SELF-CONSISTENT NORMAL FLUID AND SUPERFLUID PROFILES IN TURBULENT HE II COUNTERFLOWS

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Workshop *Interpretation of Measurements in Superfluid Turbulence of He4*

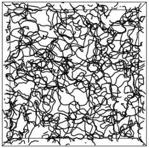
Saclay, 16 September 2015

Overview

Overview

NUMERICAL SIMULATIONS: STATE OF THE ART

- *mesoscopic lengthscales $\Delta < \ell$*



KINEMATIC SIMULATIONS $\mathbf{v}_n \Rightarrow \mathbf{v}_s$

- prescribed $\mathbf{v}_n$:
  - uniform
  - parabolic
  - vortex tubes / ABC flows
  - frozen classical turbulent channel flow
  - unsteady classical homog. and isotropic turbulence

- vortex filament method
  - Schwarz
  - Tsubota
  - Barenghi
  - Baggaley
  - Hänninen
  - L'vov

COUPLED (SELF-CONSISTENT) SIMULATIONS $\mathbf{v}_n \Leftrightarrow \mathbf{v}_s$

- simple vortex topology (single vortex lines / rings)
- decaying tangles (Kivotides)

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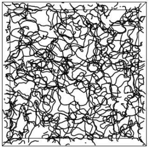

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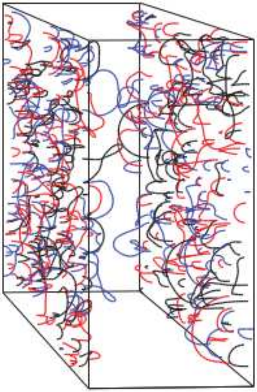
- boundaries fundamental role
  onset classical turbulence
- Plane channel counterflow
  - Baggaley & Laizet (PoF, *2013*)
  - Baggaley & Laurie (JLTP, *2015*)
  - Khomenko *et al.* (PRB,*2015*)
  - $\mathbf{v}_n$ imposed:
    Poiseuille
    laminar non–Poiseuille
    frozen turbulent DNS
- Square cross-section channel counterflow
  - Yui & Tsubota (PRB, *2015*)
  - $\mathbf{v}_n$ imposed:
    Hagen-Poiseuille
    tail-flattened



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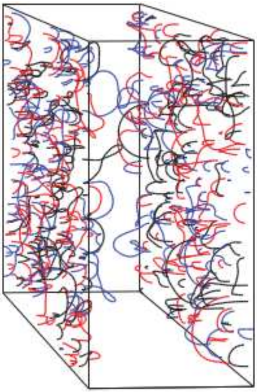


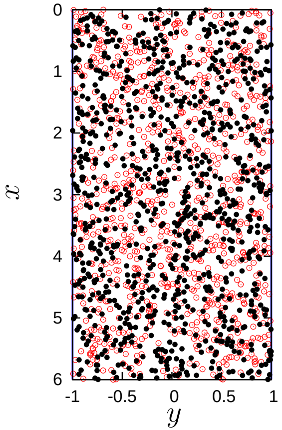
Overview

2D Plane Counterflow Channel

- 2D channel
- N vortex-points
- $\mathbf{r}_j(t) = \left(x_j(t), y_j(t)\right) \;\; j = 1, \cdots N$
- $\Gamma_j = \pm\kappa$

Connection with experiments

- $n := N/A \longleftrightarrow L$
- $L^{1/2} D = 1.03\gamma_0 \dfrac{\rho}{\rho_s} \langle u_n \rangle h_D - 1.48\beta$
- $\langle u_n \rangle = q/(T\rho S)$
- channel width $D = 9.1 \times 10^{-3}$cm
  Ladner & Tough (PRB, *1979*)
- T-I regime: $Re = 206 \ll Re_c \approx 5772$



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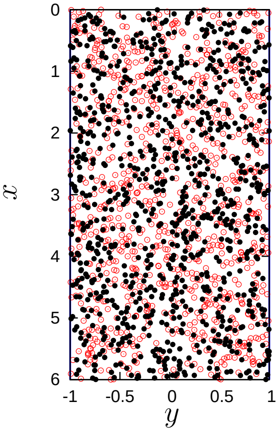

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SUPERFLUID VORTICES

- $$\frac{d\mathbf{r}_j}{dt} = \mathbf{v}_s(\mathbf{r}_j,t) + \alpha\, \mathbf{s}'_j \times \left(\mathbf{v}_n(\mathbf{r}_j,t) - \mathbf{v}_s(\mathbf{r}_j,t)\right) + \alpha' \left(\mathbf{v}_n(\mathbf{r}_j,t) - \mathbf{v}_s(\mathbf{r}_j,t)\right)$$
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- $\mathbf{v}_{si}(\mathbf{r}_j,t) = \sum_{k=1\ldots N} \mathbf{v}_{si,k}(\mathbf{r}_j,t)$

COMPLEX–POTENTIAL FORMULATION

- $v_{si,k}(z,t) = v^x_{si,k} - i v^y_{si,k} = \frac{dF_k(z)}{dz}$
- $F_k(z,t) = \mp i \frac{h}{2\pi m} \log \frac{\sinh\left[\frac{\pi}{2D}(z - z_k(t))\right]}{\sinh\left[\frac{\pi}{2D}(z - \overline{z_k}(t))\right]}$ { conformal map / inifinite images

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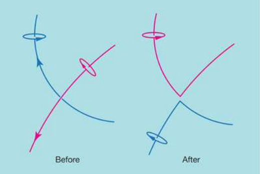
Superfluid vortices- Reconnections

- steady state ⇒ N constant
- ω_s production/destruction 3D
  - reconnections
  - mutual friction vortex stretching



Numerical Reconnection

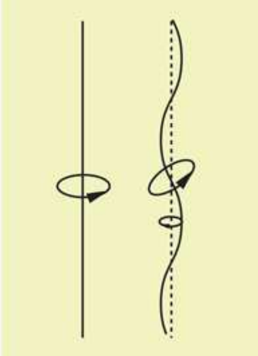
- $d(\oplus, \ominus) < \epsilon_c$
- $d(\oplus[\ominus], y = \pm 1) < \epsilon_c/2$

$$\Downarrow$$

remove vortex–points

Numerical Renucleation

- (a): RANDOM
- (b): same y



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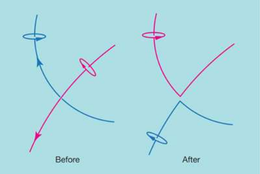

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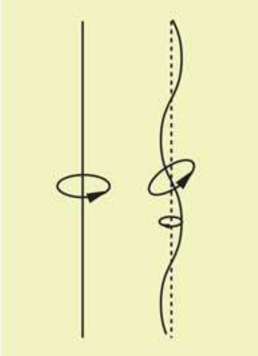

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Normal fluid

Hypotheses

- $\rho, \rho_n, \rho_s, S = \text{const}$
- $\eta_n, \lambda = \text{const}$
- incompressible , isoentropic

- $$\frac{\partial \mathbf{v}_n}{\partial t} + (\mathbf{v}_n \cdot \nabla)\mathbf{v}_n = -\frac{1}{\rho}\nabla p - \frac{\rho_s}{\rho_n} S \nabla T + \nu_n \nabla^2 \mathbf{v}_n - \frac{\rho_s}{2\rho}\nabla(\mathbf{v}_n - \mathbf{v}_s)^2 + \frac{1}{\rho_n}\widetilde{\mathbf{F}}_{ns}$$
- $\nabla \cdot \mathbf{v}_n = 0$

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- $\mathbf{v}_n' = (u_n', v_n')$ back–reaction

NORMAL FLUID

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- $\mathbf{v}'_n = (u'_n, v'_n)$ back–reaction

Normal fluid

$\Psi - \omega$ formulation

- $\mathbf{v}'_n = \left(\frac{\partial \Psi'}{\partial y}, -\frac{\partial \Psi'}{\partial x}\right)$
- $\omega'_n = \left(\nabla \times \mathbf{v}'_n\right) \cdot \hat{\mathbf{z}}$
- $\nabla^2 \Psi' = -\omega'_n$
- $\frac{\partial \omega'_n}{\partial t} + \left(u^p_n + \frac{\partial \Psi'}{\partial y}\right) \frac{\partial \omega'_n}{\partial x} - \frac{\partial \Psi'}{\partial x}\left(\frac{\partial \omega'_n}{\partial y} - \frac{d^2 u^p_n}{dy^2}\right) =$

$$\nu_n \nabla^2 \omega'_n + \frac{1}{\rho_n}\left(\frac{\partial \tilde{F}^y}{\partial x} - \frac{\partial \tilde{F}^x}{\partial y}\right)$$

Boundary conditions

- $\langle u'_n \rangle = 0 \Rightarrow \Psi'(\pm D/2) = 0$
- no–slip for $\mathbf{v}_n \Rightarrow \omega$

Fine grid

$\Delta x, \Delta y < \ell$

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$\mathbf{F}_{ns}$ Coarse–graining

Hall–Vinen formulation

- $\Delta X, \Delta Y > \ell$
- $\widetilde{\mathbf{F}}_{ns} = \alpha \rho_s \widehat{\widetilde{\boldsymbol{\omega}}}_s \times [\widetilde{\boldsymbol{\omega}}_s \times (\widetilde{\mathbf{v}}_n - \widetilde{\mathbf{v}}_s)] + \alpha' \rho_s \widetilde{\boldsymbol{\omega}}_s \times (\widetilde{\mathbf{v}}_n - \widetilde{\mathbf{v}}_s)$

- $\Theta_j(\mathbf{r}) = \frac{1}{V_j} e^{-\frac{|\mathbf{r}-\mathbf{r}_j|^2}{2\ell^2}}$ Gaussian kernel
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Overview

NUMERICAL SIMULATIONS

PARAMETERS

- $T = 1.7^{-\circ} K$
- $L^{1/2} D = 25$
- $N \simeq 2000$
- $nx = 192\,,\; ny = 64$
- $\delta_c = D/2 = 4.55 \times 10^{-3}\text{cm}$
- $u_c = \kappa/(2\pi\delta_c) = 3.49 \times 10^{-2}\text{cm/s}$
- $t_c = \delta_c/u_c = 0.13\text{s}$
- $\Delta t^* \sim 10^{-6}$

- steady–state

  $T_f = D^2/\nu_n$

VORTEX DISTRIBUTION

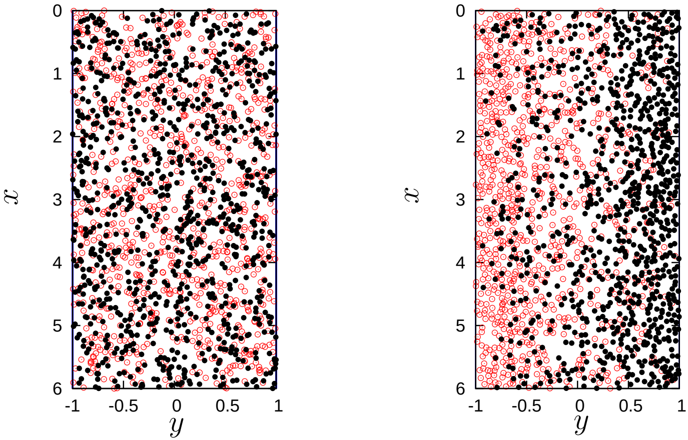

FIGURE : Initial (left) and steady–state (right) vortex configuration

$$\frac{d\mathbf{r}_j}{dt} = \mathbf{v}_s(\mathbf{r}_j,t) + \alpha\,\mathbf{s}'_j \times \left(\mathbf{v}_n(\mathbf{r}_j,t) - \mathbf{v}_s(\mathbf{r}_j,t)\right) + \alpha' \left(\mathbf{v}_n(\mathbf{r}_j,t) - \mathbf{v}_s(\mathbf{r}_j,t)\right)$$

VORTEX DENSITY PROFILES $\overline{n}(y)$

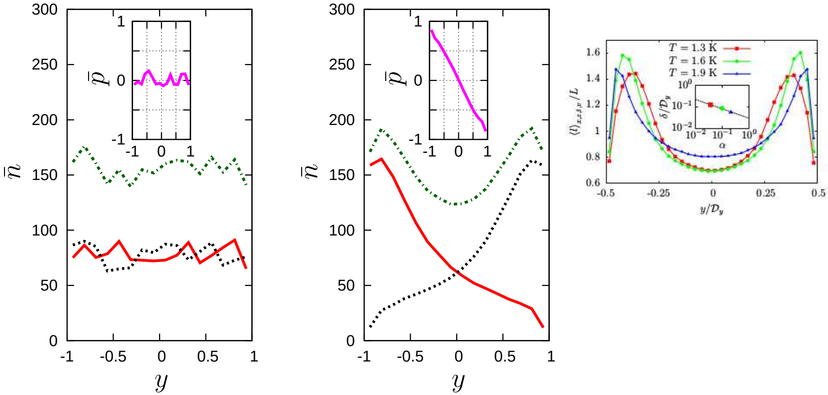

FIGURE : vortex–density profiles $\overline{n}(y)$ and vortex polarization $\overline{p}(y)$ at $t = 0$ (left) and $t > T_f$ (center). On the right normalized vortex–line density L from Baggaley & Laurie (JLTP,*2015*)

VELOCITY PROFILES $\overline{u}_s(y)$ AND $\overline{u}_n(y)$

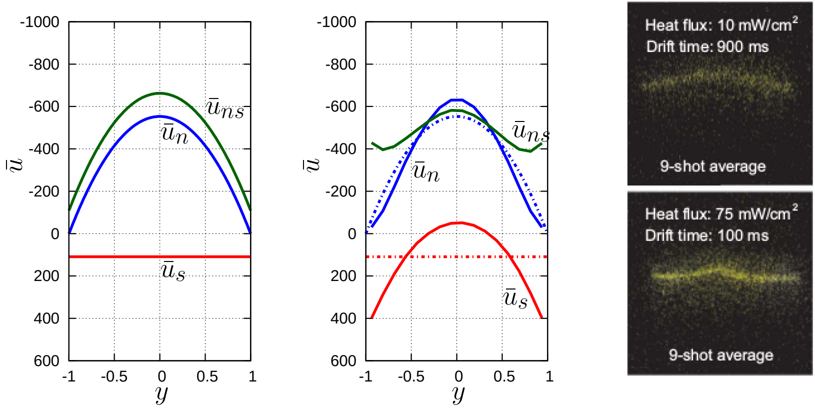

FIGURE : Superfluid and normal fluid velocity profiles $\overline{u}_s(y)$ and $\overline{u}_n(y)$ at $t = 0$ (left) and $t > T_f$ (center). On the right the experimentally measured normal fluid velocity profile by Marakov *et al.* (PRB,*2015*)

3D Analogue

- streamwise flow of expanding vortex rings
- rings lie on planes ⊥ to $\mathbf{v}_n$
- drift in opposite direction of $\mathbf{v}_n$
- circulation of rings ∥ to $\mathbf{v}_n$

- $\dot{x} = u_R = (1-\alpha')\overline{u}_s + \alpha'\overline{u}_n$
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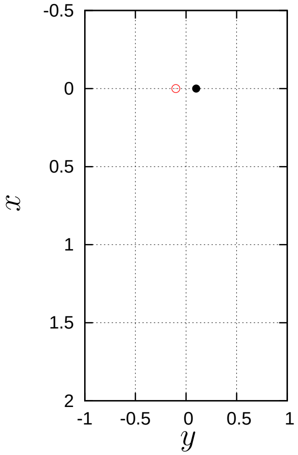

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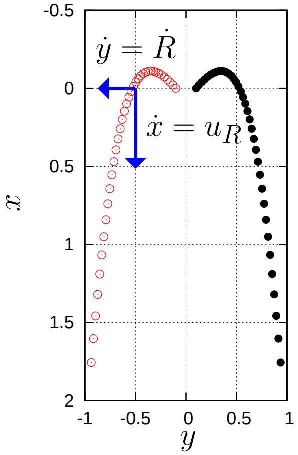
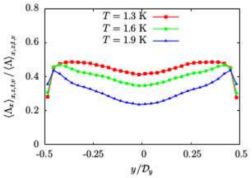
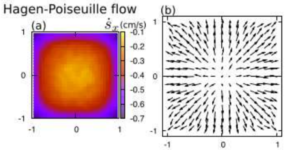


3D Analogue: is it realistic

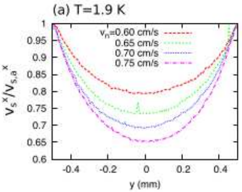
- vortex–tangle anisotropic



- vortex–lines move towards the walls
- vortex lines move faster as they approach the walls



- superflow reduced in central region



Overview

SUMMARY

- self–consistent calculation of $\mathbf{v}_n$ and $\mathbf{v}_s$
- steady–state with $L \sim$ experiment
- recover the tail–flattened $\mathbf{v}_n$ profile
  meaured by Guo (PRB, *2015*)
- recover 3D numerical vortex–density profiles
  Baggaley & Laurie (JLTP, *2015*)
- predict a $\mathbf{v}_s$ parabolic profile
  cfr. Yui & Tsubota (PRB, *2015*)

ONGOING AND FUTURE STUDIES

- Pure Superflow
- Particle tracers