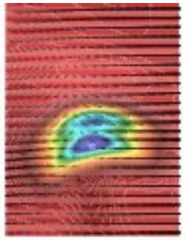

Extreme event of inertial kinetic and thermal dissipation in convective flow

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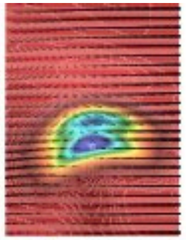
CEA Saclay France



Aim and motivations

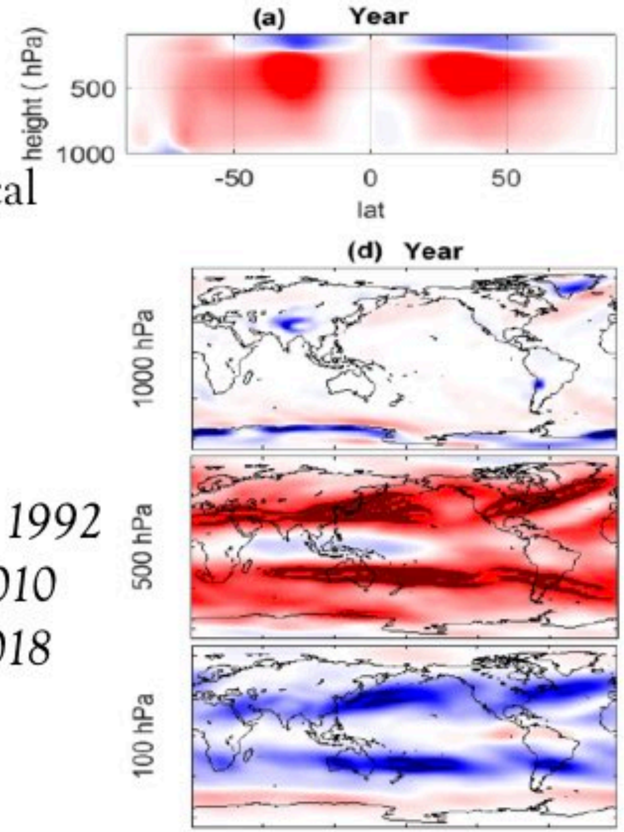
Here

- To apply to RB a new approach to analyse scaling and local properties

- To get some insights on the Kolmogorov/Bolgiano-Oboukhov scaling issue

Lvov 1991
Falkovich & Lvov 1992
Lohse & Xia 2010
Verma & al. 2018

https://arxiv.org/abs/1910.06088

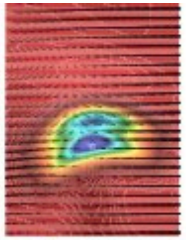


Example of energy fluxes in the atmosphere
D. Faranda *et al.*, *Journ. Atm. Sciences* (2018)

More general

- To localise and analyse extreme events

- Analyse the possible analogy with Von-Karman flow experiments (Saw et al. Nat. Comm. 2014)



Equations and numerical set-up

Cube, with imposed ΔT between the horizontal faces:

- Uniform top (T_t) and bottom (T_b) temperatures, adiabatic lateral walls;
- No slip velocity boundary conditions.

Data obtained with the open-source code Basilisk.

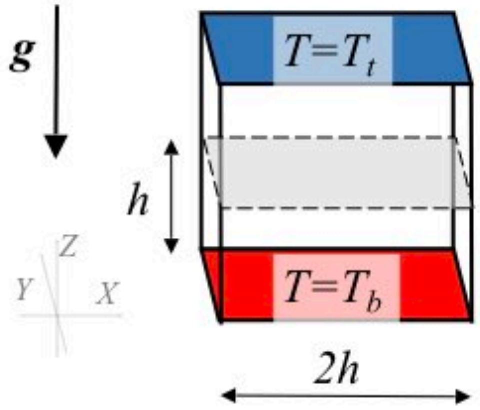
$$\frac{\partial u_i}{\partial x_i} = 0,$$

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{\partial P}{\partial x_i} + \sqrt{\frac{Pr}{Ra}} \frac{\partial^2 u_i}{\partial x_j^2} + \delta_{i,3}\theta$$

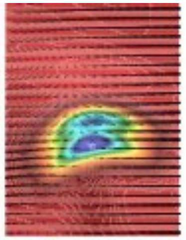
$$\frac{\partial \theta}{\partial t} + u_j \frac{\partial \theta}{\partial x_j} = \frac{1}{\sqrt{Pr Ra}} \frac{\partial^2 \theta}{\partial x_j^2},$$

$$Ra = \frac{g\alpha_0 \Delta T (2h)^3}{\nu\kappa}$$

$T_b > T_t$



Case	Ra	Pr	$N_x \times N_y \times N_z$	$\frac{\Delta}{\eta_{bulk}}$	$\frac{\Delta}{\eta_{BL}}$	Δt	N_T	Nu	Nu_{ϵ_u}	Nu_{ϵ_T}
A	10^7	1.0	$1024 \times 1024 \times 1024$	1/10	1/8	0.0015	3.3/30	15.8	16	15.9
B	10^8	1.0	$1024 \times 1024 \times 1024$	1/8	1/4	0.001	4.2/15	31.1	31.3	31.8



Small mathematical *detour*

J. Leray, "Essai sur Le Mouvement d'un fluide visqueux emplissant l'espace," Acta. Math. 63 193-248 (1934)

solutions of the INS

$$\partial_t \mathbf{u}^\nu + \nabla \cdot (\mathbf{u}^\nu \mathbf{u}^\nu) = -\nabla p^\nu + \nu \triangle \mathbf{u}^\nu, \quad \nabla \cdot \mathbf{u}^\nu = 0$$

always exist in the sense of spacetime distributions, i.e.

$$\int_{t_0}^{T} dt \int d^d\mathbf{x}[\partial_t \varphi(\mathbf{x},t) \cdot \mathbf{u}^\nu(\mathbf{x},t) + \nabla\varphi(\mathbf{x},t) : \mathbf{u}^\nu(\mathbf{x},t) \otimes \mathbf{u}^\nu(\mathbf{x},t)$$
$$+ \quad (\nabla \cdot \varphi(\mathbf{x},t)) p^\nu(\mathbf{x},t) - \nu \triangle \varphi(\mathbf{x},t) \cdot \mathbf{u}^\nu(\mathbf{x},t)] = \int d^d\mathbf{x} \;\; \varphi(\mathbf{x},t_0) \cdot \mathbf{u}_0^\nu(\mathbf{x},t),$$

Physically:
Momentum change comes from flux of momentum across the surface and no flux of mass

$$\int_{t_0}^{T} dt \int d^d\mathbf{x} \nabla\psi(\mathbf{x},t) \cdot \mathbf{u}^\nu(\mathbf{x},t) = 0$$

$$\tfrac{1}{2}||\mathbf{u}^\nu(t)||^2_{L^2(V)} + \nu \int_{t_0}^{t} d\tau ||\nabla \mathbf{u}^\nu(\tau)||^2_{L^2(V)} \le \tfrac{1}{2}||\mathbf{u}_0^\nu||^2_{L^2(V)}$$

Physically:
= ->Solutions smooth (strong)
< => Solutions singular (weak)

$$\begin{aligned}\partial_t(\frac{1}{2}\mathbf{u} \cdot \bar{\mathbf{u}}_\ell) \quad &+ \quad \nabla \cdot [(\frac{1}{2}\mathbf{u} \cdot \bar{\mathbf{u}}_\ell)\mathbf{u} + \frac{1}{2}(p\bar{\mathbf{u}}_\ell + \bar{p}_\ell \mathbf{u}) \\ &+ \quad \frac{1}{4}\overline{((|\mathbf{u}|^2\mathbf{u})}_\ell - \frac{1}{4}\overline{(|\mathbf{u}|^2)}\mathbf{u} - \nu\nabla\frac{1}{2}\mathbf{u} \cdot \bar{\mathbf{u}}_\ell] \\ &= \quad -\frac{1}{4\ell}\int d^d\mathbf{r} \;\; \nabla G_\ell(\mathbf{r}) \cdot \delta\mathbf{u}(\mathbf{r})|\delta\mathbf{u}(\mathbf{r})|^2 - \nu\nabla\mathbf{u} : \nabla\bar{\mathbf{u}}_\ell\end{aligned}$$

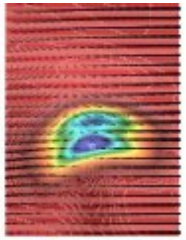
Physically:
Coarse-graining of the INS
<-> scale-by-scale analysis

$$\partial_t(\tfrac{1}{2}|\mathbf{u}|^2) + \nabla \cdot \left[(\tfrac{1}{2}|\mathbf{u}|^2 + p)\mathbf{u} - \nu\nabla(\tfrac{1}{2}|\mathbf{u}|^2)\right] = -D(\mathbf{u}) - \nu|\nabla\mathbf{u}|^2$$

Duchon & Robert (2000)

$$D(\mathbf{u}) = \lim_{\ell \to 0} \tfrac{1}{4\ell} \int d^d\mathbf{r}(\nabla G)_\ell(\mathbf{r}) \cdot \delta\mathbf{u}(\mathbf{r})|\delta\mathbf{u}(\mathbf{r})|^2$$

$$D(\mathbf{u}^\nu) + \nu|\nabla\mathbf{u}^\nu|^2 \to D(\mathbf{u}).$$



Coarse graining of Boussinesq equations

Weak equations -> Singularities and extreme events

Duchon & Robert 2000
Eyink 2003,2006
Saw & al. 2014

$$\frac{1}{2}\mathbf{u}(x,t)\cdot\tilde{\mathbf{u}}_\ell(x,t) = \int d^d r G_\ell(r)\frac{1}{2}\mathbf{u}(x,t)\cdot\mathbf{u}(x+r,t)$$

Local fluctuating form of the *Karman – Howarth – Monin equations*:

$$\textit{(1)}\ \ \partial_t E^\ell + \vec{\nabla}\cdot\vec{J}_K^\ell = -\frac{1}{4}\int d^d r\nabla G_\ell\cdot\delta\mathbf{u}(r)\,|\,\delta\mathbf{u}(r)\,|^2 - \frac{\nu}{2}\int d^d r\nabla^2 G_\ell\,|\,\delta\mathbf{u}(r)\,|^2 + \frac{1}{2}\alpha g(\mathbf{u}\tilde{\theta}_\ell + \tilde{\mathbf{u}}_\ell\theta) \equiv -\mathcal{D}_\ell - \mathcal{D}_\ell^\nu + \mathcal{D}_\ell^c$$

$$\textit{(2)}\ \ \partial_t E_T^\ell + \vec{\nabla}\cdot\vec{J}_T^\ell = -\frac{1}{4}\int d^d r\nabla G_\ell\cdot\delta\mathbf{u}(r)(\delta T)^2 - \frac{\kappa}{2}\int d^d r\nabla^2 G_\ell\,|\,\delta T(r)\,|^2 - \frac{1}{2}\alpha g(\mathbf{u}\tilde{\theta}_\ell + \tilde{\mathbf{u}}_\ell\theta) \equiv -\mathcal{D}_\ell^T - \mathcal{D}_\ell^\kappa - \mathcal{D}_\ell^c$$

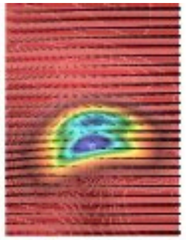
Technically:
Wavelets

$$\mathrm{D}_\ell = \frac{1}{4}\int d^d r(\nabla G_\ell(\mathbf{r}))\cdot[\mathbf{u}(\mathbf{x}+\mathbf{r},t)-\mathbf{u}(\mathbf{x},t)][\mathbf{u}(\mathbf{x}+\mathbf{r},t)-\mathbf{u}(\mathbf{x},t)]^2$$

$$\mathrm{D}_\ell^T = \frac{1}{4}\int d^d r(\nabla G_\ell(\mathbf{r}))\cdot[\mathbf{u}(\mathbf{x}+\mathbf{r},t)-\mathbf{u}(\mathbf{x},t)][\theta(\mathbf{x}+\mathbf{r},t)-\theta(\mathbf{x},t)]^2$$

$$\mathrm{D}_\ell^\nu = \frac{1}{\sqrt{RaPr}}\int d^d r(\nabla^2 G_\ell(\mathbf{r}))\left[\mathbf{u}(\mathbf{x}+\mathbf{r},t)\cdot\mathbf{u}(\mathbf{x},t) - \frac{\mathbf{u}(\mathbf{x}+\mathbf{r},t)\cdot\mathbf{u}(\mathbf{x}+\mathbf{r},t)}{2}\right]$$

$$\mathrm{D}_\ell^\kappa = \frac{1}{\sqrt{RaPr}}\int d^d r(\nabla^2 G_\ell(\mathbf{r}))\left[\theta(\mathbf{x}+\mathbf{r},t)\cdot\theta(\mathbf{x},t) - \frac{\theta^2(\mathbf{x}+\mathbf{r},t)}{2}\right]$$



Local Bolgiano-Oboukhov Length

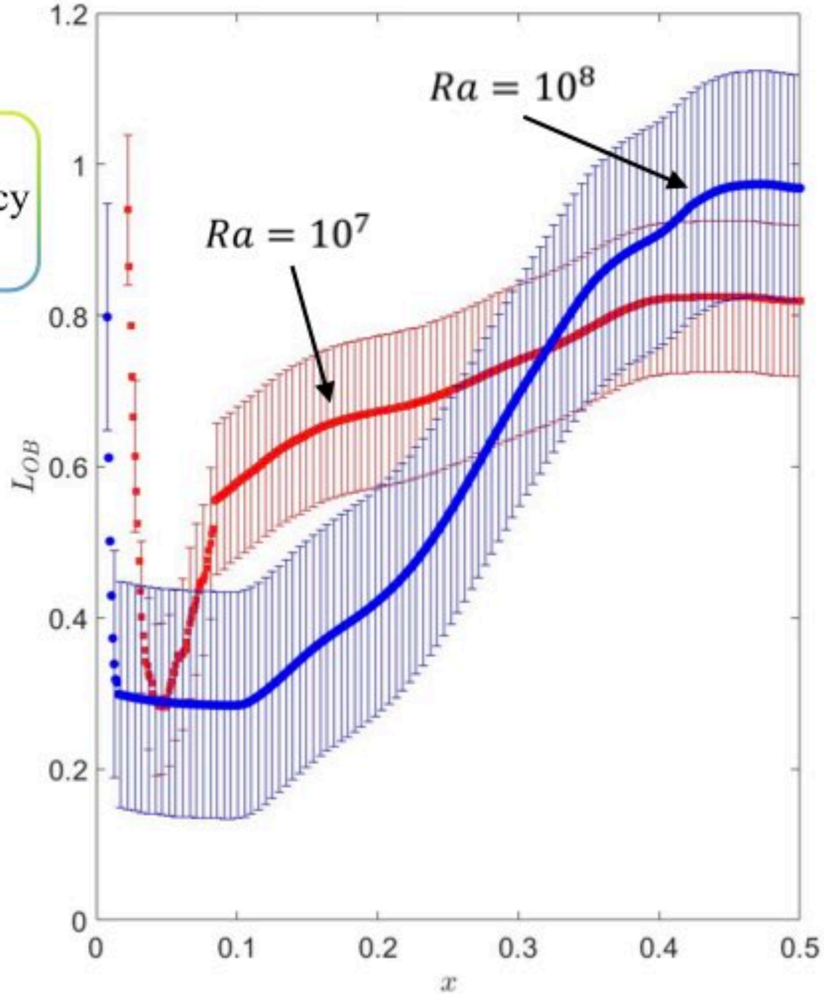
Oboukhov-Bolgiano scale, L_{OB}:
estimate of the distance at which the dissipative and buoyancy terms balance in the Boussinesq NS eqs.

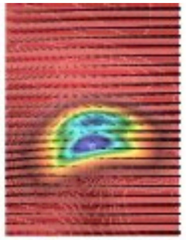
Spatial distribution at $z = h$, of L_{OB}:

$$L_{OB} = \left\langle \epsilon^{5/4} \right\rangle_t \left\langle \epsilon_T^{-3/4} \right\rangle_t$$

Observations:

- Local values of $L_{OB} < 1$, OB scaling?
- The values of L_{OB} at the center of the cell are larger for $Ra = 10^8$ than for $Ra = 10^7$;
- Spatial mean of $L_{OB} \cong 0{,}79$ for $Ra = 10^7$, and $L_{OB} \cong 0{,}72$ $Ra = 10^8$;
- Results agree with literature [Kaczorowski *et al.* (2013)]



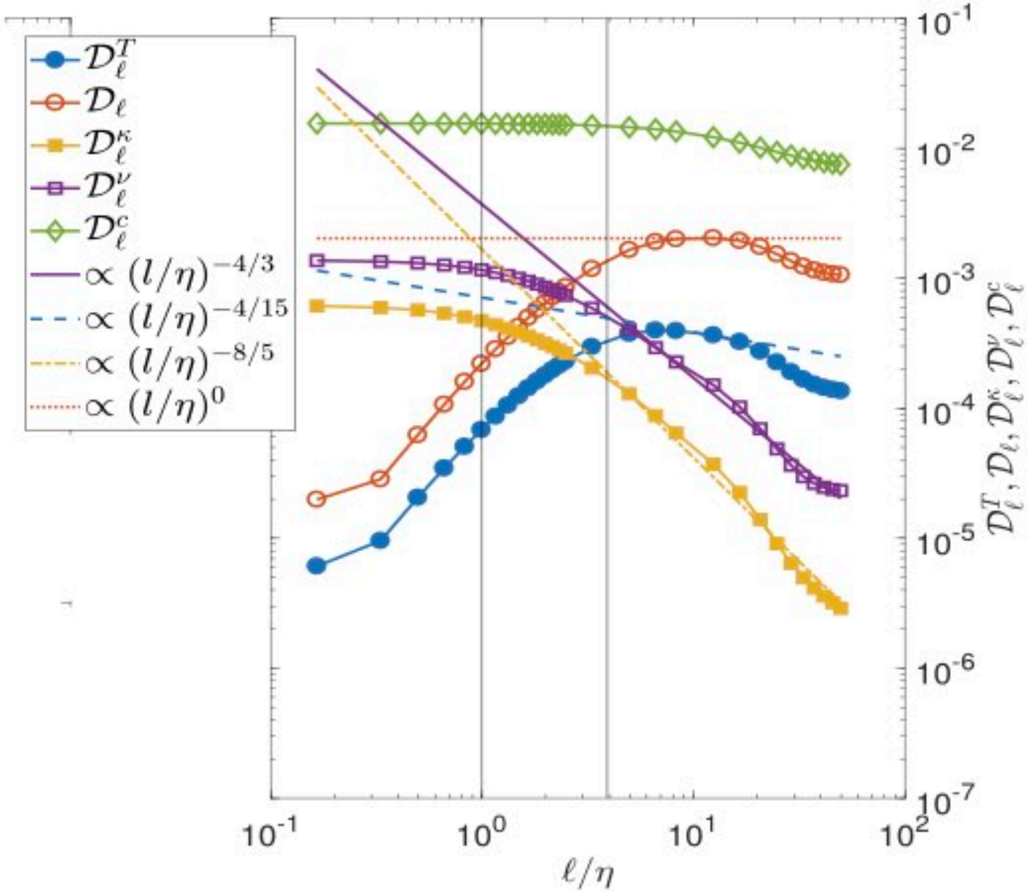


Results: Bulk

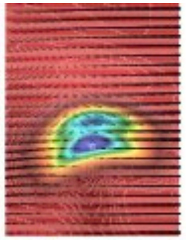
scaling hypothesis

$$\delta u_\ell \equiv \langle |u(x+l) - u(x)| \rangle \sim l^h$$

	General	Kolmogorov–41 $h_u = h_T = 1/3$	Oboukhov-Bolgiano $h_u = 3/5;\ h_T = 1/5$
D_ℓ	$\sim \ell^{3h_u-1}$	$\sim \ell^0$	$\sim \ell^{4/5}$
D_ℓ^T	$\sim \ell^{h_u+2h_T-1}$	$\sim \ell^0$	$\sim \ell^0$
D_ℓ^ν	$\sim \ell^{2h_u-2}$	$\sim \ell^{-4/3}$	$\sim \ell^{-4/5}$
D_ℓ^κ	$\sim \ell^{2h_T-2}$	$\sim \ell^{-4/3}$	$\sim \ell^{-8/5}$
D_ℓ^c	$\sim \ell^{h_u+h_T}$	$\sim \ell^{2/3}$	$\sim \ell^{4/5}$



- Scaling difficult to be assessed
- Velocity and Temperature are not Kolmogorov like
- Scaling in dissipative range
- Shear effect possibly important

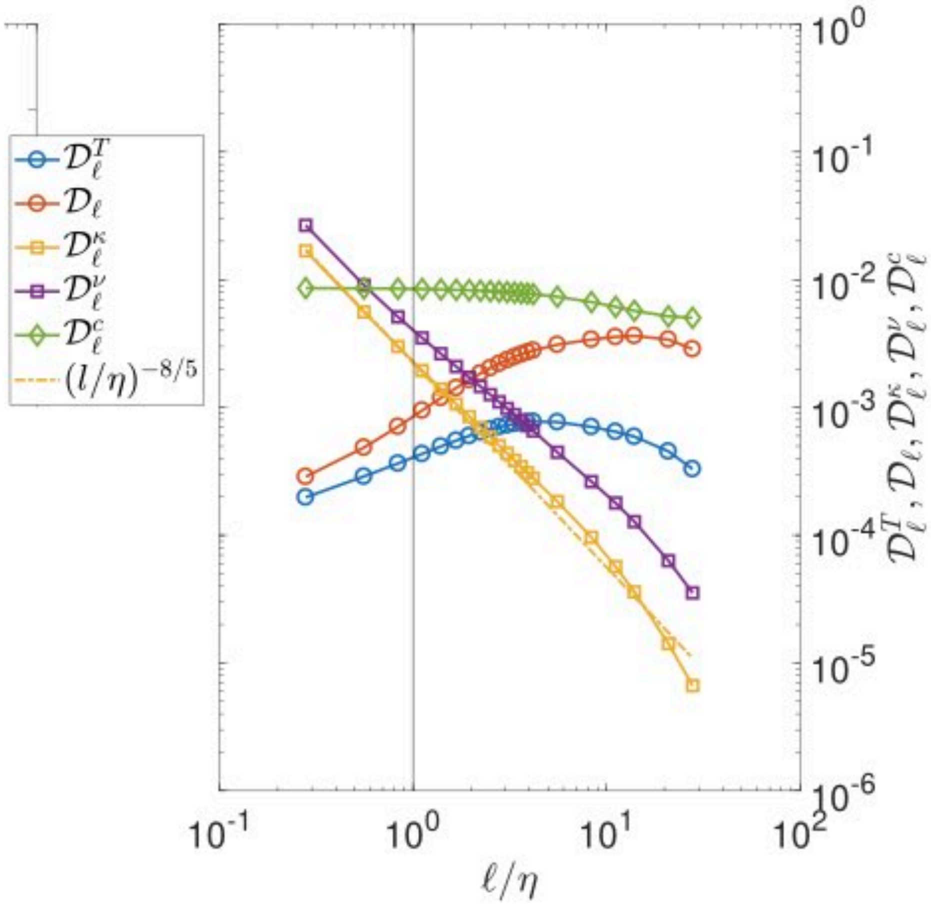


Results: Boundary layer

scaling hypothesis

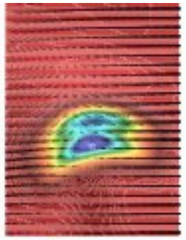
$$\delta u_\ell \equiv \langle |u(x+l) - u(x)| \rangle \sim l^h$$

	General	Kolmogorov–41 $h_u = h_T = 1/3$	Oboukhov-Bolgiano $h_u = 3/5;\ h_T = 1/5$
D_ℓ	$\sim \ell^{3h_u-1}$	$\sim \ell^0$	$\sim \ell^{4/5}$
D_ℓ^T	$\sim \ell^{h_u+2h_T-1}$	$\sim \ell^0$	$\sim \ell^0$
D_ℓ^ν	$\sim \ell^{2h_u-2}$	$\sim \ell^{-4/3}$	$\sim \ell^{-4/5}$
D_ℓ^κ	$\sim \ell^{2h_T-2}$	$\sim \ell^{-4/3}$	$\sim \ell^{-8/5}$
D_ℓ^c	$\sim \ell^{h_u+h_T}$	$\sim \ell^{2/3}$	$\sim \ell^{4/5}$



- Scaling difficult to be assessed
- Velocity and Temperature are not Kolmogorov like
- Scaling in dissipative range
- Shear effect possibly important

Transfer from kinetic to potential energy at small scales



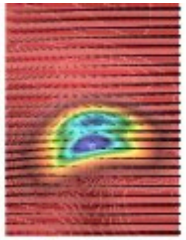
Discussion on scaling

	General	Kolmogorov–41 $h_u = h_T = 1/3$	Oboukhov-Bolgiano $h_u = 3/5;\ h_T = 1/5$	Present study $h_u = 1/3;\ h_T = 1/5$
D_ℓ	$\sim \ell^{3h_u-1}$	$\sim \ell^{0}$	$\sim \ell^{4/5}$	$\sim \ell^{0}$
D_ℓ^T	$\sim \ell^{h_u+2h_T-1}$	$\sim \ell^{0}$	$\sim \ell^{0}$	$\sim \ell^{-4/15}$
D_ℓ^ν	$\sim \ell^{2h_u-2}$	$\sim \ell^{-4/3}$	$\sim \ell^{-4/5}$	$\sim \ell^{-4/3}$
D_ℓ^κ	$\sim \ell^{2h_T-2}$	$\sim \ell^{-4/3}$	$\sim \ell^{-8/5}$	$\sim \ell^{-8/5}$
D_ℓ^c	$\sim \ell^{h_u+h_T}$	$\sim \ell^{2/3}$	$\sim \ell^{4/5}$	

- Through local analysis it is possible to access to scaling at very small scales
- Velocity is found Kolmogorov like and Temperature scales like Bolgiano
- The overall picture suggests an anisotropic distribution of energy
- Shear effect should explain the scaling near the boundaries which appear

✱ Vertical scaling remains to be assessed

✱ What about *local* scaling and extreme events in the dissipative range



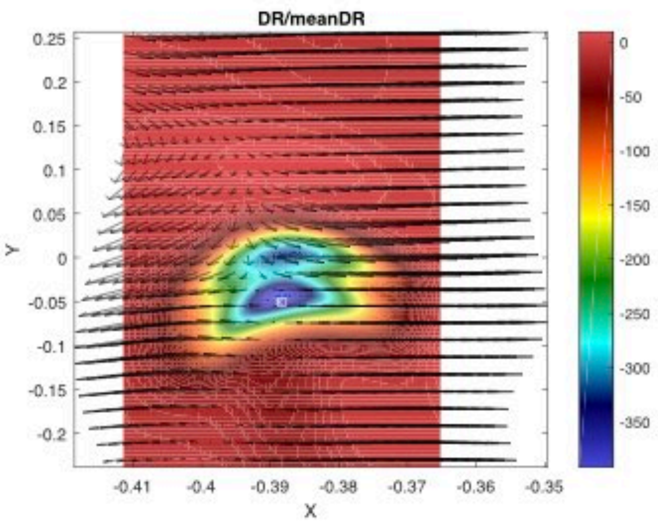
Extreme events: PDF

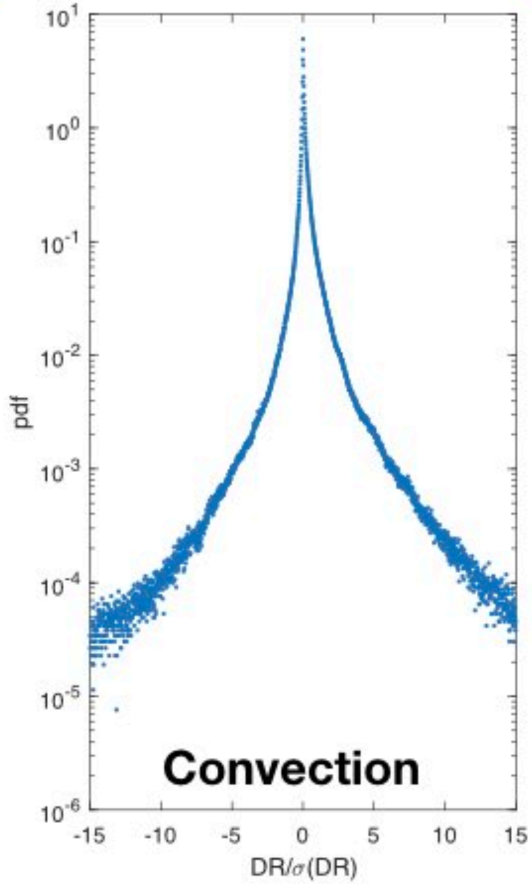
Local scaling hypothesis

$$\delta u_\ell \equiv \langle |u(x+l) - u(x)| \rangle \sim l^h$$

Multifractal picture and
Onsager conjecture
Looking for local regions where

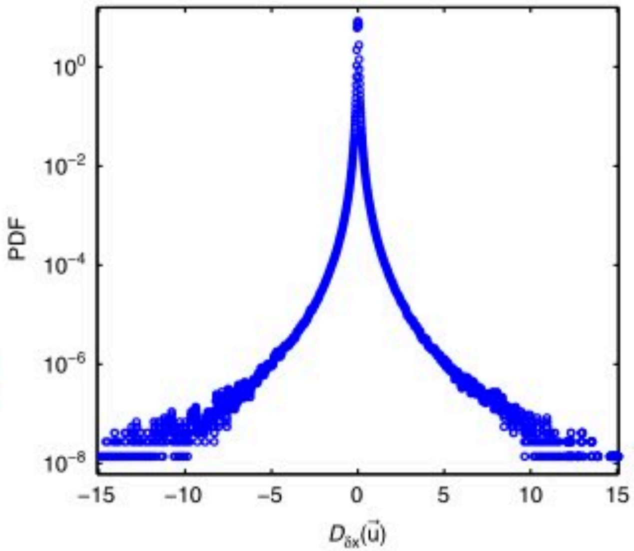
$$h < 1/3$$



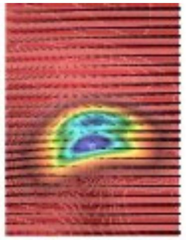


Von Karman

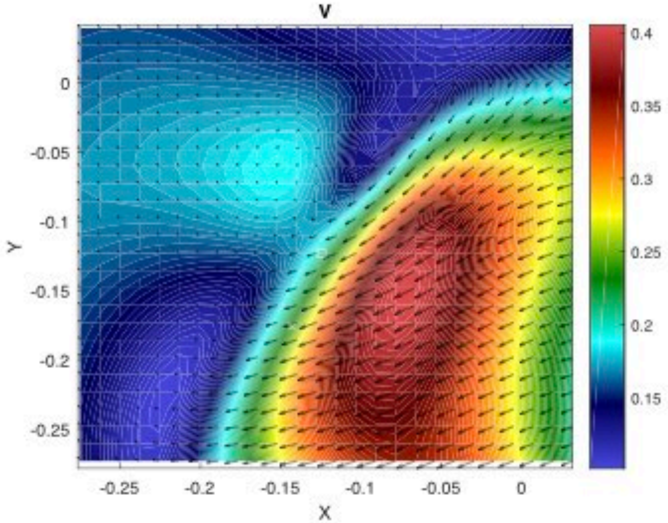
Saw et al. Nat. Comm. 2014

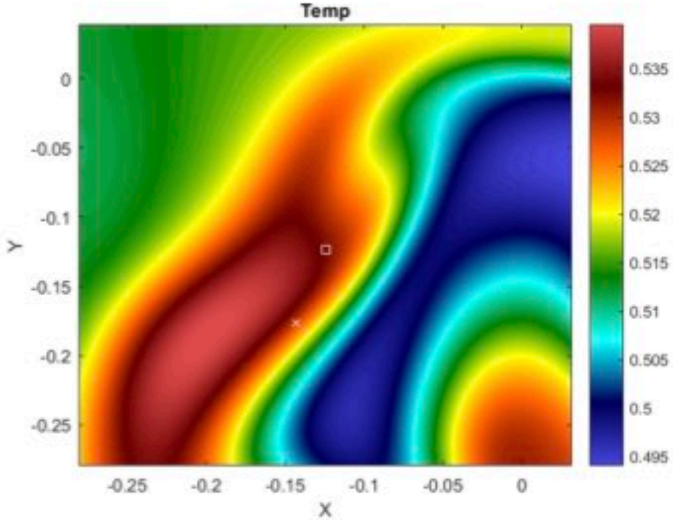


- Local inertial dissipation positive or negative (average>0)
- Dissipation locally very strong
- Some analogy with VK experiments



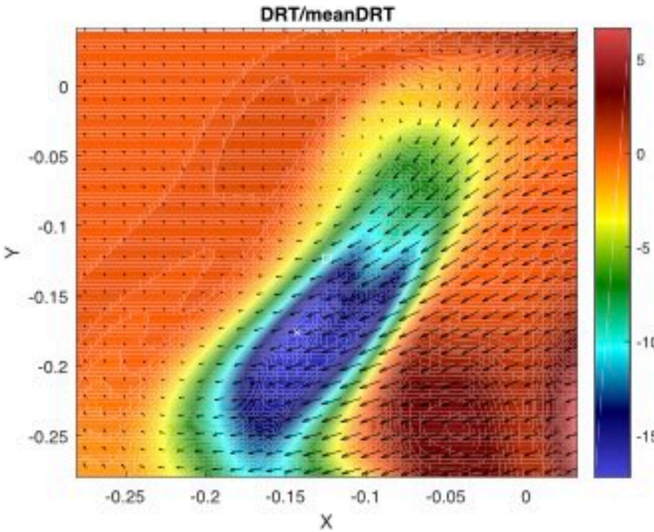
Extreme events: Bulk

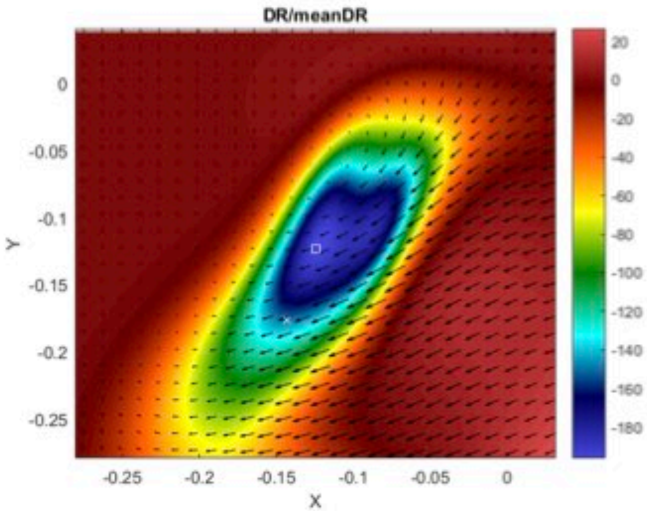


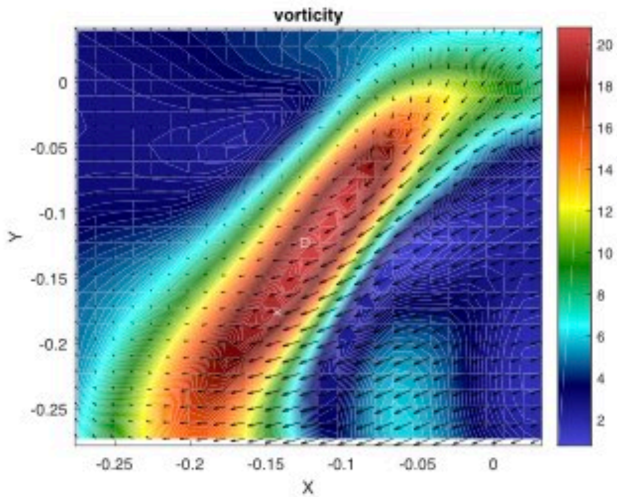


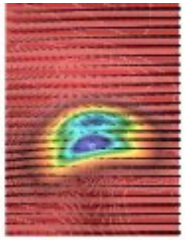
Inertial dissipation-confirmation of VK findings

Fronts patterns









Extreme events: Plumes I

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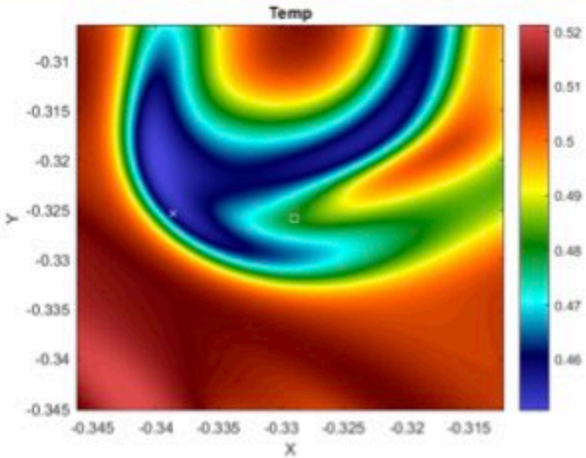
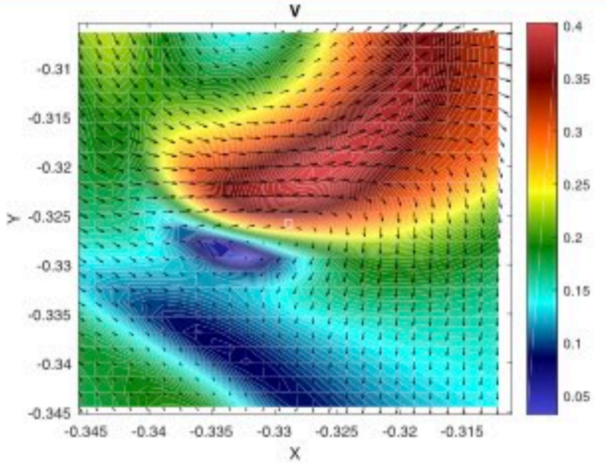
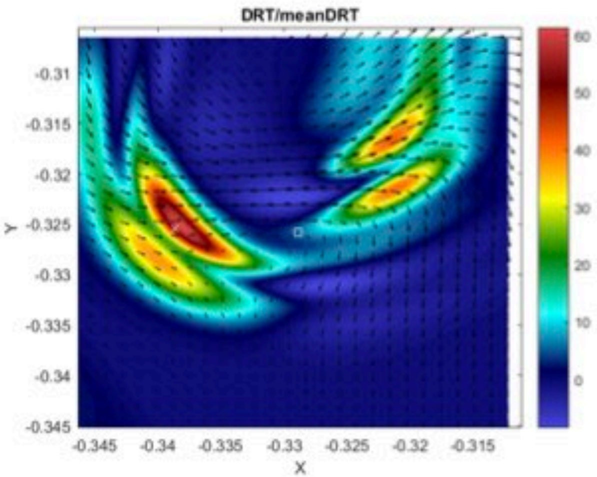
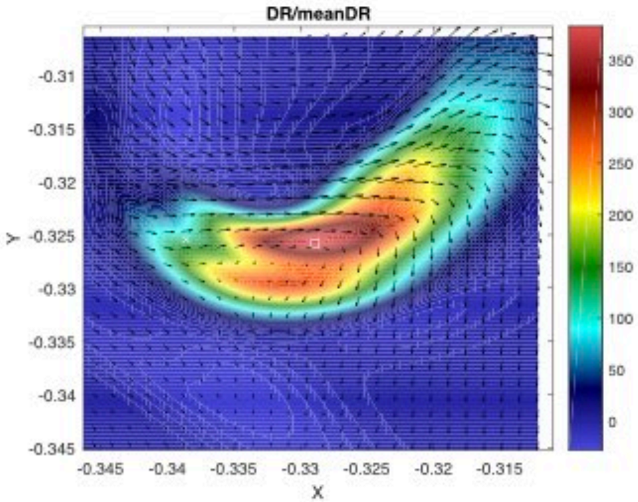
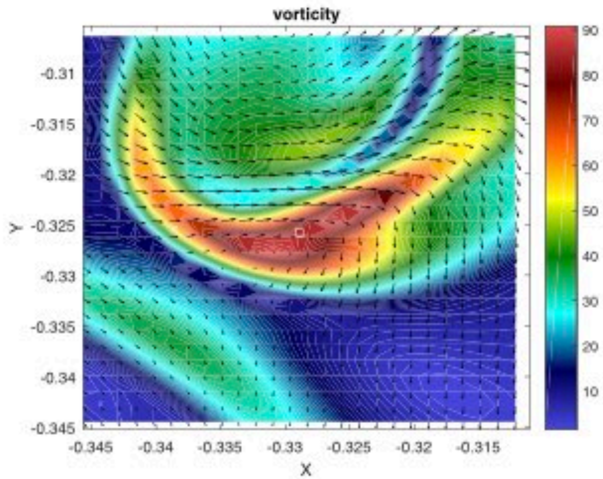


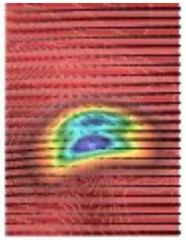


- Mechanics of plume formation near-the-wall
- Strong correlation between T and U fluxes

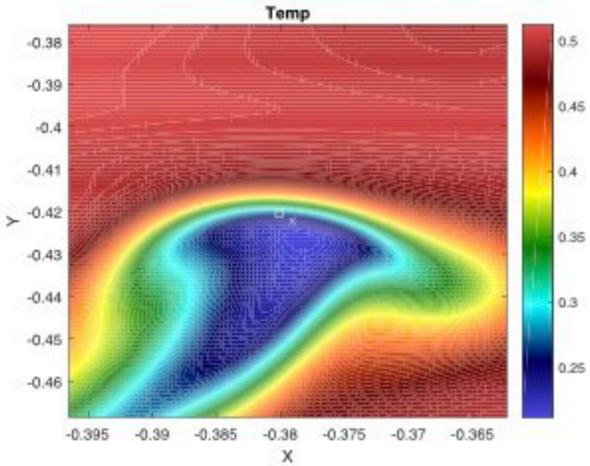


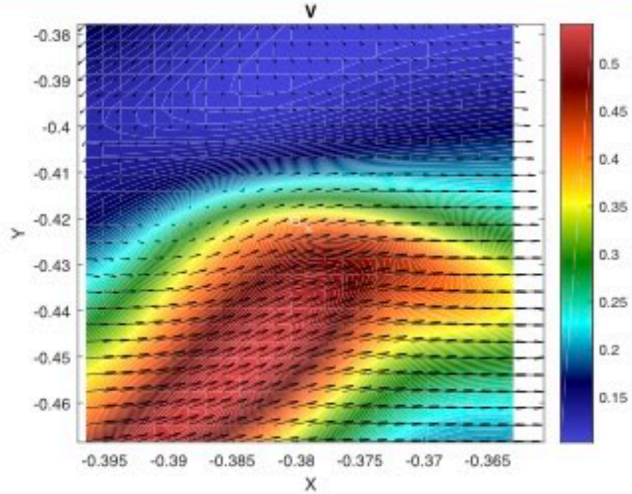




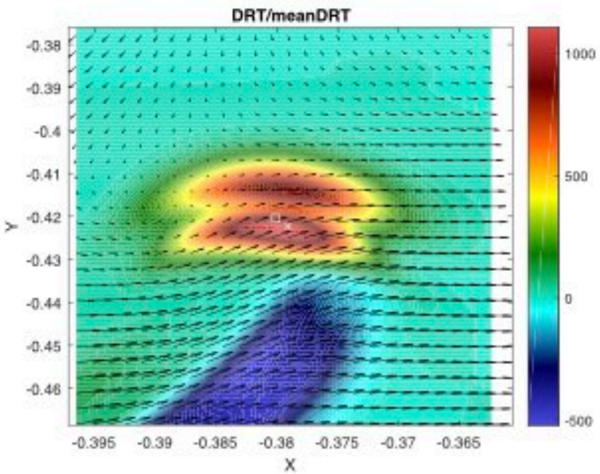


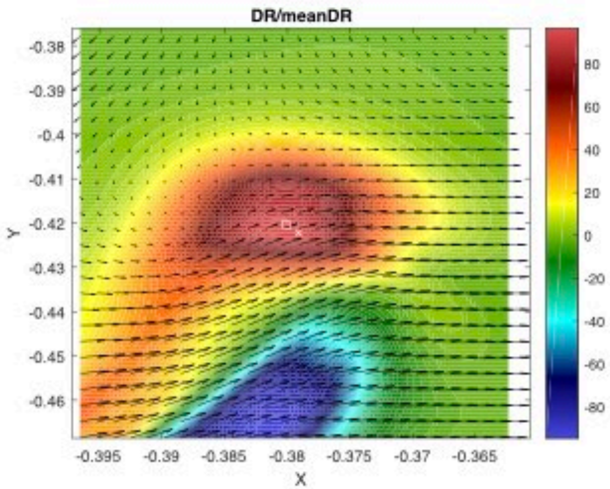
Extreme events: Plumes II

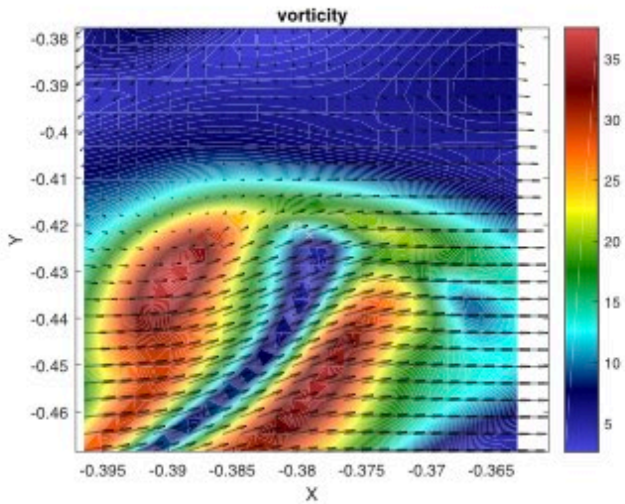


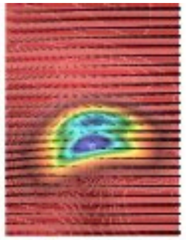


- Once formed the plume is related to extreme temperature fluxes
- Strong correlation between T and U variables
- Vorticity is partially smoothed

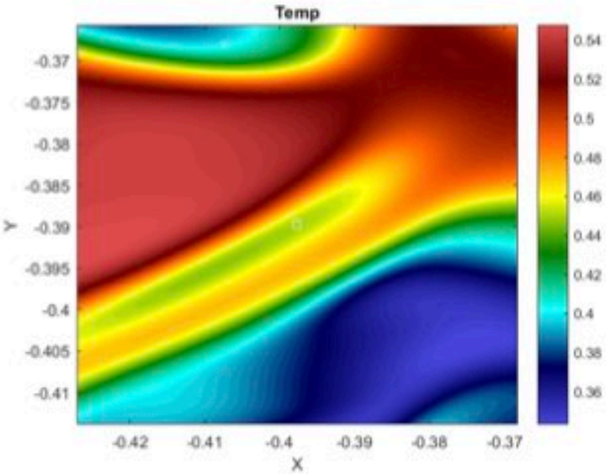


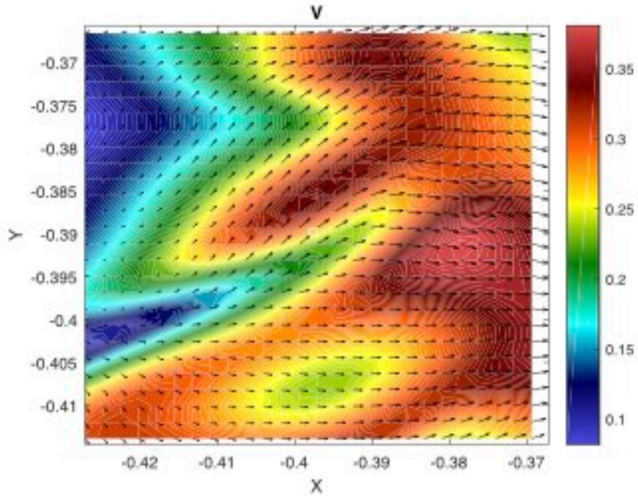




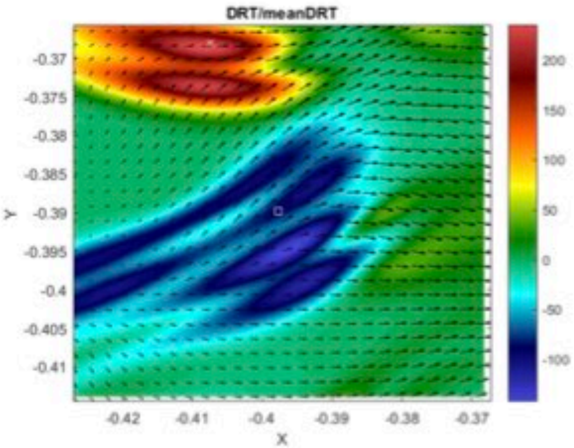


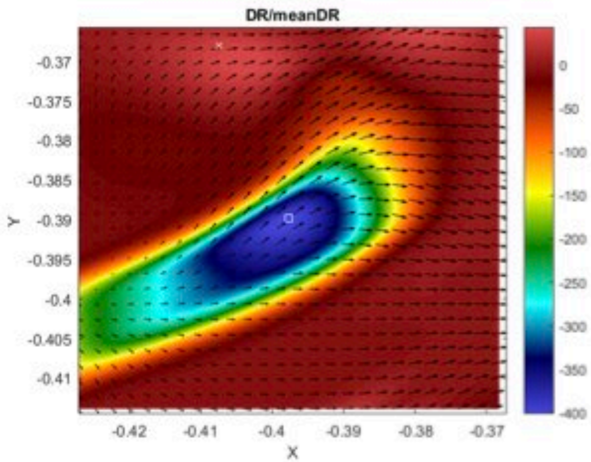
Extreme events: Plumes III

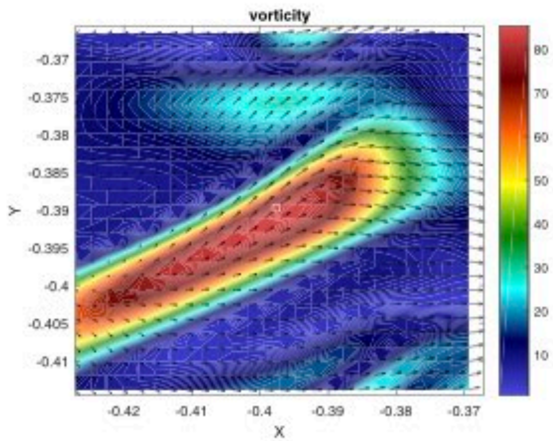


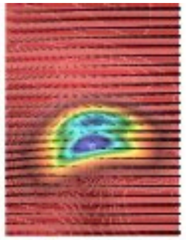


- Near-wall extreme events correlated to dissipation/vorticity
- Triggering the temperature flux and the process of plume









Conclusions

- Weak formulation of the equations of convection
- Framework useful to analyse local fluxes
- Through the use of wavelets: investigation of small-scale scaling
- Finding: Bolgiano-Oboukhov scaling in Temperature. Small-scales anisotropic
- Local analysis of extreme events at sub-Kolmogorov scale:
  - In the bulk few events related to inertial flux
  - Many events in the region near to the walls with rich dynamics and topology
  - Most common Picture: extreme inertial flux triggers temperature flux
  - In turn those fluxes triggers the appearance of plumes in V and T