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MEASURING THE TIME SPENT TRAVERSING THE BARRIER WHILE TUNNELING

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We present the principle behind an experiment that will measure the time spent traversing the potential barrier during a tunneling event. This experiment is accomplished by measuring the macroscopic quantum tunneling rate out of the zero voltage state of a current-biased Josephson junction connected to a transmission line of adjustable length. The transmission line provides a frequency-dependent damping that reduces the tunneling rate. As the length of the line is increased from zero, the depression of the tunneling rate changes from that corresponding to the termination resistor to that corresponding to a resistor equal to the line impedance. This crossover occurs at a line delay time equal to the time spent traversing the barrier.

Although quantum tunneling has been observed for many years, there exists a fundamental time scale [1, 2] for this phenomenon that has not yet been directly measured. This time scale, called by Büttiker and Landauer [2] the "traversal time for tunneling", is the time spent traversing the potential barrier during the tunneling event. In this paper we describe the theory that is the basis of an experiment, currently in progress at Saclay, in which we will measure this time.

This experiment is an outgrowth of recent theoretical understanding of how dissipation affects quantum tunneling [3]. The experimental system in which the measurement is being performed is the current-biased Josephson junction. This system is ideal because dissipation can be varied and controlled readily with all of the experimental parameters measured in situ [4].

The schematic representation of the system is given in fig. 1. A Josephson junction, of critical current I_0 and capacitance C, is connected to a current bias I through a transmission line that is terminated by a resistor Z_t. The characteristic impedance of the line is Z_1, its length is l, and the propagation velocity is v. This system can be represented [4] by a particle of coordinate δ moving in a tilted cosine potential $U(\delta) = (\Phi_0/2\pi)[I_0 \cos(\delta) - I\delta]$, where δ is the superconductor phase difference across the junction, and $\Phi_0 = h/2e$. The zero voltage state of the junction corresponds to the particle residing in a local minimum of the potential. When the particle escapes, it continues running down the potential; this corresponds to the voltage state of the junction. The experiments are usually conducted with I_0 slightly less than I. In this case the potential is very well approximated in the vicinity of one of its local minima by a cubic potential, with barrier height $\Delta U = (2\sqrt{2}\Phi_0/3\pi)(1 - I/I_0)^{3/2}$. When the transmission line length $l = 0$, the plasma oscillation frequency at the bottom of the well is $\omega_p = (2\pi I_0/C\Phi_0)^{1/2}[1 - (I/I_0)^2]^{1/4}$, of order a few GHz in our experiment.

We are interested in the escape of δ from the metastable state. At temperature $T = 0$, this escape occurs via macroscopic quantum tunneling [3]. For the case of no dissipation ($l = 0$ and $Z_t \to \infty$), a WKB calculation predicts for the exponential part of the tunneling rate $\Gamma \propto \exp(-7.2\Delta U/\hbar\omega_p)$. The prediction for the case of a resistor shunting the junction ($l = 0$) was given by Caldeira and Leggett [3]. When this dissipation is small ($l = 0$ and Z_t large), the tunneling rate is given by $\Gamma \propto \exp[-7.2(\Delta U/\hbar\omega_p)(1 + 0.87/Q + \cdots)]$, where $Q = \omega_p Z_t C$. Here we see that dissipation can strongly depress quantum tunneling.

For the case of a transmission line shunting the junction, we need Leggett's theory of quantum tunneling which takes into account frequency-dependent dissipation [5]. Using his work, one

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Fig. 1. Schematic representation of a current-biased Josephson junction connected to a transmission line of adjustable length l.

can then solve directly for the change in the tunneling exponent in perturbation theory for the case of small dissipation [6]. The result is:

$$\Gamma \propto \exp[-7.2(\Delta U/\hbar\omega_p)(1+\Delta b+\cdots)]\,,$$

$$\Delta b = \frac{15}{C\omega_p^2}\int_0^\infty \frac{Y(-\mathrm{i}\omega)\omega^3}{\sinh^2(\pi\omega/\omega_p)}\,\mathrm{d}\omega\,,$$

and where $Y(-\mathrm{i}\omega)$ is the external admittance loading the junction at negative imaginary frequencies. For the transmission line, the admittance is easily calculated to be

$$Y(\omega) = \frac{1}{Z_l}\,\frac{Z_t + \mathrm{i}Z_l\tan(\omega l/v)}{Z_l + \mathrm{i}Z_t\tan(\omega l/v)}\,.$$

Note that the admittance oscillates as a function of ω at real frequencies, but becomes an exponentially varying function at imaginary frequencies.

We plot in fig. 2 the change in the tunneling exponent Δb versus transmission line length for

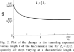

Fig. 2. Plot of the change in the tunneling exponent Δb versus length l of the transmission line for $Z_t = Z_l/2$. The quantity Δb stops varying at a characteristic length scale $l_c = (2\pi v/\omega_p)/2$. The time spent traversing the barrier is $\tau_{tt} \approx 2l_c/v$.

$Z_t = Z_l/2$. At length $l = 0$, the junction is shunted by the resistor Z_t and we recover the linear dissipation prediction $\Delta b = 0.87/\omega_p Z_t C$. For line lengths longer than about half the wavelength of radiation at frequency $\omega_p/2\pi$, Δb approaches the value $\Delta b = 0.87/\omega_p Z_l C$. In this case the reduction in tunneling is equivalent to the situation when the junction is shunted by a resistor equal to the characteristic impedance of the line [6, 7]. It is interesting to note that the same prediction for Δb is found at long transmission line lengths for any value of the termination resistance. For example, this is found even if the line is terminated with an open circuit $Z_t = \infty$. Thus, a line of finite length can look like a resistor even though it produces no physical energy loss.

We conclude that for long lengths, the tunneling process is only affected by the transmission line, and does not "see" the end of the line at all. A physical explanation of this is simple. Let us consider the correlation time τ_{tt} of the quantum fluctuations associated with the traversal of the potential barrier. If the delay time of the transmission line, given by $\tau_l = 2l/v$, is shorter than τ_{tt}, then the quantum fluctuations associated with the tunneling event interfere with themselves via reflections from the terminating impedance, and thus the termination will affect the tunneling rate. But if the delay time of the line is very long, then a tunneling event can no longer interfere with itself from a reflection off the termination. Thus, by measuring the characteristic length l_c in which the tunneling rate ceases to change with l, one can infer the time $\tau_{tt} \approx 2l_c/v \approx 2\pi/\omega_p$.

The theoretical prediction for the change in the tunneling rate versus line length is directly related to the imaginary frequency dependence of the admittance. Thus, an observation of escape rate without an oscillatory behavior will be a confirmation of this imaginary frequency dependence, as predicted by Leggett's theory. In contrast, the width and position of energy-level transitions [4] in the potential well are predicted to be proportional to real and imaginary parts of $Y(\omega)$ [6]. Since this depends on $Y(\omega)$ at real frequencies, this phenomenon will oscillate as a function of l.

In our experiment, the junction filters and measuring apparatus are similar to those described elsewhere [4]. The junction mount is of primary importance here, and is constructed as follows. The junction is positioned at one end of a coplanar transmission line of impedance 50 Ω. A microwave termination resistor is made by placing a metal bar with a thin insulating coating over the transmission line. The metal bar provides a microwave short from ground to the interior conductor, thus reducing the line's characteristic impedance. A microwave absorbing material is also included in this metal bar to suppress standing wave resonances in the bar. The end of the bar facing the junction thus looks like a termination resistance with respect to microwave radiation originating from the junction and reflecting off the bar. The bar is moved while it is at low temperatures ($T \approx 20$ mK) by a string attached between it and the top of the dilution refrigerator cryostat. Thus, we are able to measure the change in the escape rate of the junction versus line length while keeping all of the junction parameters constant.

In a separate but related experiment, we have investigated the thermal escape rate versus line length at liquid He temperatures. In this temperature regime the exponential factor of the escape rate prediction, $\exp(-\Delta U/k_B T)$, is independent of l, and the escape rate changes only because the classical prefactor changes. For a low damped system, theory predicts that the prefactor should oscillate with line length [8]. This effect has indeed been observed experimentally by our group.

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