wave turbulence → **-2.5** (for $k_\perp$)
  [SG & Bhattacharjee, PoP, 2003]
- Non-standard phenomenology → **-8/3** (for $k_\perp$)
  [Boldyrev & Perez, ApJL, 2012; Meyrand & SG, PRL, 2013]



Is it really convincing ?

Diffusion model for kinetic scales

Weak **KAW turbulence** equations $\Rightarrow$ nonlinear diffusion model in the **strongly local** interaction limit (triadic interactions)

[Passot & Sulem, JPP, 2019; Meyrand & SG, JPP, 2015]

$$\frac{\partial E(k_\perp)}{\partial t} = C\frac{\partial}{\partial k_\perp}\left[k_\perp^7 E(k_\perp)\frac{\partial (E(k_\perp)/k_\perp)}{\partial k_\perp}\right] - \eta k_\perp^6 E(k_\perp)$$

Kinetic Alfvén /oblique whistler wave turbulence

$$\begin{cases} \dfrac{\partial E(k_\perp)}{\partial t} = -\dfrac{\partial \Phi_E(k_\perp)}{\partial k_\perp} \\ E(k_\perp) = A k_\perp^x \end{cases} \Rightarrow \quad \Phi_E(k_\perp) = A^2 C(1-x)k_\perp^{5+2x}$$

x=-2.5 is the stationary constant flux solution

Let's do a **decay** simulation with C=1

$$k_{\perp i} = 2^{i/8} \quad \eta = 10^{-16}$$

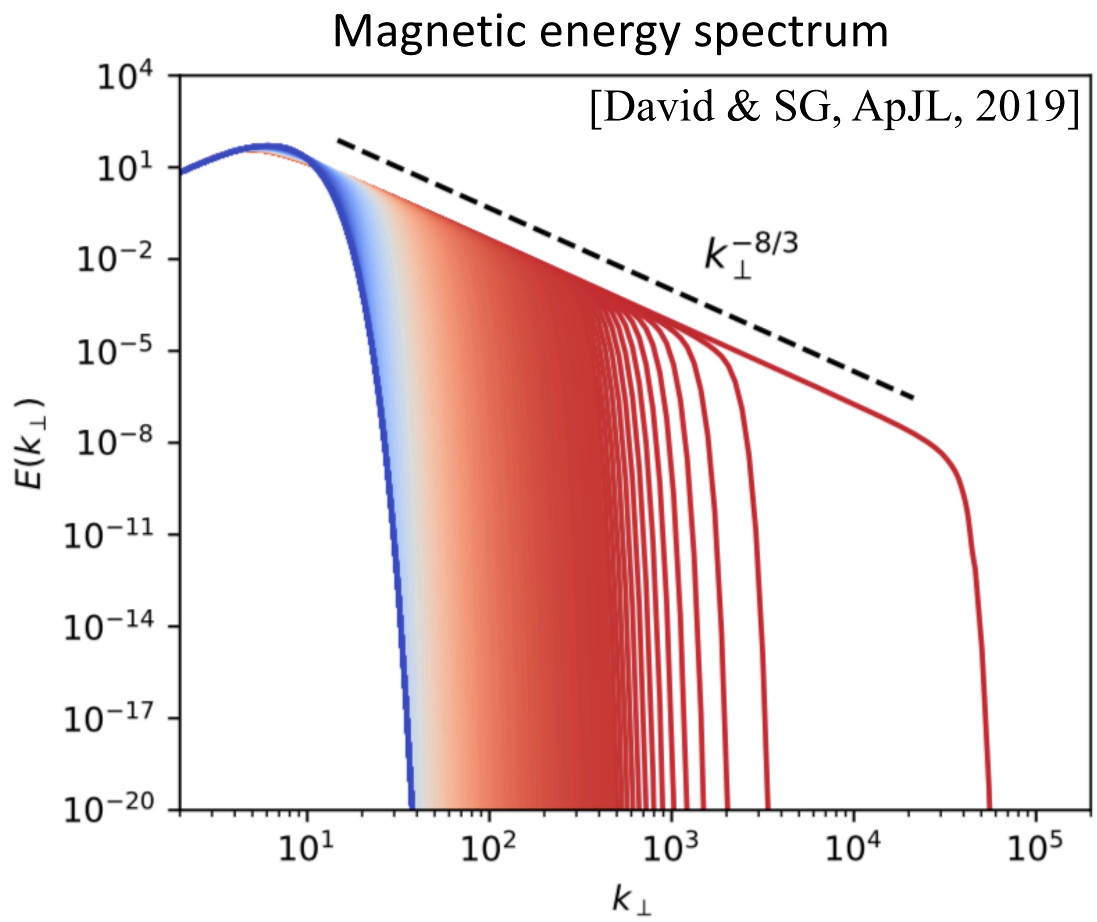
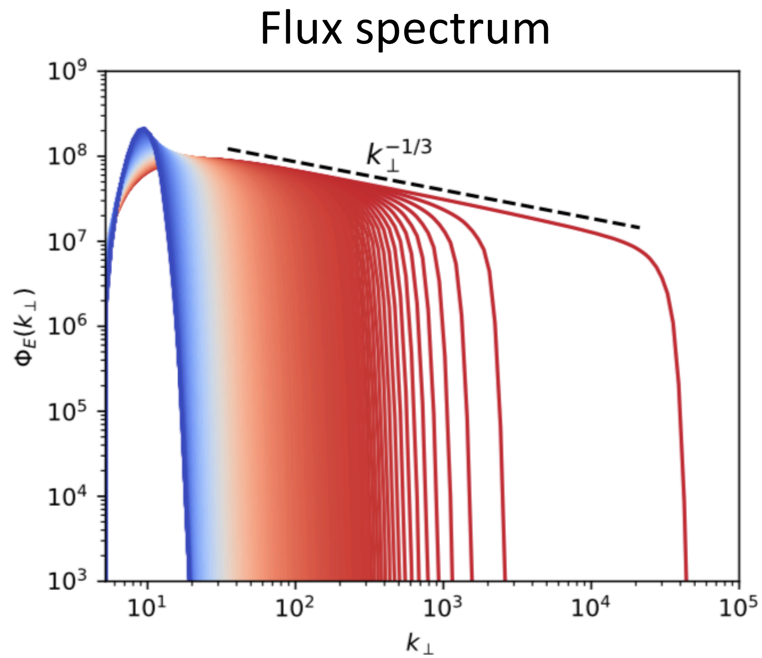


Figure 1. Time evolution (every $1000dt$) of the magnetic energy spectrum $E(k_\perp)$ from $t = 0$ (blue) to t_* (dark red). A $k_\perp^{-8/3}$ spectrum emerges over three decades.



-8/3 emerges as a non-stationary solution of KAW turbulence

[see also: Fournier & Frisch, MTA, 1983]

The 8/3–spectrum corresponds to a **self-similar solution of the second kind**

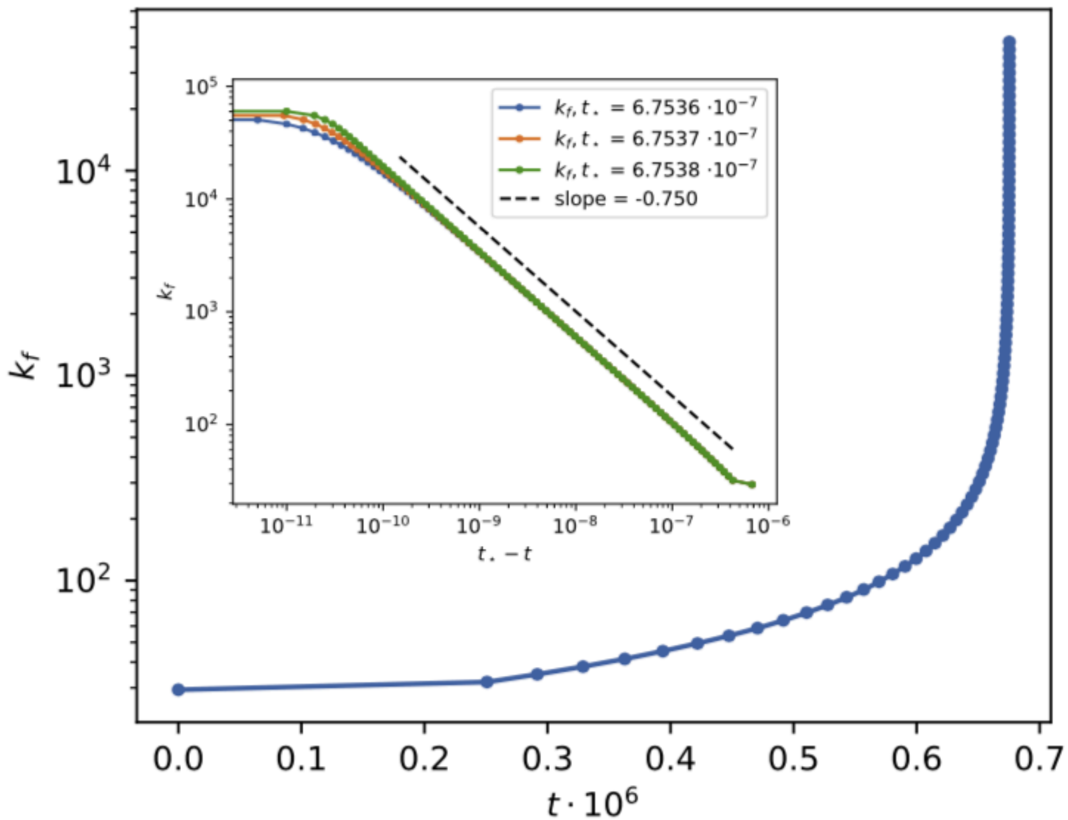
$$E(k_\perp) = \frac{1}{\tau^a} E_0\left(\frac{k_\perp}{\tau^b}\right)$$

$$\tau = t_* - t$$

$$E_0(\xi) \sim \xi^m$$

[David & SG, ApJL, 2019]

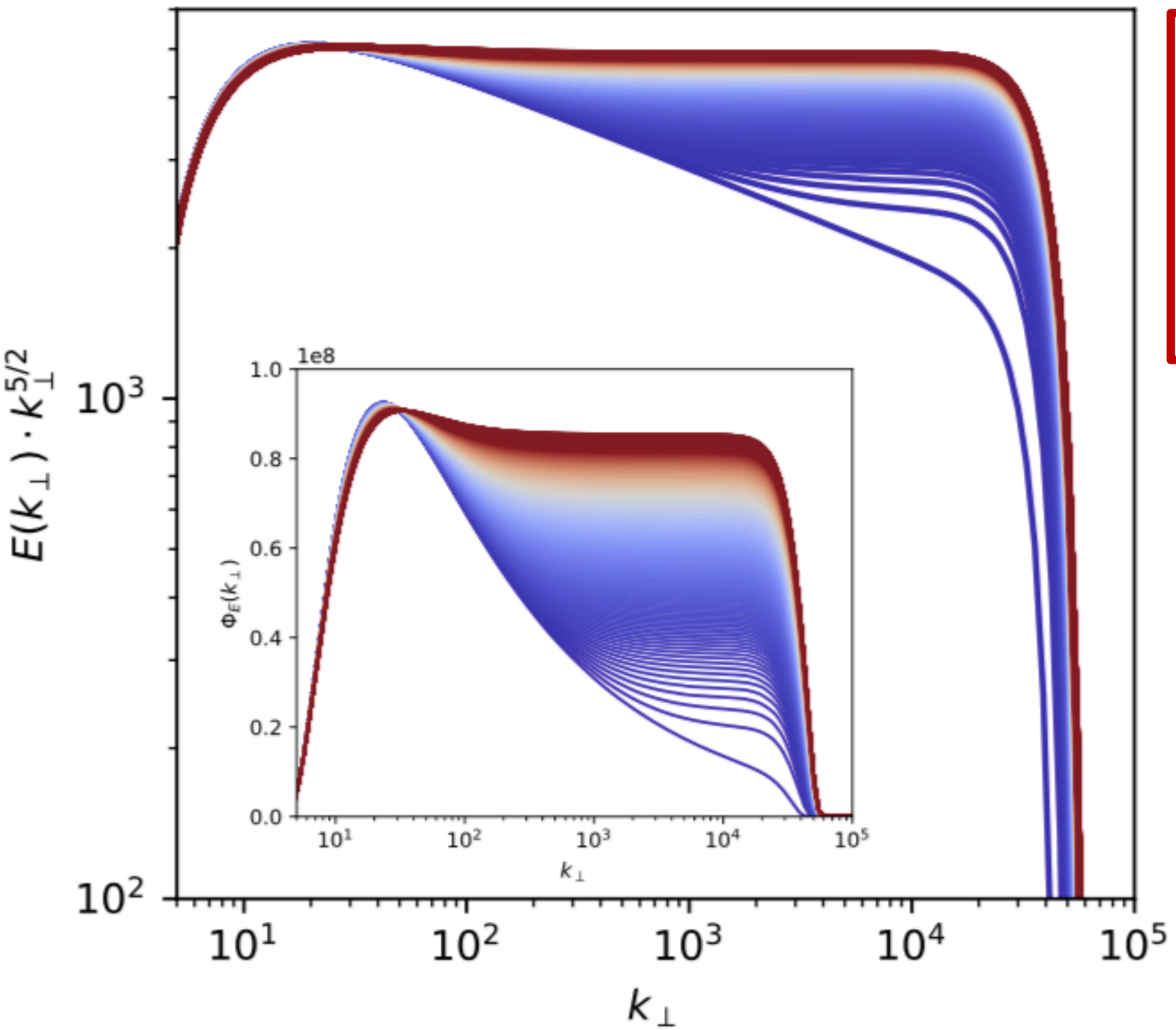
$$a = -2, \quad b = -3/4 \quad \text{and} \quad m = -8/3$$



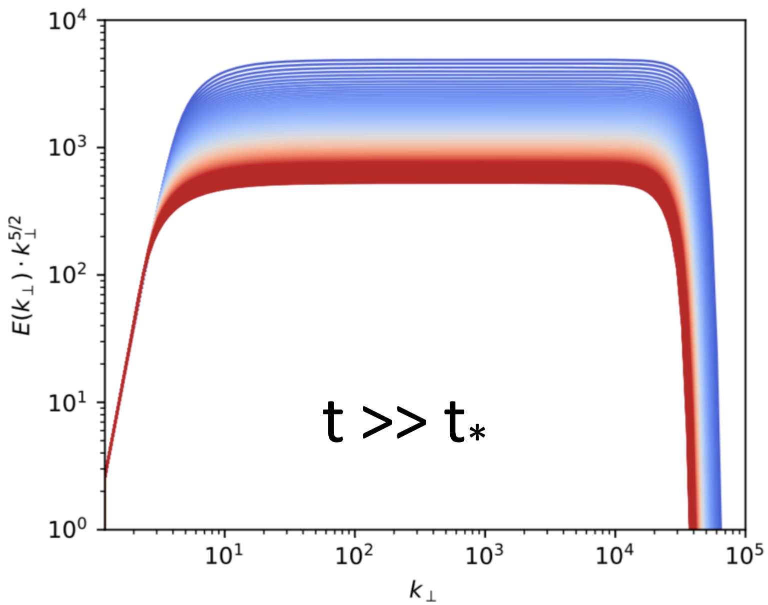
for $t < t_*$

k=+∞ is reached in a finite time

The stationary solution (-2.5) is then reached for t > t*



finite numerical box + viscous dissipation (irreversibility) affects the entire inertial range !



[David & SG, ApJL, 2019]

Interpretation for the solar wind plasma

- The solar wind is free and non-viscous

  ⇒ a **non-trivial** power law spectrum can emerge
  for which the turbulence phenomenology **is not relevant**

- Dissipation/irreversibility affects the **entire** inertial range

- Weak (KAW/whistler) turbulence leads to a **-8/3** magnetic spectrum

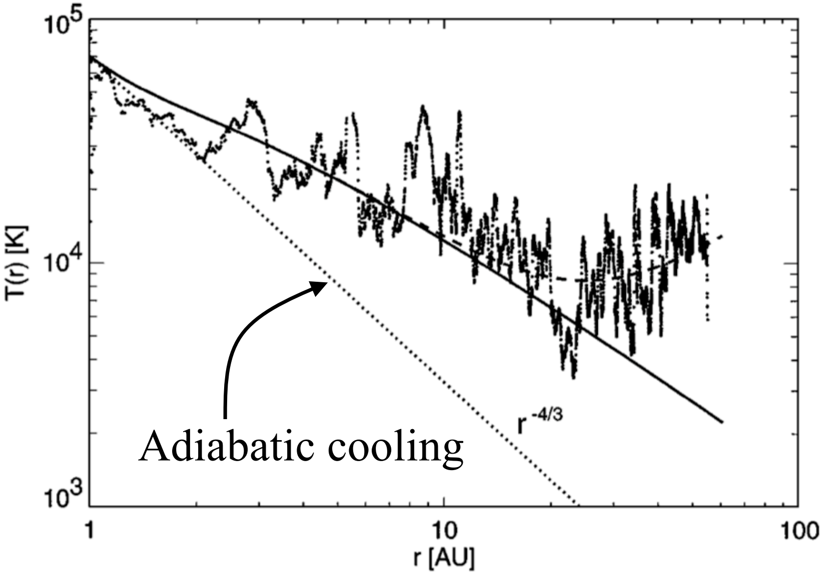
  (kinetic dissipation are neglected)

- Steep power laws can be interpreted as the **impact of kinetic effects** on the inertial range [-3.1;-8/3]

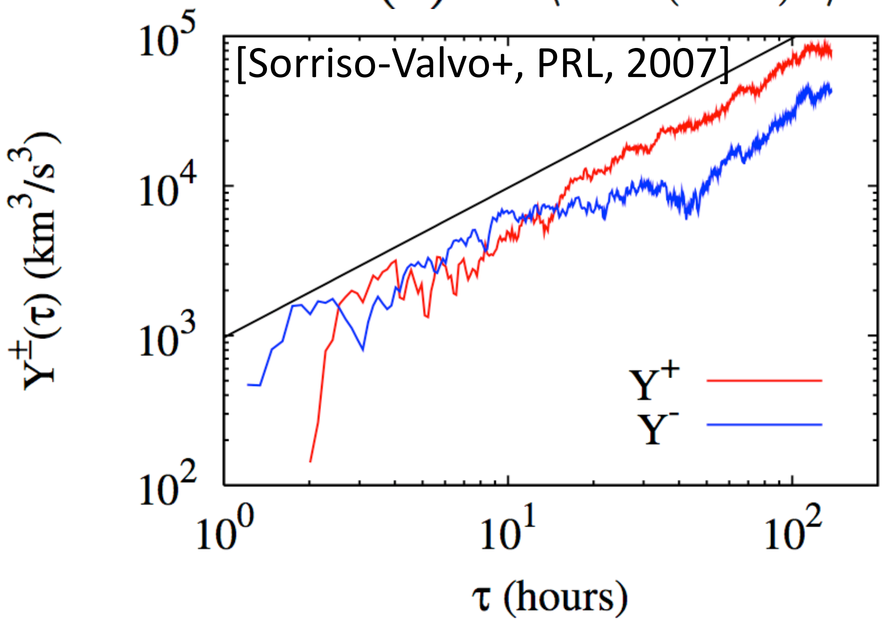
2) Exact MHD laws and plasma heating

Turbulence is a **source** of **heating** for the solar wind

[Marsch et al., JGR, 1982; Matthaeus et al., PRL, 1999]



$$Y^{\pm}(r) = \langle \delta \mathbf{z}^{\mp} (\delta \mathbf{z}^{\pm})^2 \rangle$$



$$\mathbf{z}^{\pm} \equiv \mathbf{u} \pm \mathbf{b}$$

$$-\frac{4}{3}\varepsilon^{\pm}\ell = \langle (\delta \mathbf{z}^{\pm} \cdot \delta \mathbf{z}^{\pm}) \delta z_{\ell}^{\mp} \rangle \,,$$

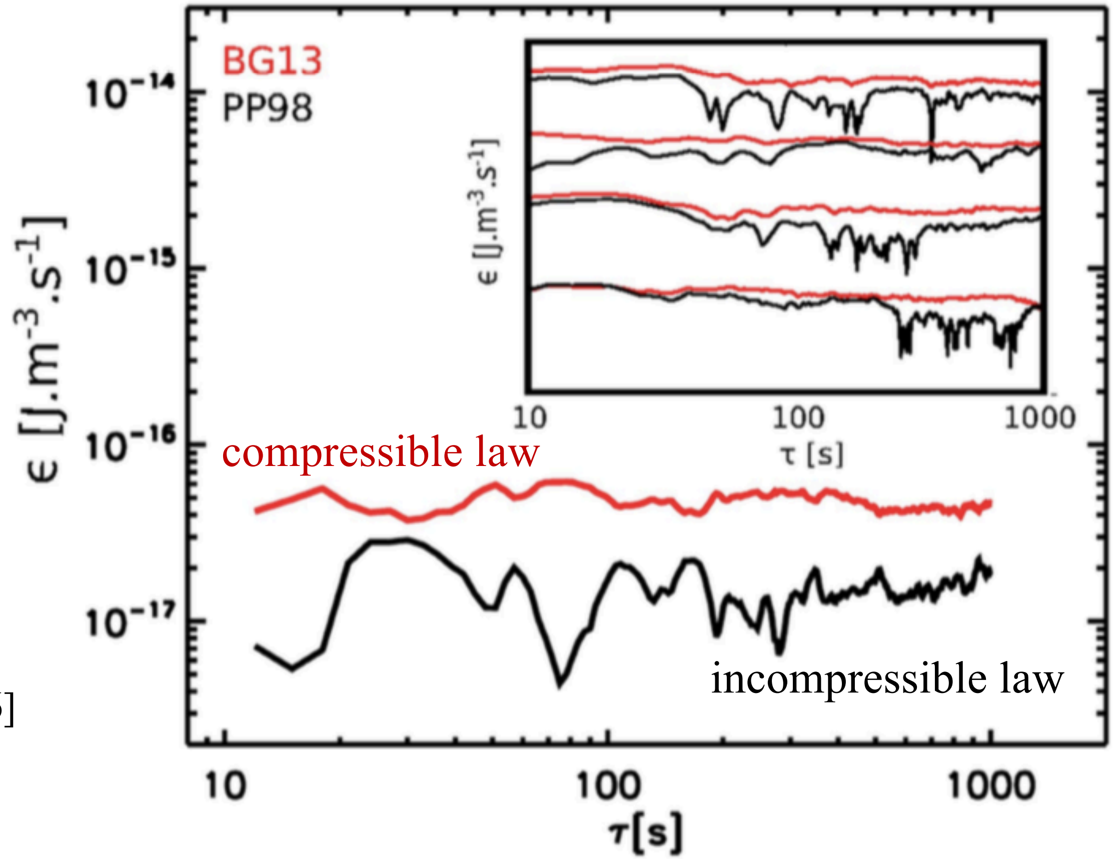
[Politano & Pouquet, PRE, 1998]

$\Longrightarrow$ $\varepsilon^{\pm} \sim 10^2 \text{ J kg}^{-1}\text{ s}^{-1}$

[Compressible laws for SW: Banerjee+, ApJL, 2016; Hadid+, ApJ, 2017; Andrés+, submitted, 2019]

[See also: Banerjee & SG, PRE, 2013; Andrés+, JPP, 2018; Ferrand+, ApJ, 2019]

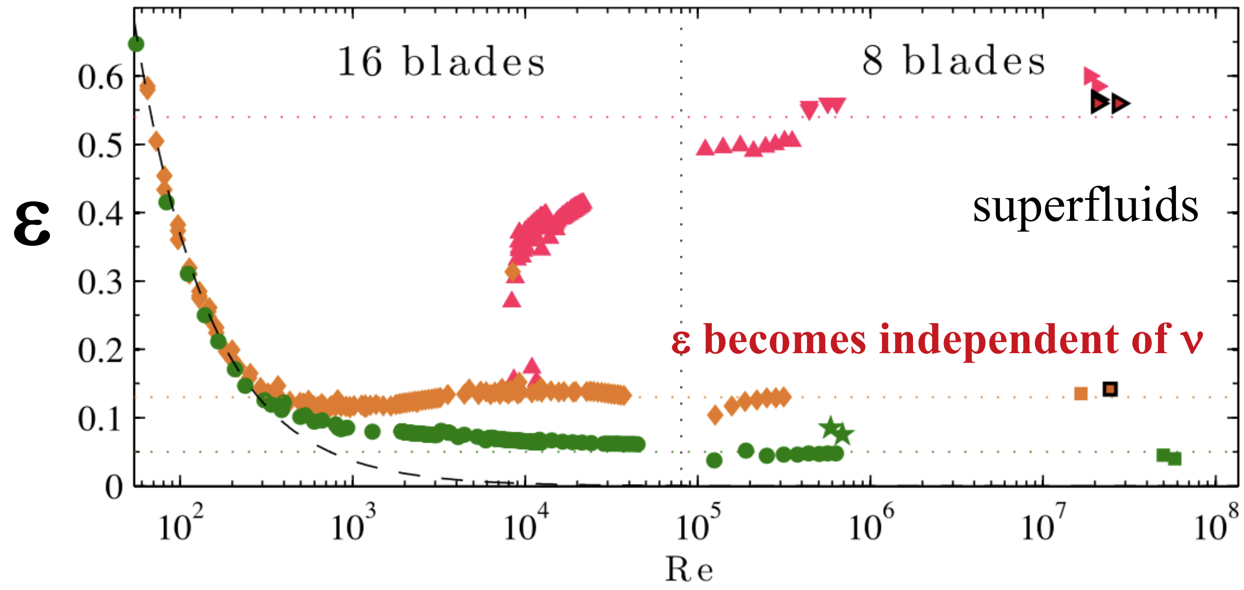
Solar wind data / THEMIS

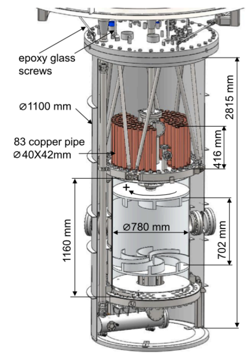


[Banerjee+, ApJL, 2016]

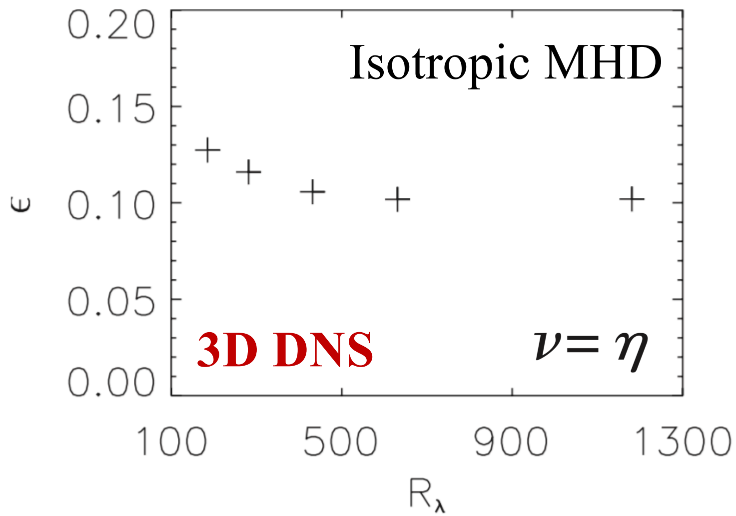
- Compressibility leads to higher turbulent heating
- The inertial range (plateau) is broader

The zeroth law of turbulence





[Ravelet et al., JFM, 2008;
Saint-Michel et al., PoF, 2014]

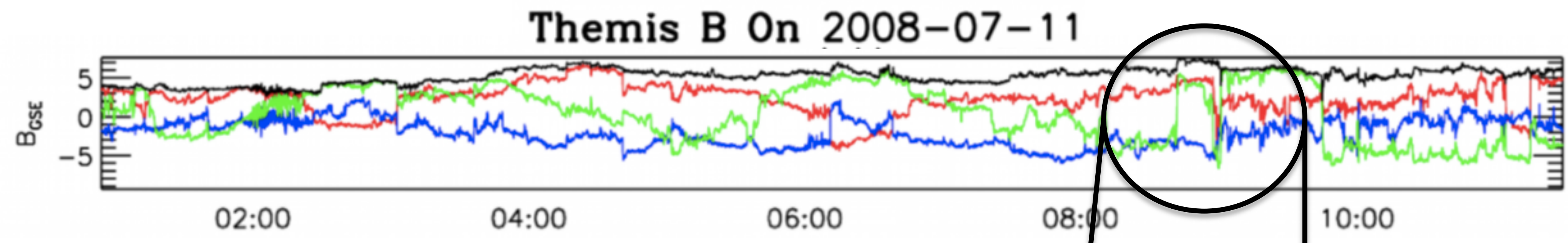


[Mininni & Pouquet, PRE, 2009]

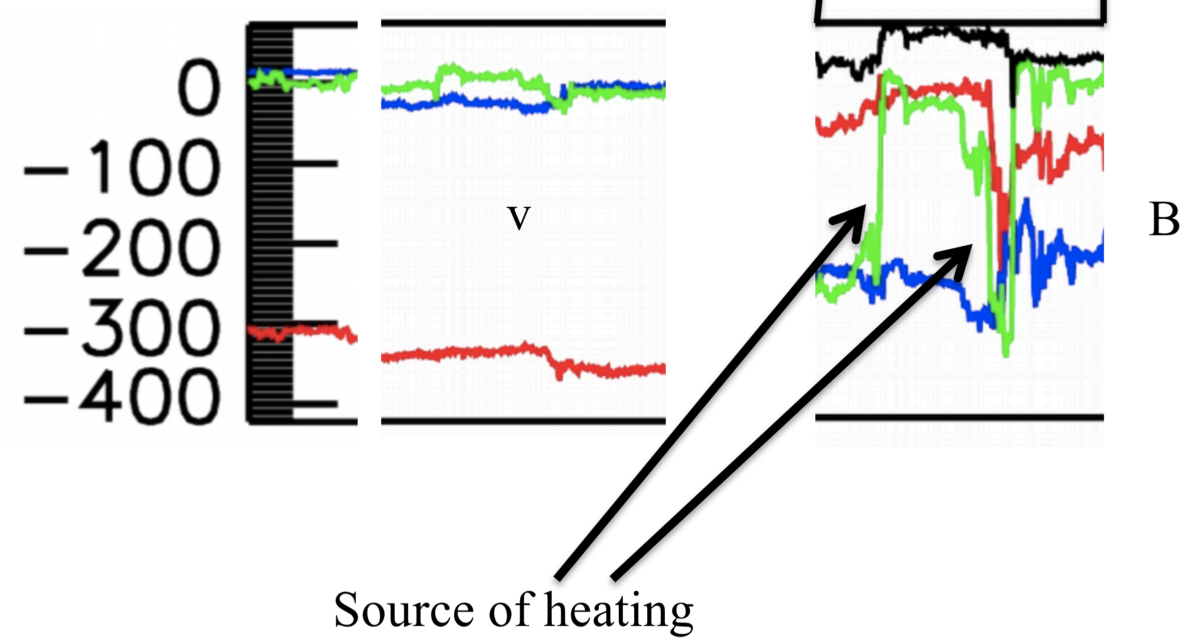
[For anisotropic MHD see: Bandyopadhyay+, PRX, 2018]

The zeroth law seems to be satisfied in incompressible HD and MHD
(Onsager's conjecture)

Fluctuations *versus* discontinuities



Solar wind



Can we estimate the heating produced by B-discontinuities ?

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Inertial energy dissipation for weak solutions of incompressible Euler and Navier–Stokes equations

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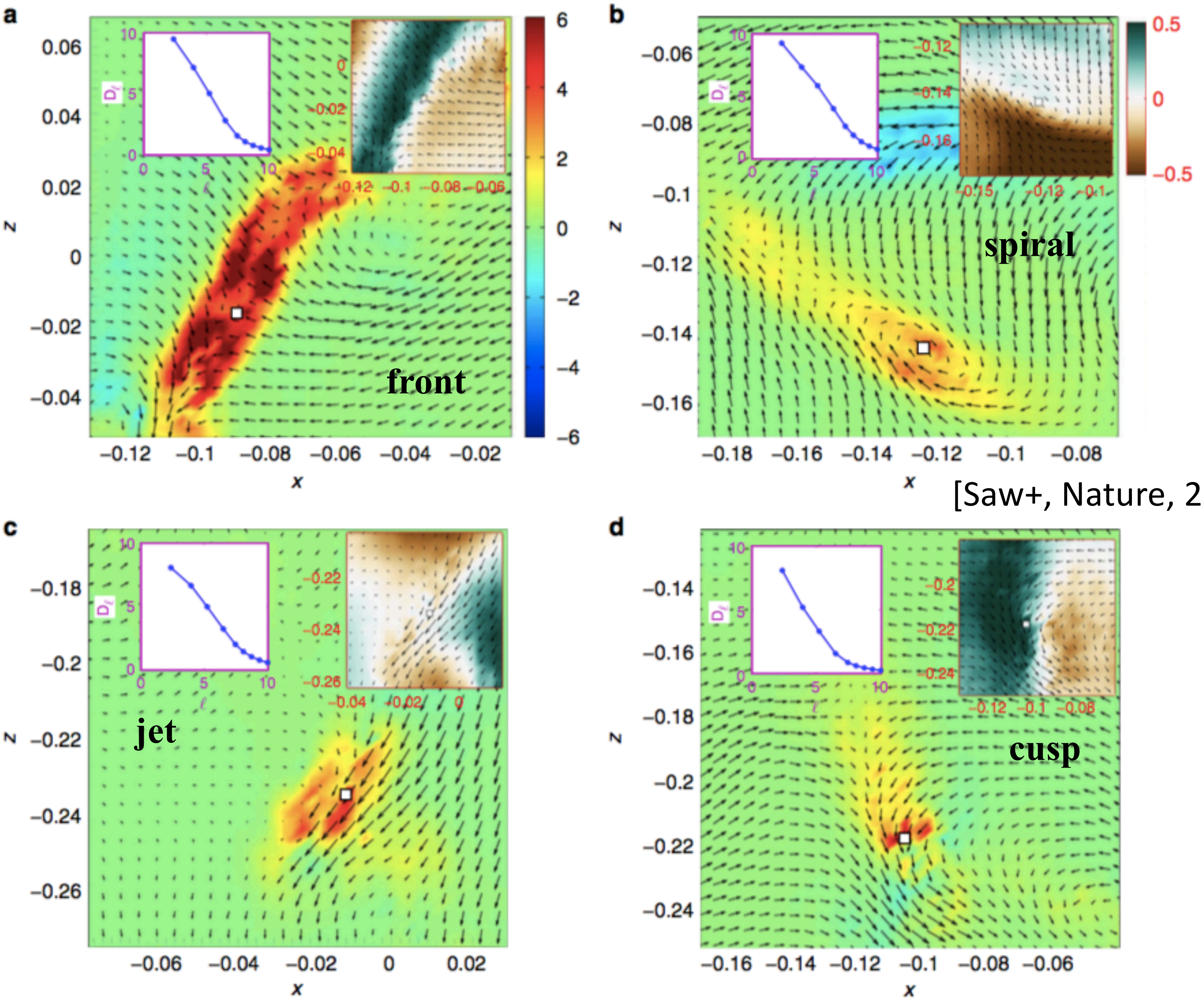
Recommended by P Constantin

Abstract. We study the local equation of energy for weak solutions of three-dimensional incompressible Navier–Stokes and Euler equations. We define a dissipation term $D(u)$ which stems from an eventual lack of smoothness in the solution u. We give in passing a simple proof of Onsager's conjecture on energy conservation for the three-dimensional Euler equation, slightly weakening the assumption of Constantin *et al*. We suggest calling weak solutions with non-negative $D(u)$ 'dissipative'.

Types of **extreme events** (28 events over 30 000 images) found experimentally

Hydro



[Saw+, Nature, 2016]

Coarse-grained approach to (Hall) MHD

Consider a **locally** space-averaged field: $\mathbf{u}^\epsilon = \varphi^\epsilon * \mathbf{u}$

with $\varphi^\epsilon(\mathbf{r}) = \varphi(\mathbf{r}/\epsilon)/\epsilon^3$, and φ a non-negative, smooth and rapidly decaying **test function**

$$\Rightarrow \quad \partial_t E^\epsilon + \nabla \cdot \mathbf{F}^\epsilon = -\mathscr{D}_\epsilon - \mathscr{D}_{\nu,\eta}$$

with $\mathscr{D}_\epsilon = \frac{1}{4} \int \nabla \varphi^\epsilon(\mathbf{r}) \cdot \left[\left((\delta \mathbf{u})^2 + (\delta \mathbf{b})^2 \right) \delta \mathbf{u} - 2(\delta \mathbf{u} \cdot \delta \mathbf{b}) \delta \mathbf{b} \right] d\mathbf{r}$

[SG, JPAMT, 2018]

$$- \frac{d_I}{2} \int \varphi^\epsilon(\mathbf{r}) \left[\delta \mathbf{j} \cdot \delta(\mathbf{j} \times \mathbf{b}) \right] d\mathbf{r}$$ ⟵ **Hall term**

- **Local version** of the Karman-Howarth relation [see Aluie, NJP, 2017 ; Kuzzay+, PRE, 2019]
- From this expression one can **recover** the exact relations [Politano & Pouquet, PRE, 1998; Banerjee & SG, JPAMT, 2017]
- This approach **is stronger** than the statistical one

Inertial energy dissipation for weak solutions

Proposition. *The local energy balance in incompressible Hall MHD can be written as:*

$$\partial_t \left(\frac{\mathbf{u}^2 + \mathbf{b}^2}{2} \right) + \nabla \cdot \left[\frac{u^2}{2}\mathbf{u} + P\mathbf{u} + \mathbf{e} \times \mathbf{b} - \nu \mathbf{u} \times \mathbf{w} \right] = -\mathscr{D}_a - \mathscr{D}_{\nu,\eta} \,, \qquad (8)$$

where $\mathbf{w}$ *is the vorticity,* $\mathscr{D}_{\nu,\eta} = \nu\, \mathbf{w}^2 + \eta\, \mathbf{j}^2$ *and* $\mathbf{e} = \eta \mathbf{j} - \mathbf{u} \times \mathbf{b} + d_I \mathbf{j} \times \mathbf{b}\,,$

$$\mathscr{D}_a = \lim_{\epsilon \to 0} \frac{1}{4} \int \nabla \varphi^\epsilon(\mathbf{r}) \cdot \left[\left((\delta \mathbf{u})^2 + (\delta \mathbf{b})^2 \right) \delta \mathbf{u} - 2(\delta \mathbf{u} \cdot \delta \mathbf{b}) \delta \mathbf{b} \right] d\mathbf{r}$$
$$- \lim_{\epsilon \to 0} \frac{d_I}{2} \int \varphi^\epsilon(\mathbf{r}) \left[\delta \mathbf{j} \cdot \delta(\mathbf{j} \times \mathbf{b}) \right] d\mathbf{r} \,, \quad \longleftarrow \textbf{Hall term} \qquad (9)$$

with $\varphi^\epsilon(\mathbf{r}) = \varphi(\mathbf{r}/\epsilon)/\epsilon^3$, φ *be an infinitely differentiable function with compact support on* $\mathbb{R}^3$ *(test function), even, non-negative with integral* 1, *and where* $\mathscr{D}_a$ *is a distribution independent of* φ.

[SG, JPAMT, 2018]

- This result generalizes the theorem by **Duchon & Robert (2000)**
- The **lack of smoothness** of the fields can lead to some dissipation in the inviscid/ideal case
- This result supports the **Onsager's conjecture** and the zeroth law

[Eyink, Physica D, 2008]

Conclusion

The solar wind *is not* a viscous fluid (it's a collisionless plasma)

⇒ **-8/3** is a solution for a dissipationless plasma

⇒ classical phenomenology **is not relevant**

Steep power laws]-8/3; -3.1] as the signature of **kinetic effects**

The **zeroth law** is well observed in incompressible HD and MHD

Onsager's conjecture is supported by a coarse-grained analysis

⇒ is it simply a mathematical curiosity? [Eyink, PRX, 2018]

A **local** expression of energy balance can be used for the solar wind

⇒ statistical homogeneity/average is not used