Coherent laminar and turbulent motion of toroidal vortex bundles in superfluid helium

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Newcastle University

In collaboration with:
Daniel Wacks
Carlo Barenghi
Donatello Gallucci
Nick Proukakis

Outline

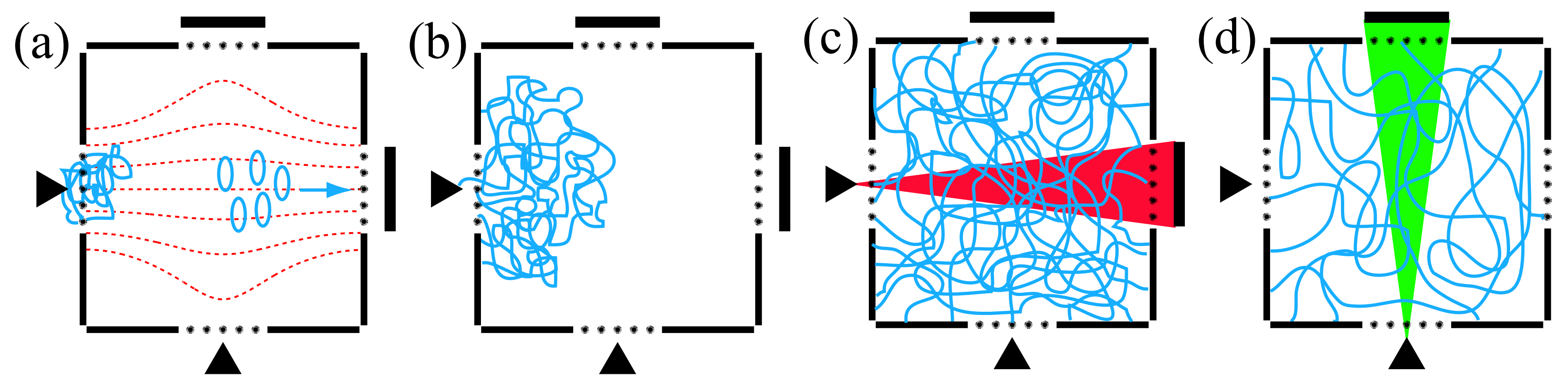
- Motivation
  - (3D) vortex bundles
  - (2D) vortex clusters
- Vortex rings in superfluid helium
- Experiment:

  Borner, Schmeling, & Schmidt (Physics of Fluids 1983)
- Numerical work:

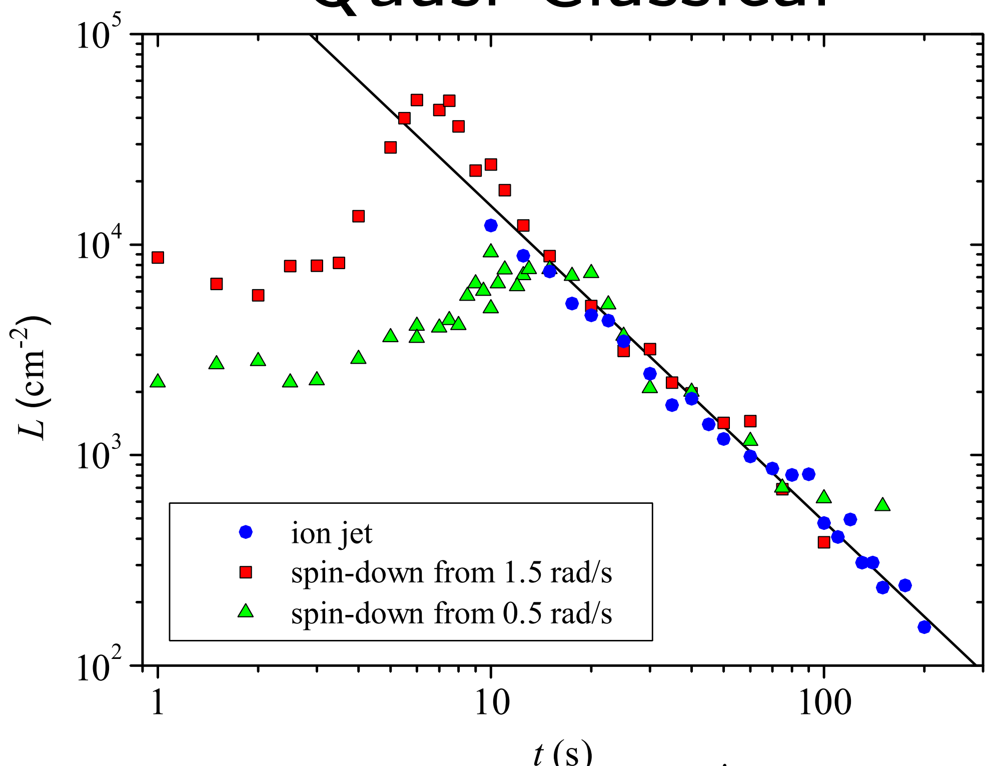
  Wacks, Baggaley, & Barenghi (PoF 2013, PRB 2014)
- Conclusions

Motivation

At T=0 two distinct regimes of quantum turbulence are observed

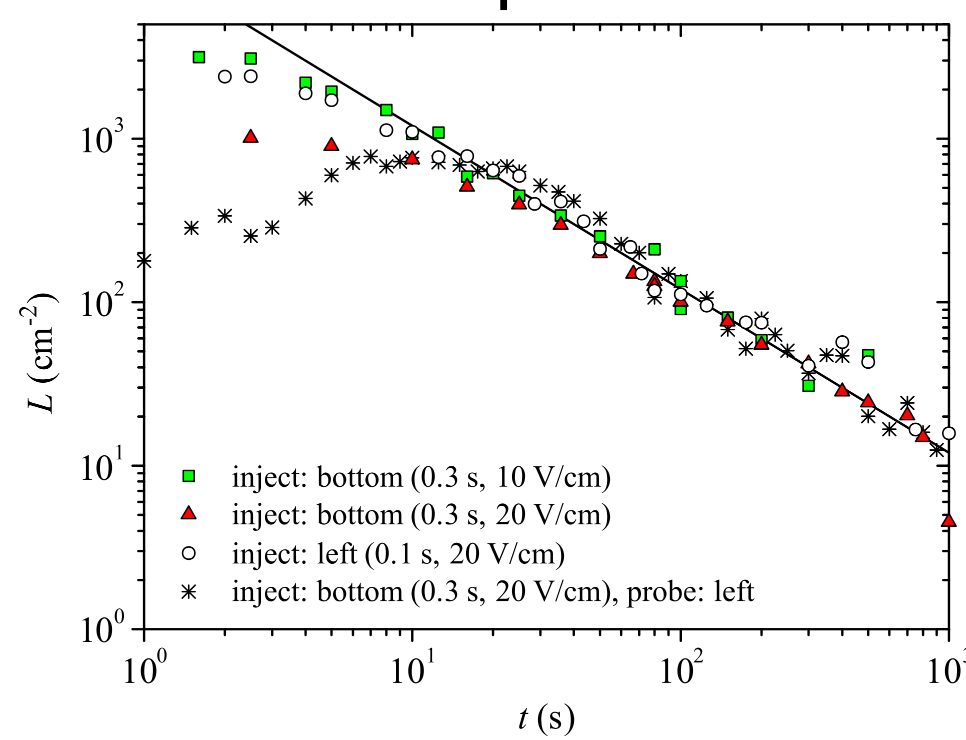


Quasi-Classical



$$L \sim t^{-3/2}$$

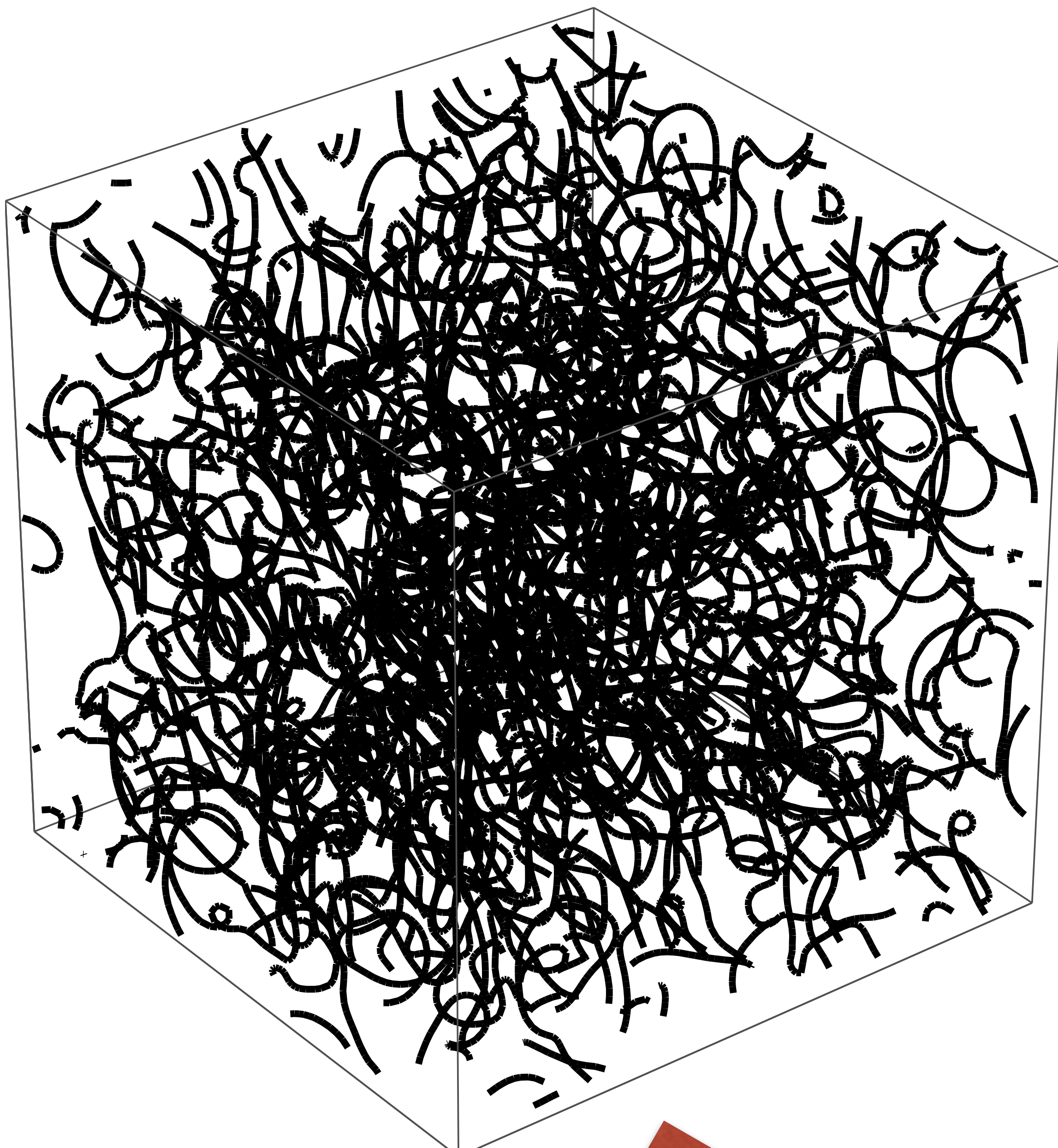
Ultra-quantum



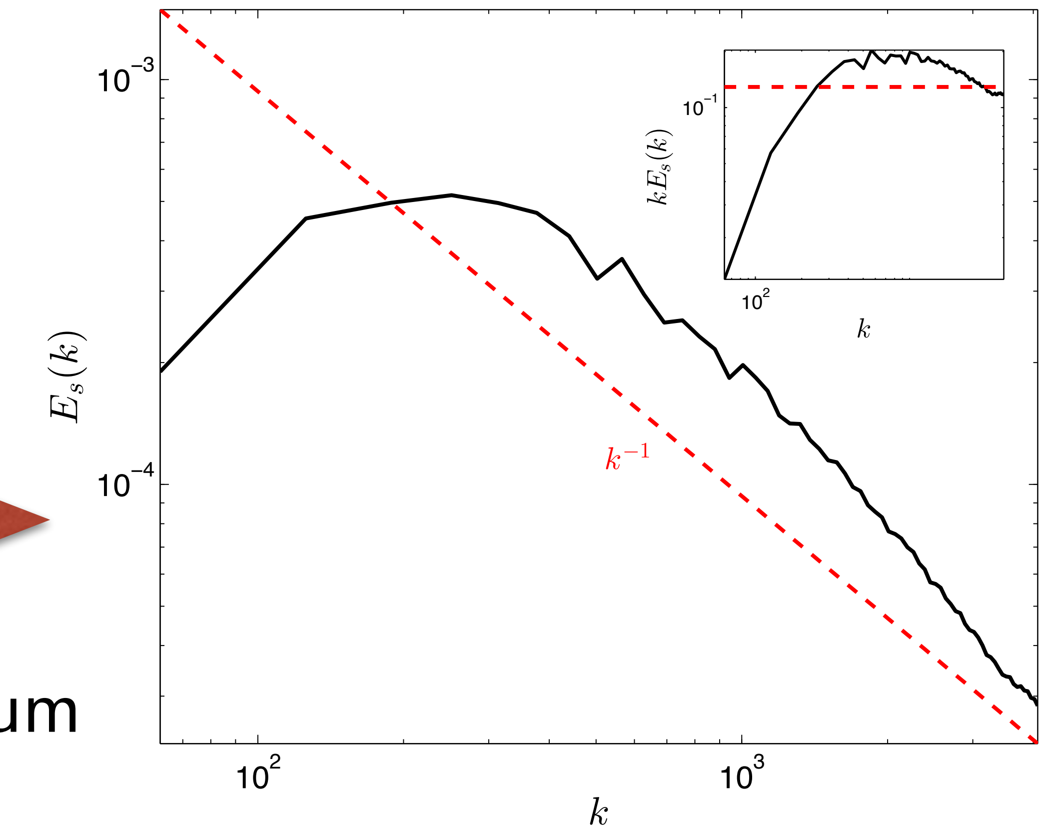
$$L \sim t^{-1}$$

Walmsley & Golov, PRL, 2008

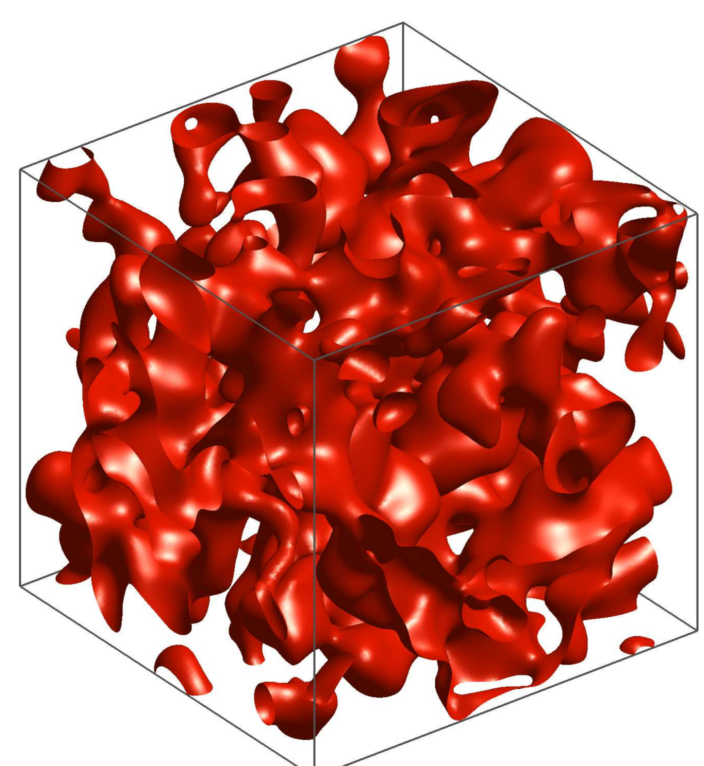
(*T*>0) Also T1 regime of thermal counterflow



Energy spectrum

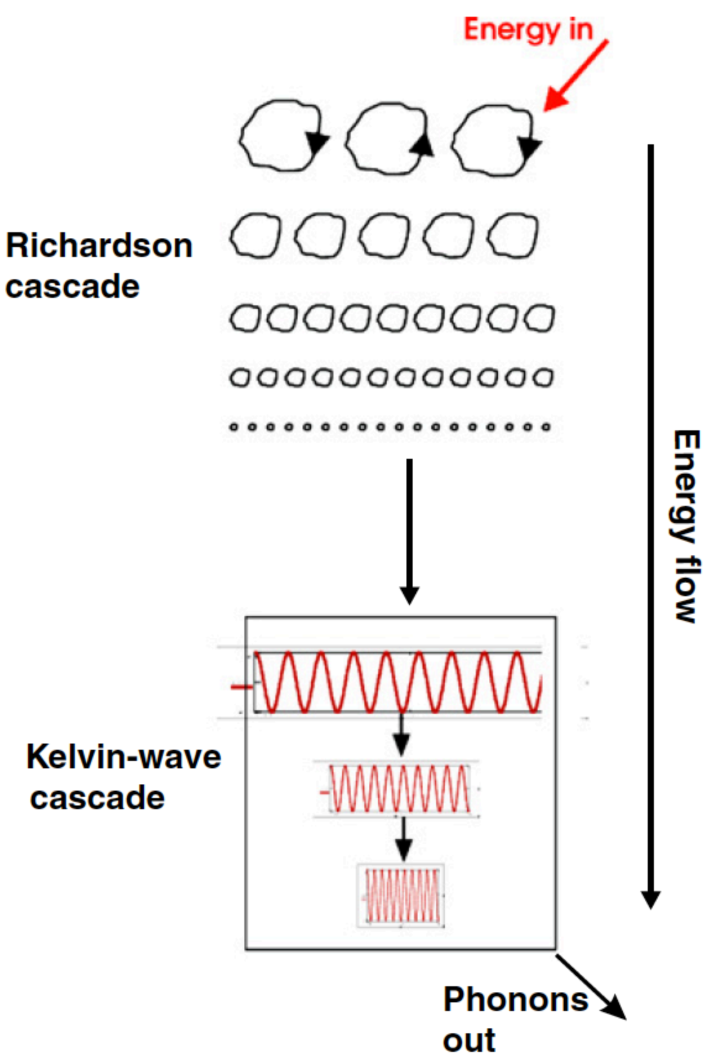


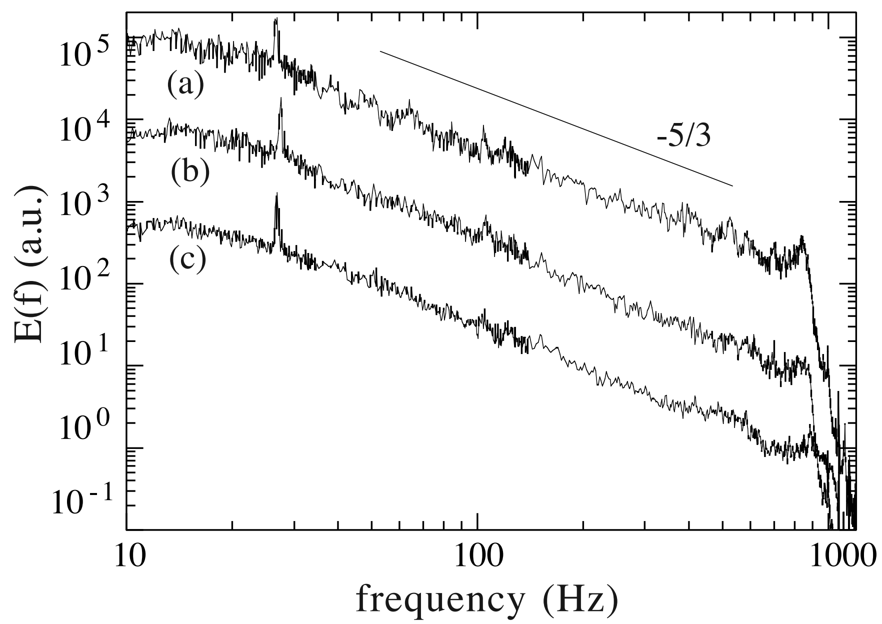
Smoothed with
cubic spline



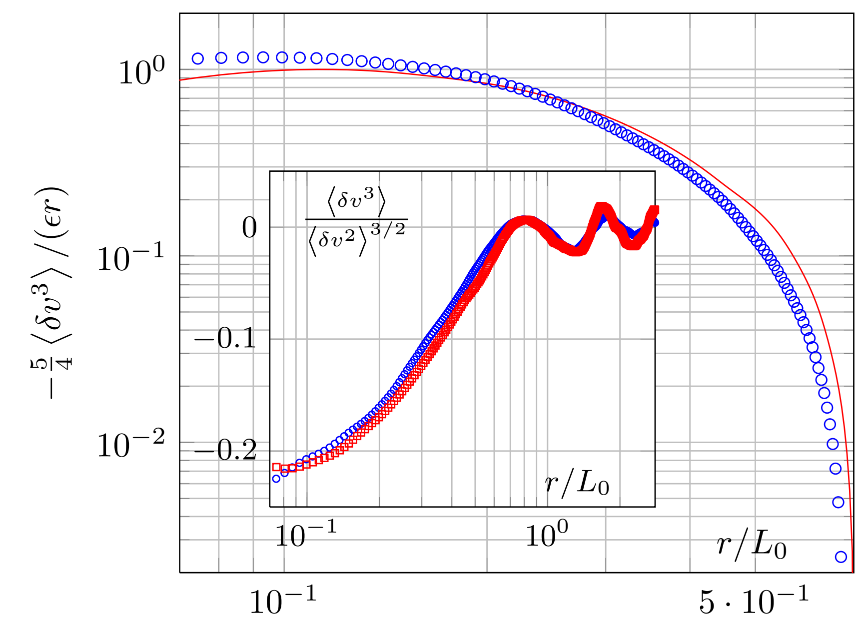
Quasi-Classical Regime

Some statistical properties of the flow are in agreement with K41

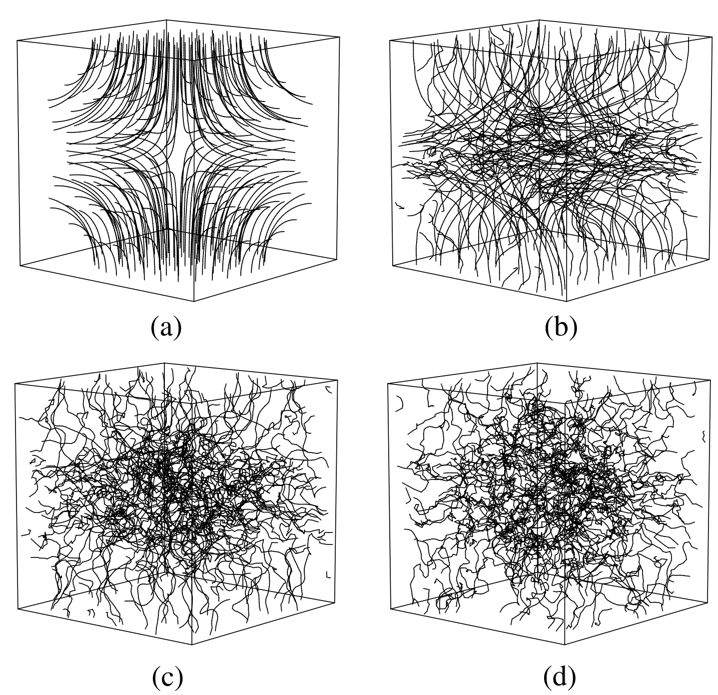


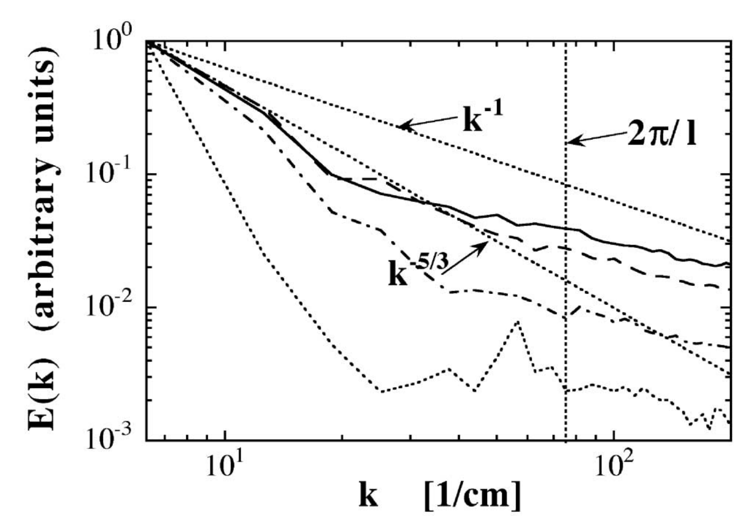


Maurer & Tabeling



Salort et al.

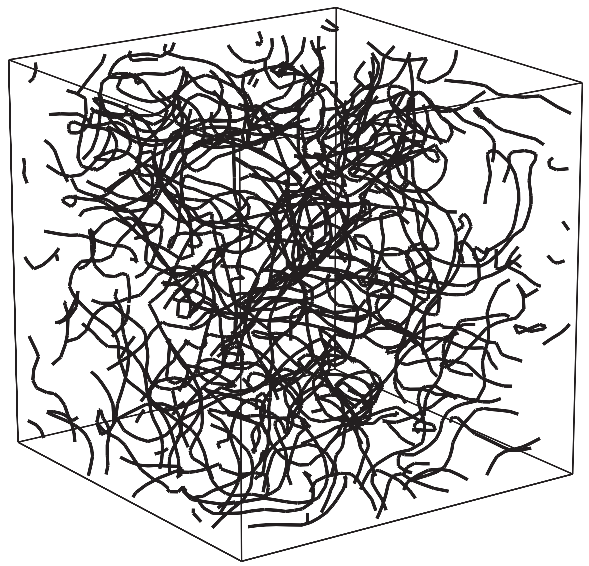


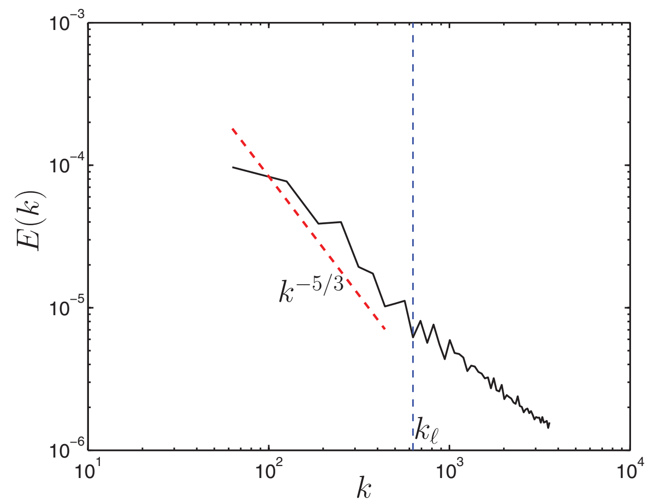


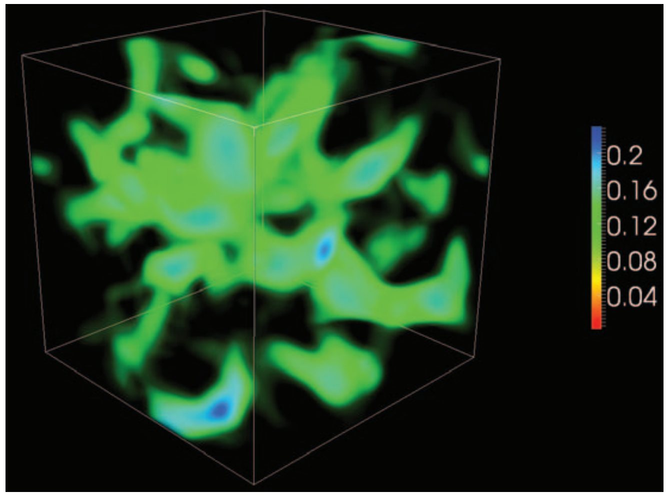
Araki et al.

Vortex bundles seem important to generate large scale flow

$T > 0$



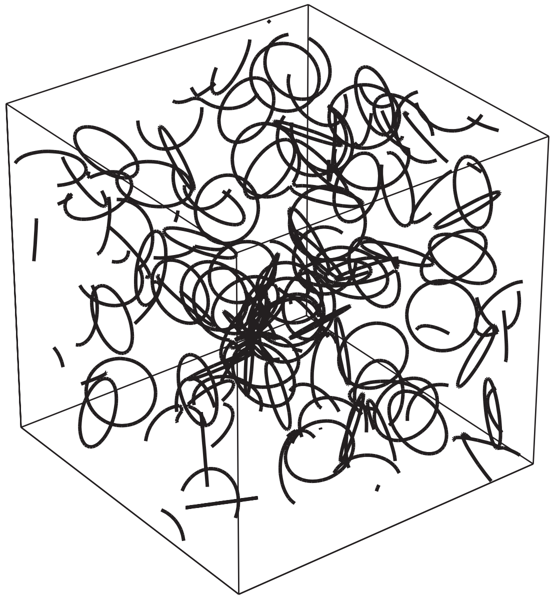


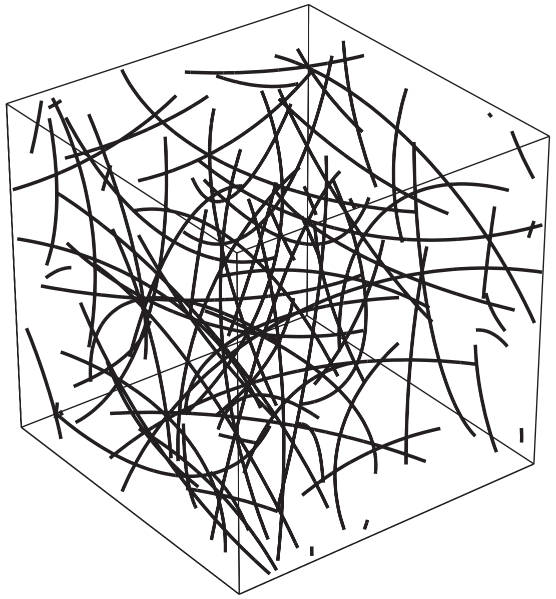


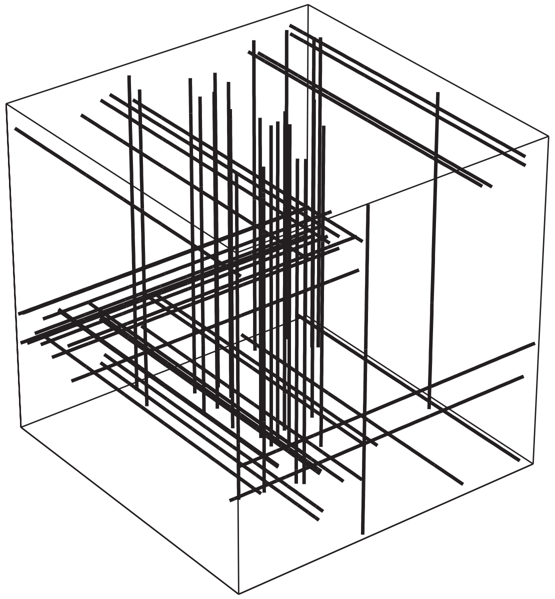
$$\mathbf{v}_n(\mathbf{s}, t) = \sum_{m=1}^{M} (\mathbf{A}_m \times \mathbf{k}_m \cos\phi_m + \mathbf{B}_m \times \mathbf{k}_m \sin\phi_m)$$

$$\boldsymbol{\omega}(\mathbf{r},\ t) = \kappa \sum_{i=1}^{N} \frac{\mathbf{s}_i'}{(2\pi\sigma^2)^{3/2}} \exp(-|\mathbf{s}_i - \mathbf{r}|^2/2\sigma^2)\Delta\xi$$

$T = 0$

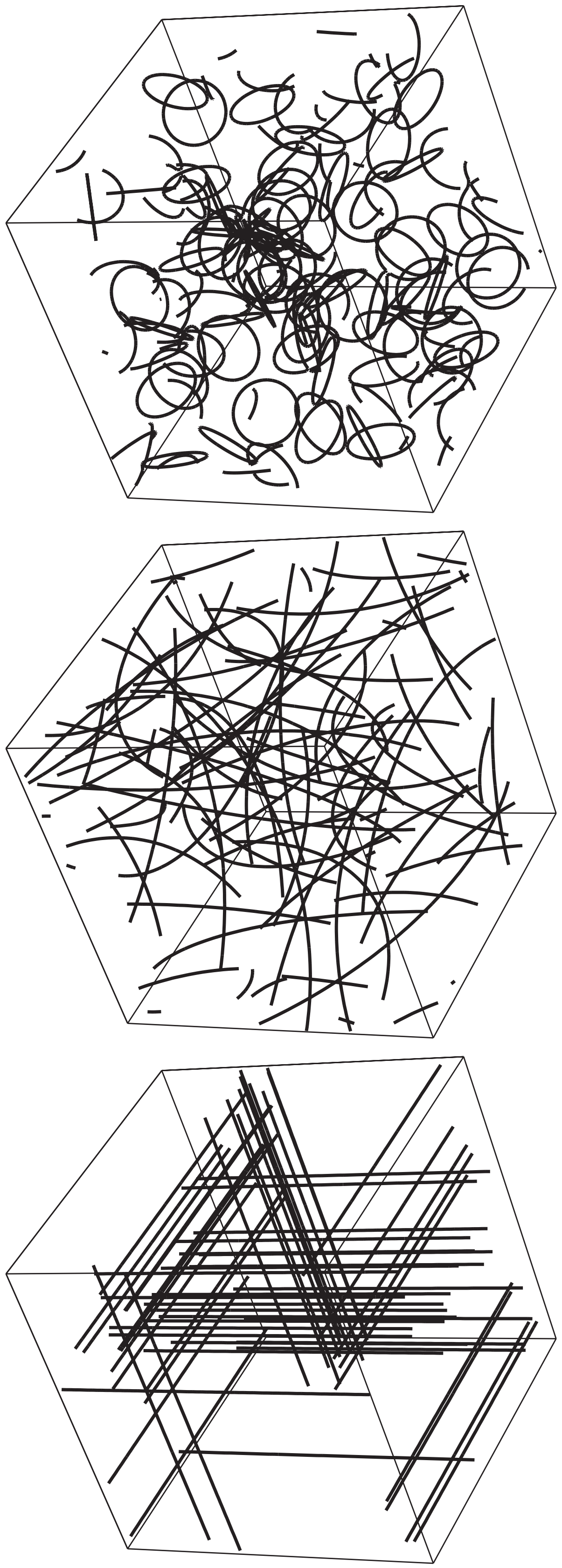






AWB, PoF, 2012

Vortex bundles seem important to generate large scale flow



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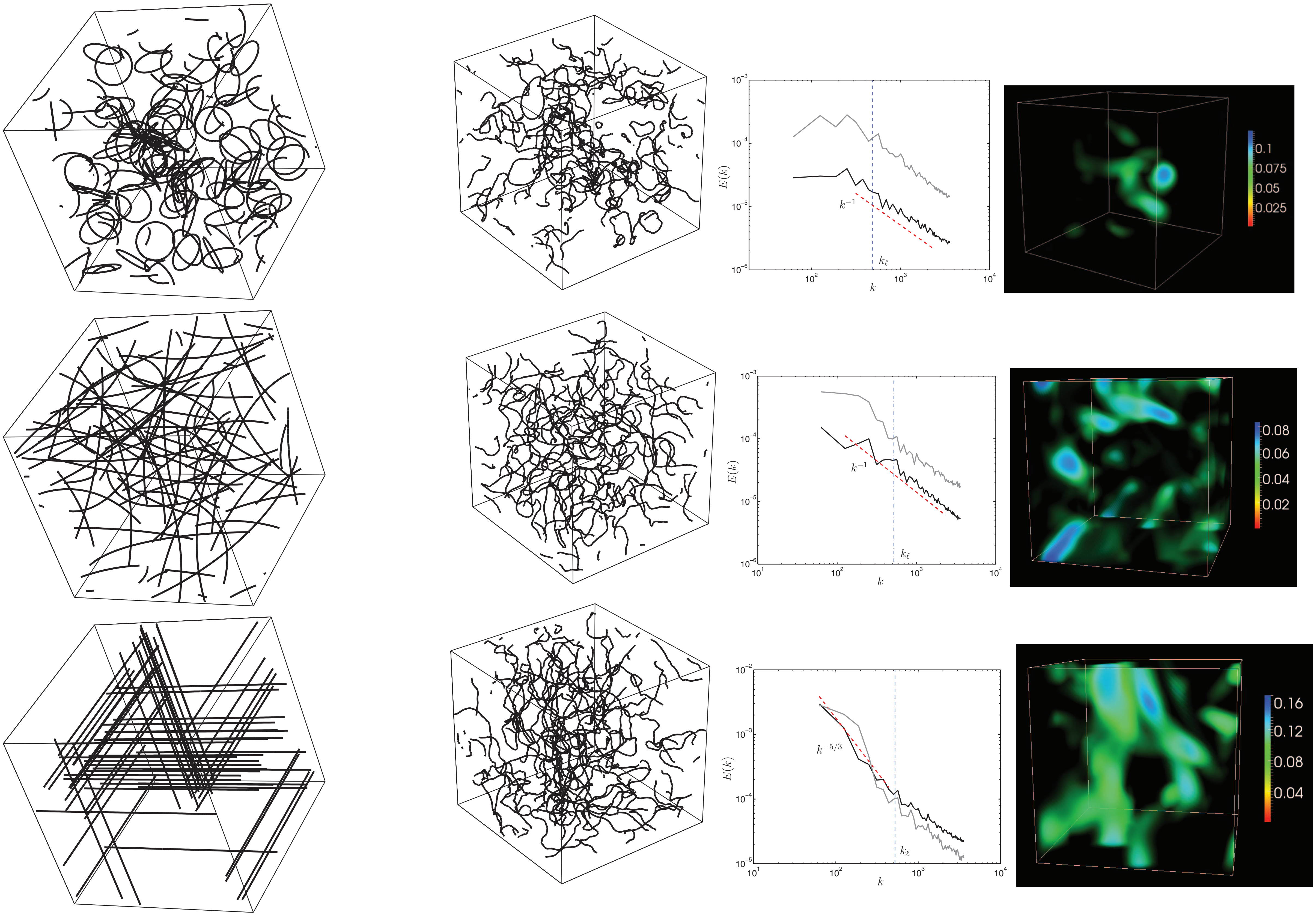

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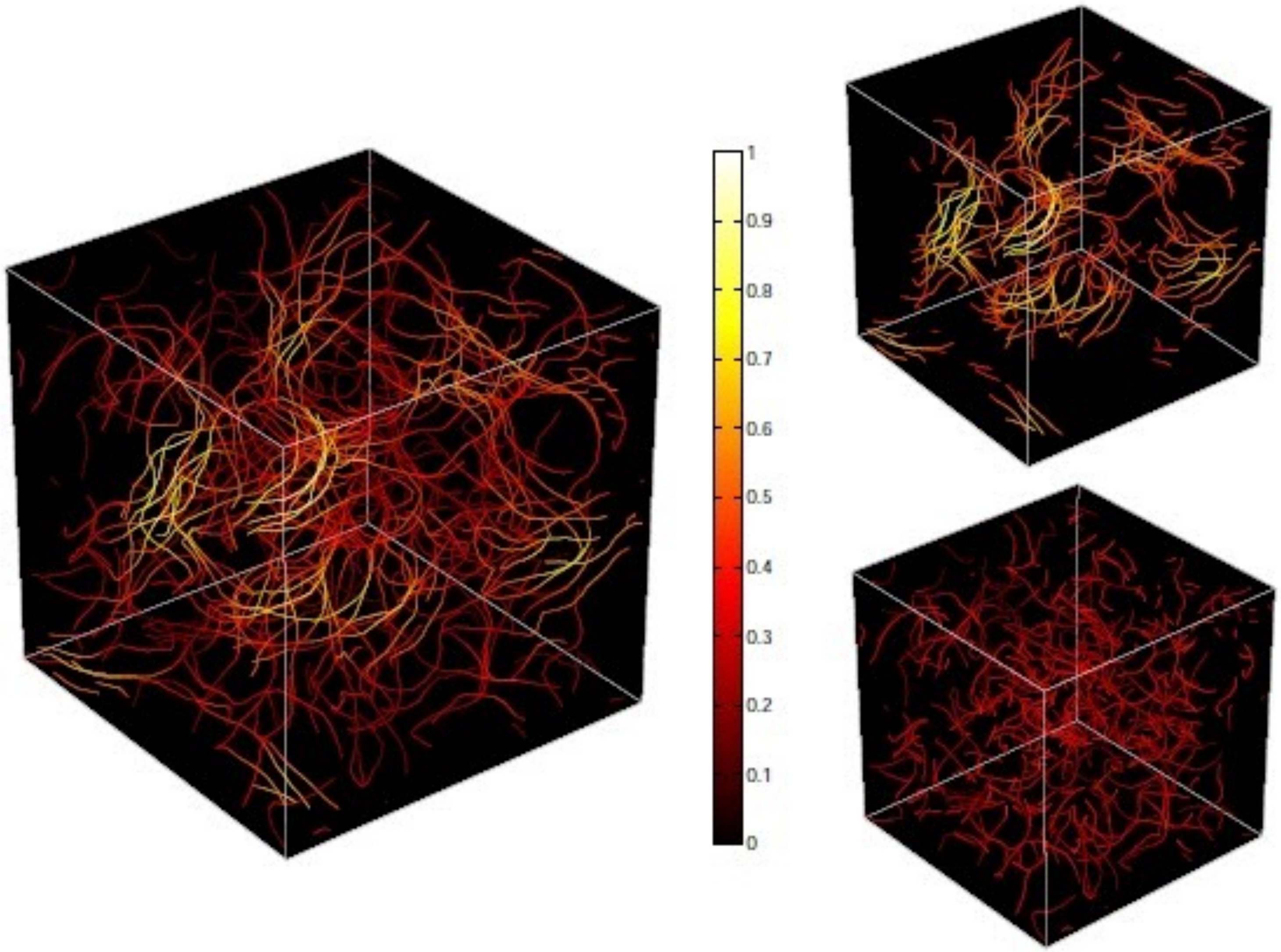

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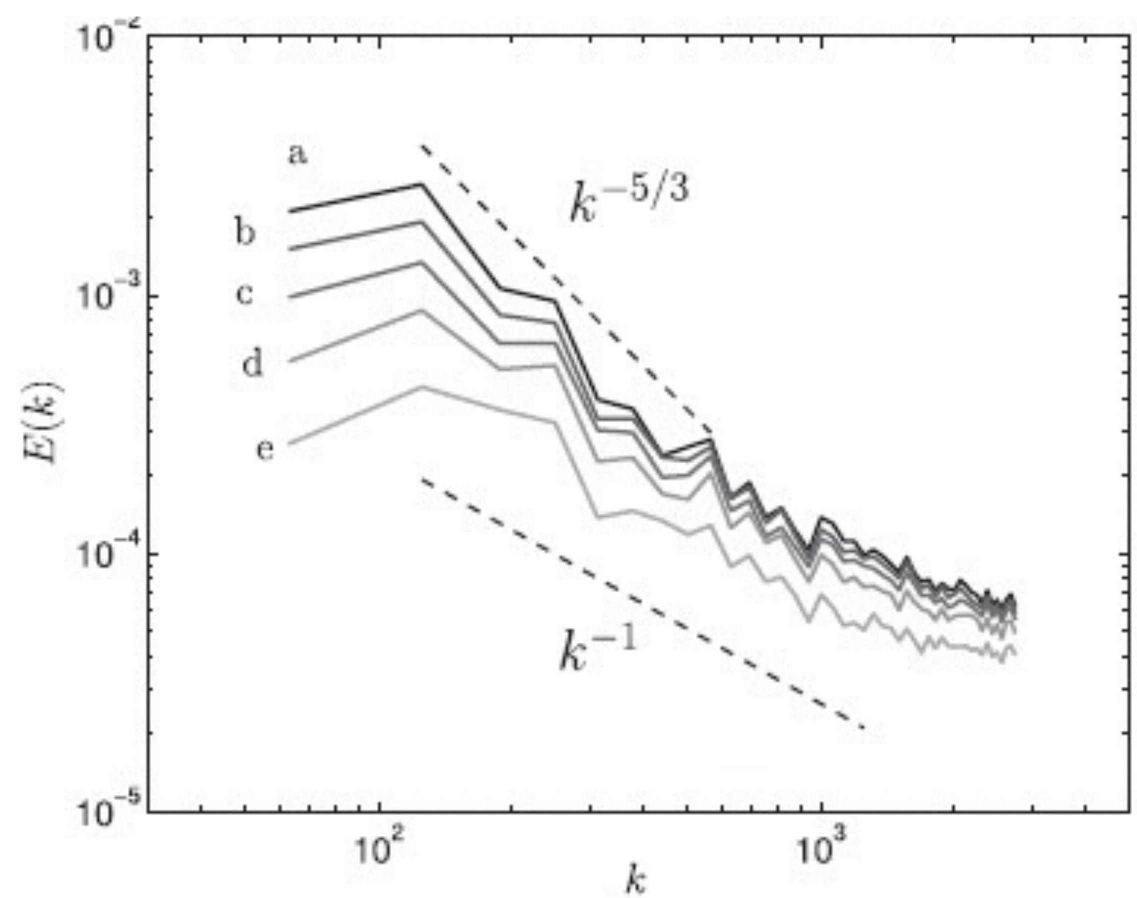
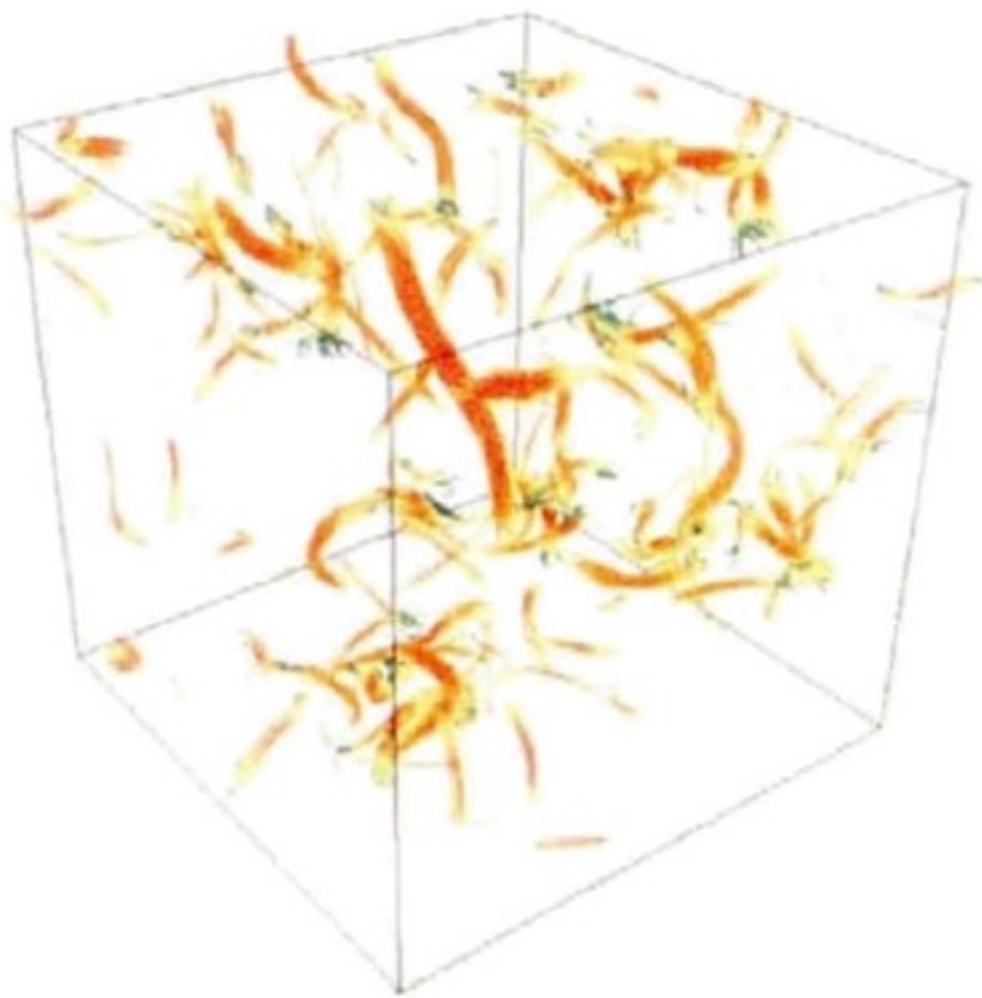


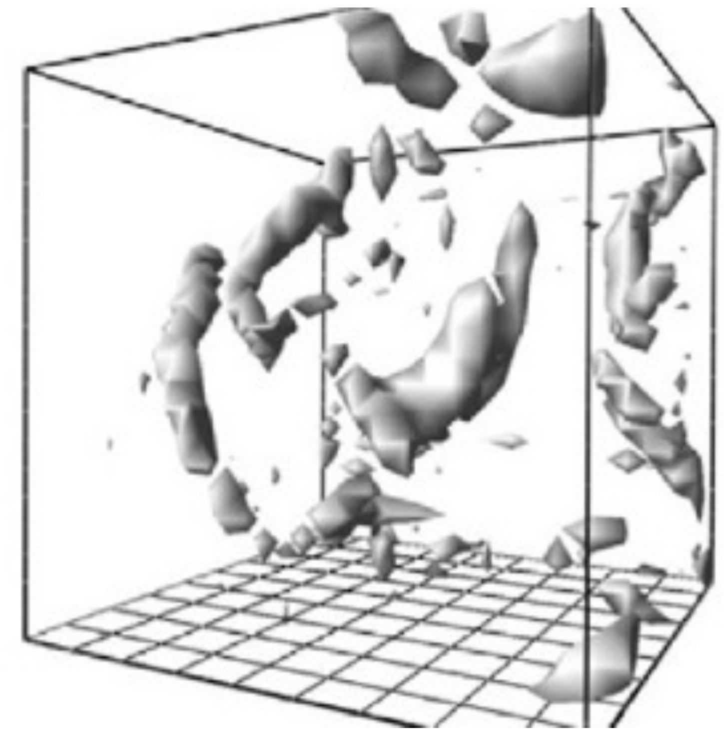
AWB, Laurie & Barenghi PRL, 2012

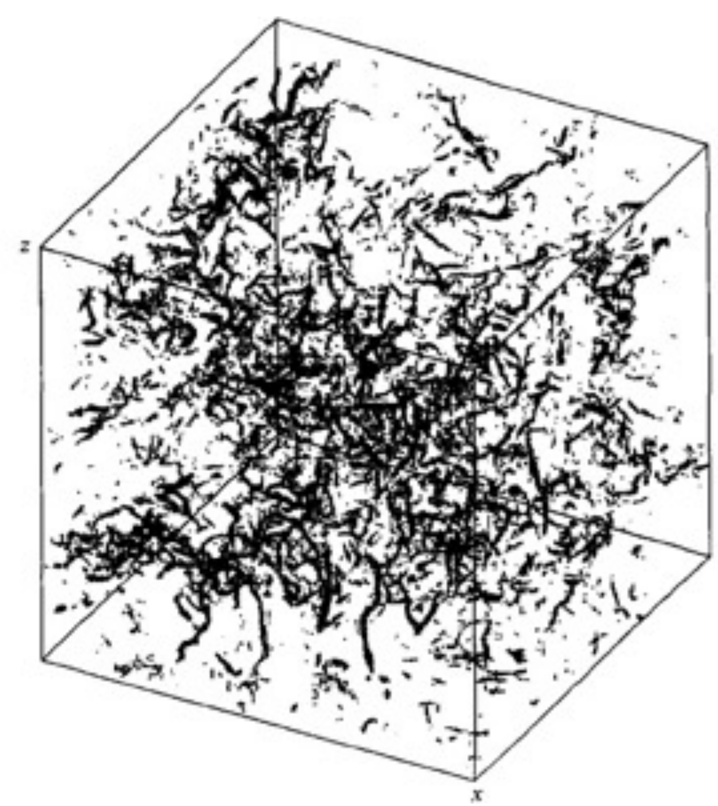
Analogy with classical turbulence

Ordinary turbulence contains metastable regions of coherent vorticity (vortex tubes, worms)



She, Jackson & Orzag, 1990





Vincent & Meneguzzi 1991

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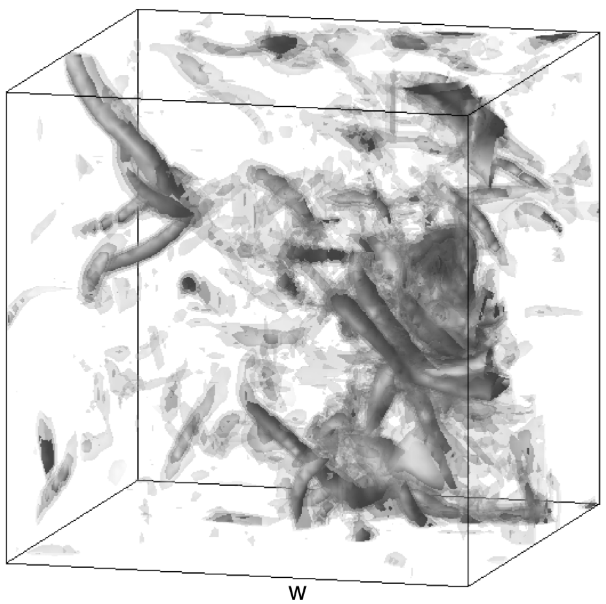
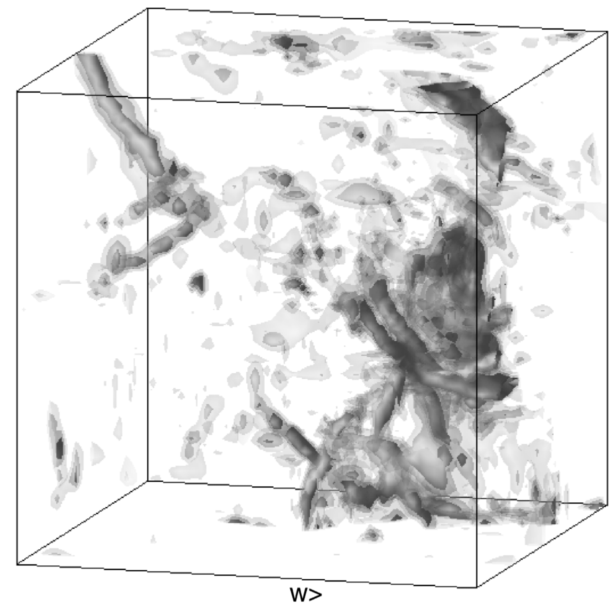
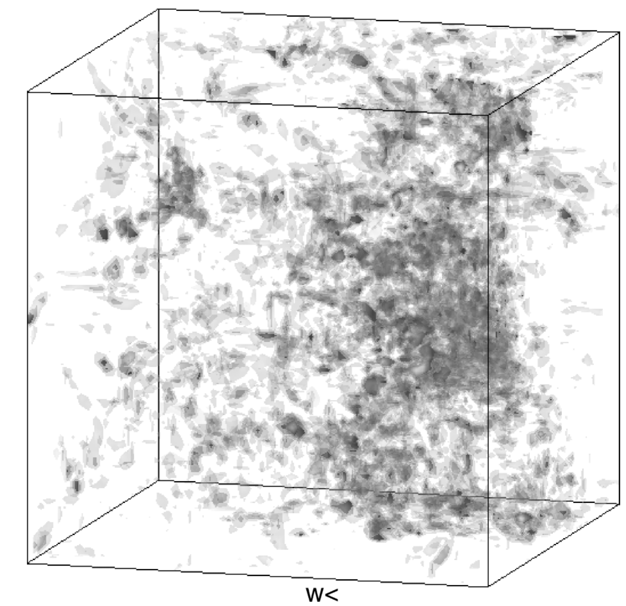


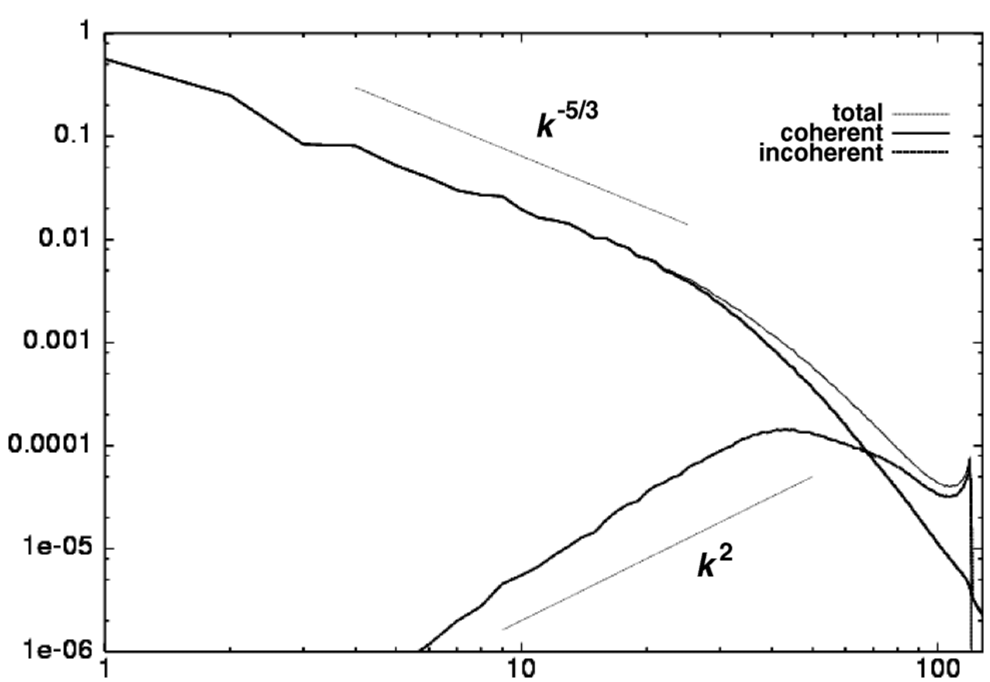
Total Vorticity



Wavelet extracted coherent vortices



Incoherent vorticity



Farge, Pellegrino & Schneider, PRL, 2001

Experimental evidence for such a picture in QT

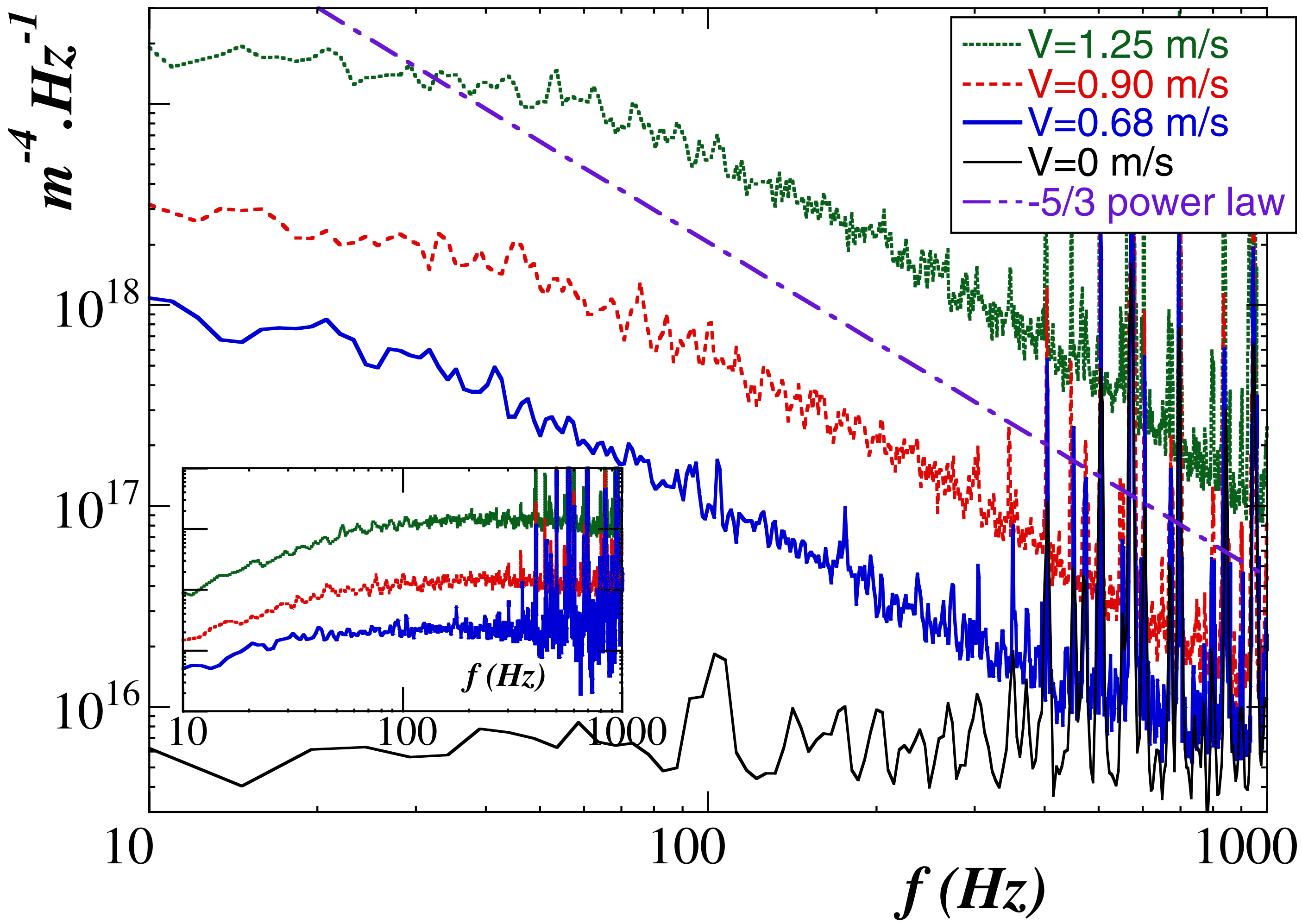


Fig. 4: Power spectrum density of the vortex line density $L_\perp$ for different mean flow velocities: from bottom to top 0, 0.68, 0.90 and 1.25 m/s. The straight line is a $(-5/3)$ power law. The insert is a $f^{-5/3}$ compensated spectrum for the 3 different mean flows after removal of a $5 \cdot 10^{15}\ \mathrm{m}^{-4}\ \mathrm{Hz}^{-1}$ white-noise floor.

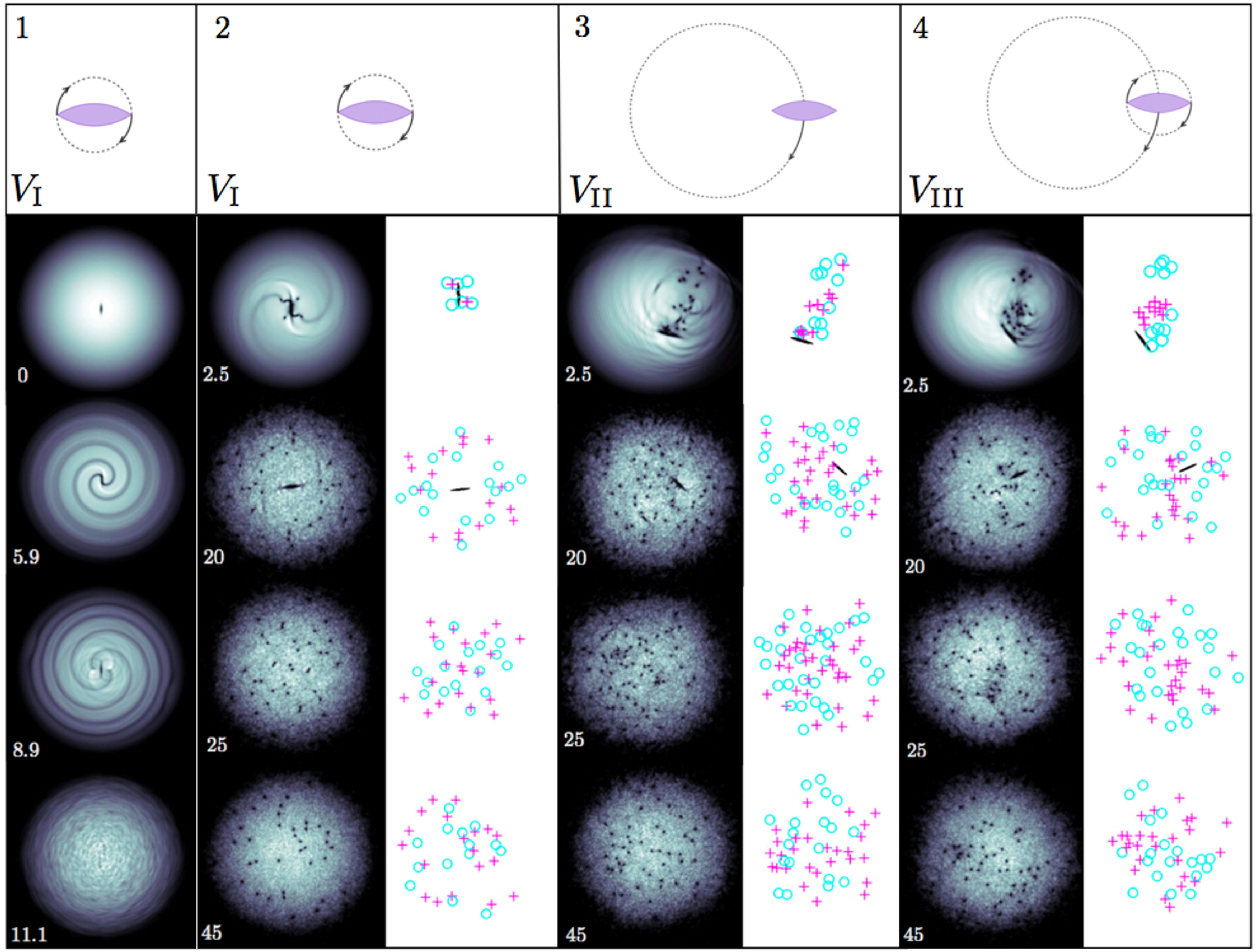
Observed frequency dependence of the spectrum, disagrees with classical vorticity spectra

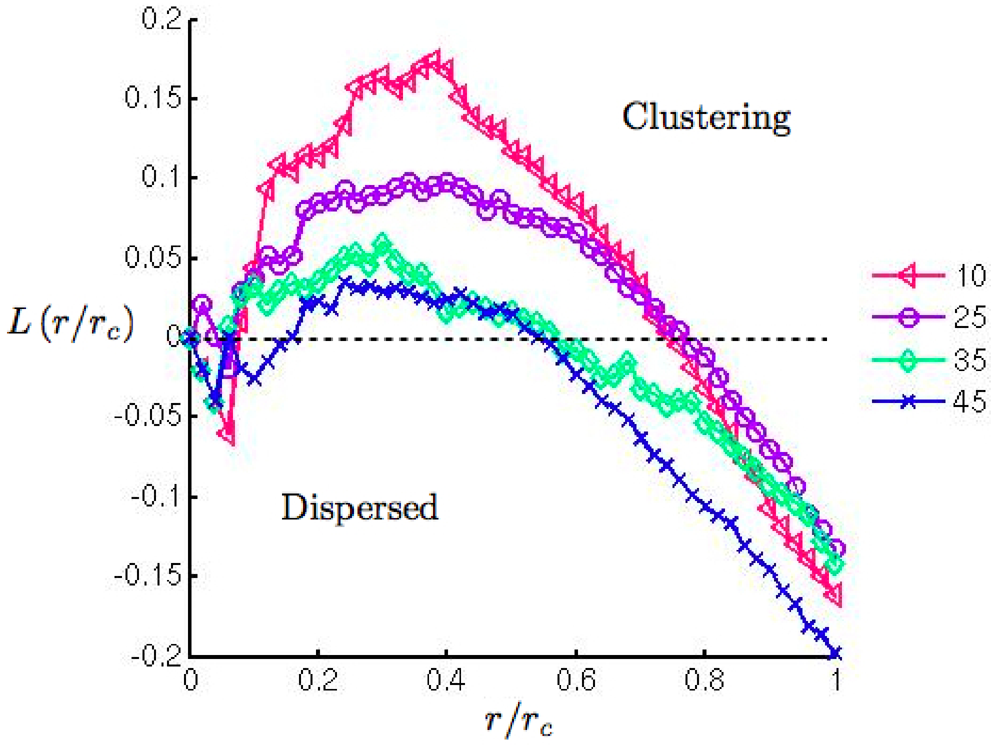
Disagreement explained if the vortex line density field is decomposed into a polarised field (which carries most of the energy) and an isotropic field (which is responsible for the spectrum)

Roche & Barenghi, EPL, 2008

In 2D Bundles -> Clusters

Received a lot of attention in the BEC community recently





White et al., 2012

2D Superfluid Wind-tunnel

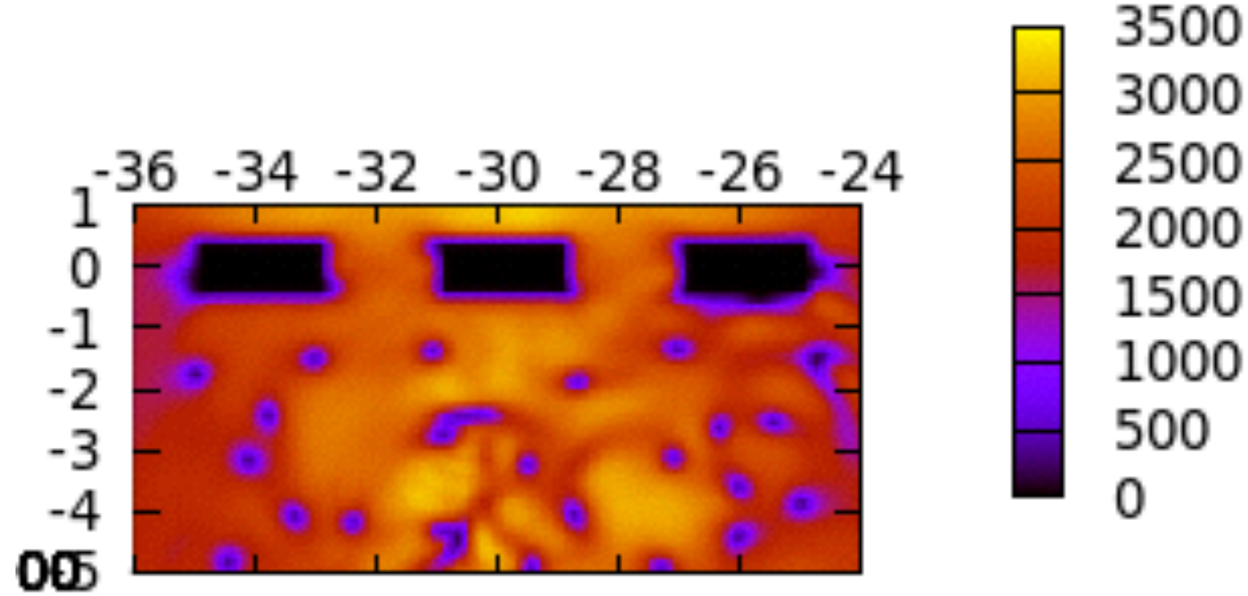
- 2D Gross-Pitaevskii equation:

$$i\hbar\frac{\partial}{\partial t}\psi(\mathbf{x},t) = \left[-\frac{\hbar^2}{2m}\nabla^2 - \mu + g|\psi(\mathbf{x},t)|^2 + V_{tot}(x)\right]\psi(\mathbf{x},t)$$

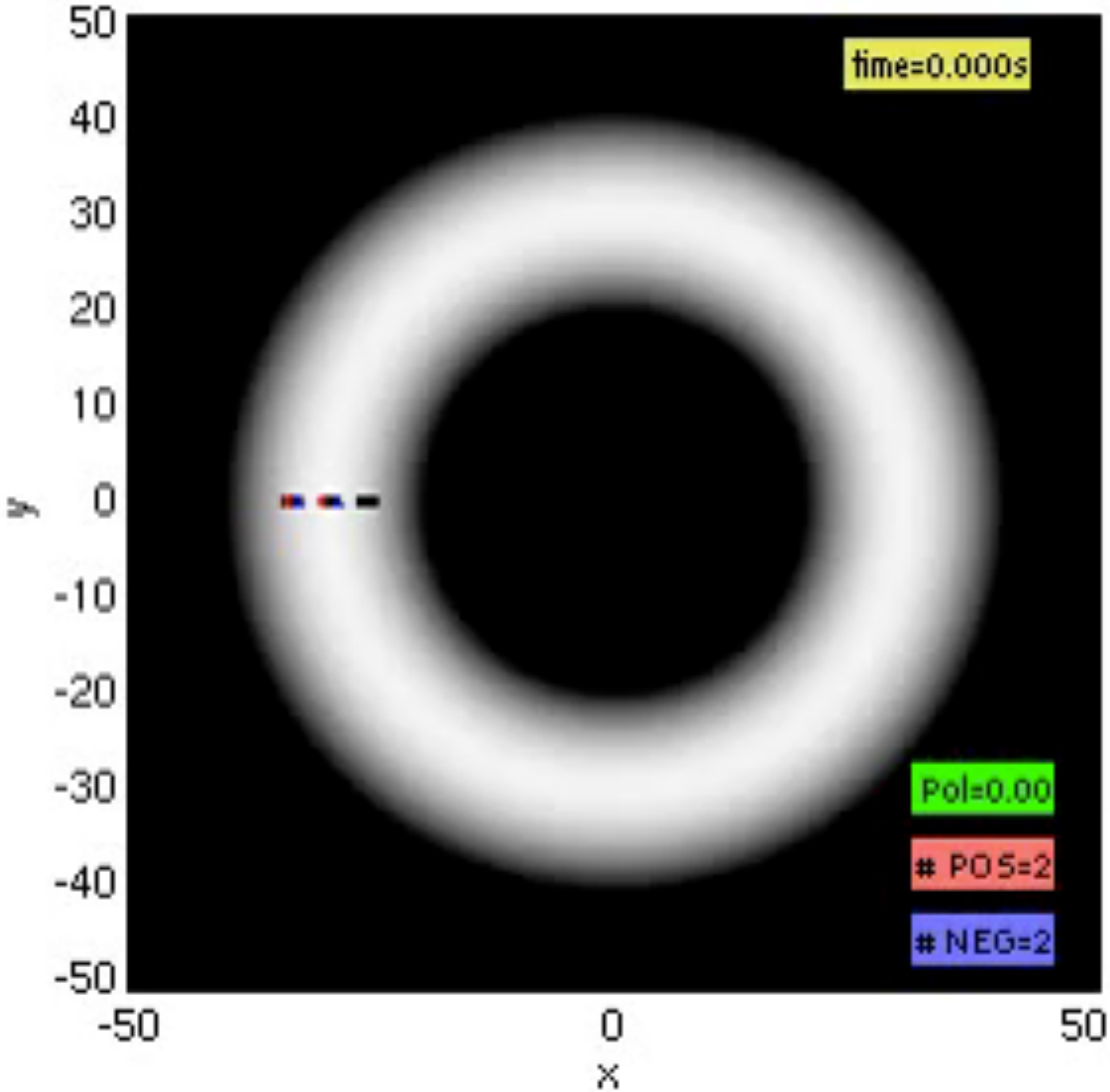
- Ring trap with a 'grid'

$$V_{tot} = V_{ring}(\rho) + V_{obst}(\rho) = V_G(1 - e^{-2(\rho-\rho_0)^2/w^2}) + \sum_{i=1}^{3} V_0\delta(\rho+\rho_i), \qquad \rho = \sqrt{x^2+y^2}$$

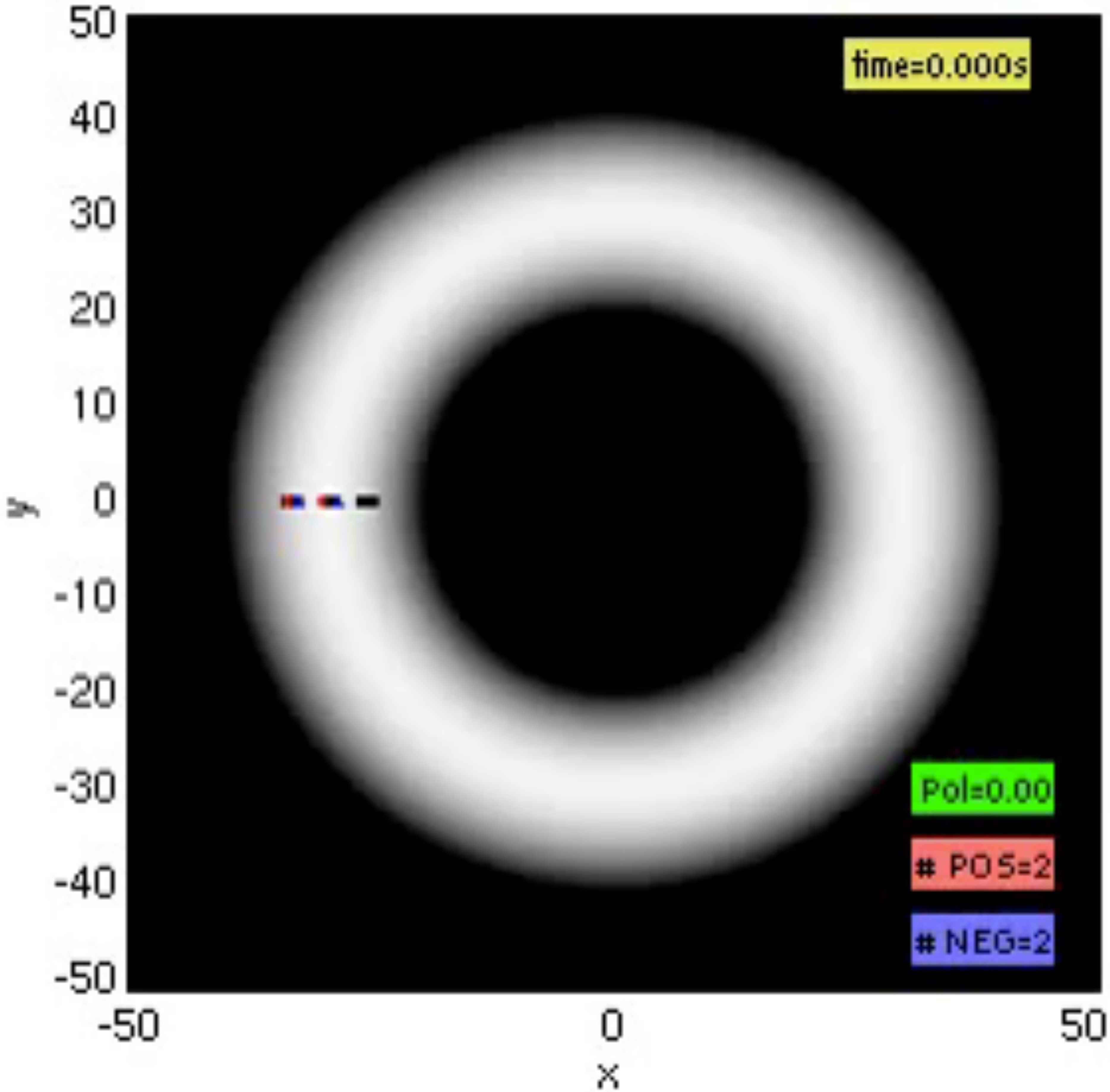
- Impose an initial phase winding, (80Γ), which in the absence of the obstacle would create a persistent current



Gallucci et al. (in prep.)



time=0.000s
Pol=0.00
# POS=2
# NEG=2
50
40
30
20
10
0
-10
-20
-30
-40
-50
y
-50
0
50
x



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-50
-50
0
50

Outline

- Motivation
  - (3D) vortex bundles
  - (2D) vortex clusters
- Vortex rings in superfluid helium
- Experiment:
  Borner, Schmeling, & Schmidt (Physics of Fluids 1983)
- Numerical work:
  Wacks, Baggaley, & Barenghi (PoF 2013, PRB 2014)
- Conclusions

A controlled setup to study bundles of quantised vortices

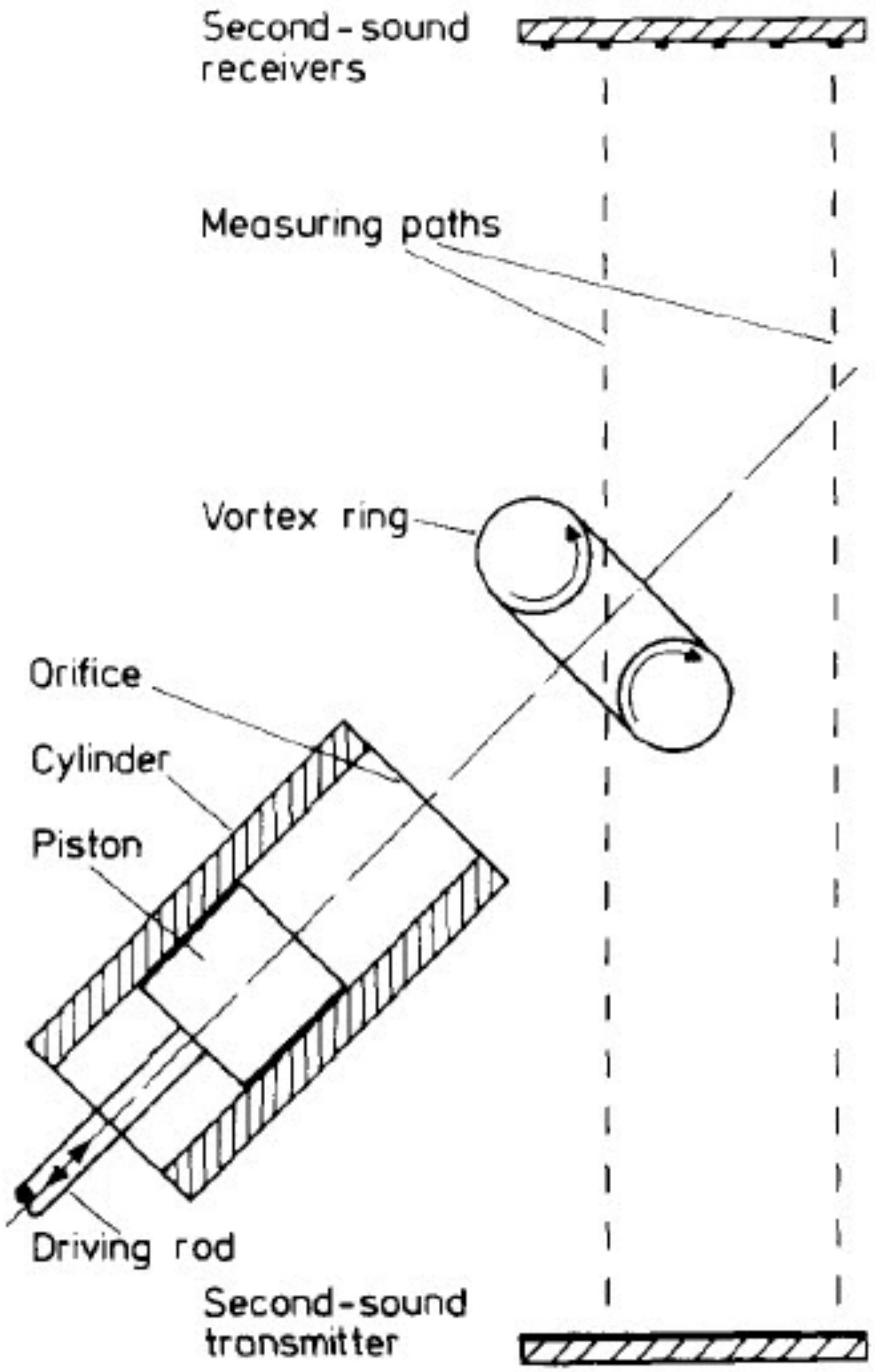
Vortex rings have a long tradition in superfluid helium, from Rayfield & Reif (1964), to Winiecki & Adams (2000) to recent work of Walmsley, Zmeev & Golov

Vortex rings are Hamiltonian objects: $v = \partial H / \partial p$

$$v = \frac{\Gamma}{4\pi R}\left[\ln(8R/a) - 1/2\right] \qquad H = \frac{\rho \Gamma^2 R}{2}\left[\ln(8R/a - 2)\right]$$

$$p = \rho \Gamma \pi R^2$$

Borner's Experiment

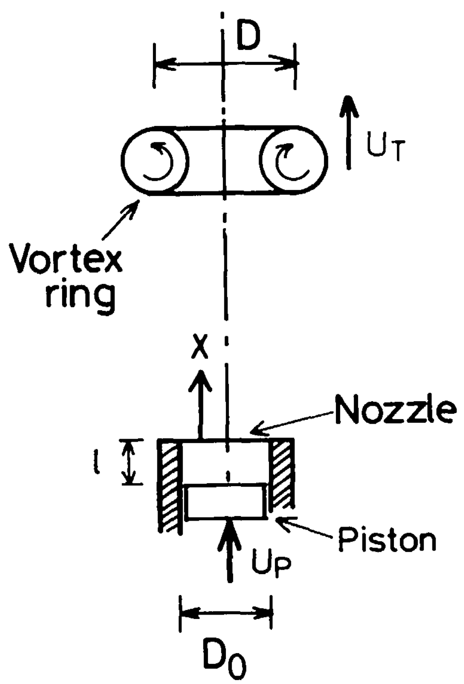


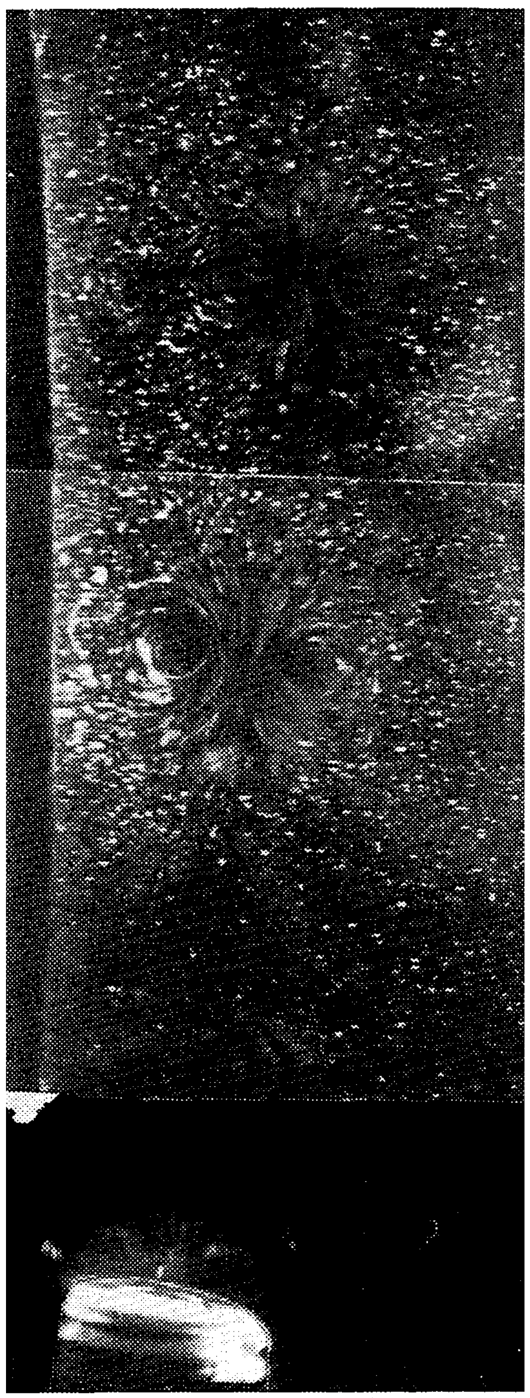
Experiment by Borner et al. 1983

- Large-scale vortex rings in superfluid helium-4
- Ring position, Γ_s, Γ_n measured acoustically vs time
- Vortex structures of larger circulation $(\sim 1000 - 2000\kappa)$ observed

Other studies

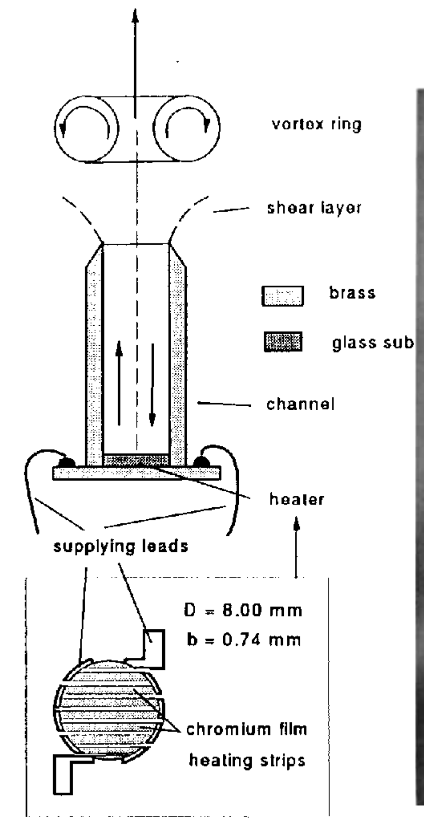
Murakami et al. (1987)

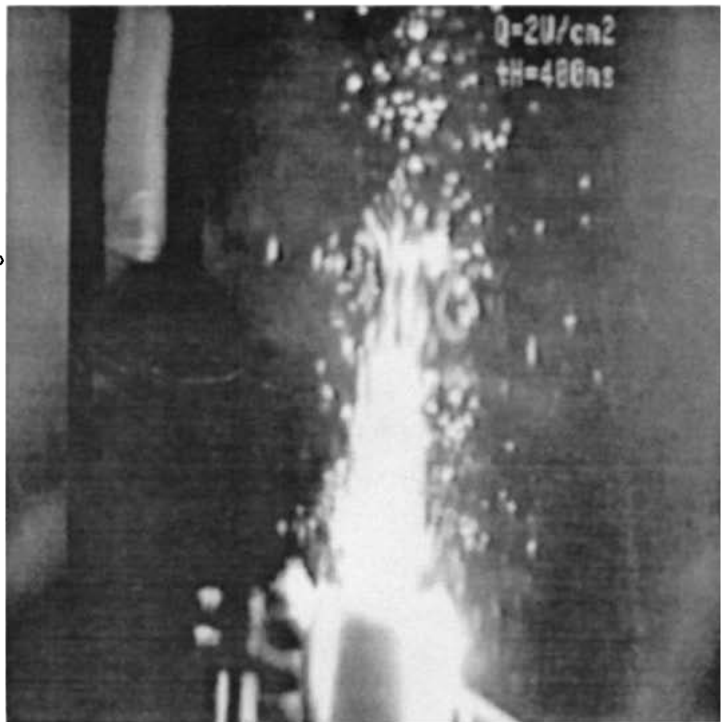




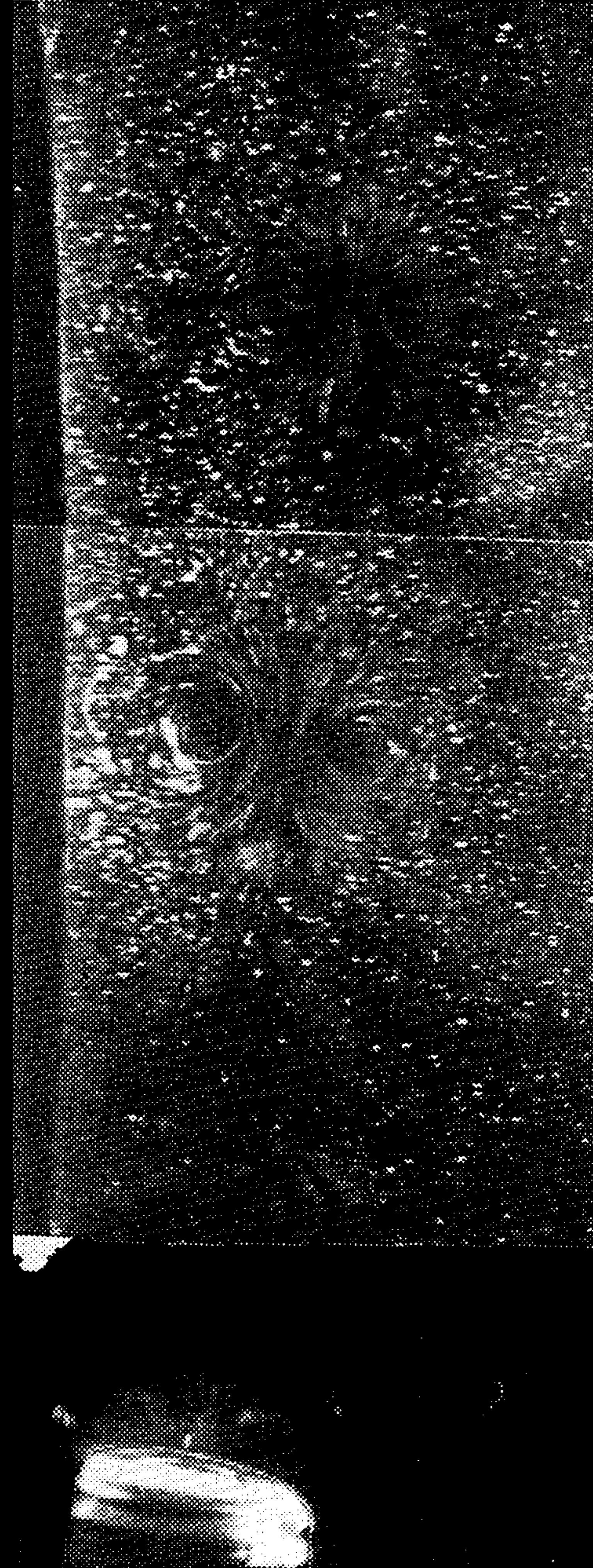
Hydrogen-Deuterium visualisation of flow filed

Stamm et al. (1993)





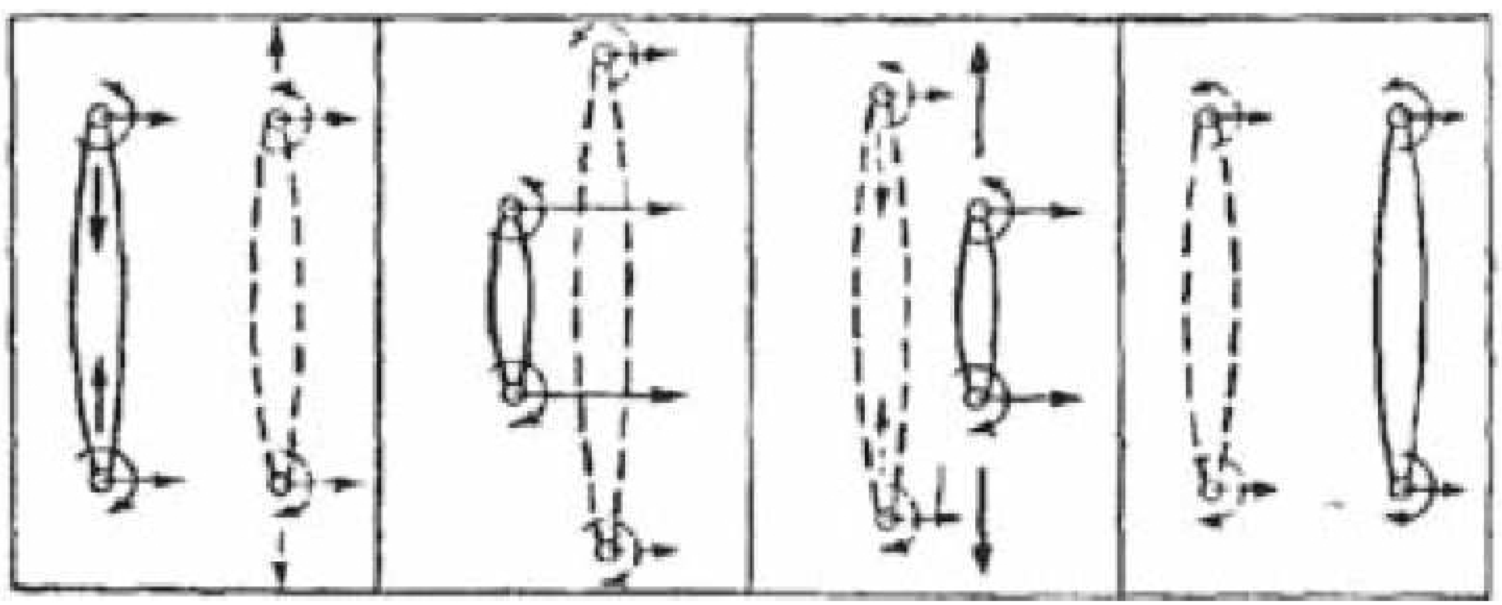
Thermally driven, visualisation with hollow glass beads



Borner's Experiment

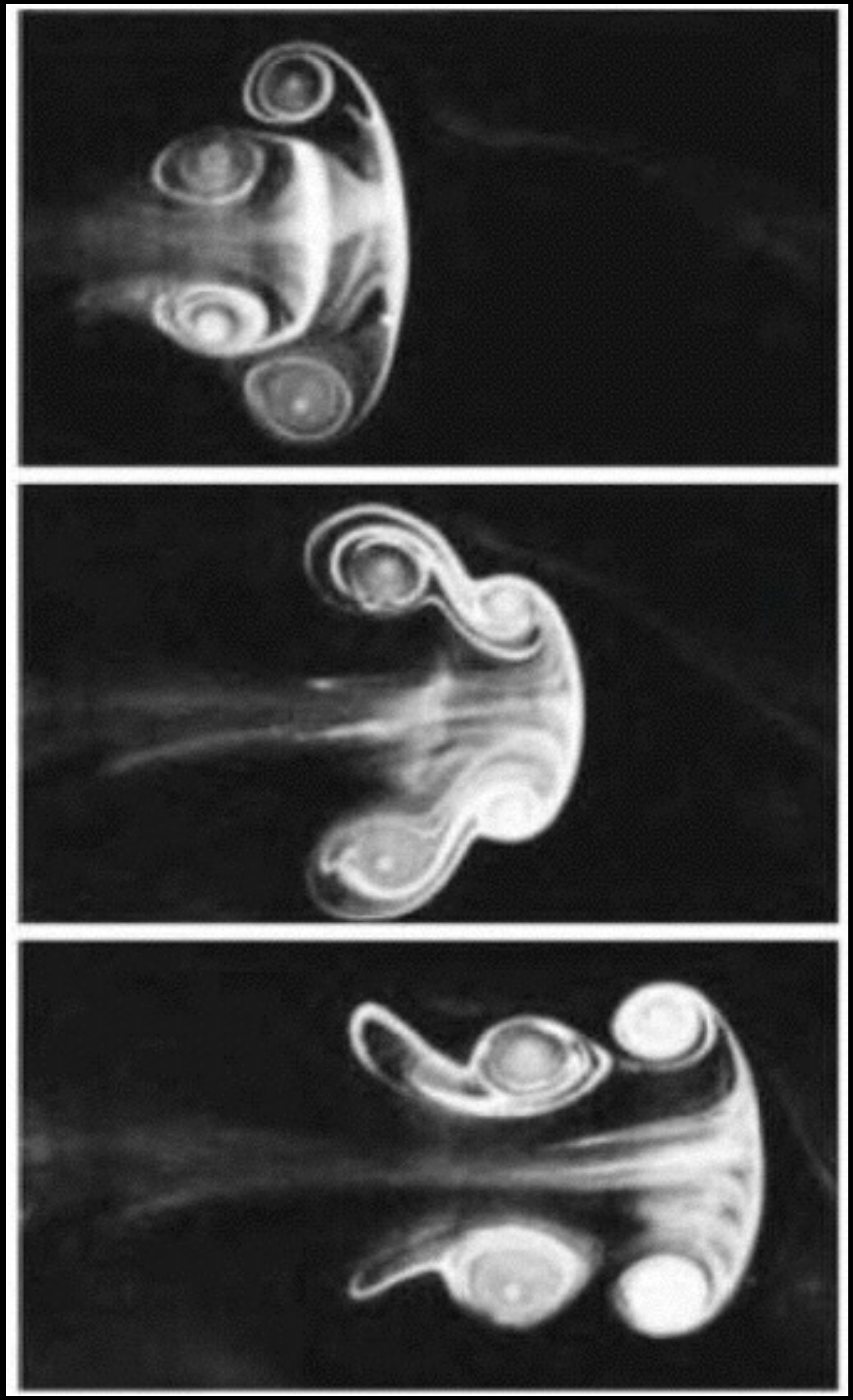
- Interpretation of the experiment:
  Bundles of $N \approx 10^3$ concentric quantised vortex rings
  Typical $\ell \approx 0.003$ cm $\gg$ core size $\xi \approx 10^{-8}$ cm

- How do vortex bundles move ?
  some kind of stable generalized ($N > 2$) **leapfrogging**



classical leapfrogging of two vortex rings,
Sommerfeld 1950

Classical leap-frogging of two vortex rings



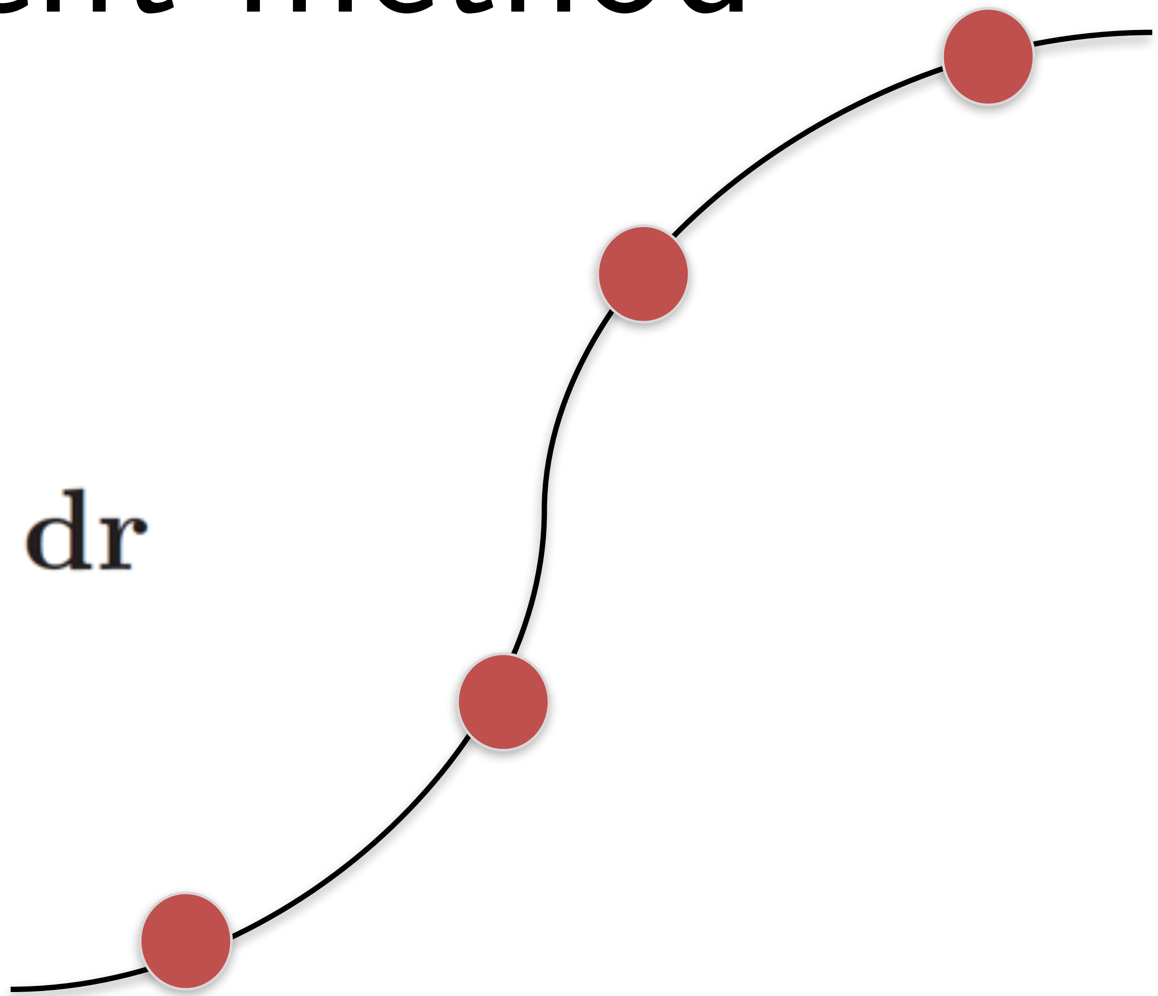
Yamadao & Matsui, 1978

6020

Vortex filament method

Biot-Savart Integral

$$\frac{d\mathbf{s}}{dt} = -\frac{\Gamma}{4\pi} \oint_{\mathcal{L}} \frac{(\mathbf{s} - \mathbf{r})}{|\mathbf{s} - \mathbf{r}|^3} \times \mathbf{dr}$$



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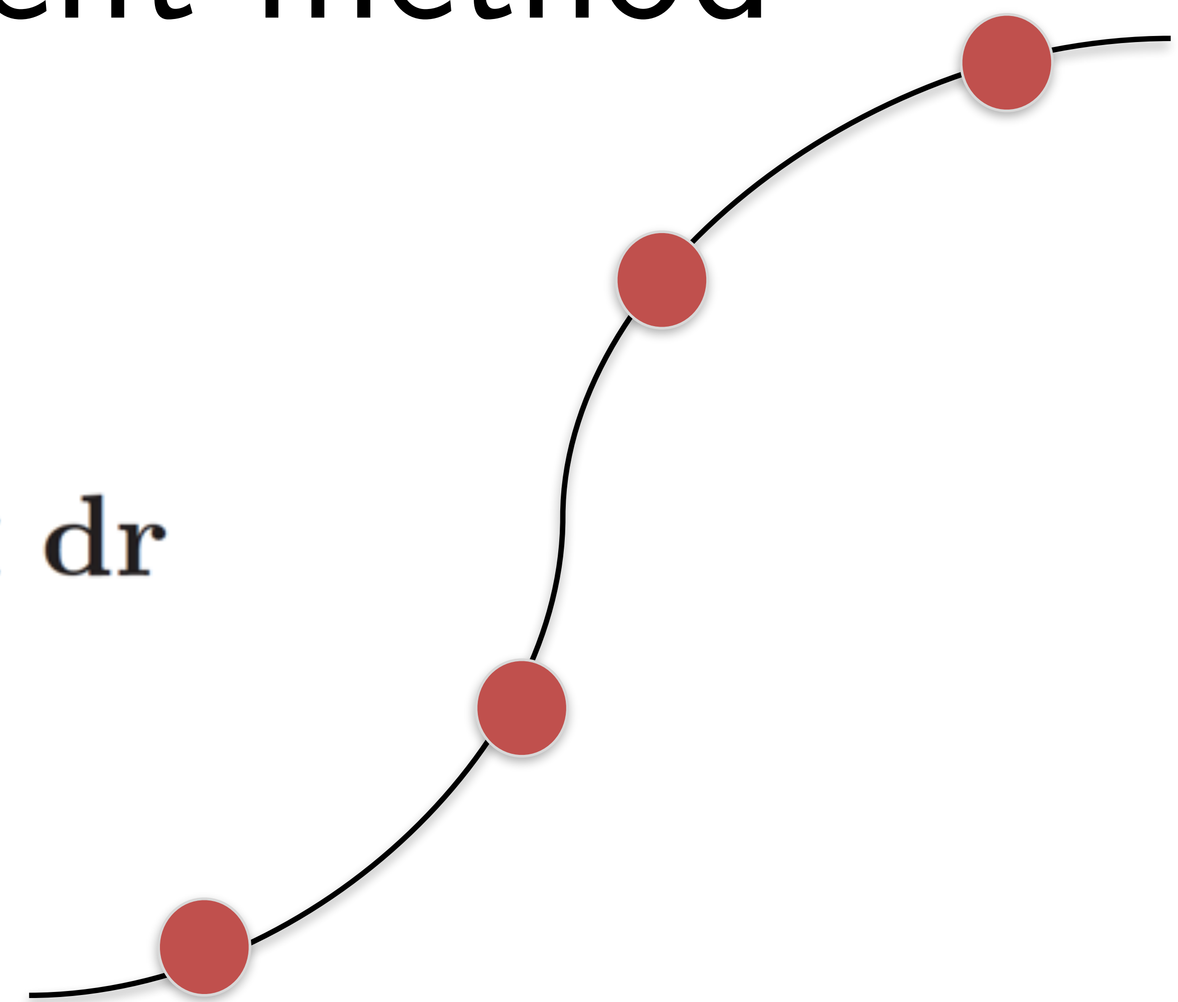
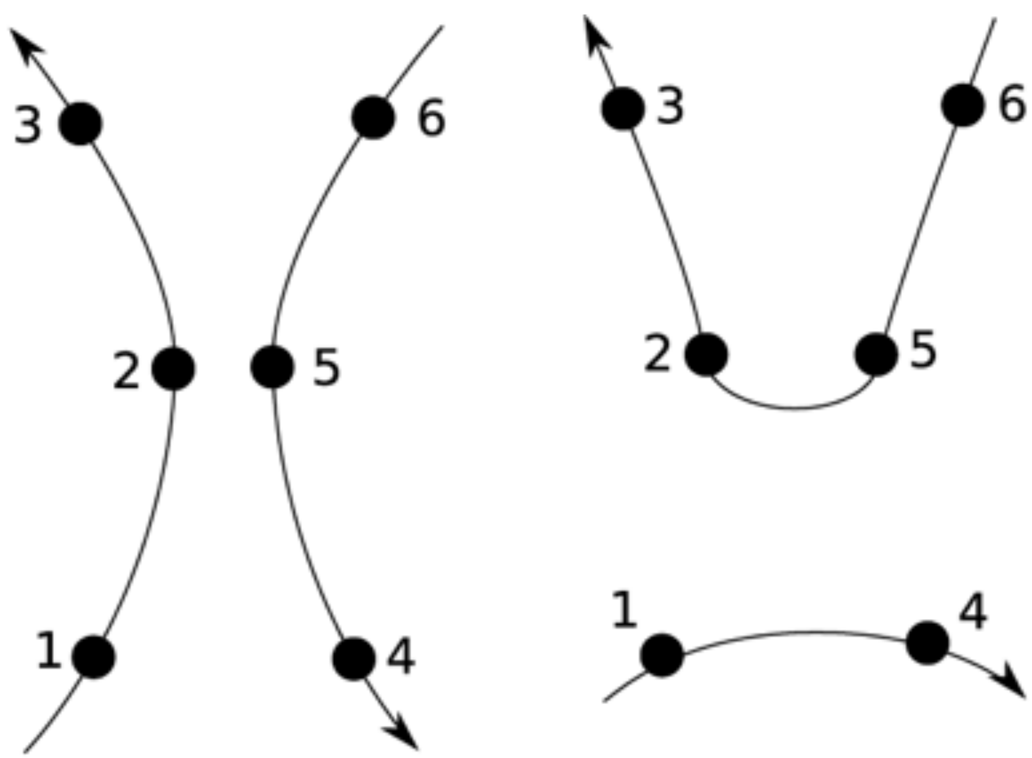
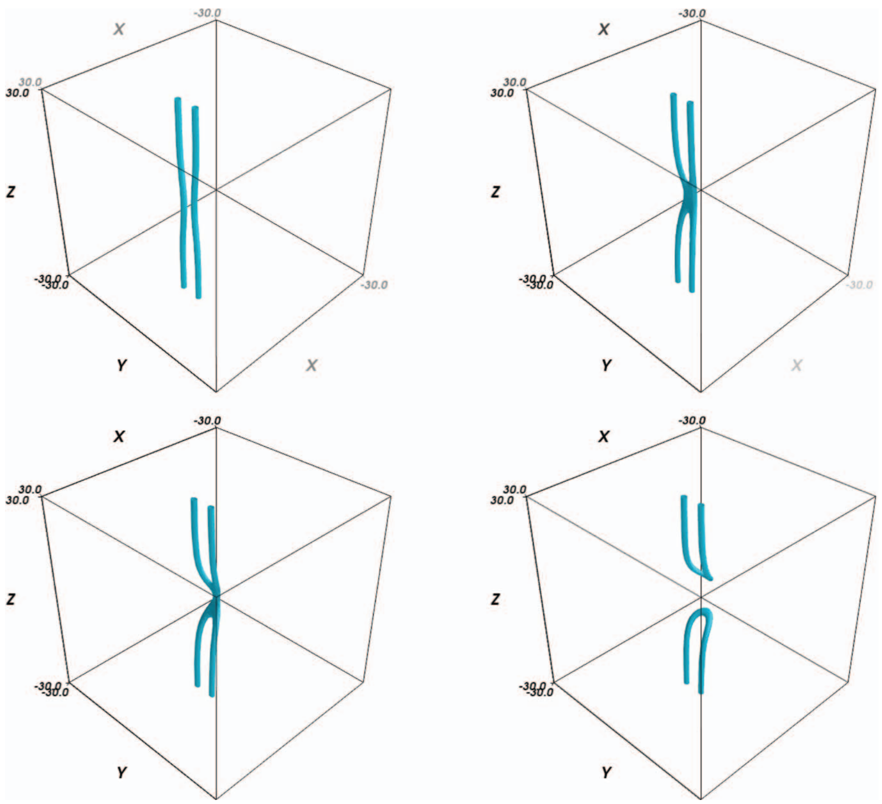


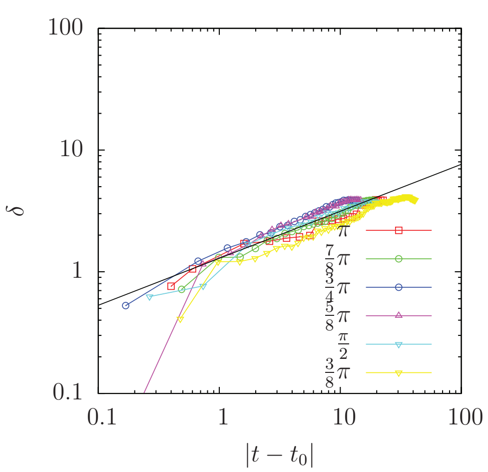


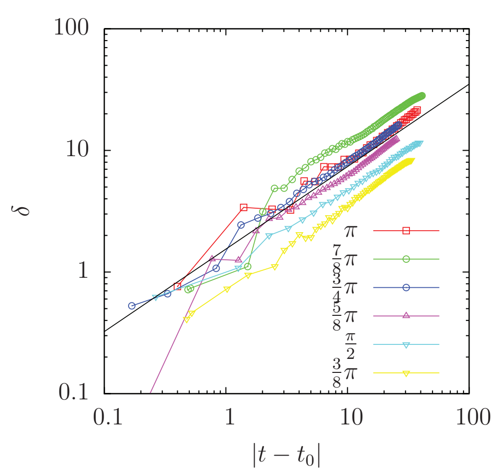
Model reconnections algorithmically 'cut and paste'

A note on reconnections

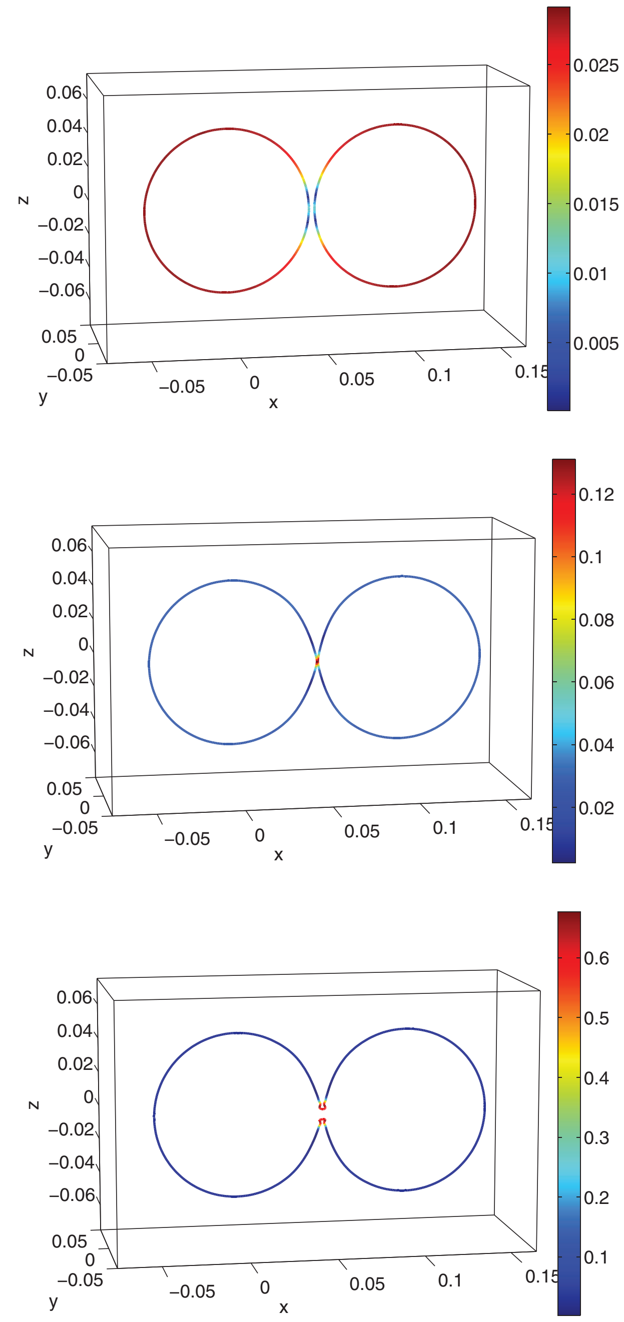
GPE

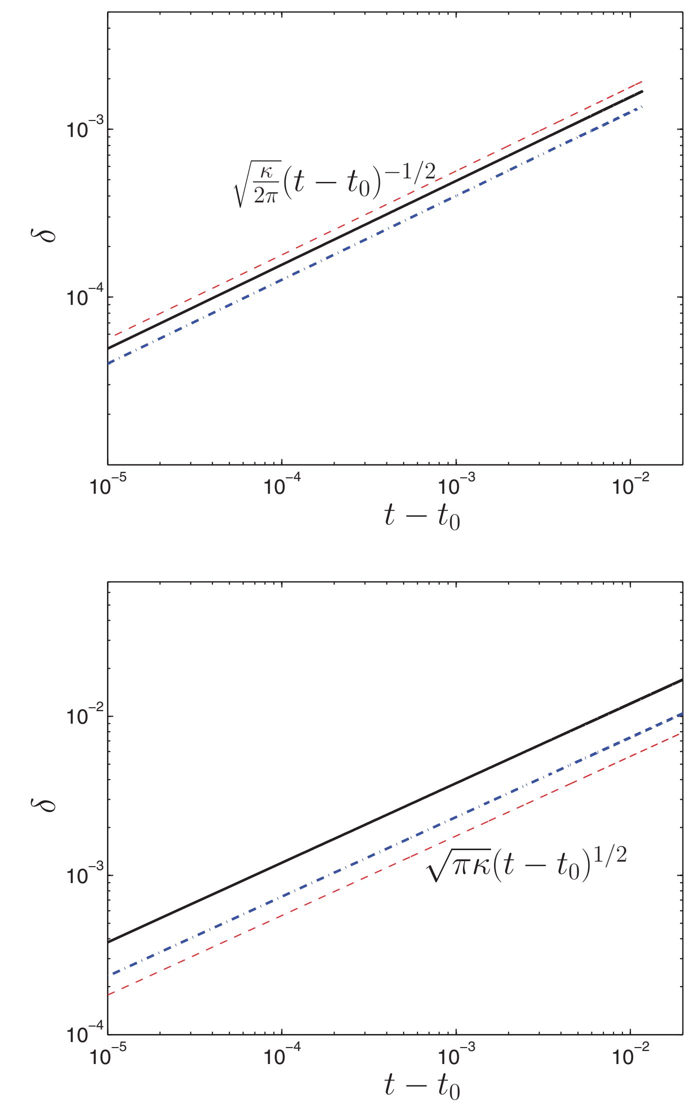






VFM





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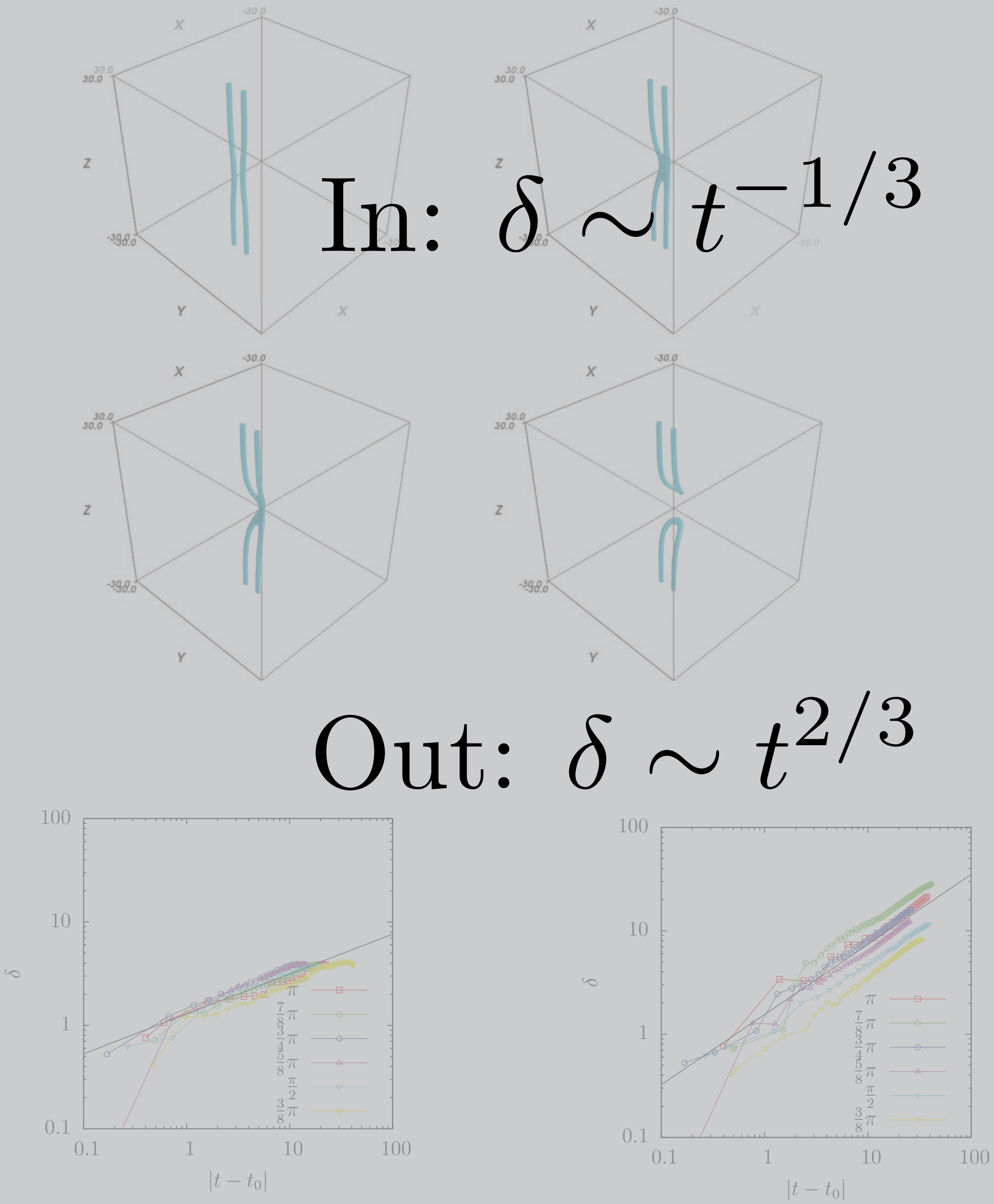
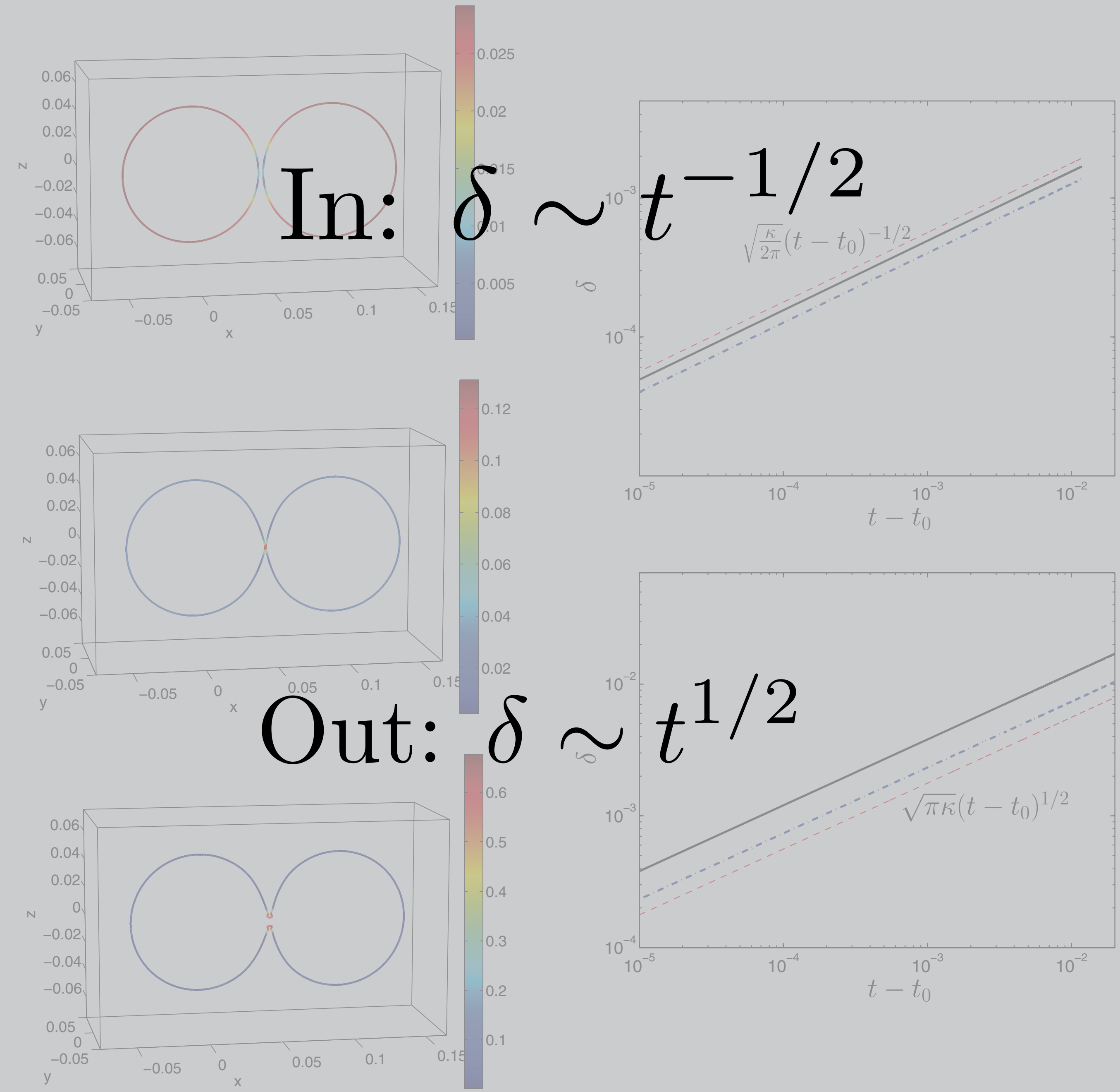


In: $\delta \sim t^{-1/3}$

Out: $\delta \sim t^{2/3}$

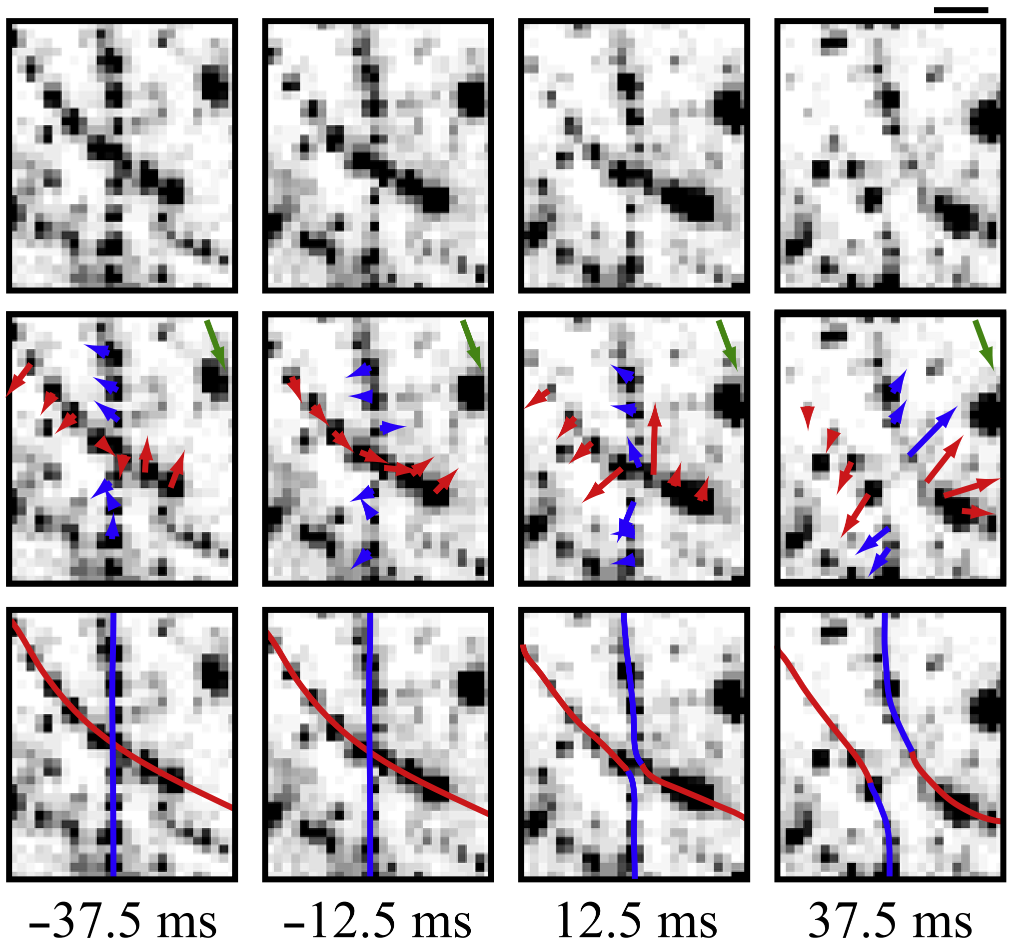
VFM

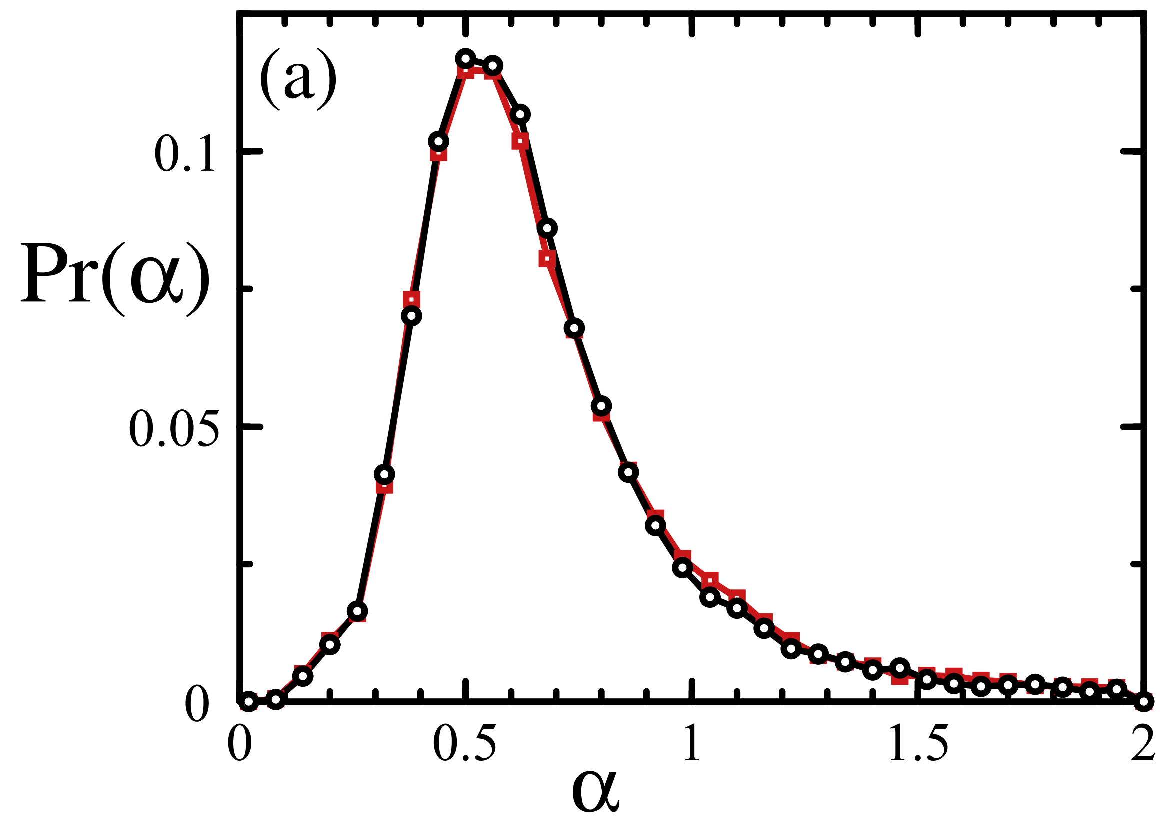


In: $\delta \sim t^{-1/2}$

Out: $\delta \sim t^{1/2}$

Experimental results





$$\delta \sim t^{\pm\alpha}$$

Paoletti et al. Physica D (2010)

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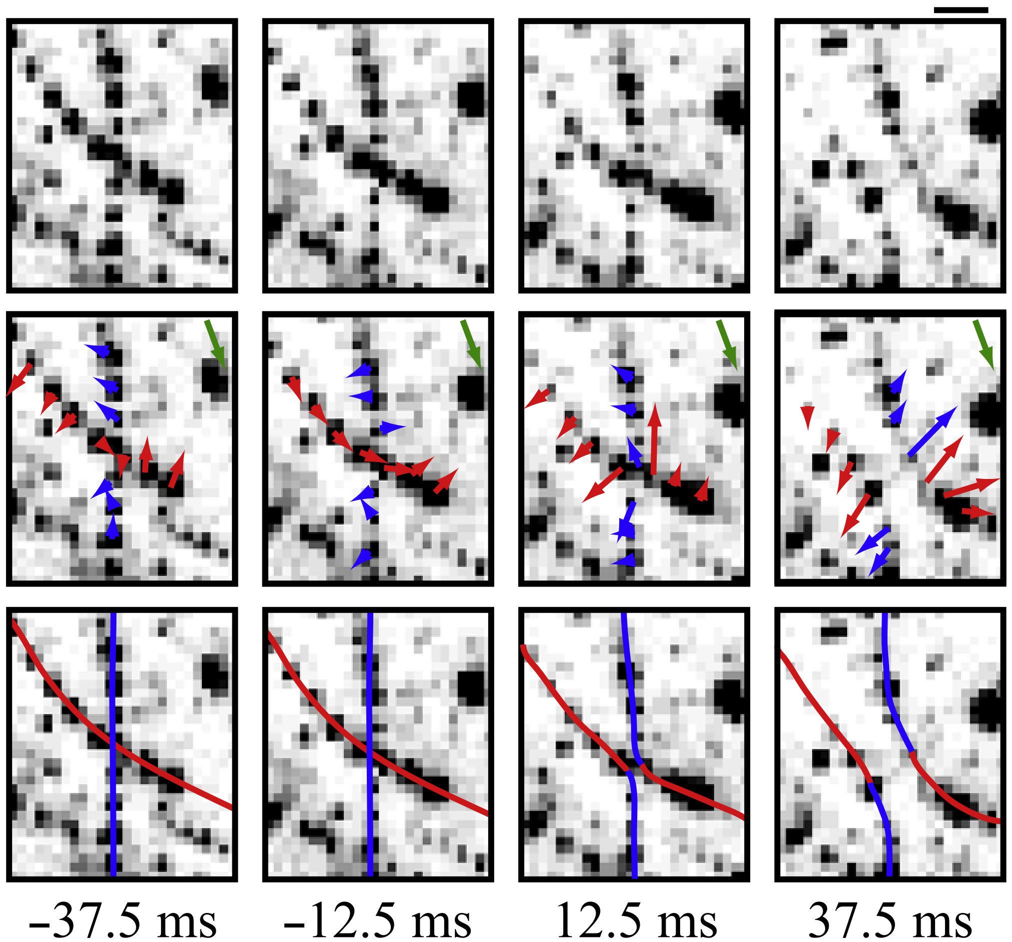

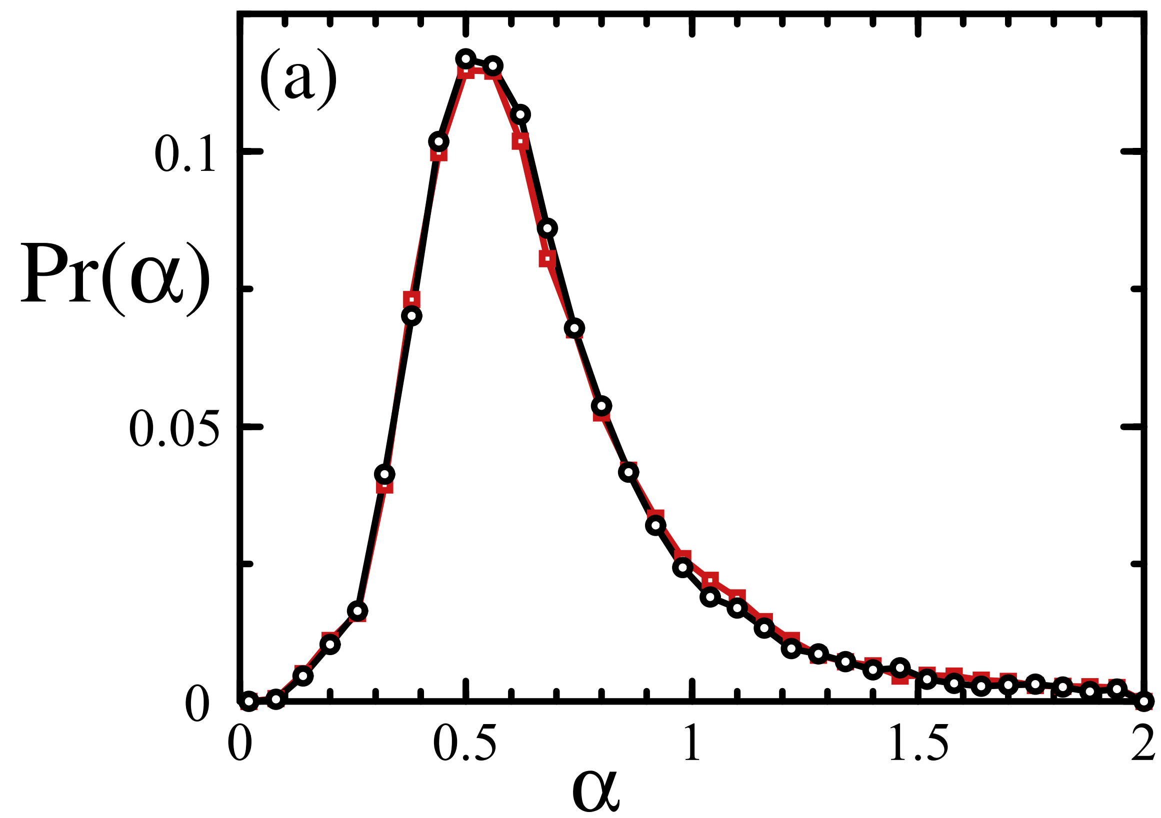

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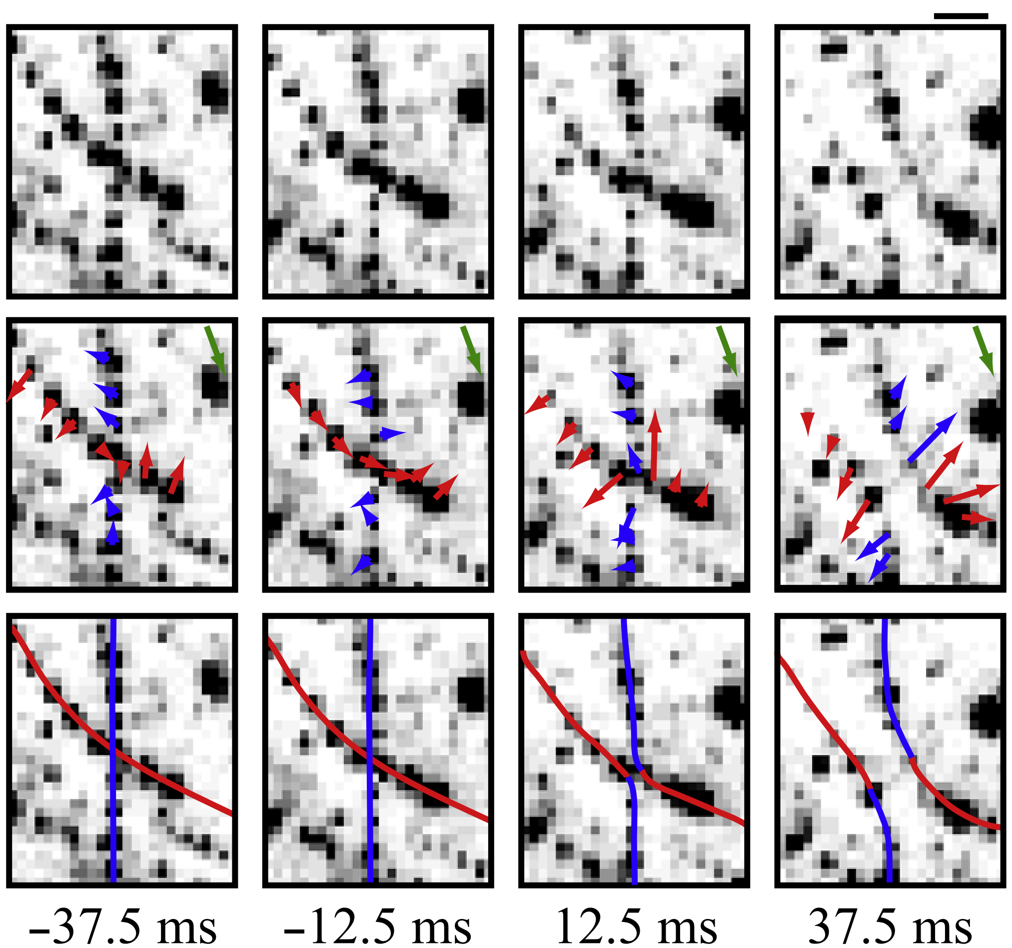
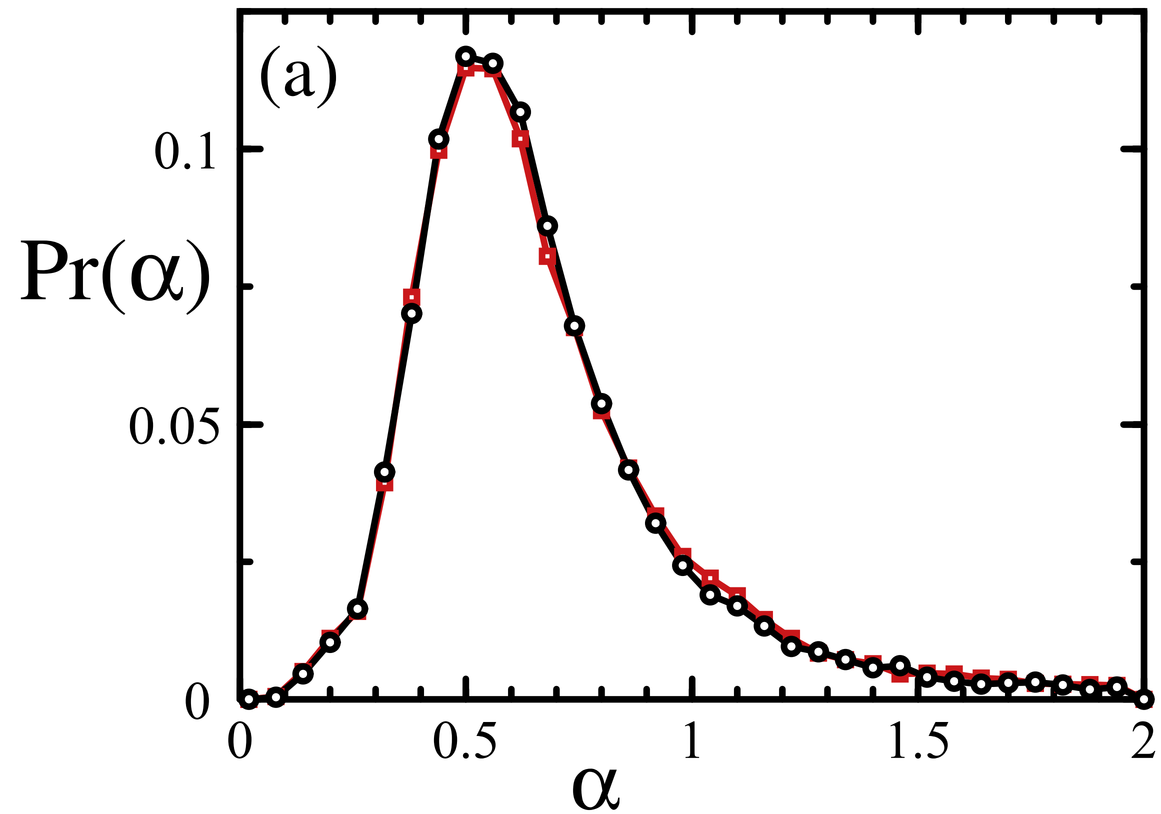
Experimental results

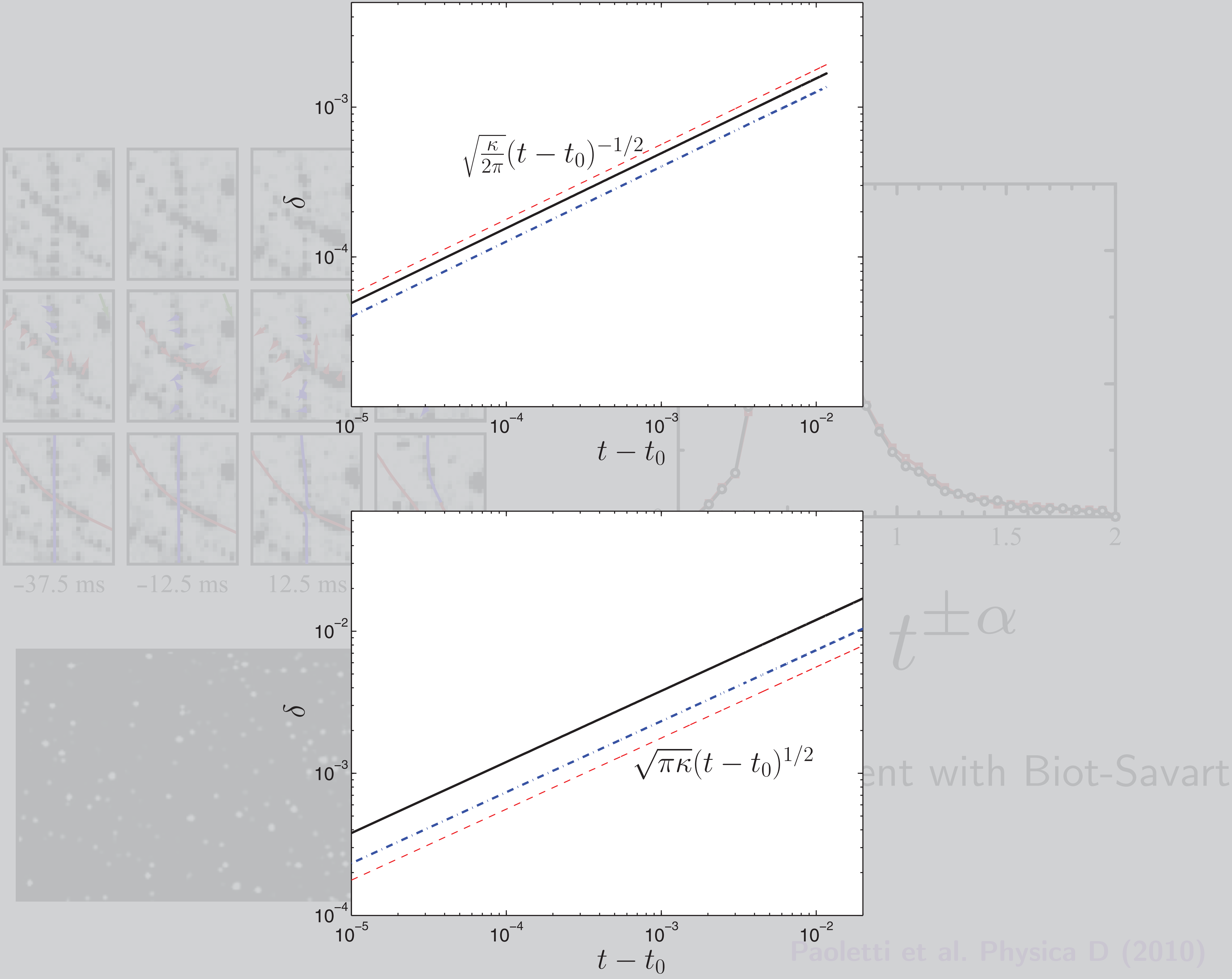




$$\delta \sim t^{\pm\alpha}$$

In agreement with Biot-Savart

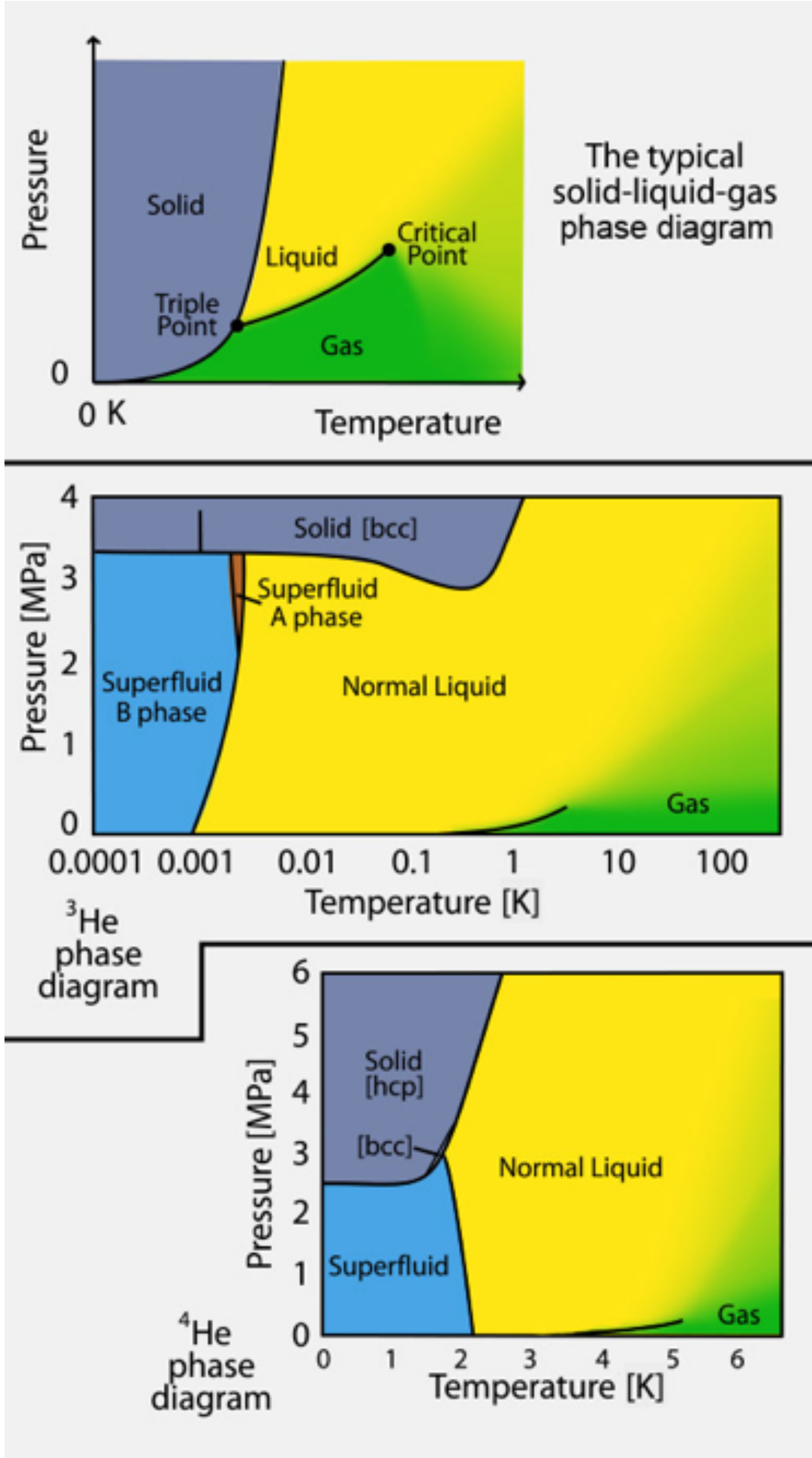
Paoletti et al. Physica D (2010)



$\sqrt{\frac{\kappa}{2\pi}}(t-t_0)^{-1/2}$
δ
10^{-3}
10^{-4}
10^{-5}
10^{-2}
$t-t_0$
$\sqrt{\pi\kappa}(t-t_0)^{1/2}$
-37.5 ms
-12.5 ms
12.5 ms
$t^{\pm\alpha}$
ent with Biot-Savart
Paoletti et al. Physica D (2010)

Finite temperature effects

- In the lab we are never at 0K
- Helium is an intimate mix of inviscid superfluid component and a viscous normal fluid.



Finite temperature effects

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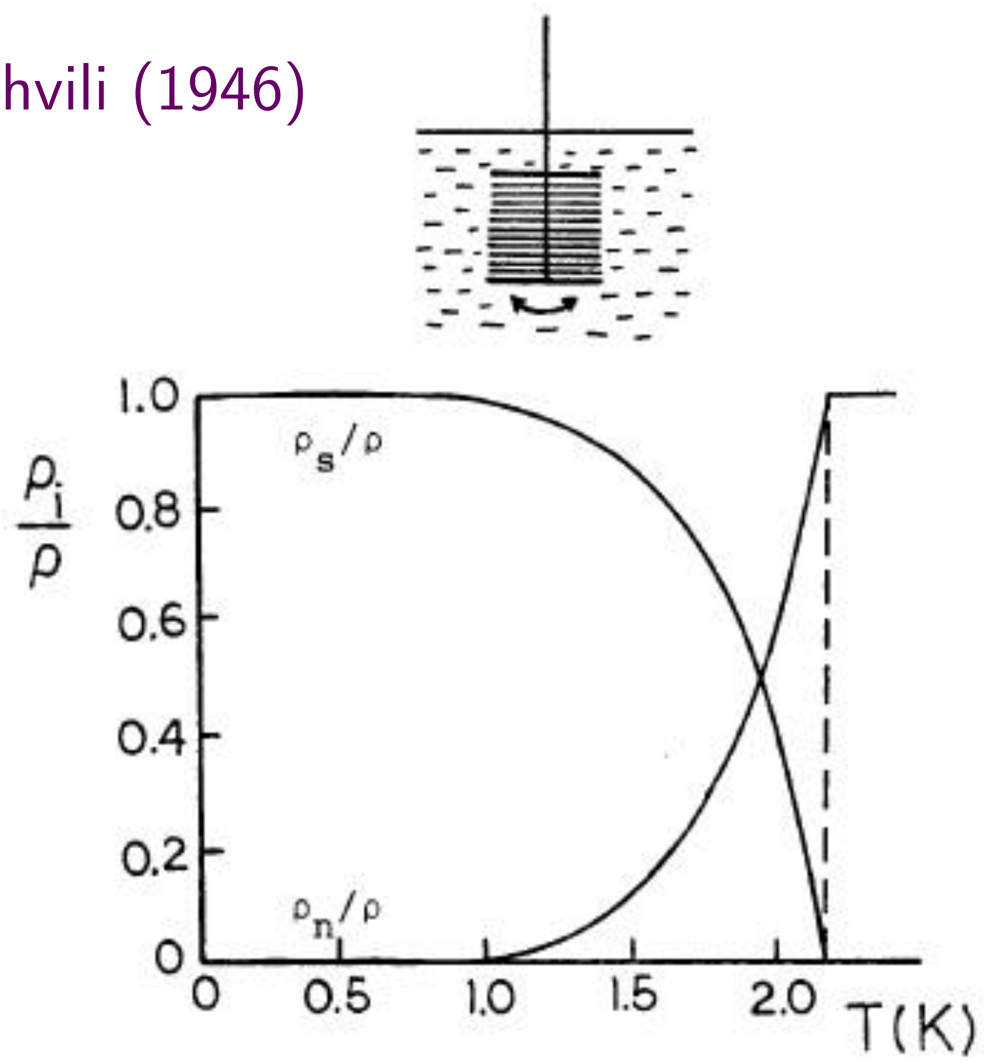
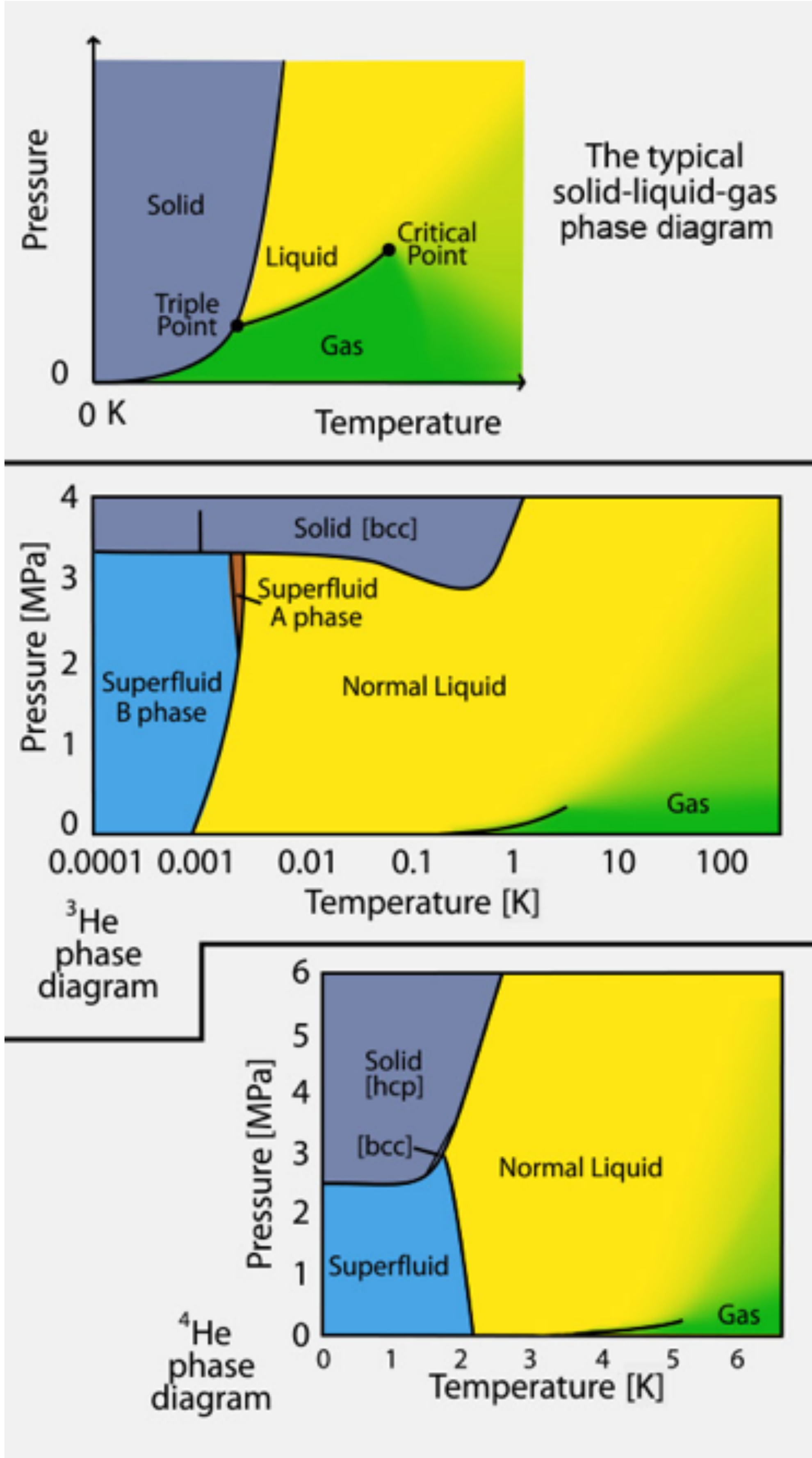
Andronikashvili (1946)





Mutual friction

Balance Magnus and drag forces

$$\frac{d\mathbf{s}}{dt} = \mathbf{v}_s^{\text{tot}} + \alpha \mathbf{s}' \times (\mathbf{v}_n^{\text{ext}} - \mathbf{v}_s^{\text{tot}}) - \alpha' \mathbf{s}' \times [\mathbf{s}' \times (\mathbf{v}_n^{\text{ext}} - \mathbf{v}_s^{\text{tot}})]$$

Counterflow Turbulence

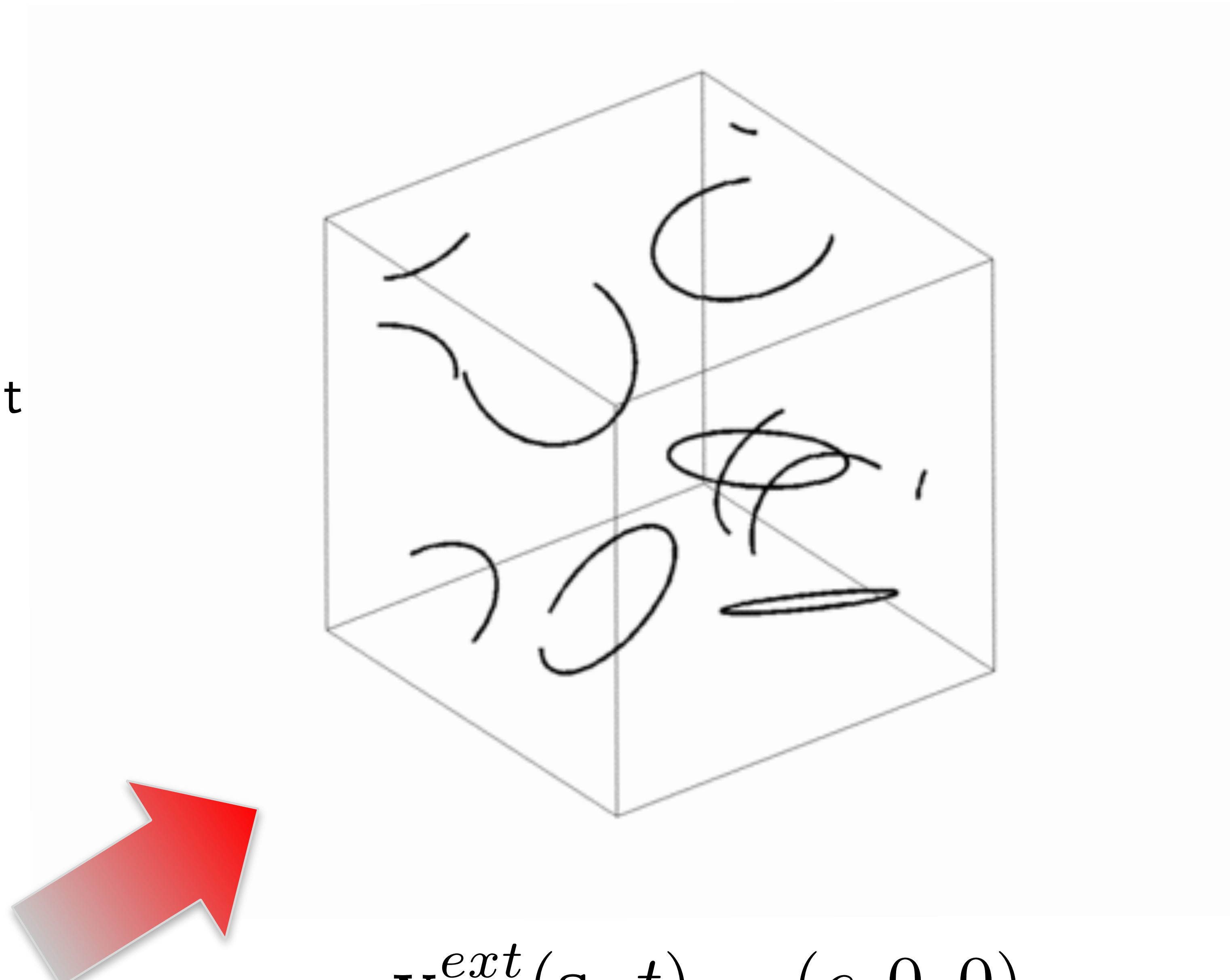
Normal viscous fluid coupled to inviscid superfluid via mutual friction.

Superfluid component extracts energy from normal fluid component via Donelly-Glaberson instability, amplification of Kelvin waves.

Kelvin wave grows with amplitude

$$A(t) = A(0)e^{\sigma t}$$

$$\sigma(k) = \alpha(kV - \nu' k^2)$$



$$\mathbf{v}_n^{ext}(\mathbf{s}, t) = (c, 0, 0)$$

Mutual friction

Balance Magnus and drag forces

$$\frac{d\mathbf{s}}{dt} = \mathbf{v}_s^{\text{tot}} + \alpha \mathbf{s}' \times \left(\mathbf{v}_n^{\text{ext}} - \mathbf{v}_s^{\text{tot}}\right) - \alpha' \mathbf{s}' \times \left[\mathbf{s}' \times \left(\mathbf{v}_n^{\text{ext}} - \mathbf{v}_s^{\text{tot}}\right)\right]$$

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$$\mathbf{v}_n^{ext}(\mathbf{s}, t) = (c, 0, 0)$$

Numerical simulations of Borner's experiment

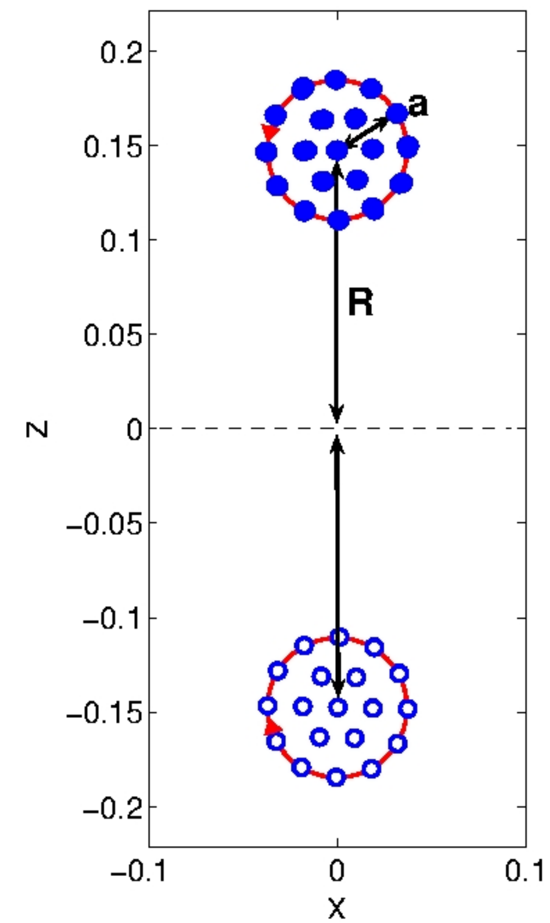
- Vortex lines = space curves $\mathbf{s}(t)$:
  Biot–Savart law

$$\frac{d\mathbf{s}}{dt} = \mathbf{v}_{si} = -\frac{\kappa}{4\pi}\oint_{\mathcal{L}} \frac{(\mathbf{s}-\mathbf{r})}{|\mathbf{s}-\mathbf{r}|^3} \times \mathbf{dr}$$

  plus reconnection *Ansatz*

- ^{4}He parameters:
  Circulation $\kappa = 10^{-3}\ \mathrm{cm}^2/\mathrm{s}$
  Vortex core size $\xi = 10^{-8}\ \mathrm{cm}$

- Initial condition:
  (arbitrary) vortex lattice of N rings
  $(N = 7, 19, 37, 61, 91, \cdots)$

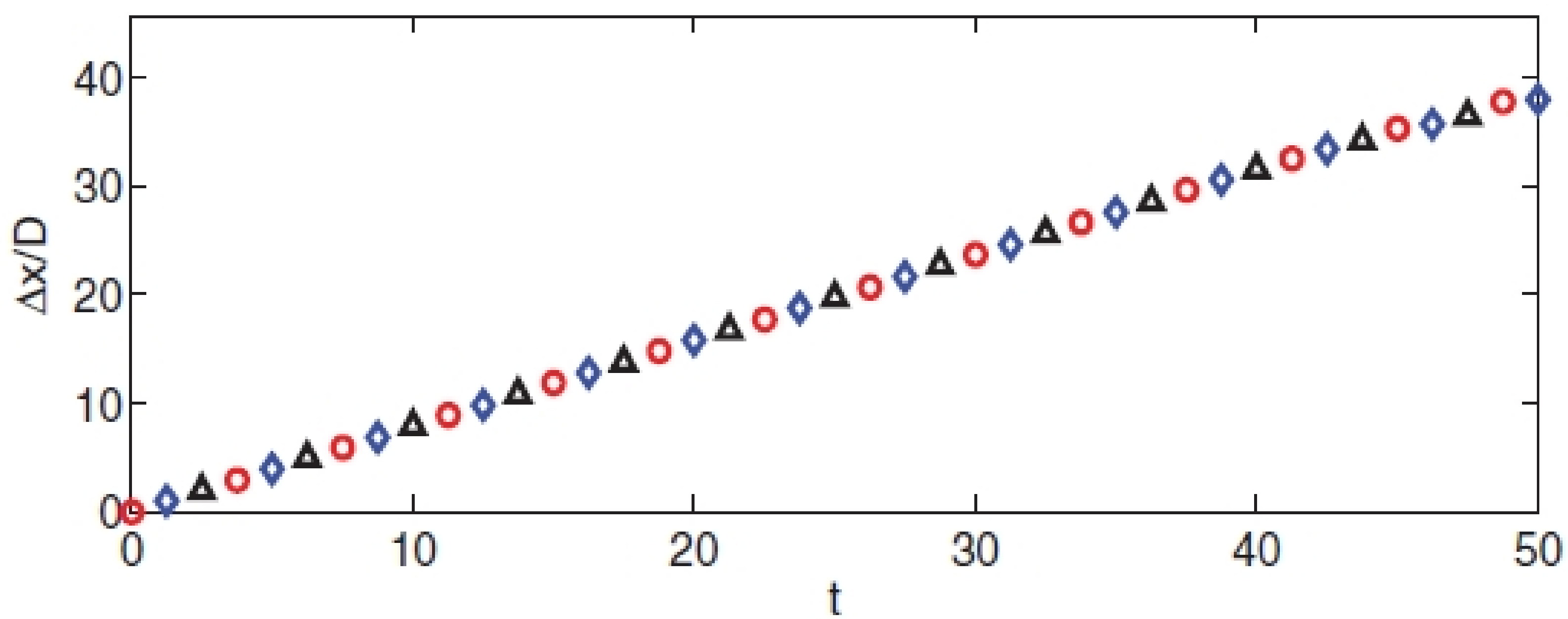


Initial condition
$N = 19$ rings

Begin at T=0

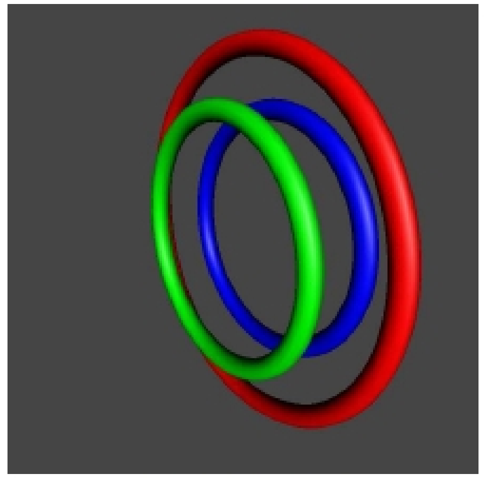
Tests

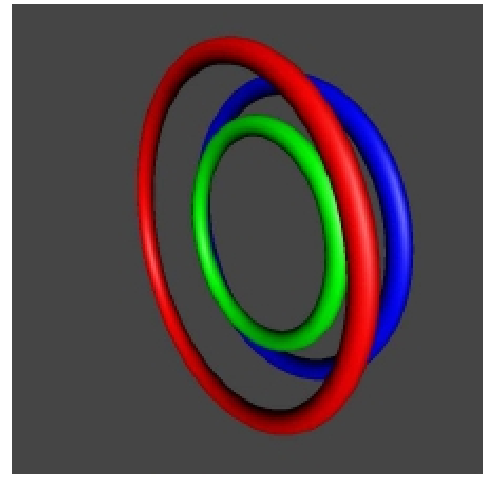
$$\text{Energy}: \quad E = \frac{1}{2}\int_V \mathbf{v}^2 dV = \kappa \oint \mathbf{v}\cdot\mathbf{s}\times\mathbf{s}' d\xi_0$$

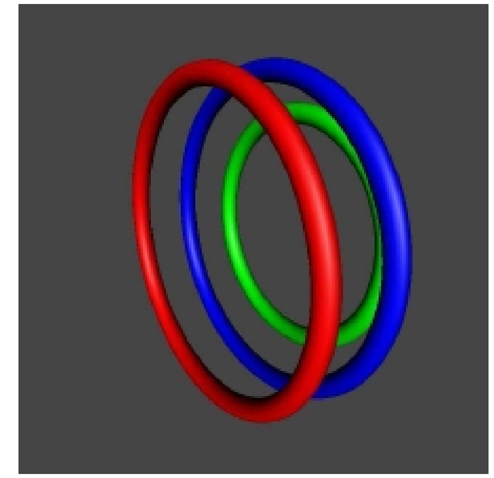


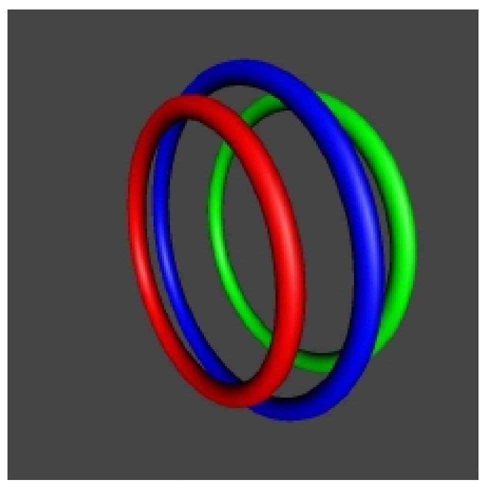
Leapfrogging of $N = 2$ rings travelling up to 40 diameters
Symbols refer to different numerical resolutions $\Delta\xi_0$
$\Delta E/E$ within 0.5%, 1% for largest bundles

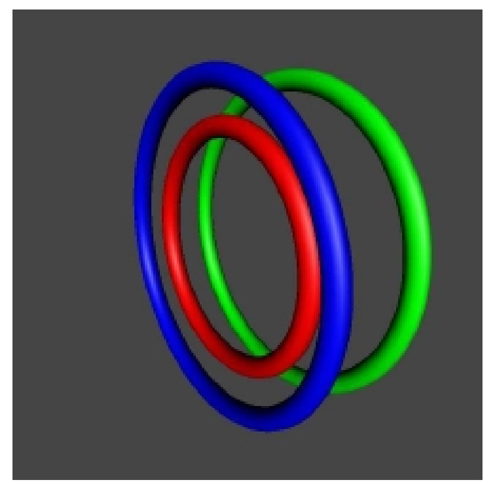
N=3 rings: Generalised Leapfrogging

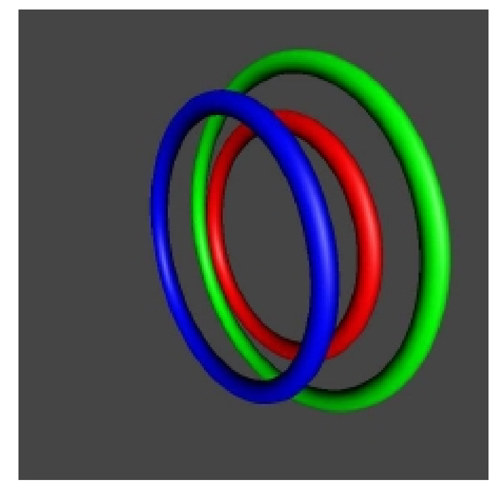

(a) $t = 0.005$

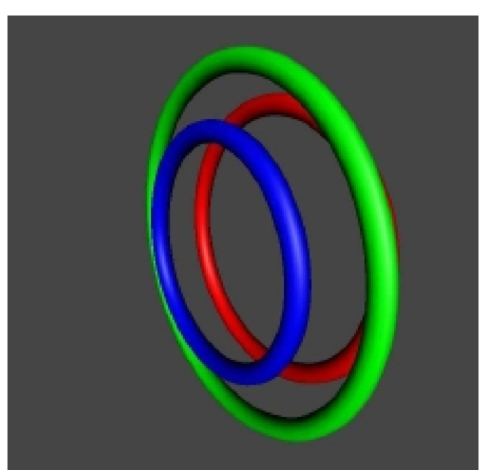

(b) $t = 0.4$

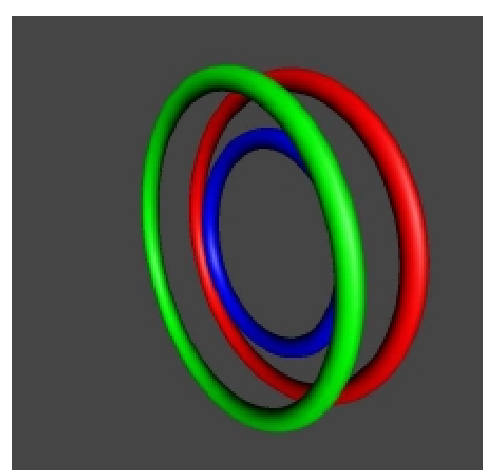

(c) $t = 0.8$

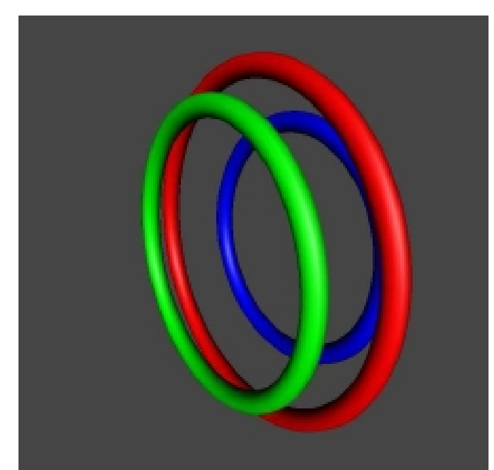

(d) $t = 1.2$


(e) $t = 1.6$

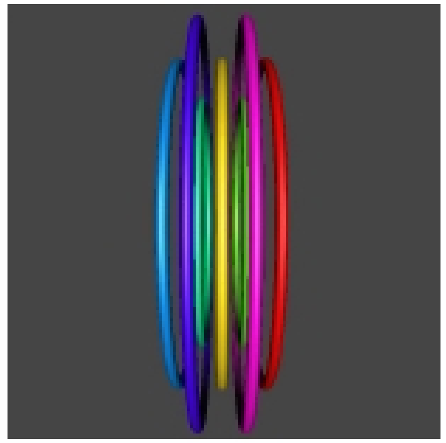

(f) $t = 2.0$


(g) $t = 2.4$

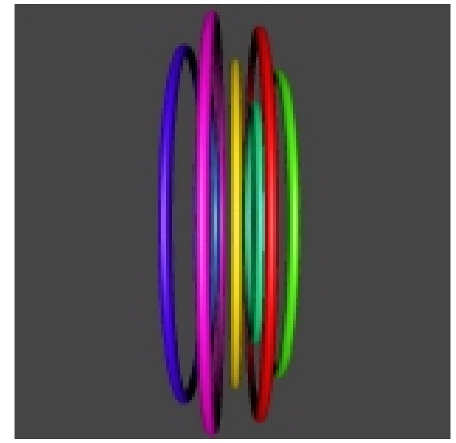

(h) $t = 2.8$


(i) $t = 3.2$

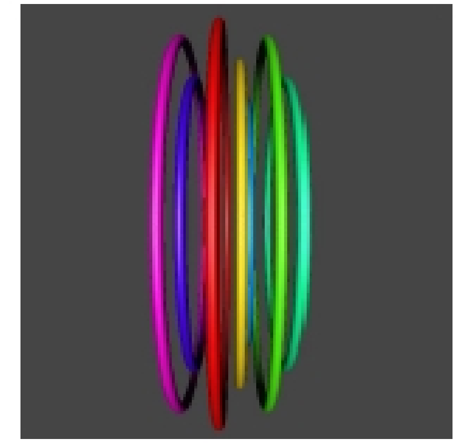
N=3 rings: Generalised Leapfrogging



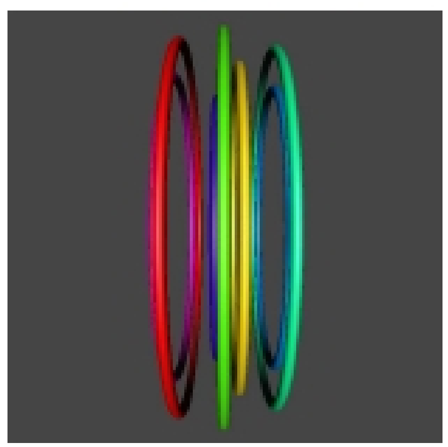
(a) $t = 0.0075$ s



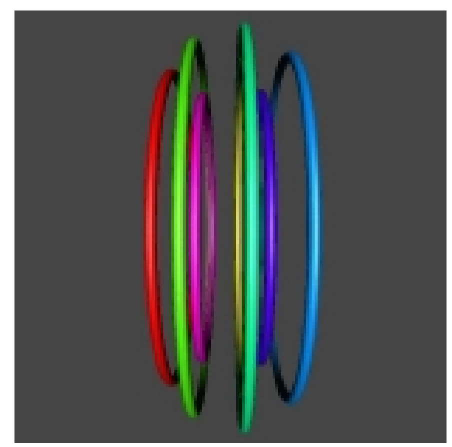
(b) $t = 0.75$ s



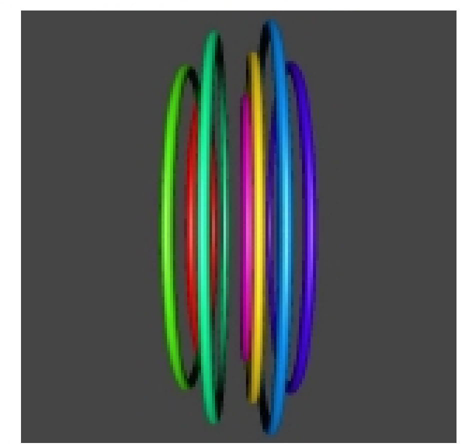
(c) $t = 1.5$ s



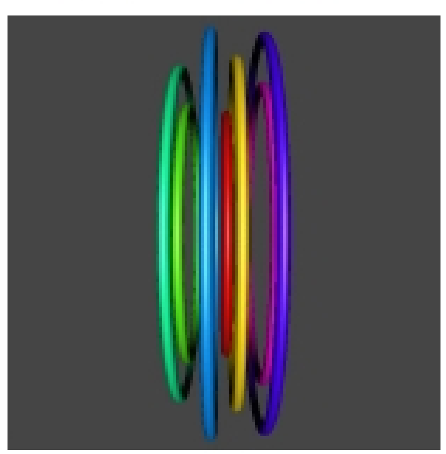
(d) $t = 2.25$ s



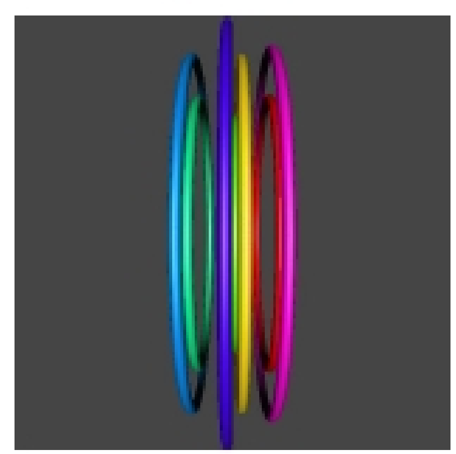
(e) $t = 3.0$ s



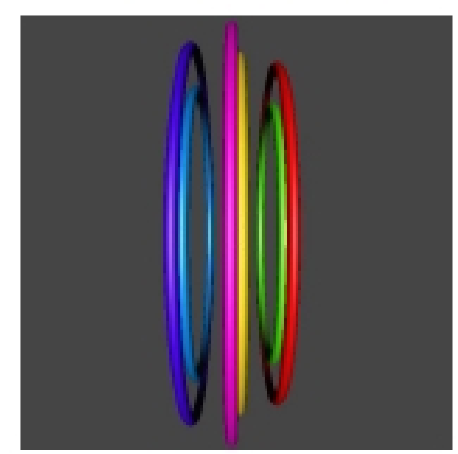
(f) $t = 3.75$ s



(g) $t = 4.5$ s



(h) $t = 5.25$ s



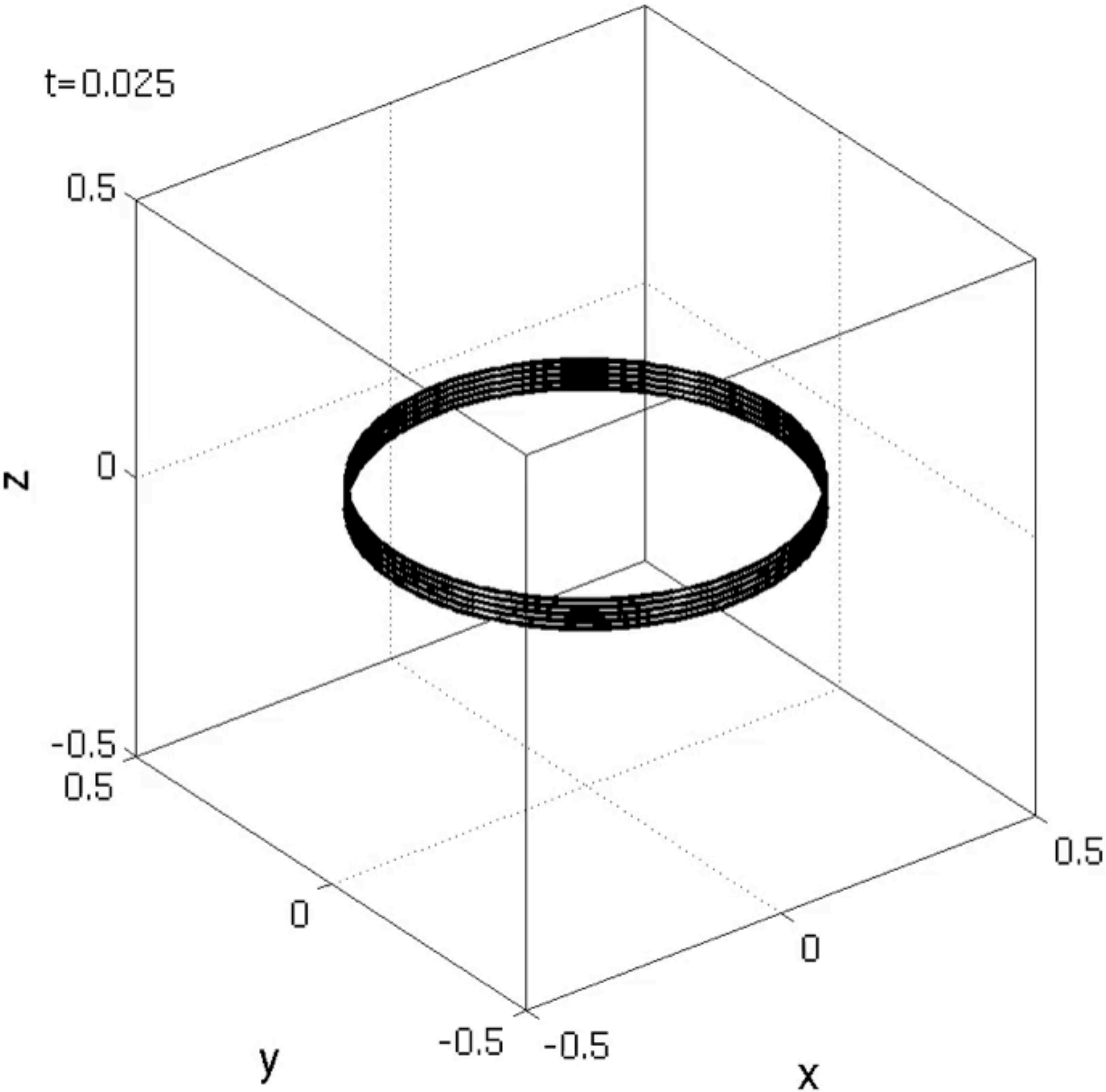
(i) $t = 6.0$ s

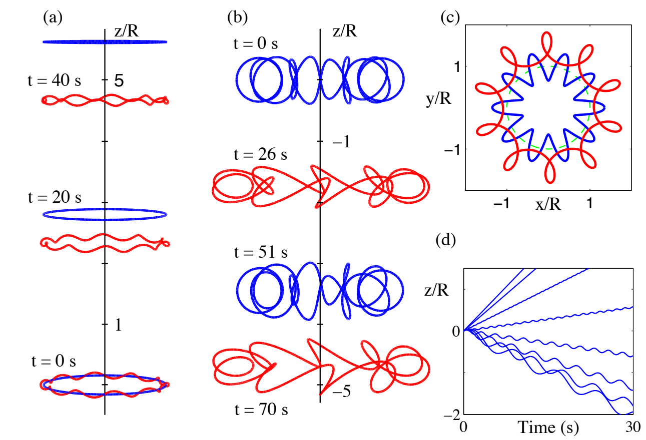
Larger N

• Large N bundles of rings, over long time/distance, tend to develop long wave perturbations

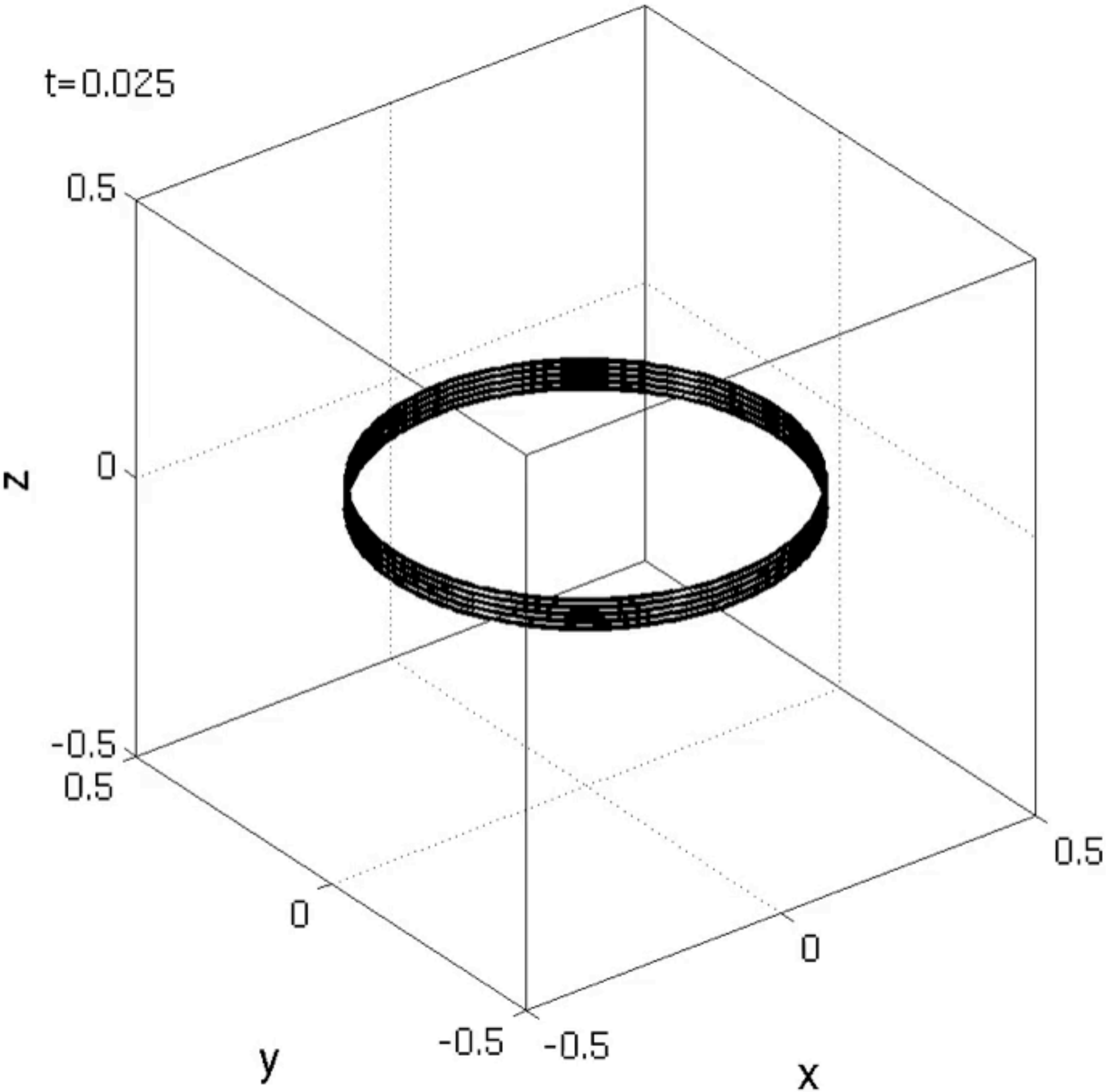
• These perturbations eventually induce vortex reconnections, hence short waves perturbations which travel around the rings, which induce further short wave perturbations

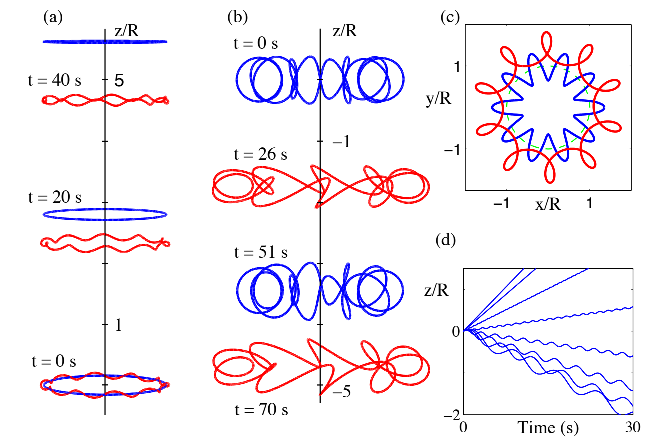
• Due to reconnections, the number N becomes ill defined, but vortex bundles are robust, and travel at essentially constant speed over a large distance even if turbulent



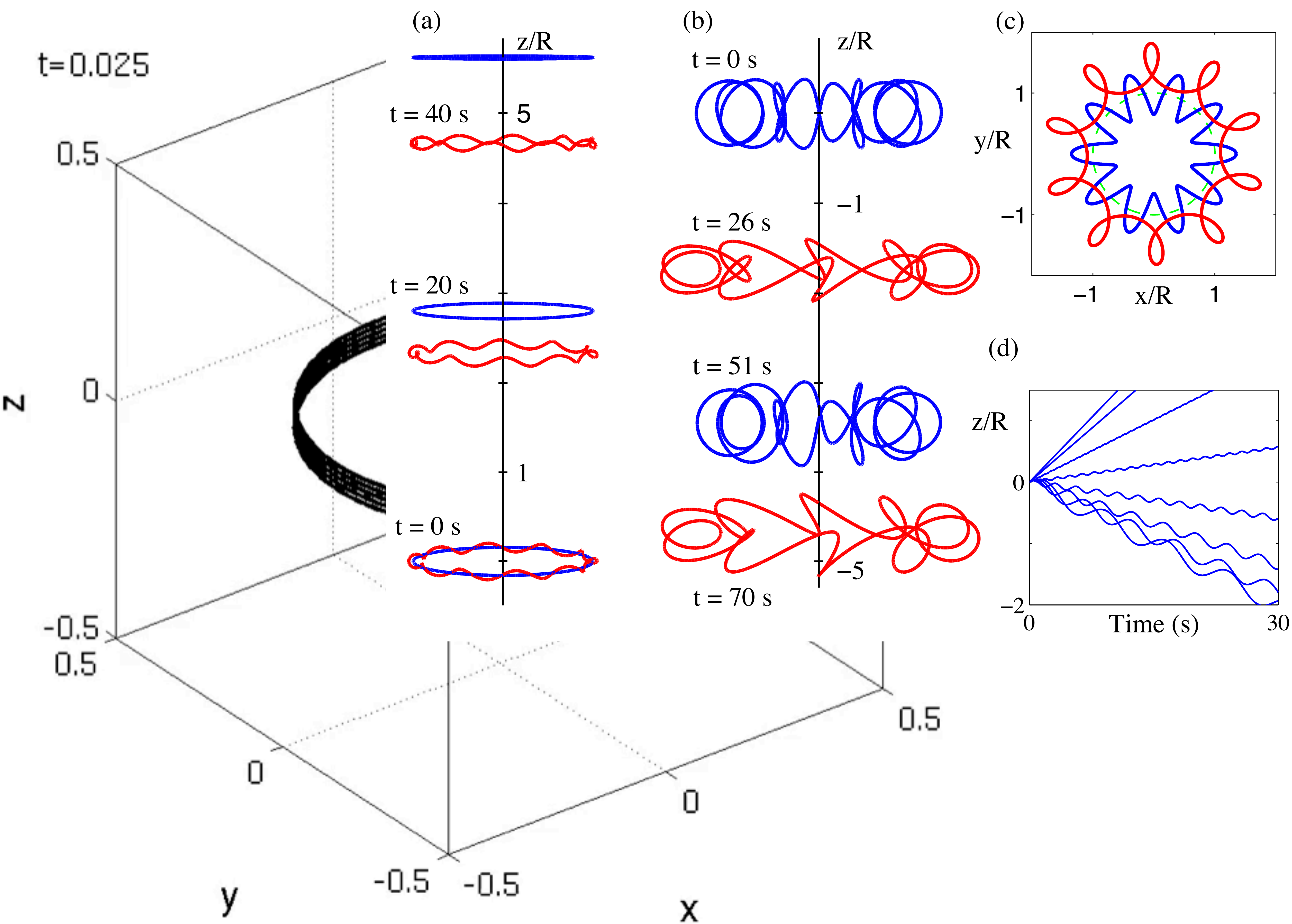


Barenghi at al., 2008



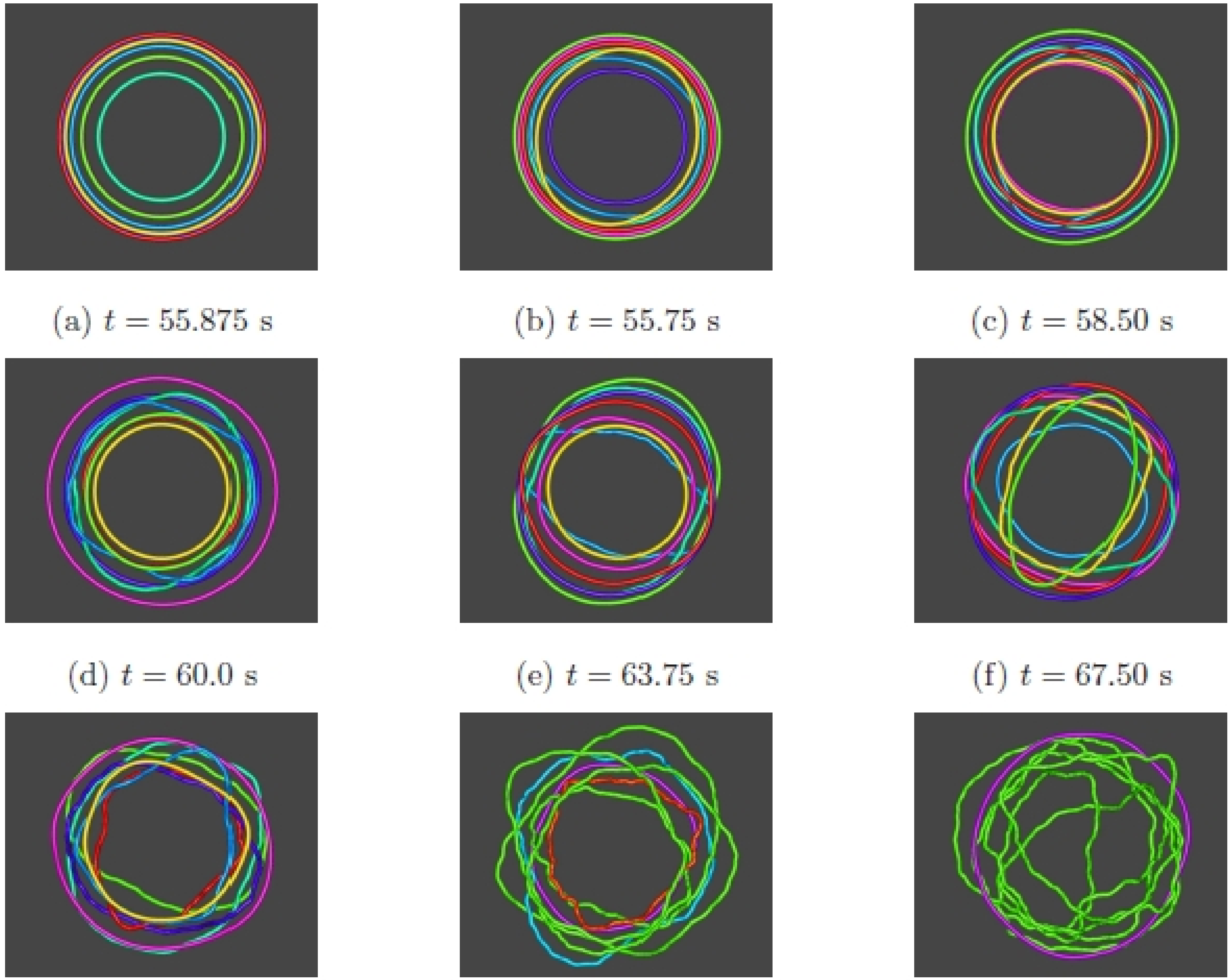


Barenghi at al., 2008



Barenghi at al., 2008

N=7 rings: Instability



(a) $t = 55.875$ s (b) $t = 55.75$ s (c) $t = 58.50$ s

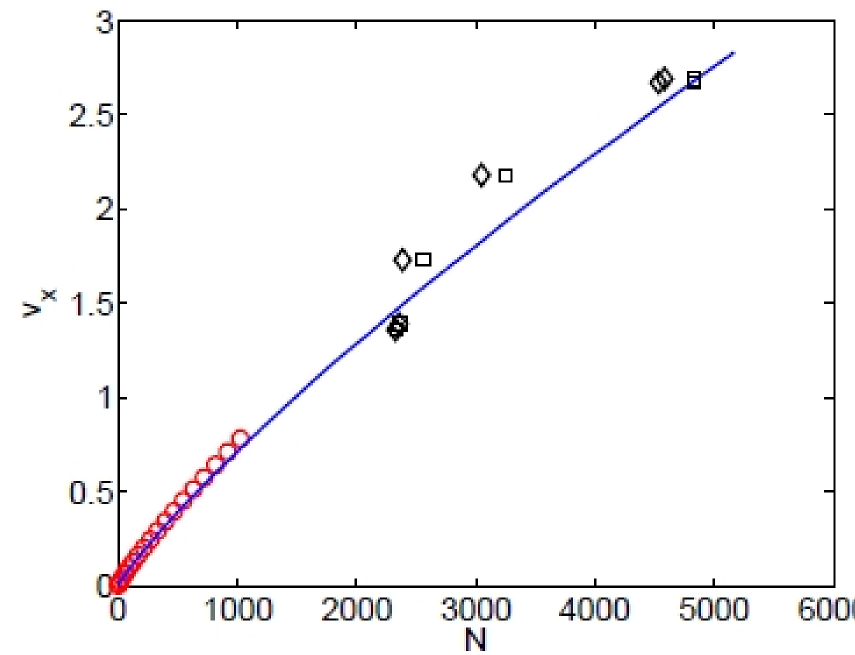
(d) $t = 60.0$ s (e) $t = 63.75$ s (f) $t = 67.50$ s

(g) $t = 71.25$ s (h) $t = 75.0075$ s (i) $t = 78.75$ s

Velocity

One vortex ring:

$$v = \frac{\kappa}{4\pi R}\left(\ln\left(8R/\xi\right) - 1/2\right)$$

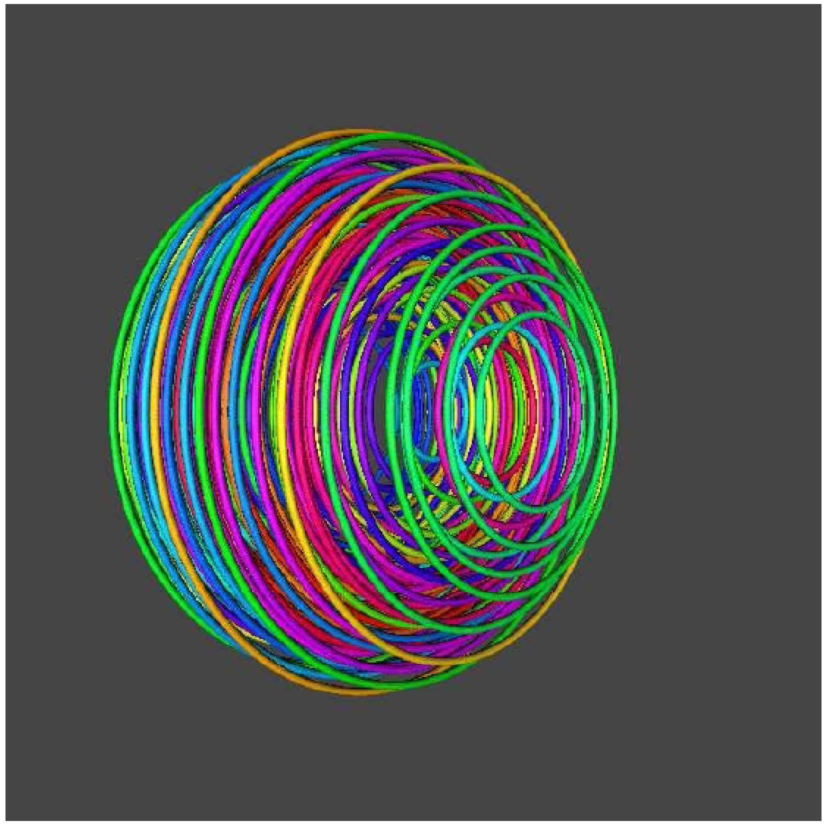


Model bundle of N rings by
$\kappa \to N\kappa$ and $\xi \to a$

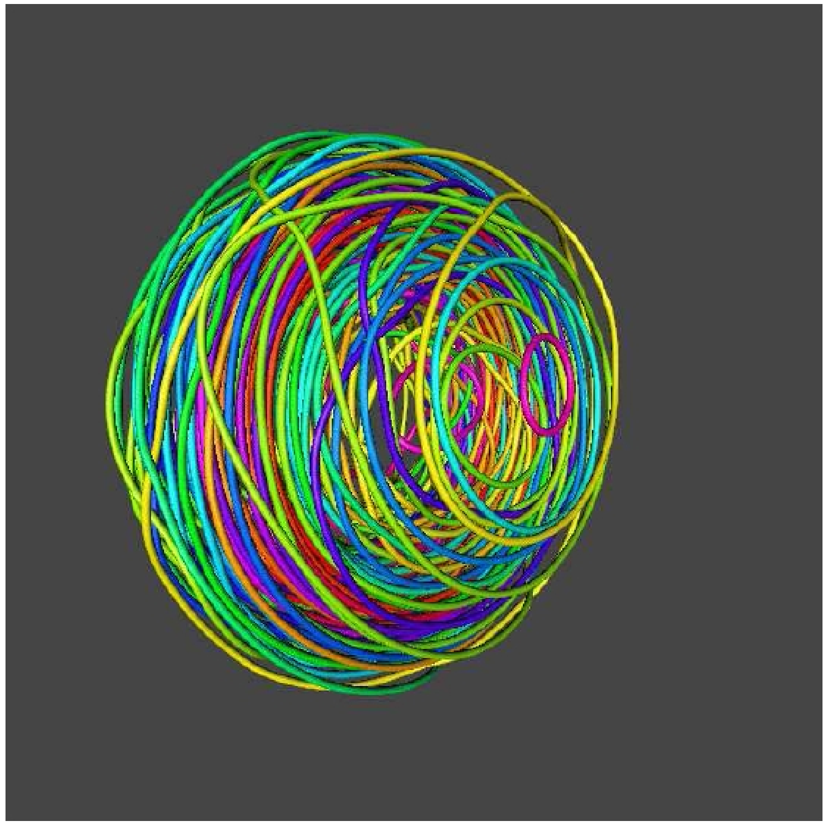
$$v = \frac{N\kappa}{4\pi R}\left(\ln\left(8R/a\right) - 1/2\right)$$

Black squares and diamonds: Borner's experiments
Red circles: numerical simulations $N \leq 1027$
Blue line: model

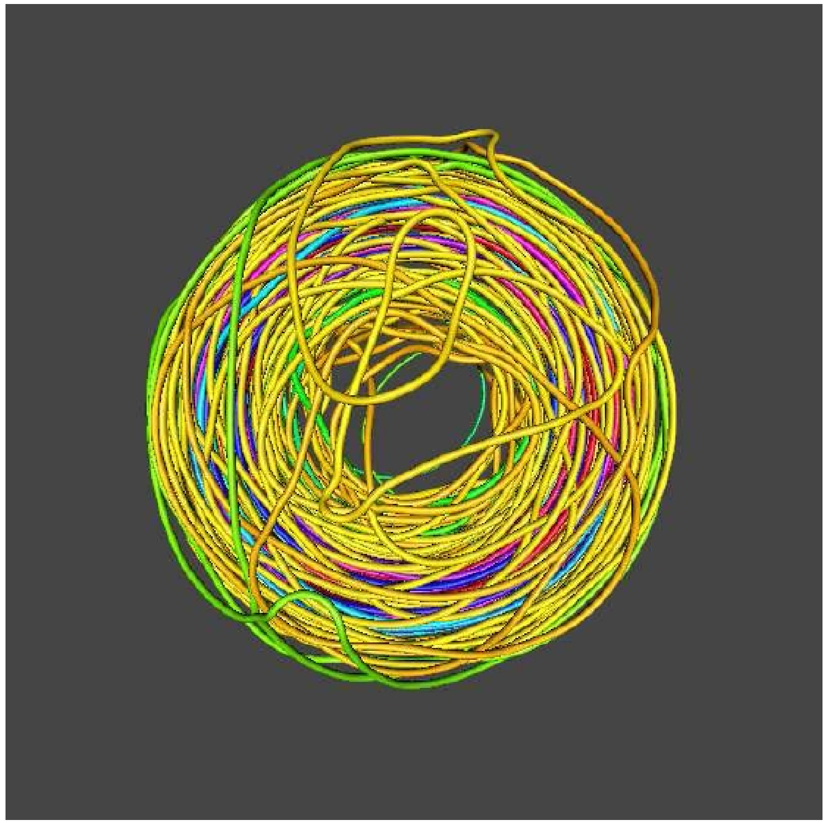
N=91 Bundle



$\Delta x/D = 2.65$



$\Delta x/D = 5.17$

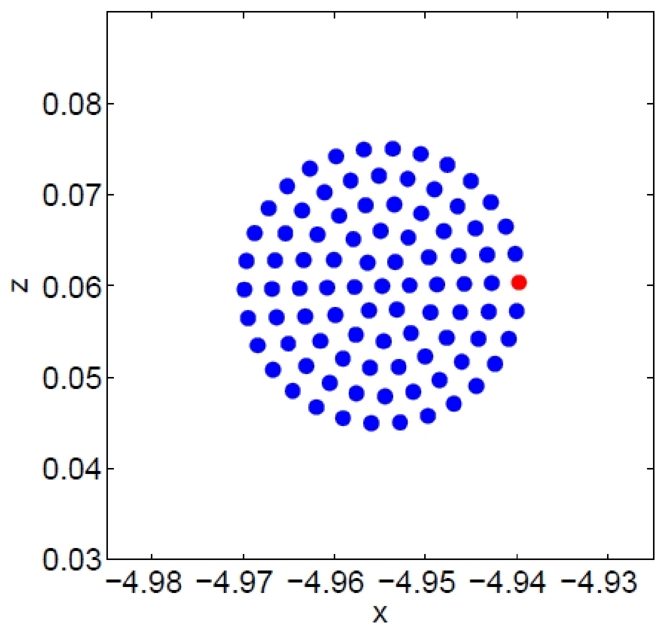


$\Delta x/D = 10.04$

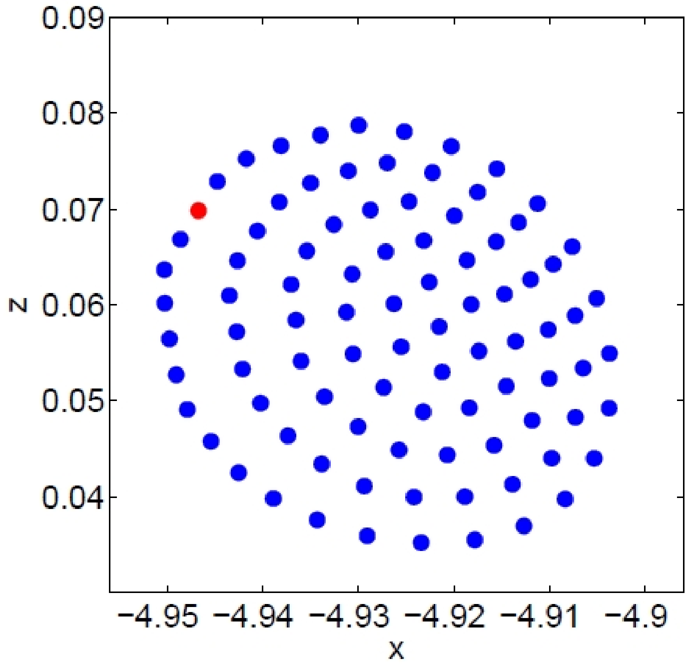
Cross-section

The long-term instability is probably due to "core" deformation: the bundle does not remain circular, but acquires a D-shape and becomes stretched

$N = 91$



Initial



After half a leapfrog

See classical stability of vortex rings
(Moore, Saffman, Widnall, Fukumoto, Moffatt, etc)

What about the effect of friction?

At $T > 0$ friction modifies vortex dynamics:

$$\frac{d\mathbf{s}}{dt} = \mathbf{v}_s^{ext} + \mathbf{v}_{si} + \alpha \mathbf{s}' \times (\mathbf{v}_n^{ext} - \mathbf{v}_s^{ext} - \mathbf{v}_{si}) - \alpha' \mathbf{s}' \times [\mathbf{s}' \times (\mathbf{v}_n^{ext} - \mathbf{v}_s^{ext} - \mathbf{v}_{si})]$$

Motion of a single ring (Barenghi, Donnelly, Vinen 1983):

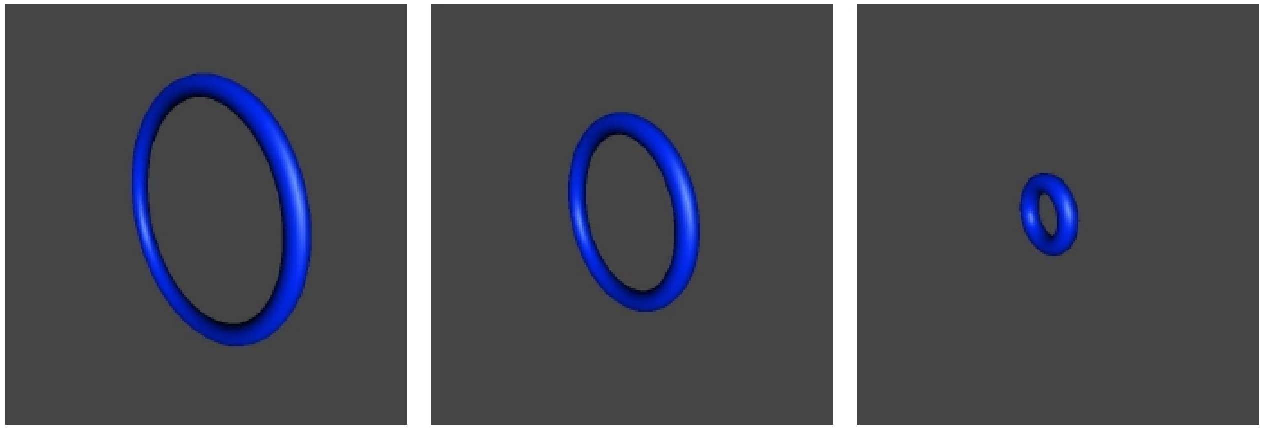
$$\frac{dR}{dt} = \frac{\gamma}{\rho_s \kappa}(v_s^{ext} - v_n^{ext} - v_{si})$$

where $\gamma = \gamma(\alpha, \alpha')$ and

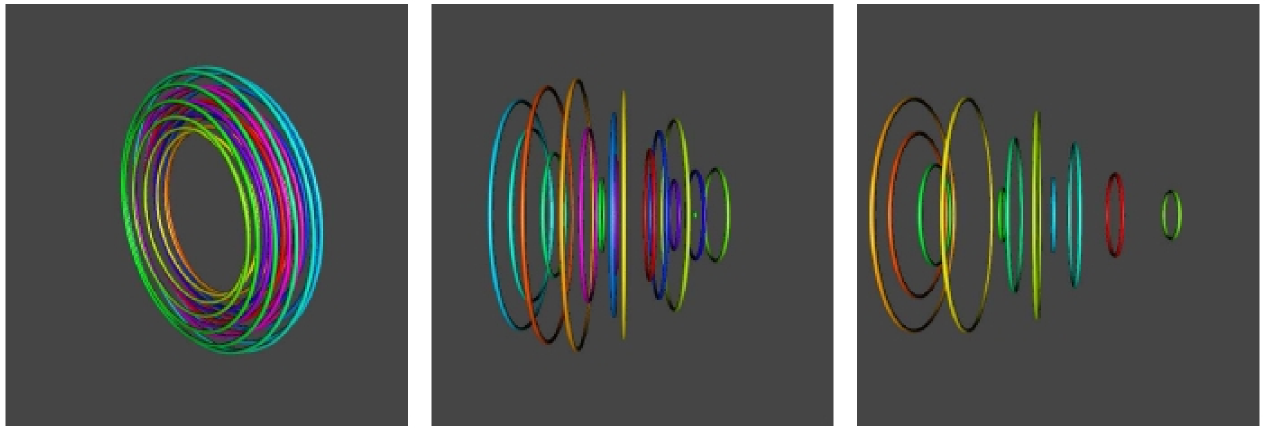
$$v_{si} = \frac{\kappa}{4\pi R}[\ln(8R/\xi) - 1/2]$$

Effect of friction

Assume $\mathbf{v}_n^{ext} = \mathbf{v}_s^{ext} = \mathbf{0}$



Decay of a single ring



Decay of $N = 19$ bundle

Effect of friction

- Clearly not in agreement with Borner's and others results at $T>0$
- A solution of the puzzle:
- The piston which creates the superfluid bundle must generate a normal fluid vortex ring too !
- We add to the equation of motion a normal fluid ring (with solid body rotating core) placed at the moving centre of the superfluid bundle. This term $\mathbf{v}_n \neq 0$ effectively cancels the friction and stabilizes the bundle.

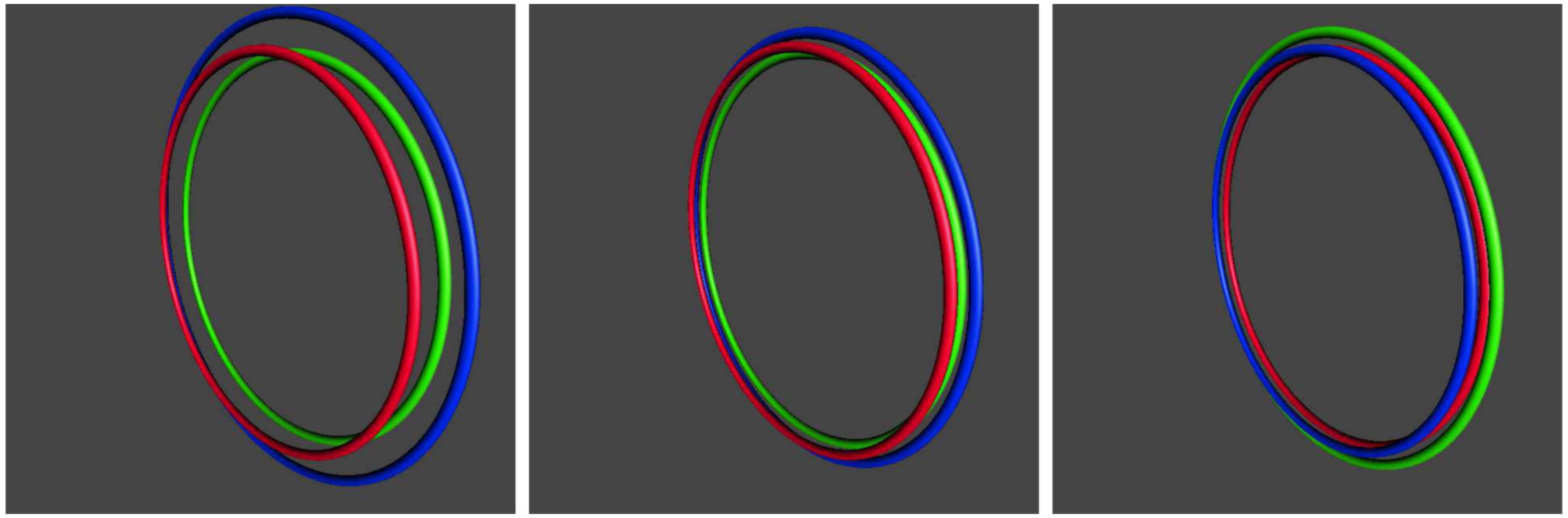


FIG. 3. (Color online) Motion of 3 vortex rings in the presence of friction at $T = 2.02$ K and normal-fluid ring ($\mathbf{v}_n \neq 0$). The initial condition is the same as in Fig. 1. Left: $t = 0$; middle: $t = 3.6$; right: $t = 7.2$ s. It is apparent that the superfluid vortex bundle moves in a stable way as in the absence of friction (Fig. 1); the individual rings leapfrog around each other.

TABLE III. Evolution at $T = 2.02$ K in the presence of friction and $\mathbf{v}_n$.

N	t (s)	$\Delta z/D$	Λ/Λ_0	$\bar{c}/\bar{c}_0$	v/v_0
2	50	11.87	0.97	1.02	0.99
3	40	11.37	0.96	1.07	1.10
7	30	10.13	1.52	1.18	0.69
19	60	10.21	0.80	1.08	0.64

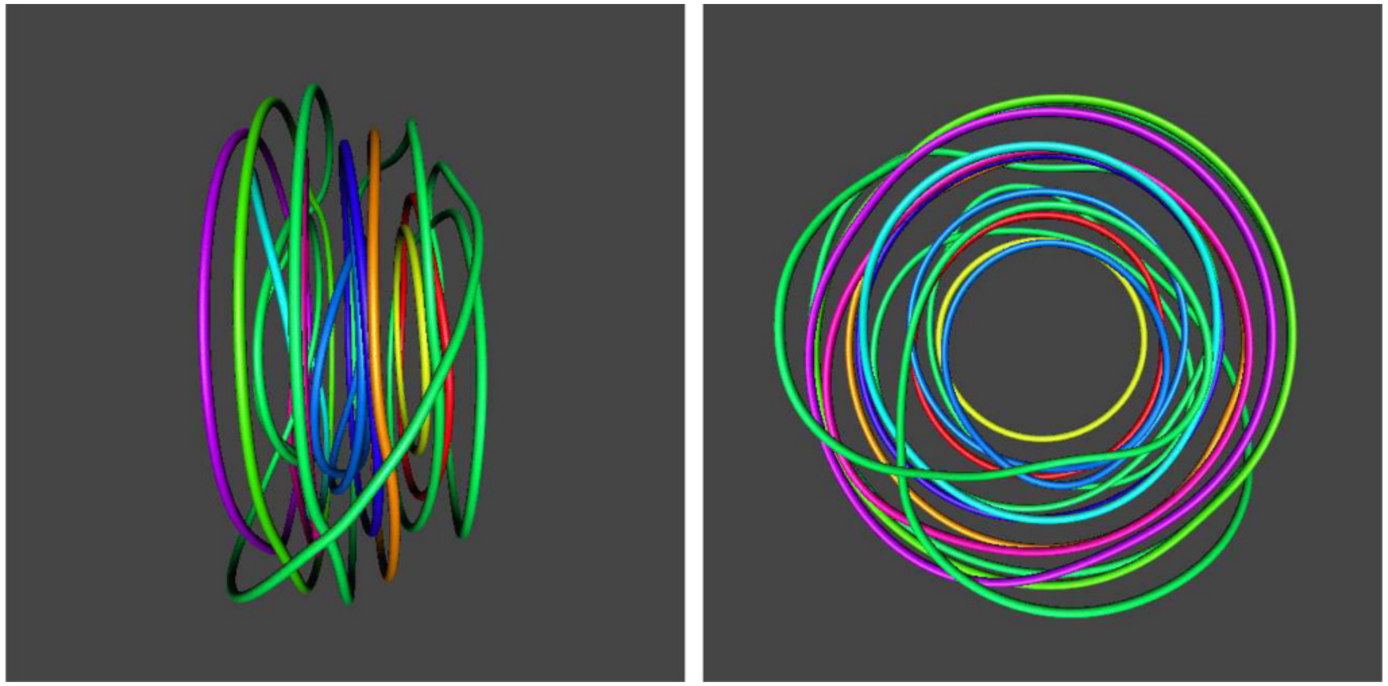


FIG. 5. (Color online) Vortex bundle at $T = 2.02$ K in the presence of friction and normal-fluid ring $\mathbf{v}_n$. Top: Side (left) and rear (right) view of vortex bundle with $N = 19$ rings at time $t = 80$ s.

To summarise

• Large-scale bundles of superfluid vortex rings are indeed sufficiently robust to travel a distance of at least 10 diameters as observed in experiments

• Generalised leapfrogging motion

• Results hold true in the presence of friction (high T):
the corresponding (continuous) large-scale normal fluid structure prevents the superfluid rings from shrinking.

• Implications for quantum turbulence ?

Conclusions

- Our results suggest that bundles of superfluid vortex rings can travel coherently a significant distance, at least one order of magnitude larger than their diameter at both zero and finite temperatures.
- In the absence of any direct experimental observation, the existence (and the nonexistence) of quantised vortex bundles has generated much discussion, particularly with respect to the quasi-classical regime.
- The vortex rings generated by the piston-cylinder setup provide a "forced" but controlled method to study the coupling of the normal and super fluid components.
- The detailed mechanism of the generation of the double vortex ring structure at the hole of the cylinder is an interesting problem of two-fluid hydrodynamics which would be worth studying.

Conclusions (ctd): Perhaps it is time to revisit macroscopic vortex rings experimentally again