Singularities and Irreversibility

Main SPONSOR:

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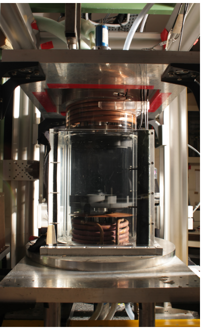
Symmetries and Navier-Stokes



$$\vec{\nabla} \bullet \vec{u} = \vec{0}$$

$$\partial_t \vec{u} + (\vec{u} \bullet \vec{\nabla})\vec{u} = -\frac{1}{\rho}\vec{\nabla} p + \nu \Delta \vec{u}$$

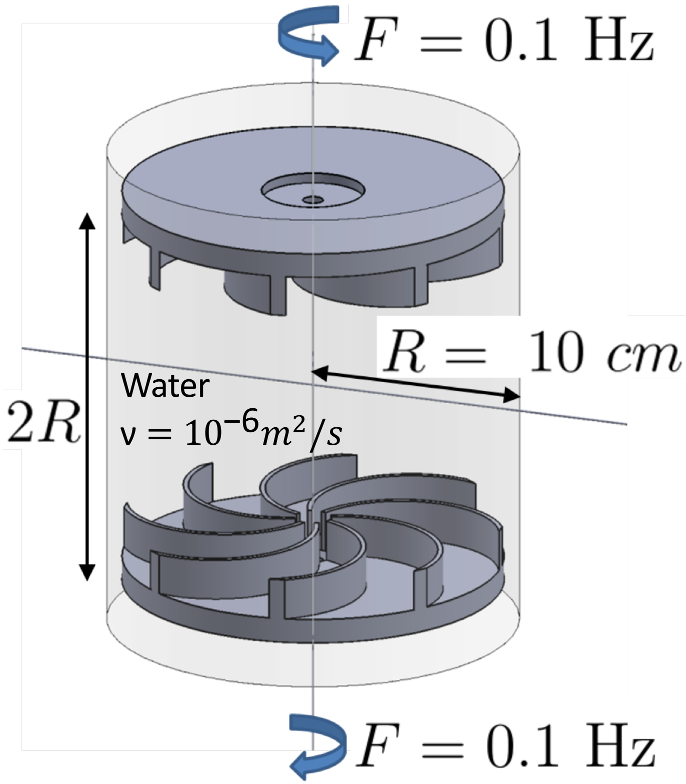
Broken Symmetry (ii)

Time-reversal $t \rightarrow -t$ Only for $\nu = 0$

u → -u

Time-reversal breaking by viscosity

Experimental set-up : the von Kármán flow



- Revnolds number:

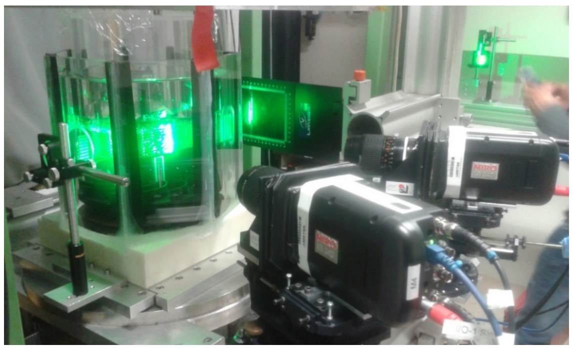
$$Re = \frac{2\pi R^2 F}{\nu} = 6.3 \times 10^3$$

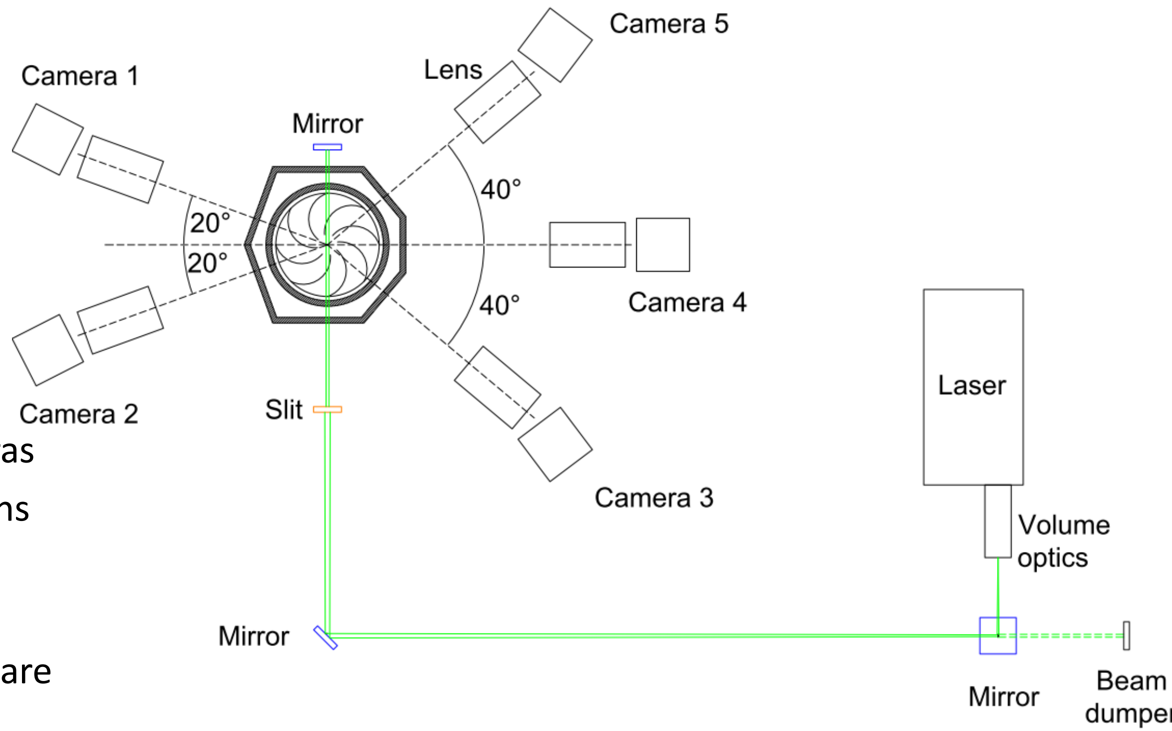
- Average dissipation rate ε computed from torque measurements

- Kolmogorov scale :

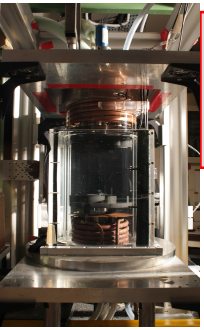
$$\eta = (\nu^3/\epsilon)^{1/4} = 0.3 \text{ mm}$$

Experimental set-up : velocity measurement



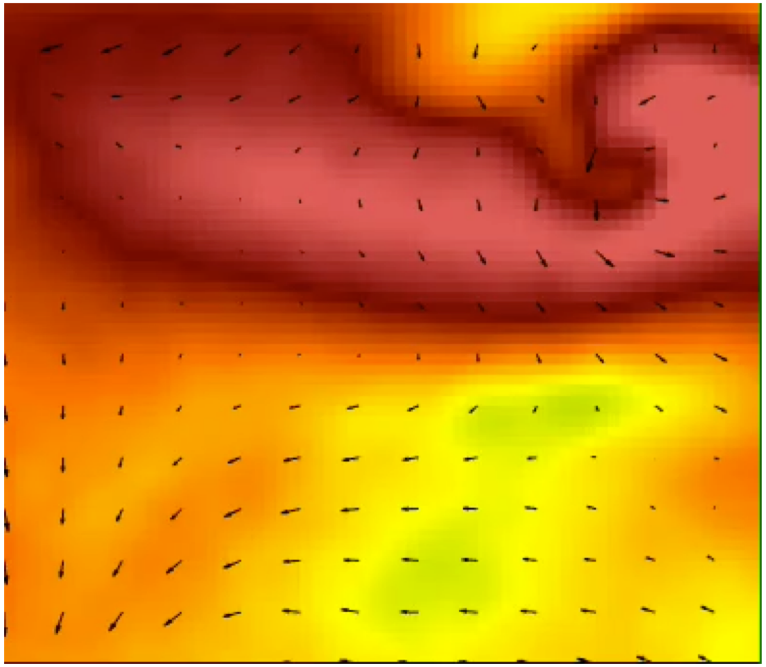


- Tomographic PIV with 5 cameras
- Outer tank to reduce distortions
- Measurement volume : 4cmx4cmx0,5cm
- Processing with Lavision software
- 75% overlap
- Vector spacing : dx=0,35mm=1,2η
- 30000 frames collected

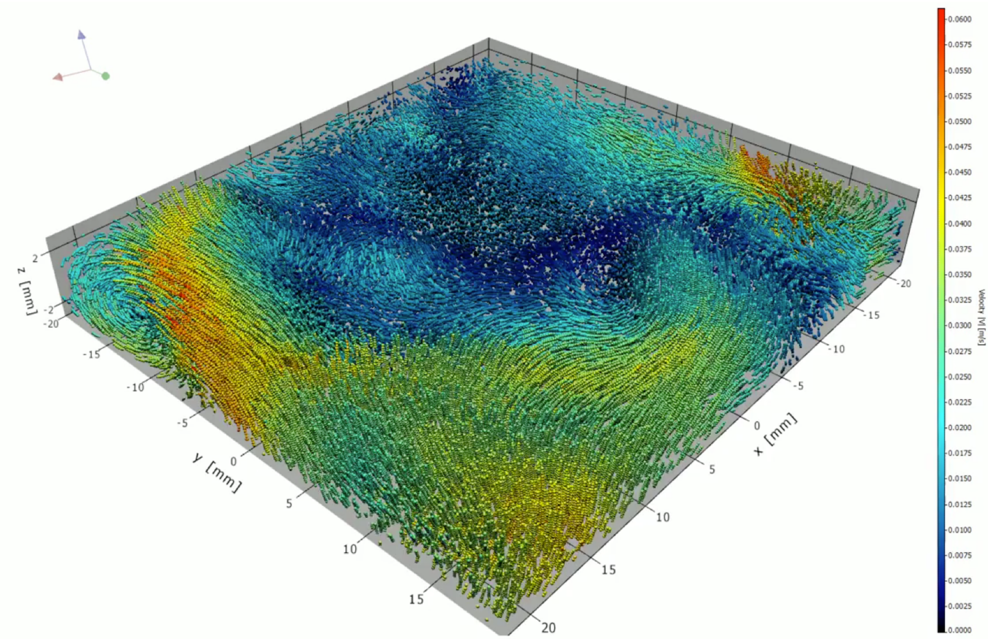


Velocity measurements in von Karman flow

Turbulent flow viewed in PIV



Turbulent flow viewed in PTV



TPIV velocity fields

$V_x(x, y, z), V_y(x, y, z), V_z(x, y, z)$

Time and space resolved Lagrangian trajectories

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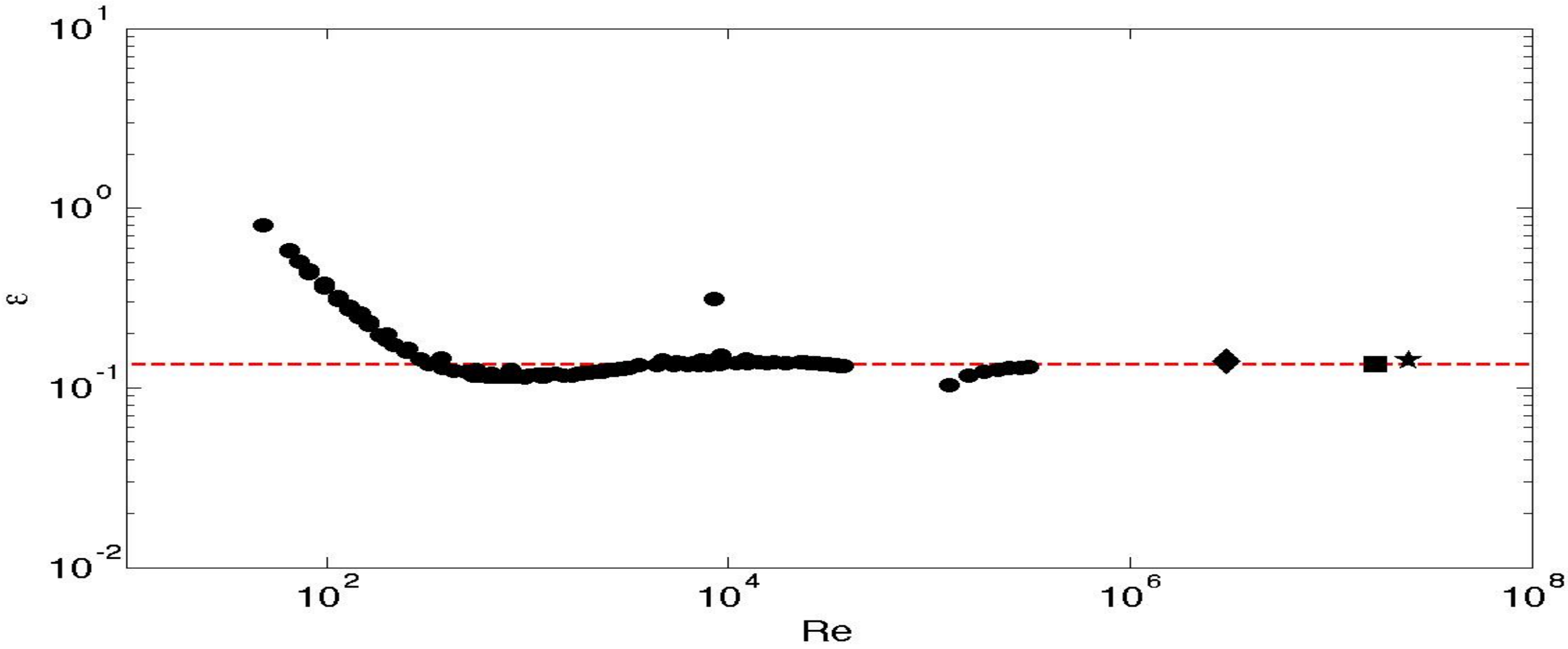
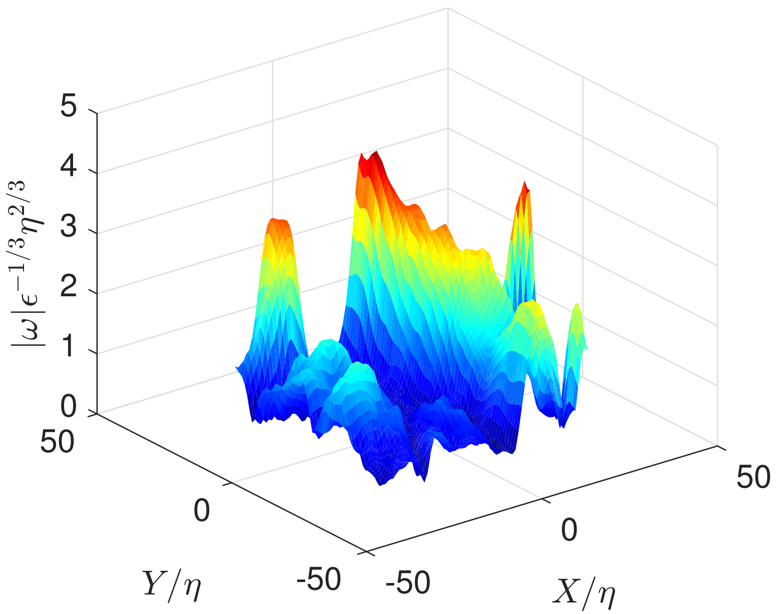
Energy dissipation in von Karman flow

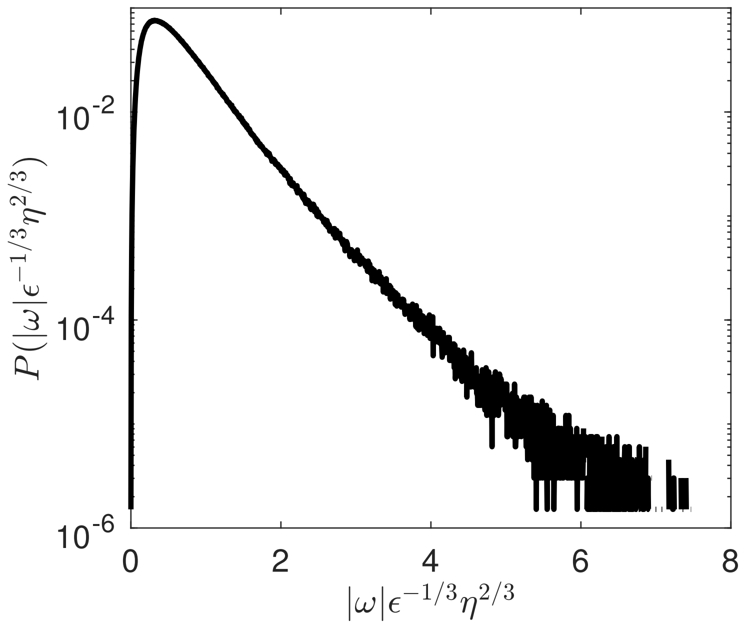


Non-dimensional Energy dissipation per unit mass is constant at large Reynolds

Independent of viscosity?

Enstrophy blow-up





$$\epsilon = \nu < (\nabla u)^2 >= \nu < \omega^2 > \Longrightarrow < \omega^2 > \approx \frac{\epsilon}{\nu}$$

$$lim_{\nu \longrightarrow 0} < \omega^2 >= \infty$$

Building of very large gradients at small scale… Singularity?
How to measure them/quantify them in experiments?

Singularities and Eulerian energy dissipation

$$-\mathcal{D} \quad -\mathcal{D}_\nu$$

$$\frac{1}{2}\partial_t \mathbf{u}^2 + \mathrm{div}\left(\mathbf{u}\left(\frac{1}{2}\mathbf{u}^2 + p\right) - \nu\nabla\mathbf{u}\right) = D(u) - \nu\ \nabla\mathbf{u})^2$$

Inertial dissipation:

$$D(u) = \lim_{\ell\to 0}\frac{1}{4}\int_{r\leq\ell} d^3r\ \nabla\phi_\ell(r)\cdot\delta u_r|\delta u_r|^2$$

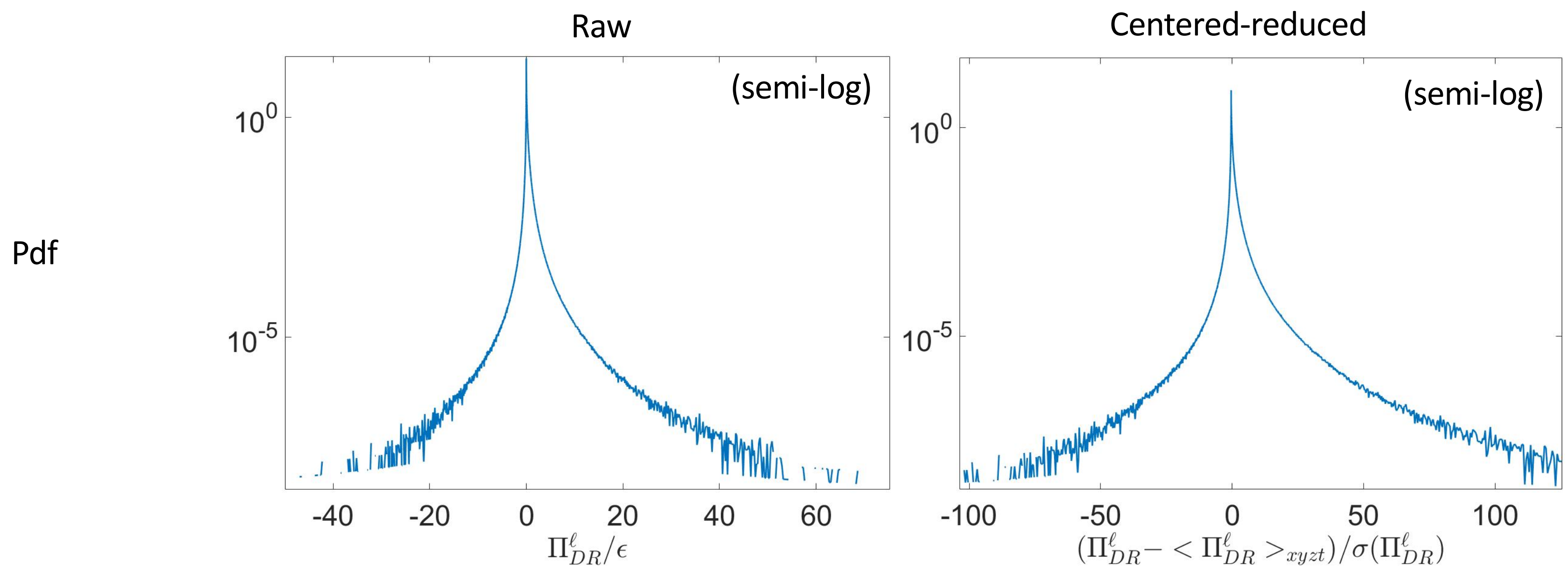
$\delta u(\ell) \sim \ell^h$ In the limit of $\ell \approx 0$

$$D(u)[x] \propto \lim_{\ell\to 0}\ell^{3h-1}$$

If $h \leq 1/3$ → Dissipation through singularities

Duchon&Robert. Nonlinearity (2000)

Extreme events of the Duchon-Robert term



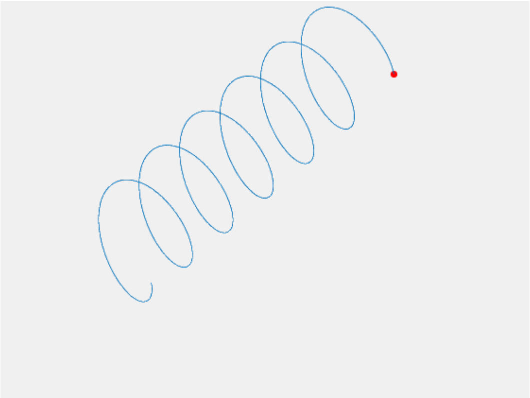
Largest $\lvert\Pi^{\ell}_{DR}\rvert$	97ε	266σ
10^{th} largest $\lvert\Pi^{\ell}_{DR}\rvert$	58ε	159σ
1000^{th} largest $\lvert\Pi^{\ell}_{DR}\rvert$	14ε	38σ

Debue PhD Thesis (2019); ANR EXPLOIT

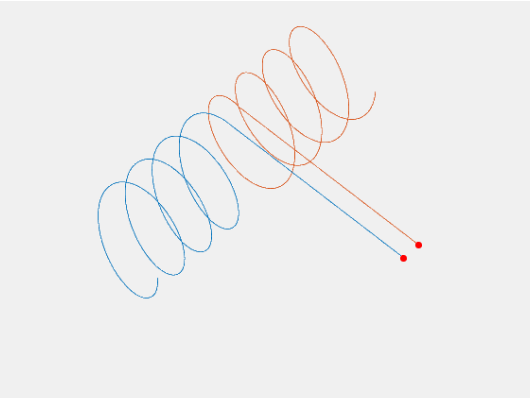
Direct observation of extreme events

- We observed three kinds of structures, based on the velocity streamlines :

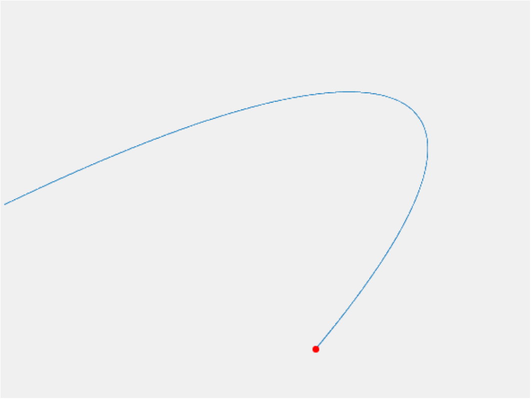
« Screw vortices »



« Roll vortices »



« U-turns »



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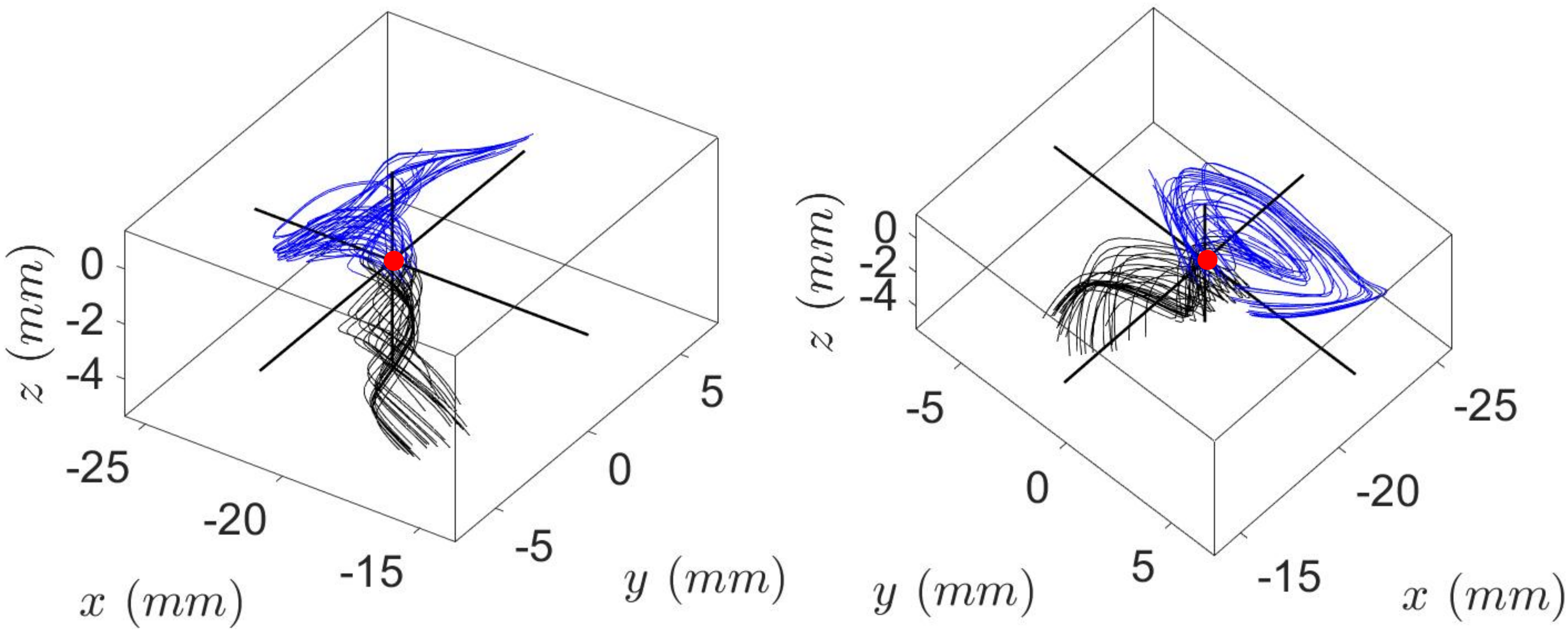
Screw vortices : velocity streamlines



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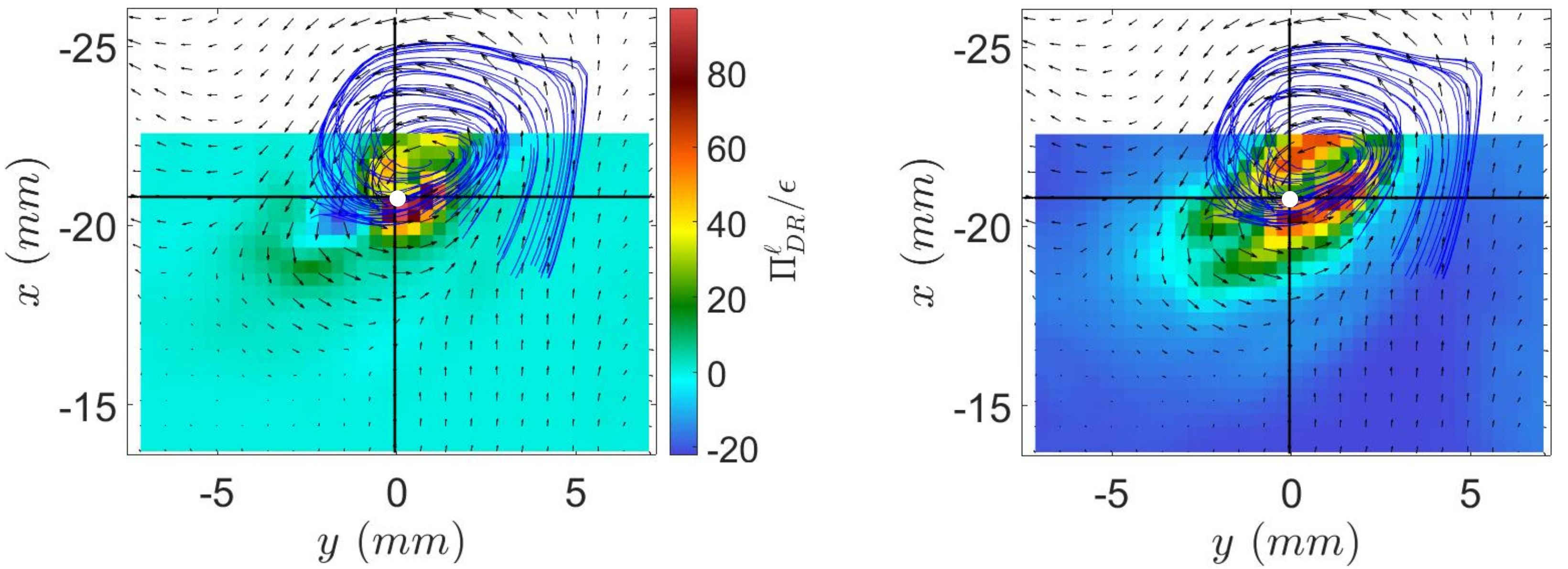
Screw vortices : Π^{ℓ}_{DR} and $\mathscr{D}^{\nu}_{\ell}$



Π^{ℓ}_{DR} $\mathscr{D}^{\ell}_{\nu}$

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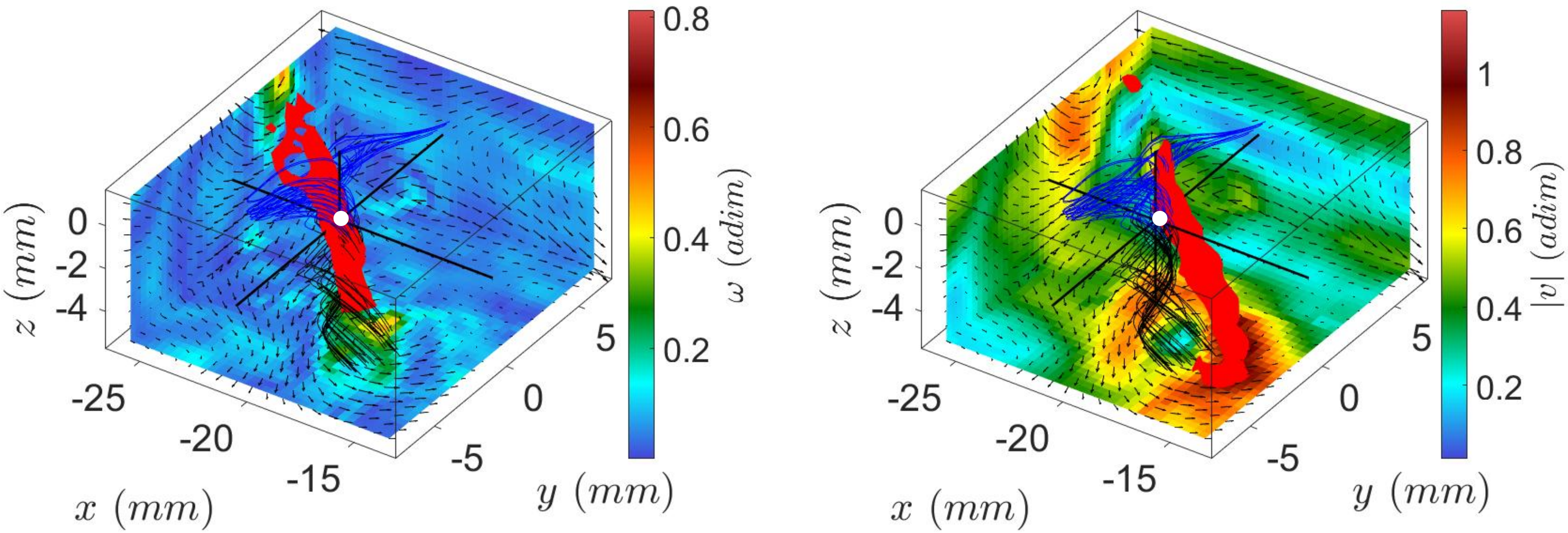
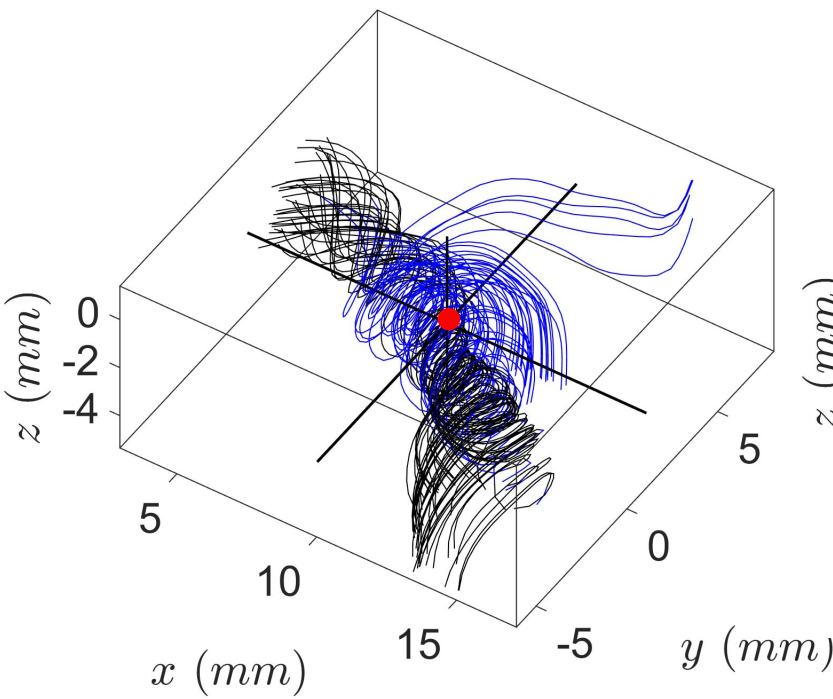
Screw vortices : vorticity and velocity norms

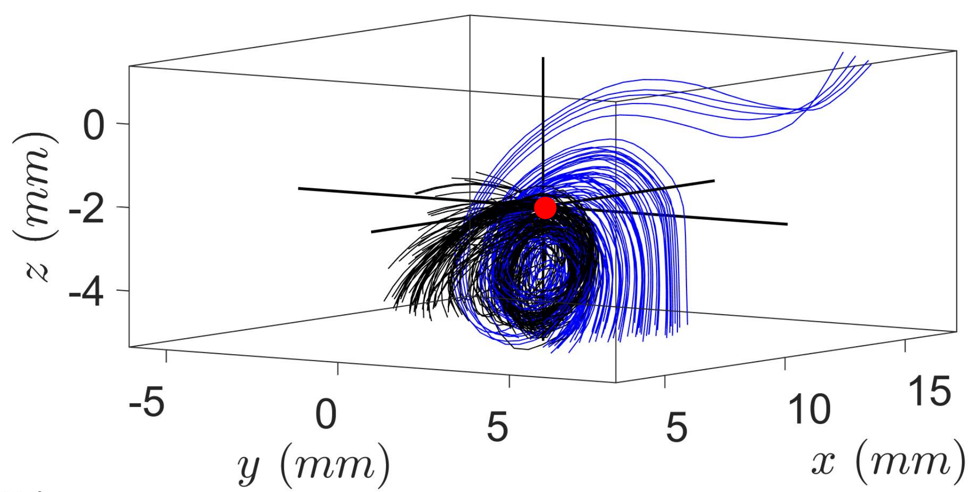


ω |v|

Debue PhD Thesis (2019); ANR EXPLOIT

Roll vortices : velocity streamlines





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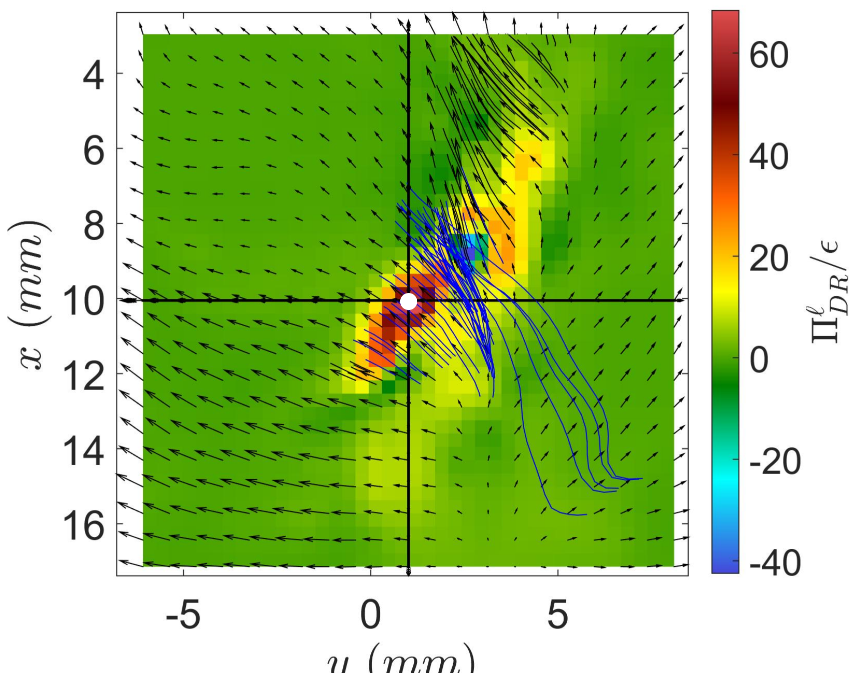
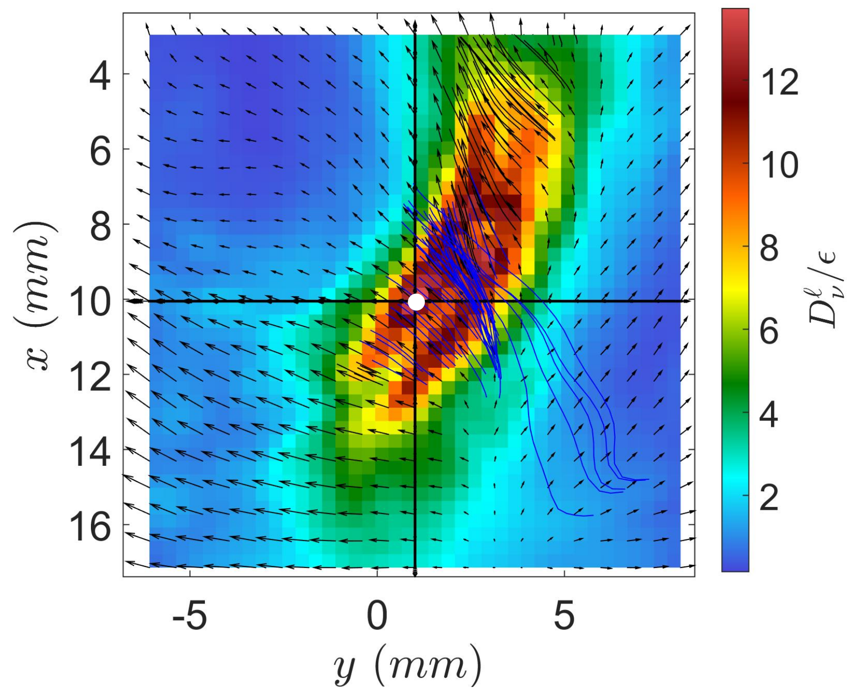
Roll vortices : Π^{ℓ}_{DR} and $\mathscr{D}^{\nu}_{\ell}$



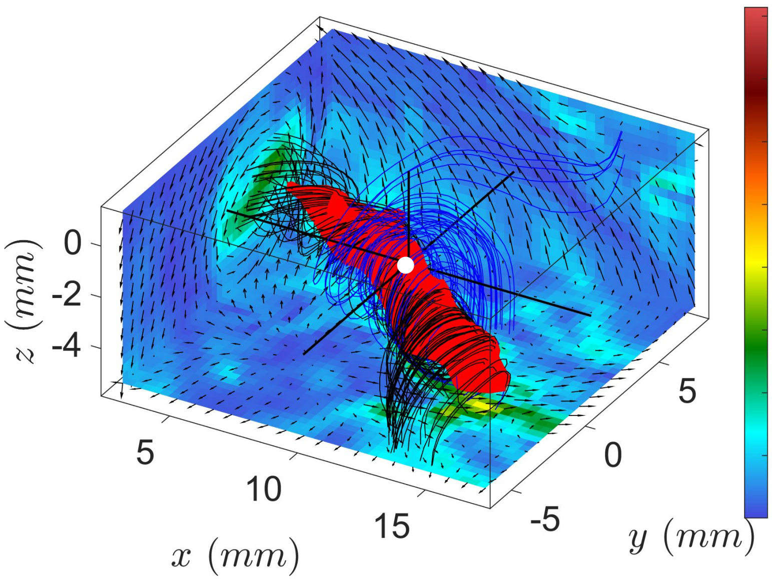
Π^{ℓ}_{DR}



$\mathscr{D}^{\ell}_{\nu}$

Debue PhD Thesis (2019); ANR EXPLOIT

Roll vortices : vorticity and velocity norms



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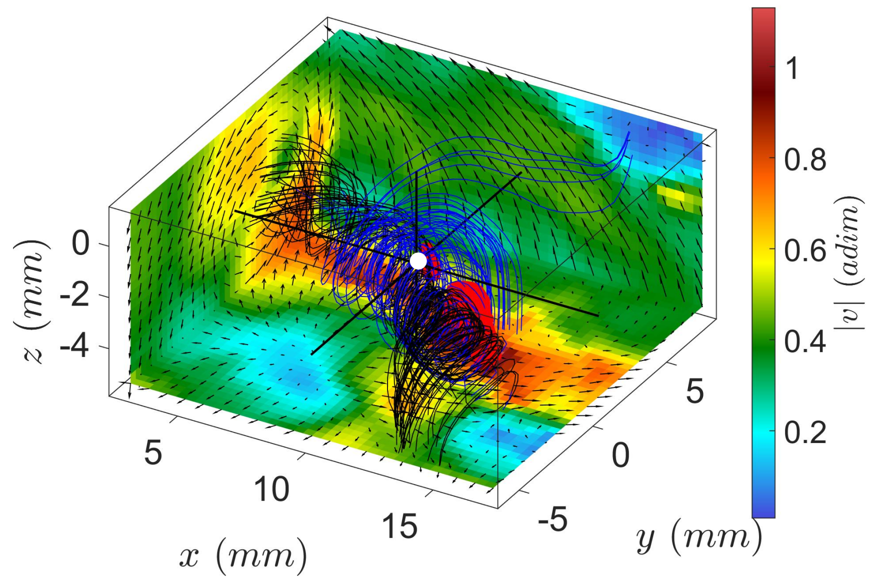

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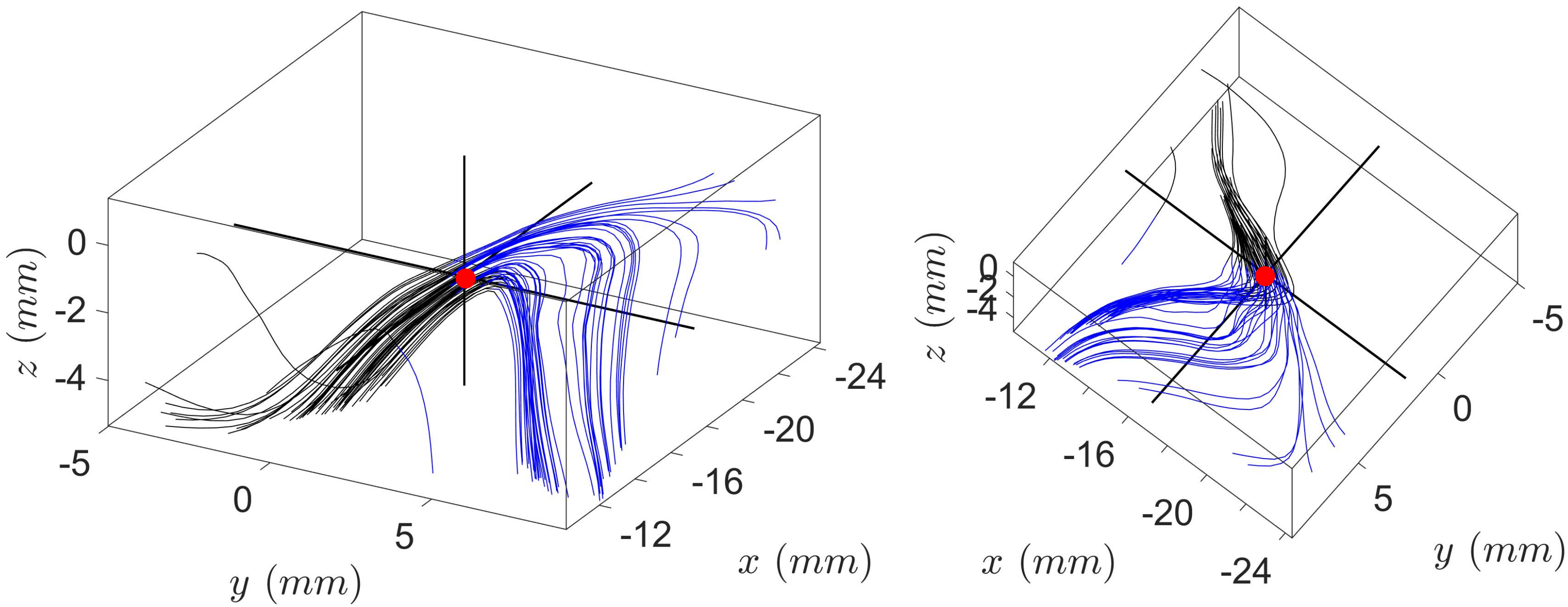
U-turns : velocity streamlines



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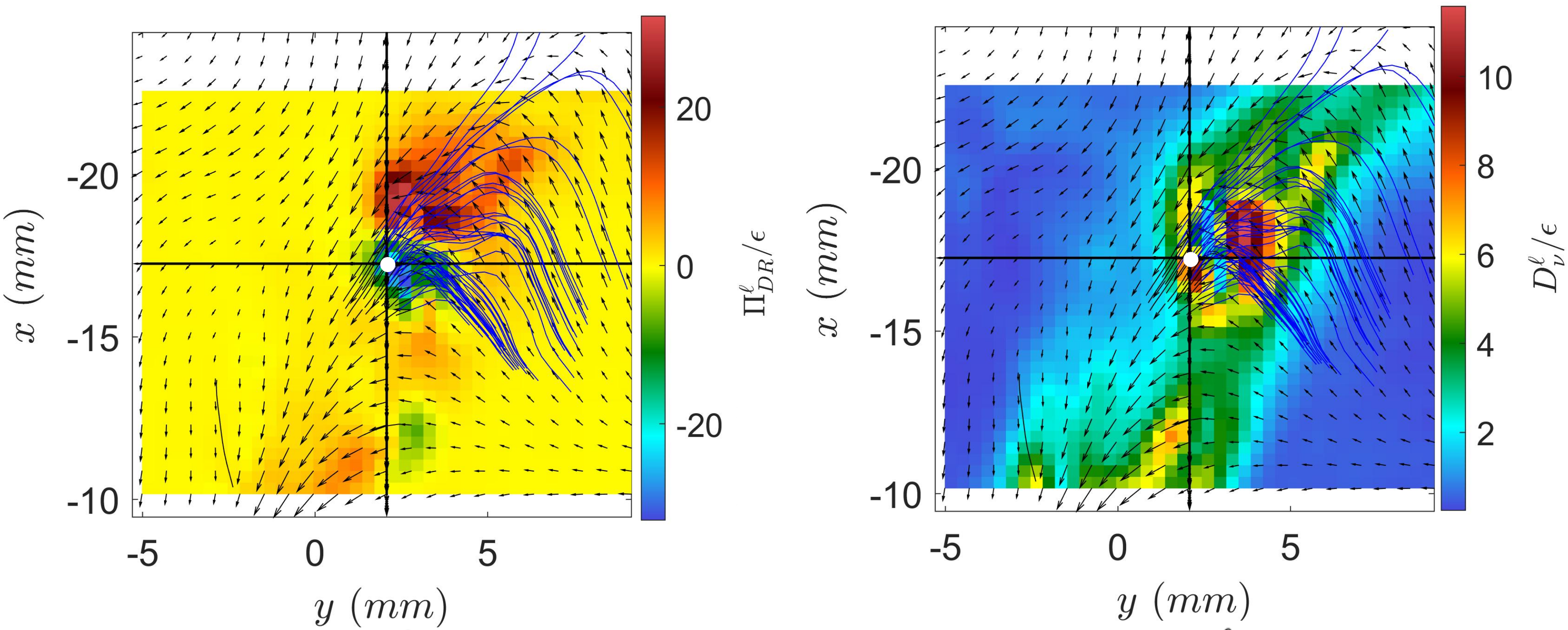
U-turns : Π^{ℓ}_{DR} and $\mathscr{D}^{\nu}_{\ell}$



Π^{ℓ}_{DR} $\mathscr{D}^{\ell}_{\nu}$

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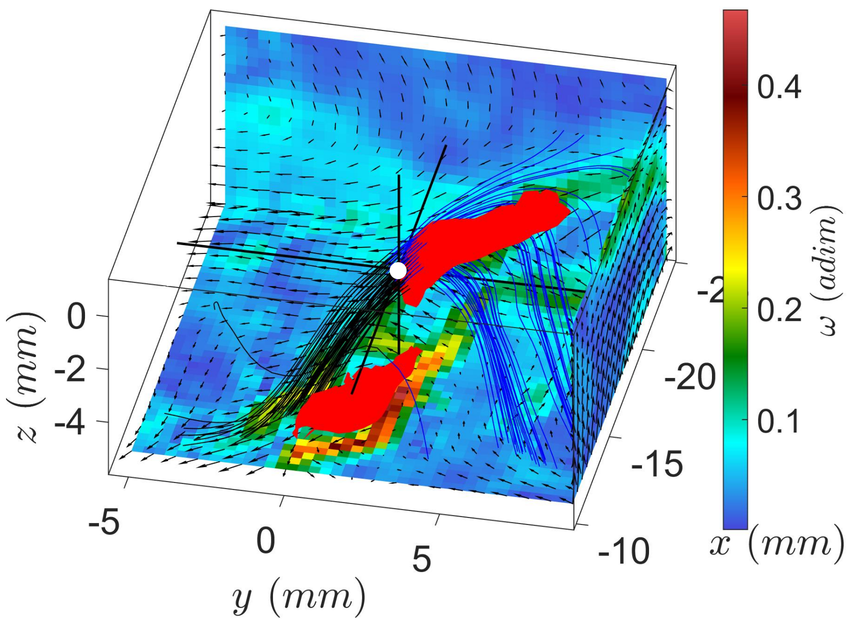
U-turns : vorticity and velocity norms



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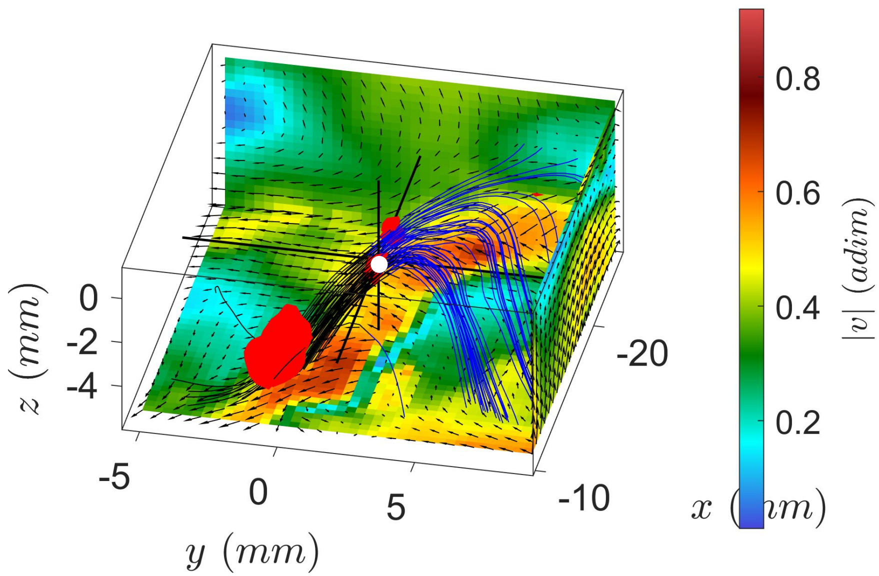

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Debue PhD Thesis (2019); ANR EXPLOIT

Euler vs Lagrangian local energy dissipation

$$-\mathcal{D} \quad -\mathcal{D}_\nu$$

$$\frac{1}{2}\partial_t \mathbf{u}^2 + \mathrm{div}\left(\mathbf{u}\left(\frac{1}{2}\mathbf{u}^2 + p\right) - \nu\nabla\mathbf{u}\right) = D(u) - \nu\,\nabla\mathbf{u})^2$$

Richardson law :

$$\left\langle\left(\delta X_r^{t\mp\tau}\right)^2\right\rangle \sim a_\mp \tau^3$$

Inertial dissipation:

$$D(u) = \lim_{\ell\to 0}\frac{1}{4}\int_{r\leq\ell} d^3r\ \nabla\phi_\ell(r)\cdot\delta u_r|\delta u_r|^2$$

Irreversibility

$$a_+ \neq a_-$$

$\delta u(\ell) \sim \ell^h$ In the limit of $\ell \approx 0$

$$D(u)[x] \propto \lim_{\ell\to 0}\ell^{3h-1}$$

If $h \leq 1/3$ → Dissipation through singularities

Duchon&Robert. Nonlinearity (2000)

$$D_L = \lim_{r,\tau\to(0,0)}(\langle(\delta X_r^{t+\tau})^2\rangle - \langle(\delta X_r^{t-\tau})^2\rangle)/6\tau^3$$

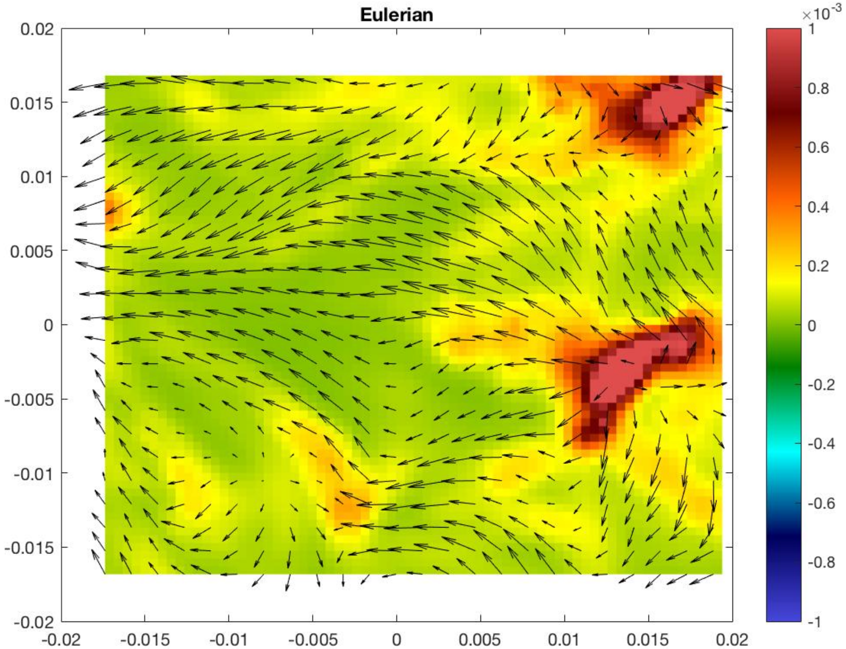
J. Jucha et al. PRL (2014)

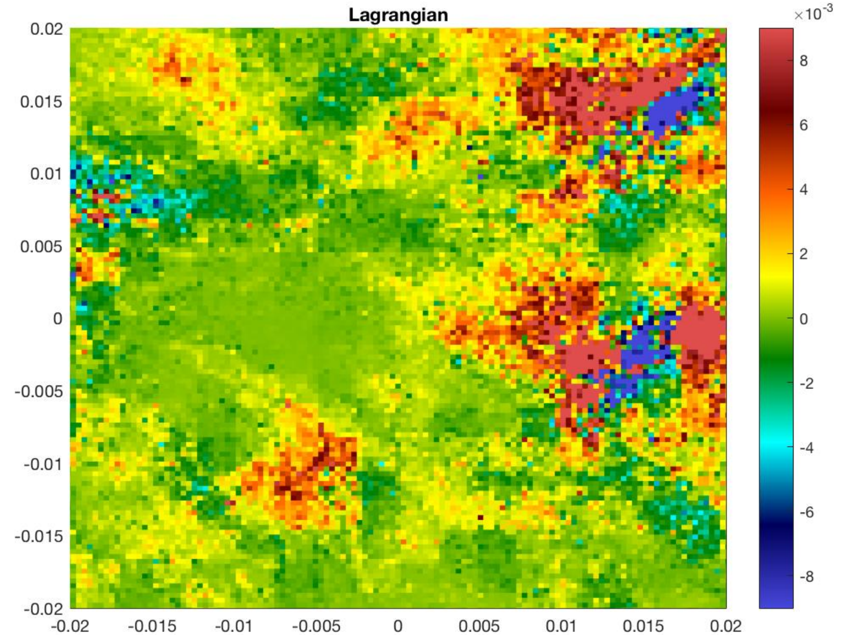
Theory

$$D_L = \varepsilon = \mathcal{D} + \mathcal{D}_\nu$$

Th. Drivas, 2019

Eulerian vs Lagrangian local dissipation





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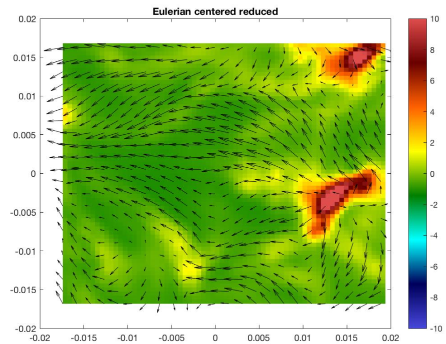
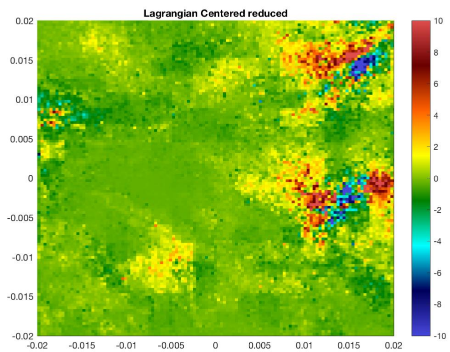
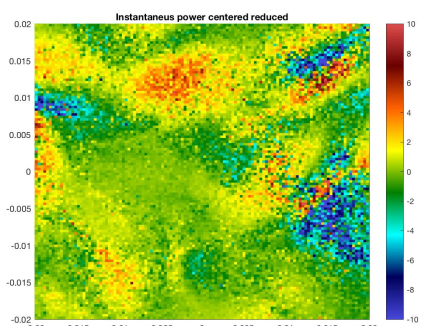
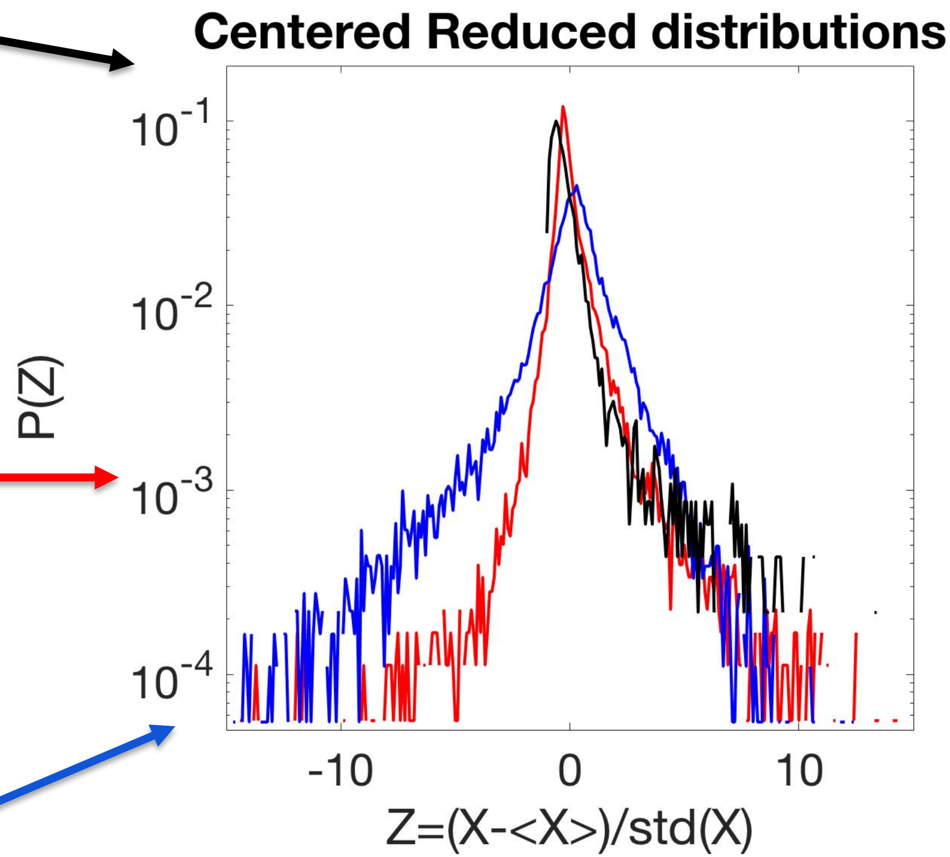
Eulerian vs Lagrangian local dissipation







Instanteneous power different from local dissipation



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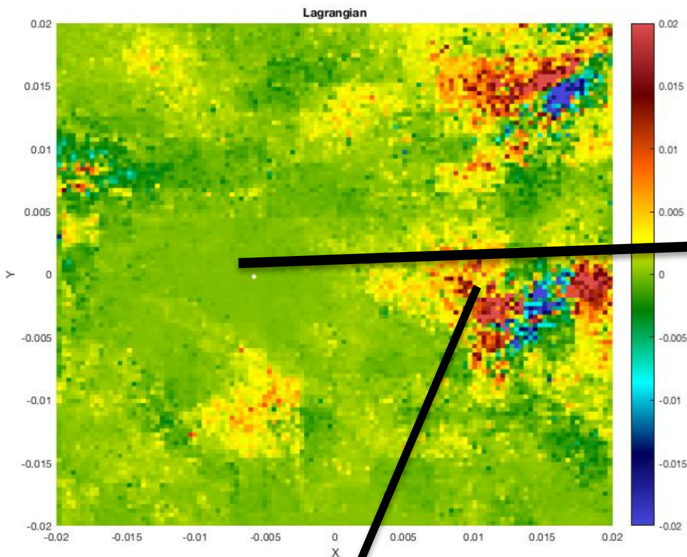
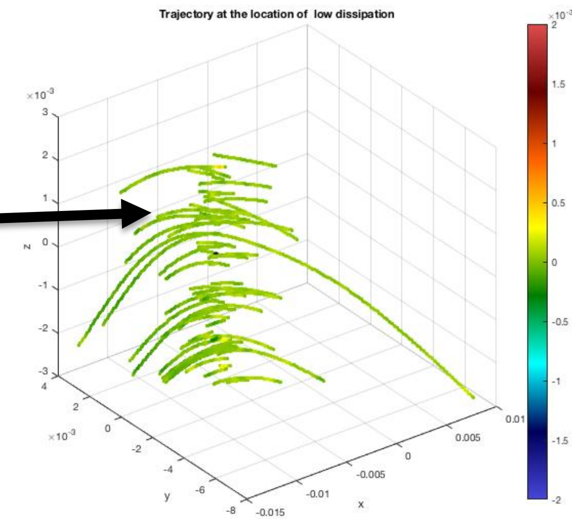
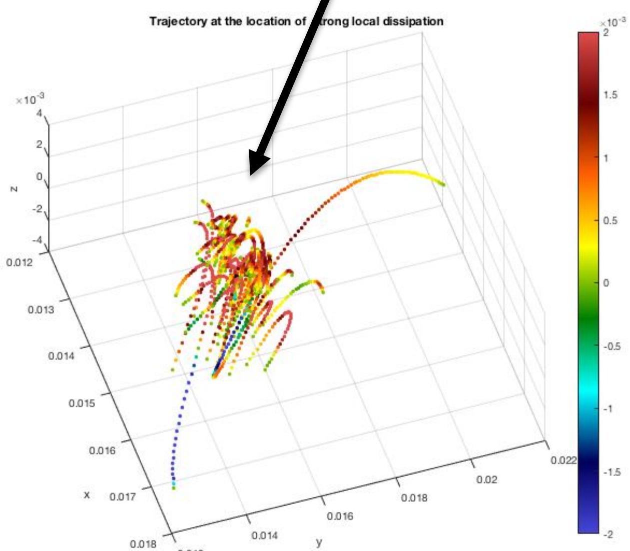
Geometry of trajectories at special points







Chaotic trajectories at high dssipation
Regular trajectories at low dissipation

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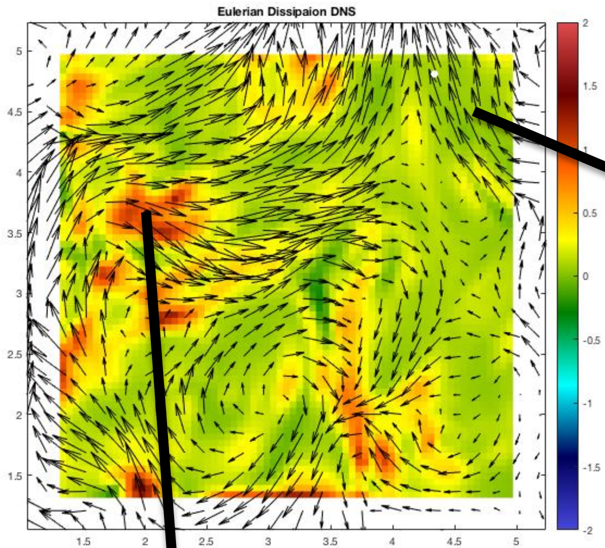
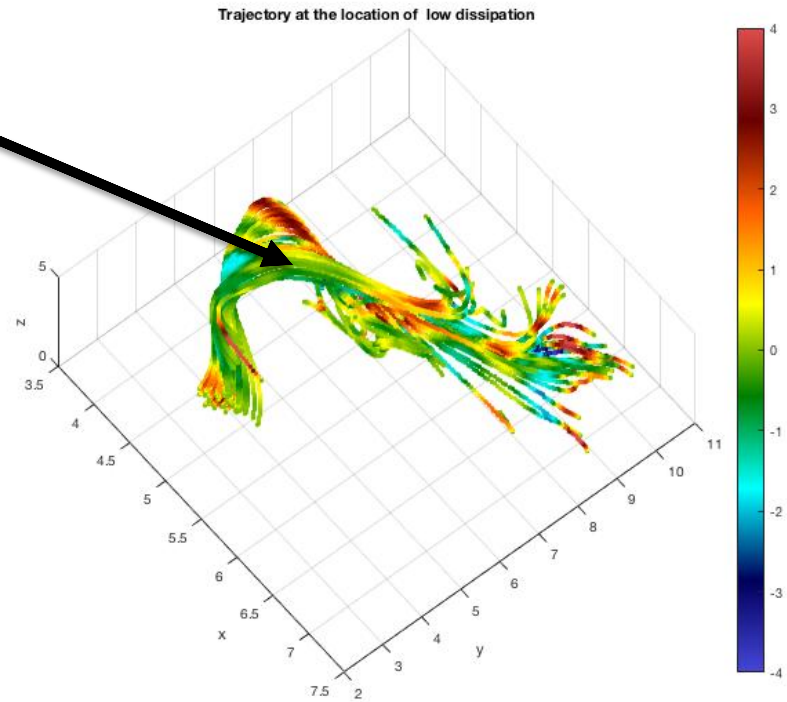
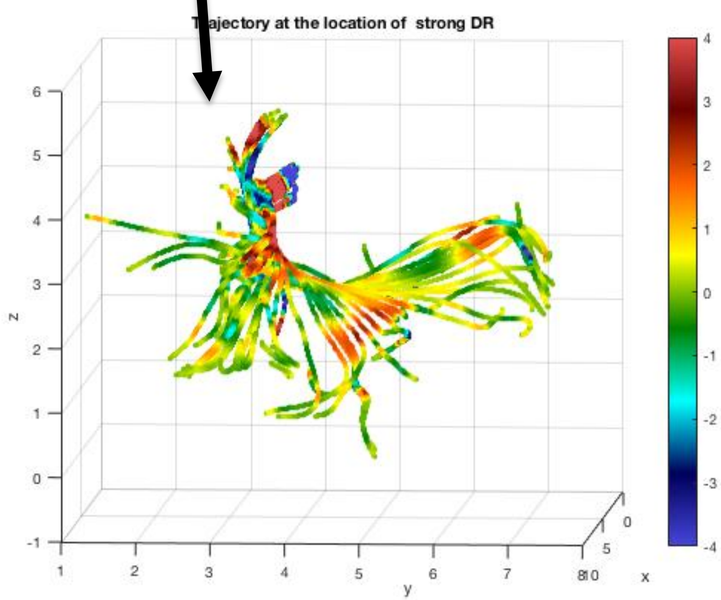
Geometry of trajectories at special points (DNS)







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Conclusion

Lagrangian and Eulerian irreversibility provides **complementary** view

Eulerian irreversibility points to **quasi singularities** of Eulerien Velocity Field
Lagrangian irreversibility points to **chaotic points** of Lagrangian trajectories

OPEN Questions: signatures of non-unicity?
Structure of quasi-singularities?
Dynamics of quasi-singularities?