Quantum Shot noise

probing interactions and magic

properties of the Fermi sea

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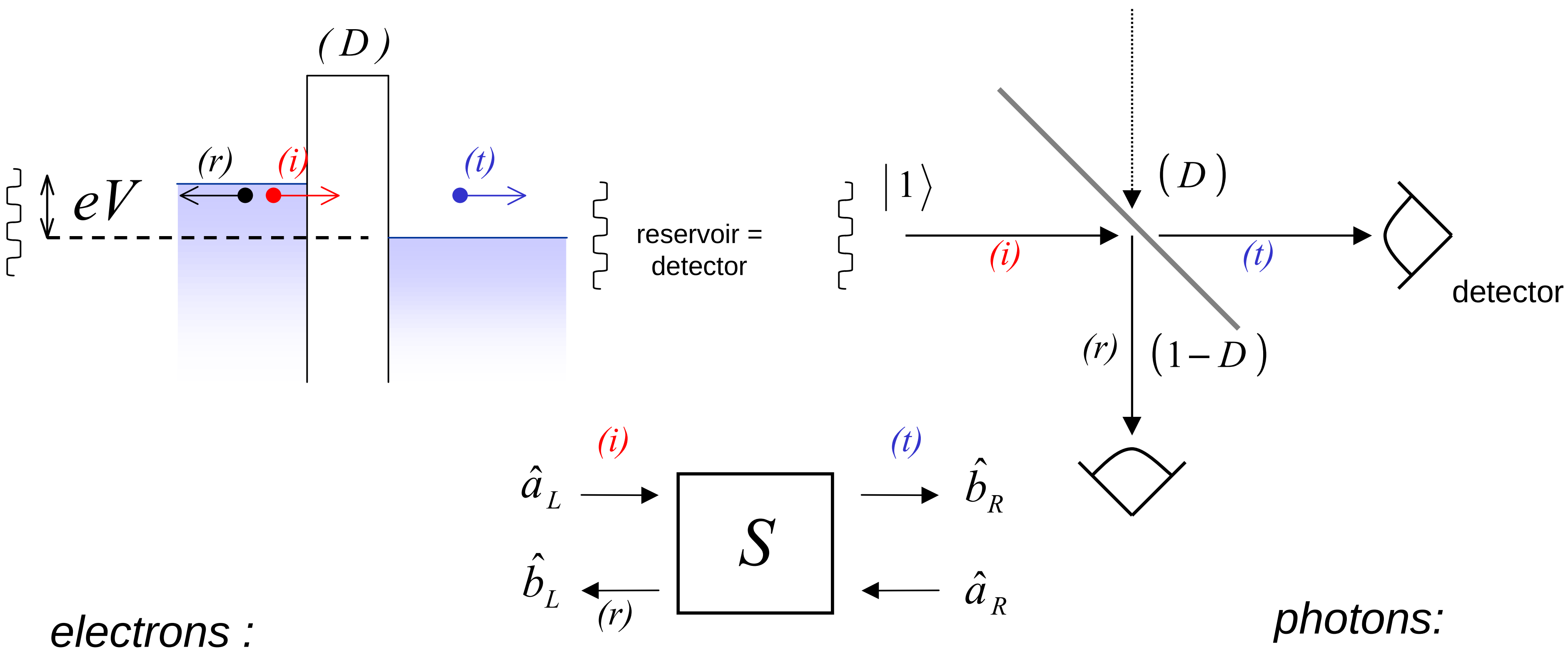
WHY STUDY SHOT NOISE ?



electrons :

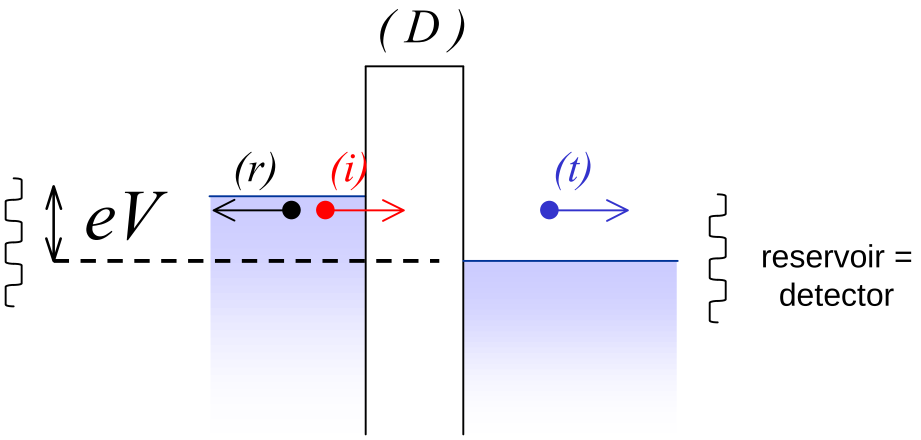
photons:

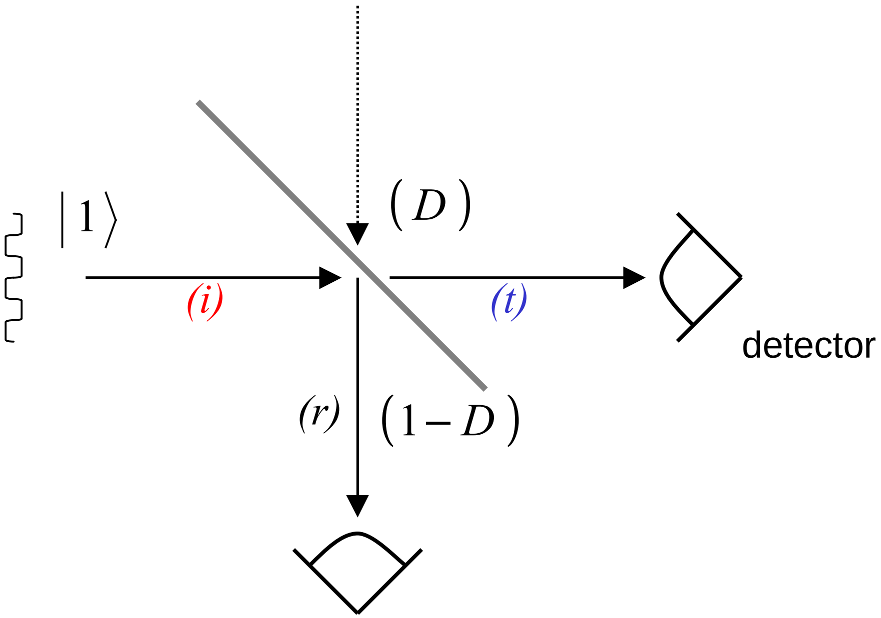
electrons interference

(wave)
scattering description

light interference

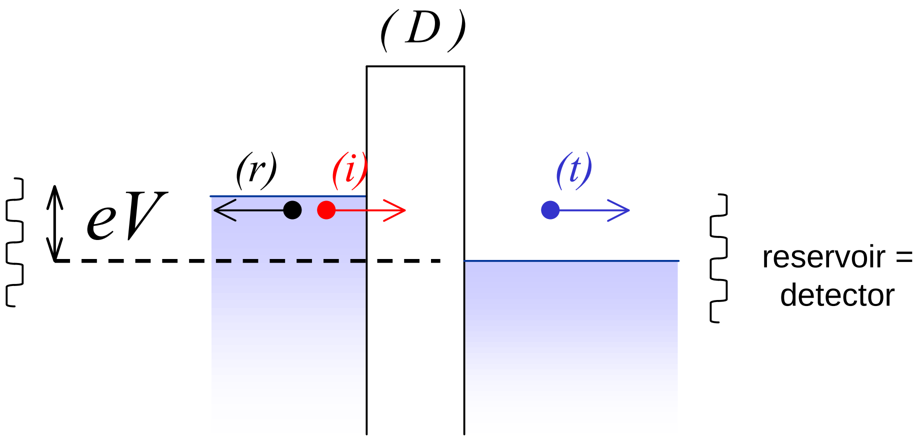
from de 70’s to 90’s (and beyond) mesoscopic physics addressed *single particle* coherence properties via *conductance* measurements

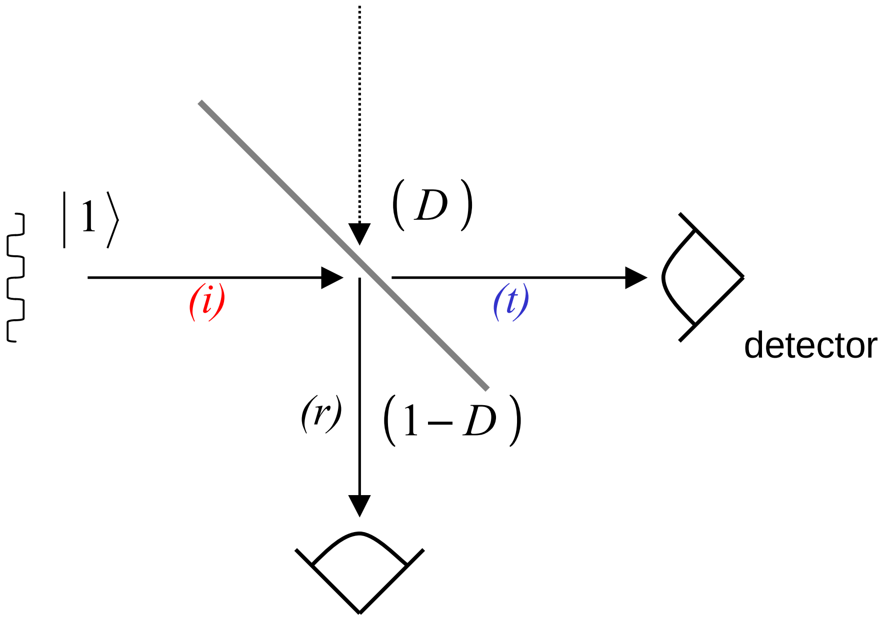




electrons :		*photons:*
electrons interference	(wave)	light interference
shot noise	(particle)	photon noise

while in the 60's, optics addressed *two* photons properties (Hanbury-Brown Twiss correlations)
this is only beginning of the 90's (mid 90's for experiments) that two-electron correlations where considered
via *quantum shot noise*





electrons : *photons:*

electrons interference (wave) light interference

shot noise (particle) photon noise

different quantum noise results for different quantum statistics (Bose versus Fermi)
Fermi sea gives noiseless electron generation while photons are fundamentally noisy

electronic quantum shot noise studies revealed yet unregarded beautiful properties of the Fermi sea

the magic Fermi sea

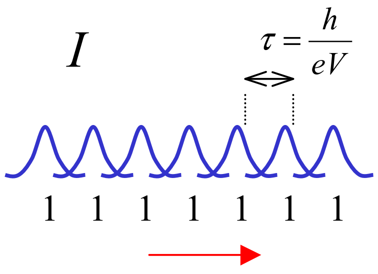
$dI_{el.} = \frac{e}{h} d\varepsilon$ or $d\dot{N}_{el.} = \frac{1}{h} d\varepsilon$

$\Rightarrow$ quantum of conductance : $\frac{e^2}{h}$

$d\dot{N}_{ph.} = \frac{N_{occ.}}{h} d\varepsilon$ or $dI_{ph.} = (h\nu) N_{occ.} d\nu$

(no equivalent, no known continuous generation of photon number states)

$$I = e.\frac{eV}{h}$$



noiseless injection of electrons

noiseless electrons

electron entanglement
using linear ‘ electron optics ’
and equilibrium thermodynamic sources

noiseless electrons may
generate low noise photons

more with shot noise :

current spectral density :

$$S_I(f) = \left\langle \Delta I^2 \right\rangle / \Delta f$$

proportional to the *charge* of the quasi-particle carrying current (...but only in the Poissonian regime)

$$\boxed{S_I = 2\,q\,I}$$

$q = e$ already in the 20's attempt to determine the electron charge in vacuum diodes (but less accurate that Millikan's experiments, due to space charge effect)

(repulsive interactions reduce shot noise)

$q = e/3$ in 1997, the Laughlin fractional charge of the Fractional Quantum Hall Effect was unambiguously established via shot noise. The last (but not least) proof definitely establishing the FQH effect.

later :

$q = 2e$ the Cooper pair charge observed at mesoscopic superconducting-normal interfaces.

Future :

$q = g\,e$ in Luttinger liquids, such as long single wall carbon nanotubes (requires f > THz)

I. Electronic scattering (a brief introduction)

II. Quantum Shot noise

- 1 - Quantum partition noise
  - - one and two particle partitioning :electrons/ photons
  - - electronic shot noise
- 2- scattering derivation of quantum shot noise
  - a- $S(\omega)$ for an ideal one mode conductor
  - b- quantum shot noise for a single mode
  - c-zero frequency shot noise and multimode case
- 3- experimental examples
- 4- current noise cross-correlations

III Shot Noise and Interactions:

1. Fractional Quantum Hall effect

2.. Superconducting-Normal mesoscopic interfaces

3. Interactions in a QPC : 0.7 structure

IV. Shot noise: a tool to detect entanglement

1. Entanglement with the Fermi statistics
2. Coincidence measurements using shot noise correlations

V. Shot noise and high frequencies

1. Photo-assisted Shot Noise
2. High frequency Shot Noise
3. Photon Noise emitted by a Conductor

Introduction

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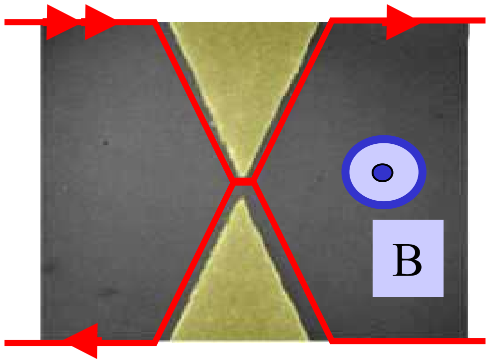
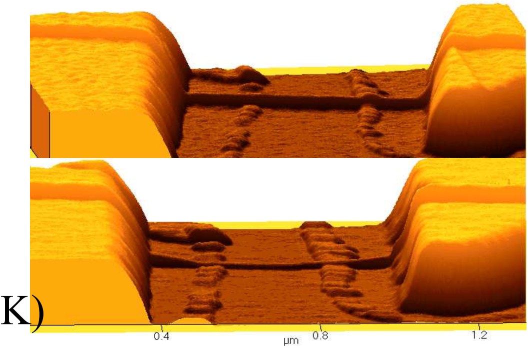
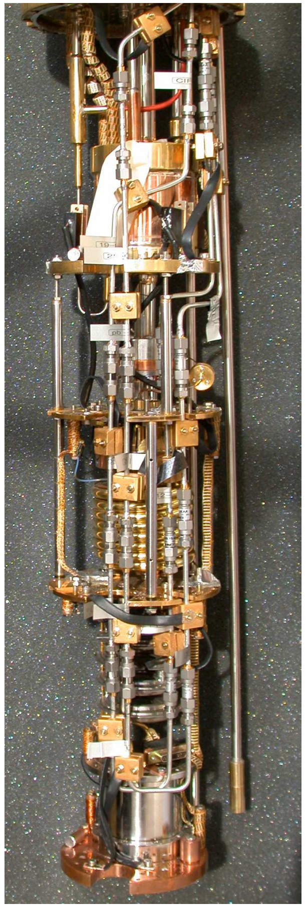
Conclusion



- quantum point contact : shot noise, edge states, co-tunneling of Q.-Dots
- ballistic qubits
- mesoscopic capacitor
- carbone nanotube
- Fractional Quantum Hall effect

- high frequency (40 GHz)
- ultra low noise measurements
- high magnetic field (18T) and low T (20mK)
- lithography
- cryo-electronics





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II. 1 Quantum Partition Noise

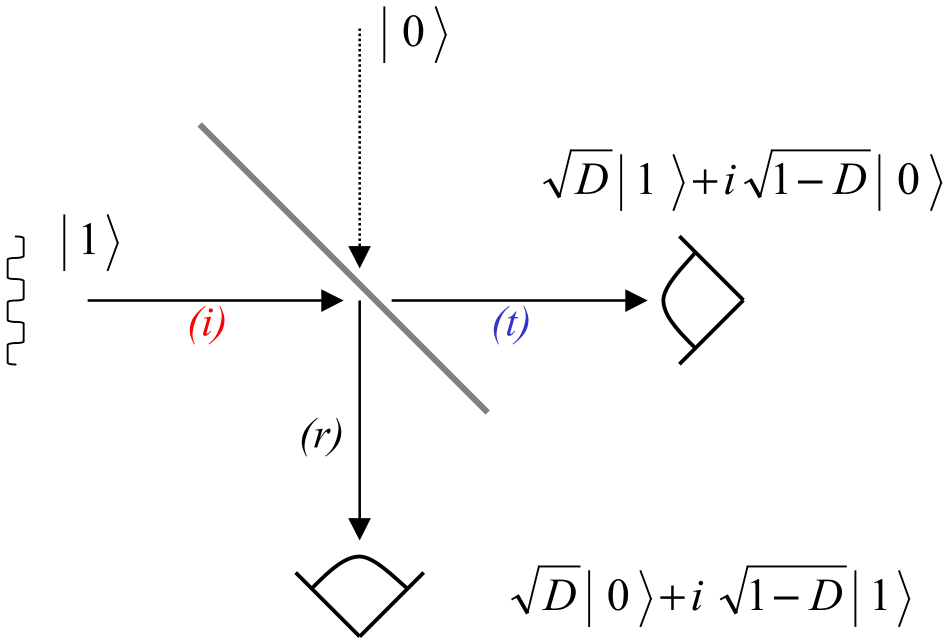
single -particle partitioning :

assume only one particle in a wave incident on a scatter

- scattering ⇒ the wave is diffracted
- diffraction+reservoirs ⇒ the particle is randomly partitioned

⇒ quantum partition noise is *diffraction* (there is no classical analog) + particle-wave duality

(quantum noise exemplifies particle-wave duality)



$$n_i = 1,1,1,1,1,1,1,1,\dots$$

$$n_t = 1,1,0,1,0,1,0,0,\dots$$

$$n_r = 0,0,1,0,1,0,1,1,\dots$$

$$\overline{n_t} = D \qquad \overline{n_r} = 1-D$$

$$\overline{(\Delta n_t)^2} = \overline{n_t}(1-\overline{n_t}) = D(1-D) = \overline{(\Delta n_r)^2}$$

binomial = Poisson X reduction factor (1 - *D*)

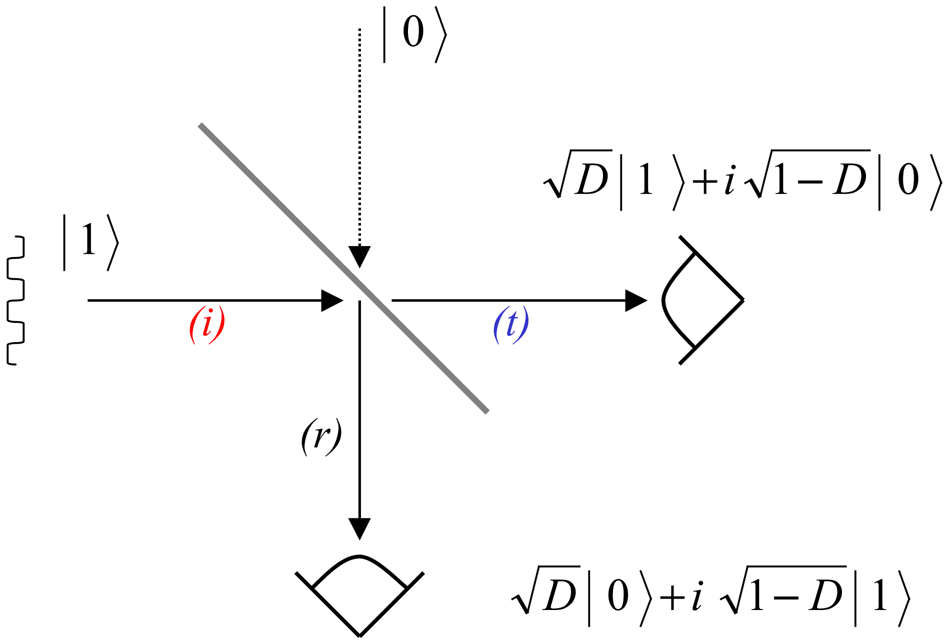
$$\overline{\Delta n_t . \Delta n_r} = -D(1-D)$$

assume only one particle in a wave incident on a scatter

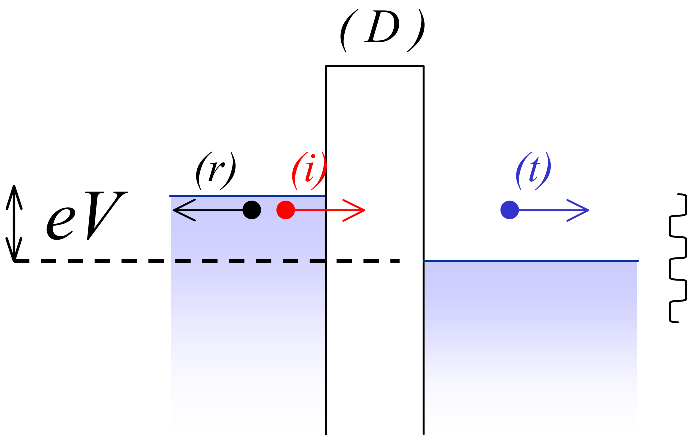
- scattering ⇒ the wave is diffracted
- diffraction+reservoirs ⇒ the particle is randomly partitioned

⇒ quantum partition noise is *diffraction* (there is no classical analog)

(quantum noise exemplifies particle-wave duality)

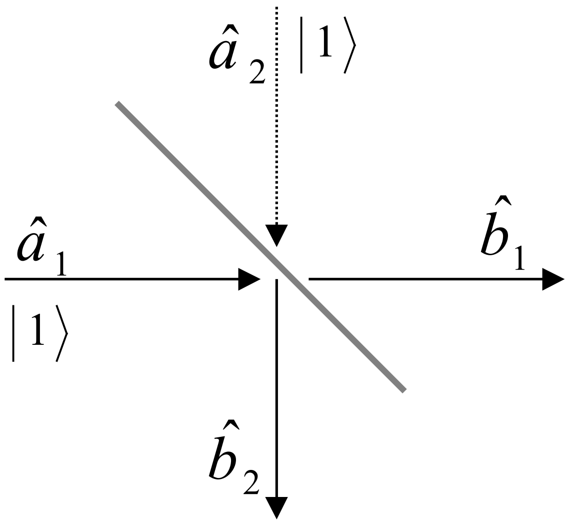


this applies also to electronic waves:



⇒ responsible for current fluctuations
or quantum shot noise

two-particle partitioning : difference between Bosons and Fermions



$$\begin{pmatrix}\hat b_1\\ \hat b_2\end{pmatrix} = S\begin{pmatrix}\hat a_1\\ \hat a_2\end{pmatrix} \qquad S=\begin{pmatrix}\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\end{pmatrix}$$

$$\Rightarrow \qquad \begin{pmatrix}\hat a_1^{+}\\ \hat a_2^{+}\end{pmatrix} = S\begin{pmatrix}\hat b_1^{+}\\ \hat b_2^{+}\end{pmatrix}, \quad \text{as } S^{+}S=1$$

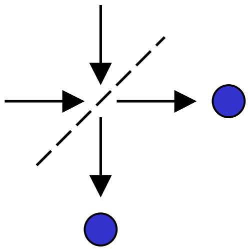
initial state : $|i\rangle = \hat a_1^{+}\hat a_2^{+}|0\rangle_{in}$

final state: $|f\rangle = \frac{1}{2}\left(\hat b_1^{+}\hat b_1^{+} - \hat b_2^{+}\hat b_2^{+}\right) + \frac{1}{2}\left(\hat b_1^{+}\hat b_2^{+} - \hat b_2^{+}\hat b_1^{+}\right)|0\rangle_{out}$

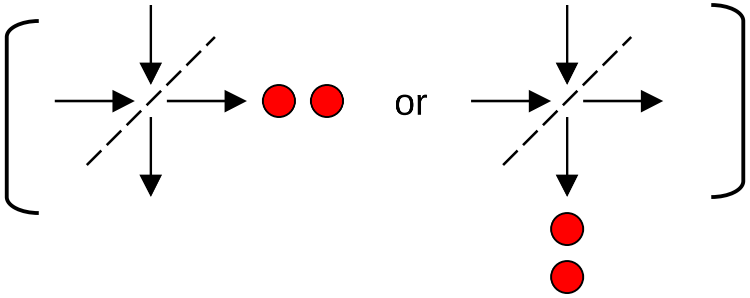
= 0 (Fermion)

= 0 (Boson)

Fermions:



Bosons:



no bunching, no noise

bunching, binomial two-particle partition noise

electron sources versus photon source : reservoir

electrons

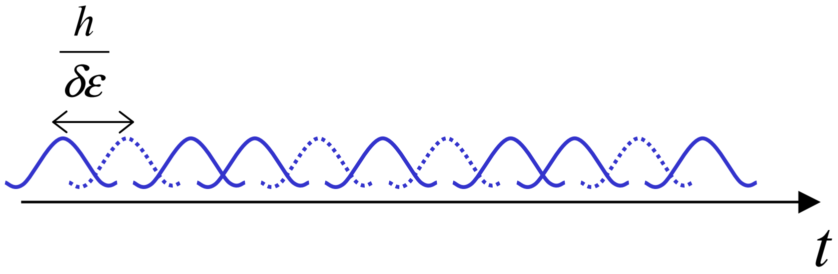
$$dI_{el.} = \frac{e}{h} f_{F.D.}(\varepsilon)\, d\varepsilon$$

$$\overline{N}_{\tau} = f_{F.D.}(\varepsilon)\frac{\delta\varepsilon}{h}\tau$$

$$\overline{(\Delta N)^2}_{\tau} = \overline{\left(N - \overline{N}_{\tau}\right)^2}_{\tau}$$

$$\overline{(\Delta N)^2}_{\tau} = f_{F.D.}(1 - f_{F.D.})\frac{\delta\varepsilon}{h}\tau = \overline{N}_{\tau}(1 - f_{F.D.})$$

sub - poissonian
(anti-bunching)



in particular : noiseless Fermi sea at T=0

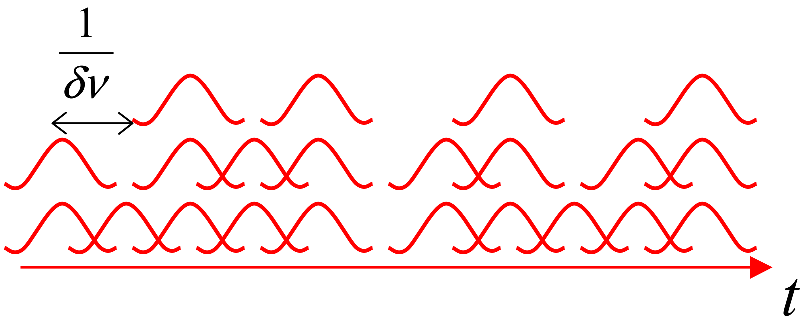
photons

$$dI_{ph.} = h\nu\, f_{B.E.}(h\nu)\, d\nu$$

$$\overline{N}_{\tau} = f_{B.E.}(\varepsilon)\,\delta\nu\,\tau$$

$$\overline{(\Delta N)^2}_{\tau} = f_{B.E.}(1 + f_{B.E.})\,\delta\nu\,\tau = \overline{N}_{\tau}\left(1 + \frac{\overline{N}_{\tau}}{\delta\nu\,\tau}\right)$$

super - poissonian
(thermal photon bunching)



simple derivation of the electronic quantum shot noise for a single mode conductor

incoming current :

$$I_0 = e\left(eV/h\right)$$

(noiseless thanks
to Fermi statistics)

transmitted current :

$$I = D\ I_0 = D\frac{e^2}{h}V$$

current noise in B.W. Δf :

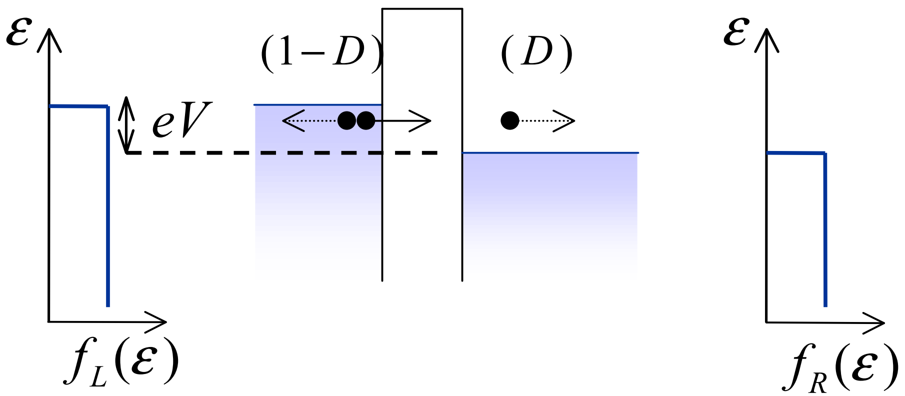
$$\left\langle (\Delta I)^2 \right\rangle = 2eI_0\ \Delta f \cdot D(1-D)$$

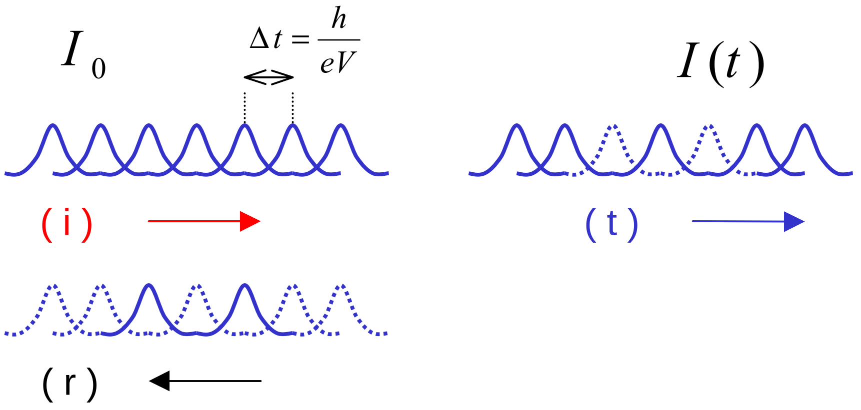
Variance of partioning
binomial statistics

$$S_I = 2eI\ (1-D) = 2eI\,.F$$

Poisson (Schottky)

reduction factor
(Fano)

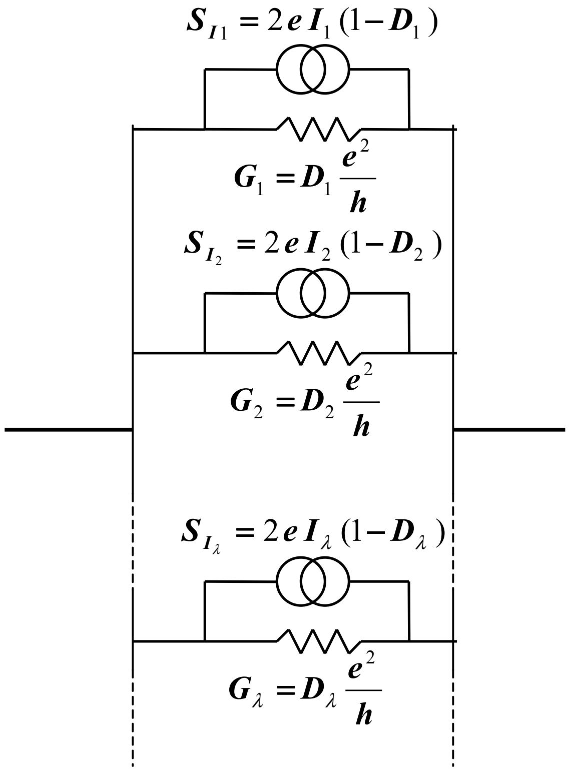




anti-correlation of transmitted
and reflected current fluctuations

V. A. Khlus, Zh. Eksp. Teor. Fiz. 93 (1987) 2179 [Sov. Phys. JETP 66 (1987) 1243].
G. B. Lesovik, Pis'ma Zh. Eksp. Teor. Fiz. 49 (1989) 513 [JETP Lett. 49 (1989) 592].

electronic shot noise for a multi-mode conductor



$S_{I1} = 2eI_1(1-D_1)$

$G_1 = D_1 \frac{e^2}{h}$

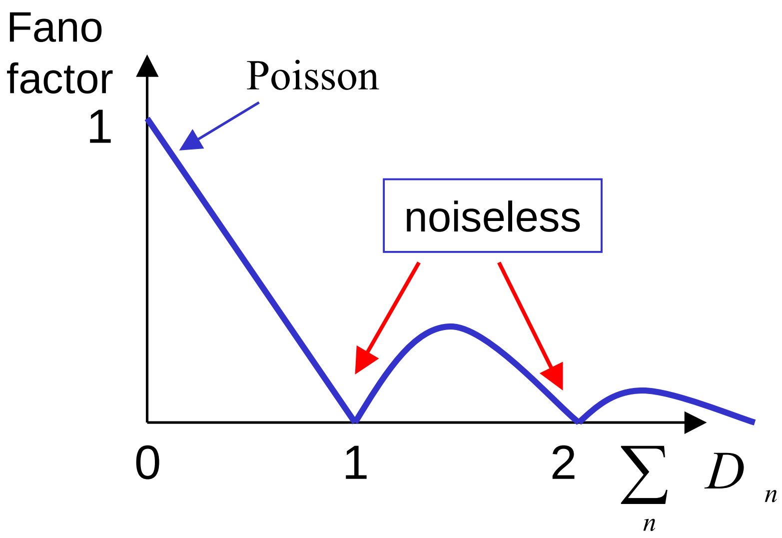
$S_{I_2} = 2eI_2(1-D_2)$

$G_2 = D_2 \frac{e^2}{h}$

$S_{I_\lambda} = 2eI_\lambda(1-D_\lambda)$

$G_\lambda = D_\lambda \frac{e^2}{h}$

$$\boxed{S_I = 2eI \frac{\sum_n D_n(1-D_n)}{\sum_n D_n} = 2eI.F}$$



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3- experimental examples

4- current noise cross-correlations
  - scattering derivations
  - electronic analog of the optical Hanbury-Brown Twiss experiment
  - electronic quantum exchange

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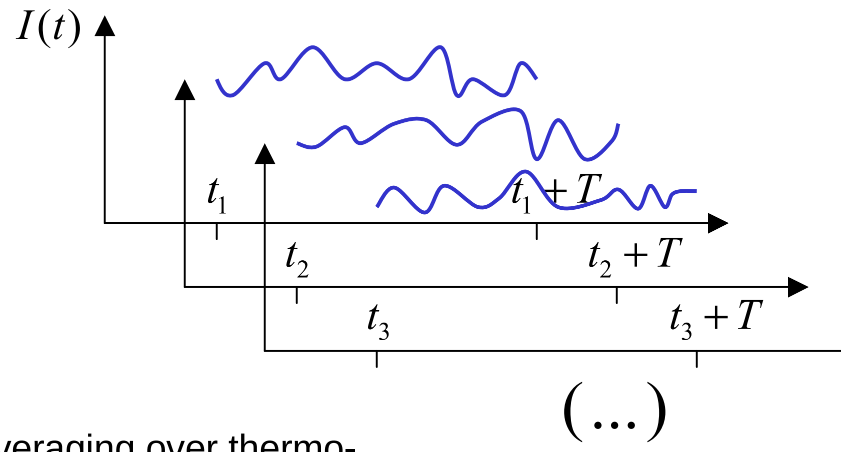
IV. Shot noise: ***the*** tool to detect entanglement

V. Shot noise and high frequencies

II. 2 Scattering derivation of quantum shot noise



some classical definitions to start:

$$I(t)=\sum_{n=-\infty}^{+\infty} x_n e^{-i2\pi n\frac{t}{T}} \quad ; \quad T\to\infty \quad ; \quad x_n{}^* = x_{-n}$$

$$\overline{I(t)}=\sum_{n=-\infty}^{+\infty} \overline{x_n} e^{-i2\pi n\frac{t}{T}} \quad \text{and} \quad \overline{I}=\overline{x_0}$$

(ensemble averaging over thermo-dynamically equivalent realizations)

$$\overline{I^2(t)}=\sum_n \sum_m \overline{x_n x_m}\, e^{-i2\pi\frac{(n+m)t}{T}} \qquad \overline{x_n x_m}=0 \text{ if } m\neq -n$$

(stationary condition)

$$=\overline{x_0{}^2}+2\sum_{n=1}^{\infty}\overline{x_n x_n{}^*}$$

$$\overline{\Delta I^2(t)}=\overline{(I-\bar{I})^2}=2\sum_{n=1}^{\infty}\overline{x_n x_n{}^*} \qquad \overline{\Delta I^2}=\int_0^{\infty} S_I(\nu)\,d\nu$$

with $\nu\equiv\frac{n}{T}$ and $d\nu\equiv\frac{1}{T}$

$$\boxed{S_I(\nu)=\lim_{T\to\infty} 2.T.\overline{x_n x_n{}^{2}}}$$

spectral density of the current

$$\overline{I(t)I(t+\tau)} = \sum_{n} \sum_{m} \overline{x_n x_m}\, e^{-i2\pi\frac{(n+m)t}{T}}\, e^{-i2\pi\frac{n\tau}{T}} \qquad \overline{x_n x_m} = 0 \text{ if } m \neq -n$$

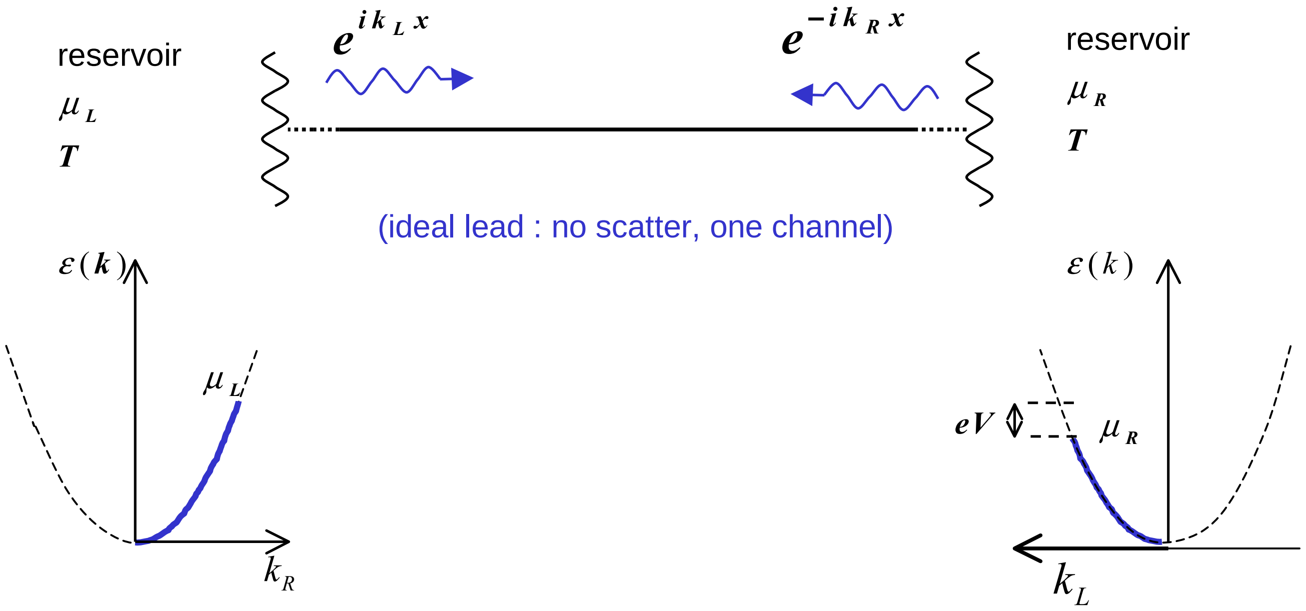
$$= 2\sum_{n=1}^{\infty} \overline{x_n x_n^{*}} \cos 2\pi \frac{n\tau}{T} = \int_0^{\infty} d\nu\, S_I(\nu) \cos 2\pi\nu\tau$$

$$\boxed{S_I(\nu) = 2\int_{-\infty}^{\infty} d\tau\, \overline{I(t)I(t+\tau)}\, e^{i2\pi\nu\tau}}$$

this classical expression will be used to define the quantum noise spectral density

second quantification representation

(to be ready to go further than simple scattering: … shot noise, ac transport, entanglement …)



(ideal lead : no scatter, one channel)

$$\hat{\psi}_L(x,t) = \frac{e}{h}\int d\varepsilon \frac{1}{\sqrt{2\pi\hbar v_L(\varepsilon)}}\,\hat{a}_L(\varepsilon)\,e^{i(k_L x-\varepsilon t)}$$

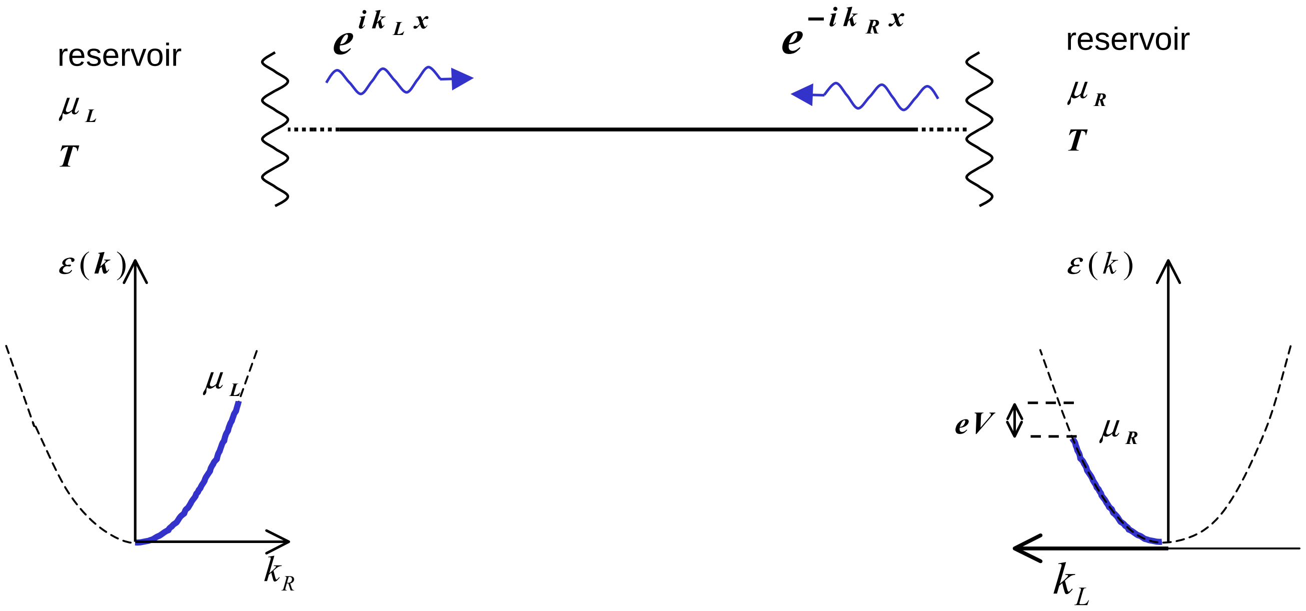
$$\hat{\psi}_R(x,t) = \frac{e}{h}\int d\varepsilon \frac{1}{\sqrt{2\pi\hbar v_R(\varepsilon)}}\,\hat{a}_R(\varepsilon)\,e^{i(-k_R x-\varepsilon t)}$$

$$e\frac{\partial\left(\hat{\psi}^+\hat{\psi}\right)}{\partial t} + \frac{\partial}{\partial x}\hat{I} = 0$$

$$\boxed{\begin{aligned}&\left\{\hat{a}_\beta(\varepsilon'),\hat{a}_\alpha{}^+(\varepsilon)\right\} = \delta_{\alpha,\beta}\,\delta(\varepsilon'-\varepsilon)\\ &\left\{\hat{a}_\beta(\varepsilon'),\hat{a}_\alpha(\varepsilon)\right\} = 0\\ &\left\langle\hat{a}_\alpha{}^+(\varepsilon).\hat{a}_\beta(\varepsilon')\right\rangle = f_\alpha(\varepsilon)\,\delta_{\alpha,\beta}\,\delta(\varepsilon'-\varepsilon)\end{aligned}}$$

$\hat{a}_{\alpha(\beta)}$, act on the Fock space of the reservoirs

$\left|L\right\rangle = \prod_{\varepsilon=0,\mu_L}\hat{a}^+{}_L(\varepsilon)\left|0\right\rangle_L$ and $\left|R\right\rangle = \prod_{\varepsilon=0,\mu_R}\hat{a}^+{}_R(\varepsilon)\left|0\right\rangle_R$

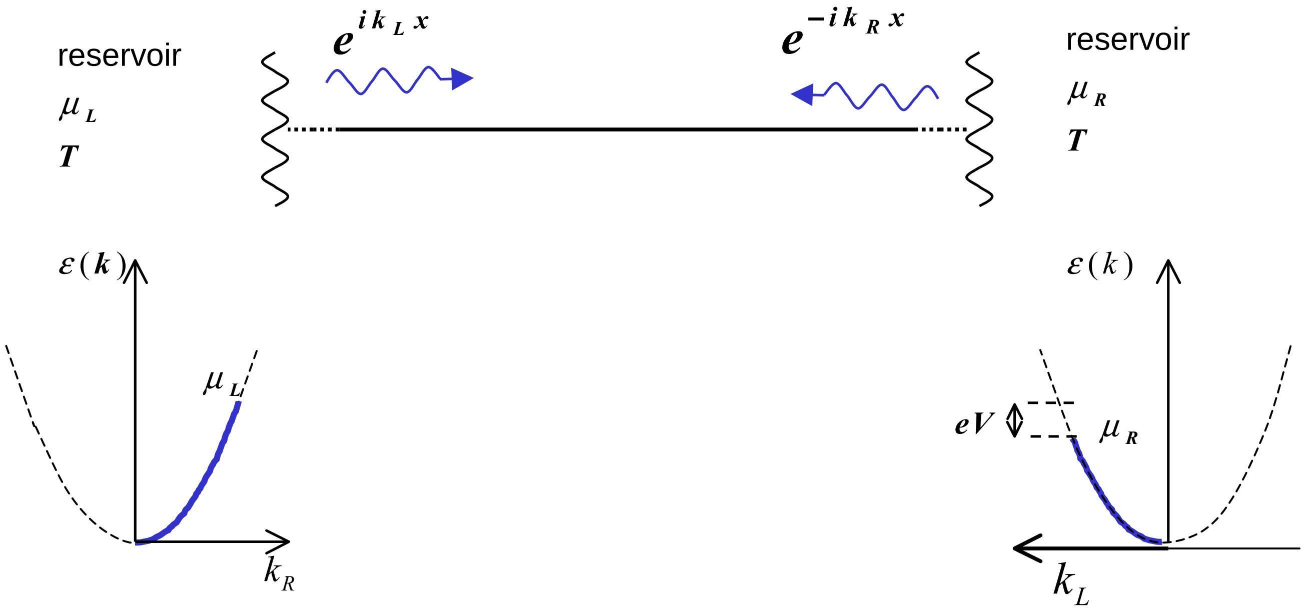


$$\hat{\psi}_L(x,t) = \frac{e}{h}\int d\varepsilon \frac{1}{\sqrt{2\pi\hbar v_L(\varepsilon)}}\ \hat{a}_L(\varepsilon)\, e^{i(k_L x - \varepsilon t)}$$

$$\hat{\psi}_R(x,t) = \frac{e}{h}\int d\varepsilon \frac{1}{\sqrt{2\pi\hbar v_R(\varepsilon)}}\ \hat{a}_R(\varepsilon)\, e^{i(-k_R x - \varepsilon t)}$$

$$\widehat{I}_L(x,t) = e\frac{\hbar}{i2m}\left(\widehat{\Psi}_L^\dagger(x,t)\frac{\partial\widehat{\Psi}_L(x,t)}{\partial x} - \frac{\partial\widehat{\Psi}_L^\dagger(x,t)}{\partial x}\widehat{\Psi}_L(x,t)\right)$$

$$= \frac{e}{h}\int d\varepsilon d\varepsilon' \widehat{a}_L^\dagger(\varepsilon')\widehat{a}_L(\varepsilon)\frac{v(\varepsilon)+v(\varepsilon')}{2\sqrt{v(\varepsilon)}\sqrt{v(\varepsilon')}}e^{i(k(\varepsilon)-k(\varepsilon'))x}e^{i(\varepsilon'-\varepsilon)t} \longrightarrow \left\langle \hat{I}_L(x,t)\right\rangle = \boxed{I_L = \frac{e}{h}\int d\varepsilon\, f_L(\varepsilon)}$$



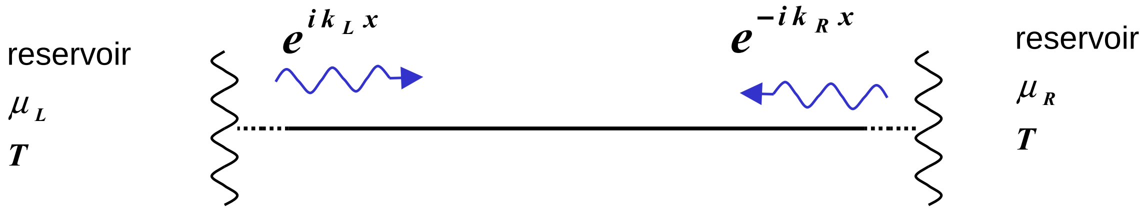
$$\boxed{\begin{aligned} I &= \int_0^\infty \left(f_L(\varepsilon) - f_R(\varepsilon)\right)\frac{e}{h}\,d\varepsilon \\ I &= \frac{e^2}{h}V \qquad \mu_L = \mu_R + eV \qquad \forall\ V,\ T \end{aligned}}$$

the conductance : $\dfrac{e^2}{h}$

does not depend on temperature and voltage

$$I = e\frac{eV}{h} \quad \text{Pauli } (1 \times e) \ + \ \text{Heisenberg}\,(eV/h)$$

II. 2.a. quantum noise of an ideal one mode wire



- *no scattering* (no shot noise, here *only reservoir noise* is considered)
- *useful* to point out some specific and general properties of quantum noise

- first consider the noise contribution coming from the left reservoir : $\hat{\psi}_L(x,t) = \frac{e}{h}\int d\varepsilon \frac{1}{\sqrt{2\pi\hbar v_L(\varepsilon)}}\,\hat{a}_L(\varepsilon)\,e^{i(k_L x-\varepsilon t)}$

$$\hat{I}_L(x,t)=\frac{e}{h}\int d\varepsilon d\varepsilon' \frac{v_L(\varepsilon')+v_L(\varepsilon)}{2\sqrt{v_L(\varepsilon')v_L(\varepsilon)}}\,\hat{a}_L^+(\varepsilon')\hat{a}_L(\varepsilon)\,e^{i(k_L(\varepsilon)-k_L(\varepsilon'))x}e^{-i(\varepsilon-\varepsilon')t}$$

$$\langle I_L\rangle=\frac{e}{h}\int d\varepsilon\, f_L(\varepsilon) \qquad \langle \hat{a}_L{}^+(\varepsilon')\hat{a}_L(\varepsilon)\rangle = f_L(\varepsilon)\,\delta(\varepsilon'-\varepsilon)$$

we want to compute :

$$\boxed{S_I(\nu) = 2\int_{-\infty}^{\infty} d\tau \langle \hat{I}(0)\hat{I}(\tau)\rangle e^{i2\pi\nu\tau}}$$

$$\left\langle \hat{I}(x_L,0)\hat{I}(x_L,\tau)\right\rangle = \left(\frac{e}{h}\right)^2 \int d\varepsilon''' d\varepsilon'' d\varepsilon' d\varepsilon \, \frac{v_L(\varepsilon''')+v_L(\varepsilon'')}{2\sqrt{v_L(\varepsilon''')v_L(\varepsilon'')}} \frac{v_L(\varepsilon')+v_L(\varepsilon)}{2\sqrt{v_L(\varepsilon')v_L(\varepsilon)}} \; e^{i\left(k_L(\varepsilon''')-k_L(\varepsilon'')\right)x} \; e^{i\left(k_L(\varepsilon)-k_L(\varepsilon')\right)x} \ldots$$

$$\ldots \left\langle \hat{a}_L^+(\varepsilon''')\hat{a}_L(\varepsilon'')\hat{a}_L^+(\varepsilon')\hat{a}_L(\varepsilon) \right\rangle e^{-i(\varepsilon-\varepsilon')\tau}$$

normal pairing : $\varepsilon''' = \varepsilon''$ and $\varepsilon' = \varepsilon$

gives the contribution: $\left\langle \hat{I}(x_L,0)\right\rangle \left\langle \hat{I}(x_L,\tau)\right\rangle = \left\langle I_L \right\rangle^2$

→ the fluctuations : $\left\langle \Delta\hat{I}(x_L,0).\Delta\hat{I}(x_L,\tau)\right\rangle = \left\langle \hat{I}(x_L,0)\,\hat{I}(x_L,\tau)\right\rangle - \left\langle \hat{I}(x_L,0)\right\rangle \left\langle \hat{I}(x_L,\tau)\right\rangle$

only come from the

exchange pairing term : $\varepsilon''' = \varepsilon$ and $\varepsilon' = \varepsilon''$

$$\left\langle \hat{a}_L^+(\varepsilon''')\hat{a}_L(\varepsilon'')\hat{a}_L^+(\varepsilon')\hat{a}_L(\varepsilon) \right\rangle_{exchange} \equiv f_L(\varepsilon)\,\delta(\varepsilon'''-\varepsilon)\left(1-f_L(\varepsilon')\right)\delta(\varepsilon''-\varepsilon')$$

$$\left\langle \hat{I}(x_L,0)\hat{I}(x_L,\tau)\right\rangle = \left(\frac{e}{h}\right)^2 \int d\varepsilon' d\varepsilon \,\frac{\left(v_L(\varepsilon')+v_L(\varepsilon)\right)^2}{4v_L(\varepsilon')v_L(\varepsilon)}\, f_L(\varepsilon)\left(1-f_L(\varepsilon')\right)e^{i2\left(k_L(\varepsilon)-k_L(\varepsilon')\right)x}\; e^{-i\left(\varepsilon-\varepsilon'\right)\tau/\hbar}$$

spectral density of the current fluctuations:

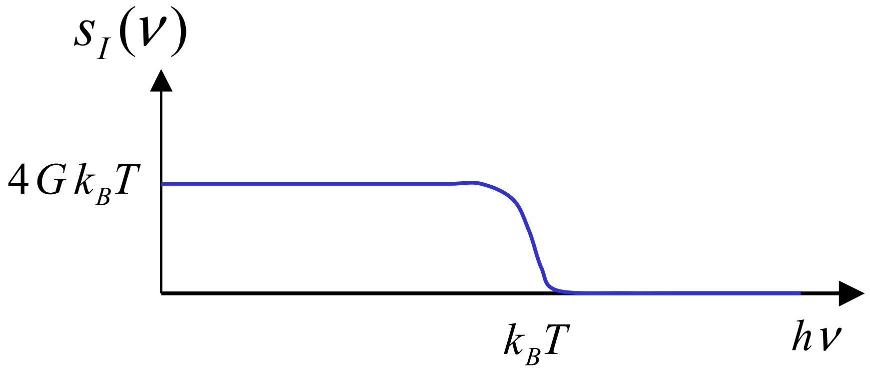
$$S_I(\nu) = 2\int_{-\infty}^{\infty} d\tau \left\langle \hat{I}(0)\hat{I}(\tau)\right\rangle e^{i2\pi\nu\tau}$$

$$S_{I_L}(\nu) \cong 2\frac{e^2}{h}\int d\varepsilon\, f_L(\varepsilon)\left(1-f_L(\varepsilon-\hbar\omega)\right)$$

$(\hbar\omega << E_F$ and $v_F/\omega >> L)$

adding contribution of right the reservoir:

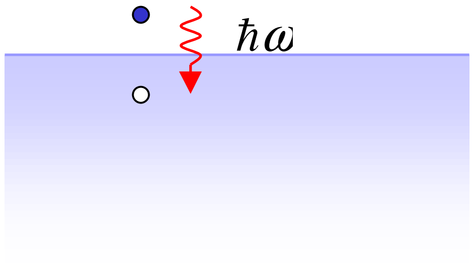
$$\boxed{\begin{aligned} S_I(\nu) &= 4\frac{e^2}{h}\int d\varepsilon\, f_L(\varepsilon)\left(1-f_L(\varepsilon-\hbar\omega)\right) \\ &= 4\frac{e^2}{h}\frac{\hbar\omega}{e^{\frac{\hbar\omega}{k_BT}}-1} \\ &= 4\frac{e^2}{h}\hbar\omega\, N(\omega) \end{aligned}}$$



no (detectable) reservoir noise at zero temperature

$$\begin{aligned} S_I(\nu) &= 4\frac{e^2}{h}\int d\varepsilon\, f_L(\varepsilon)\big(1-f_L(\varepsilon-\hbar\omega)\big) \\ &= 4\frac{e^2}{h}\frac{\hbar\omega}{e^{\frac{\hbar\omega}{k_BT}}-1} \\ &= 4\frac{e^2}{h}h\nu\, N(\nu) \end{aligned}$$

('spontaneous' fluctuations)



only fluctuations corresponding to electronic transitions *down in energy* are considered in this definition of S$_I$

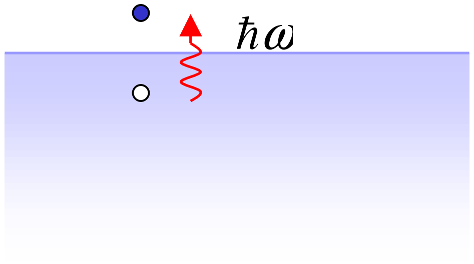
$$S_I(\nu) = 2\int_{-\infty}^{\infty} d\tau \left\langle \hat{I}(0)\hat{I}(\tau)\right\rangle e^{i2\pi\nu\tau}$$

does not commute

$$S_I^{'}(\nu) = S_I(-\nu) = 2\int_{-\infty}^{\infty} d\tau \left\langle \hat{I}(0)\hat{I}(\tau)\right\rangle e^{-i2\pi\nu\tau} = 2\int_{-\infty}^{\infty} d\tau \left\langle \hat{I}(\tau)\hat{I}(0)\right\rangle e^{+i2\pi\nu\tau}$$

$$\begin{aligned} S'_I(\nu) &= 4\frac{e^2}{h}\int d\varepsilon\, f_L(\varepsilon)\big(1-f_L(\varepsilon+\hbar\omega)\big) \\ &= 4\frac{e^2}{h}h\nu\big(N(\nu)+1\big) \end{aligned}$$

('stimulated' fluctuations)



transitions up in energy: these fluctuations are revealed when connecting to an 'active' detector able to excite the Fermi sea

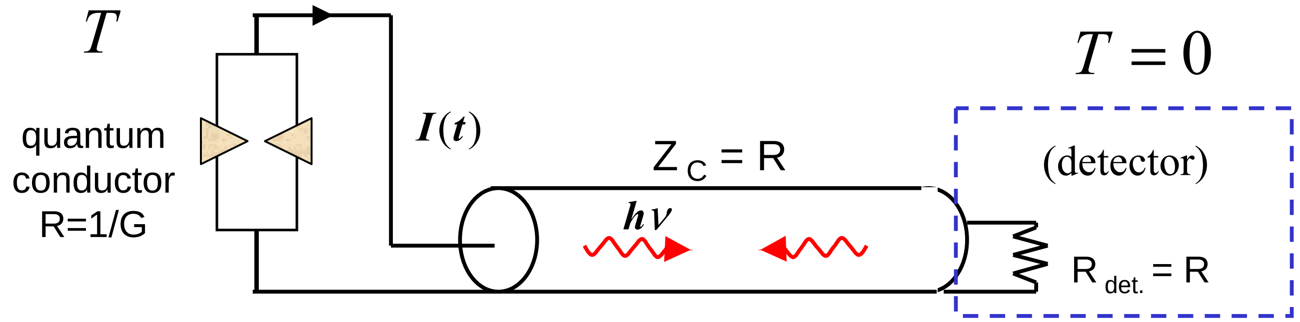
-fluctuation -dissipation (here calculated in the frame of the scattering approach)

$$S_I(-\nu)-S_I(\nu)=2Gh\nu$$

(Kubo)

(see poster : Pierre Billangeon
also : Deblock/Kouwenhoven : noise of a supercond. charge qubit using SIS junctions)

- meaning of $S_I(\nu)$:



$$P=RS_I(\nu)d\nu$$

(noise detectable with detector in ground state) (also Nyquist 1928)

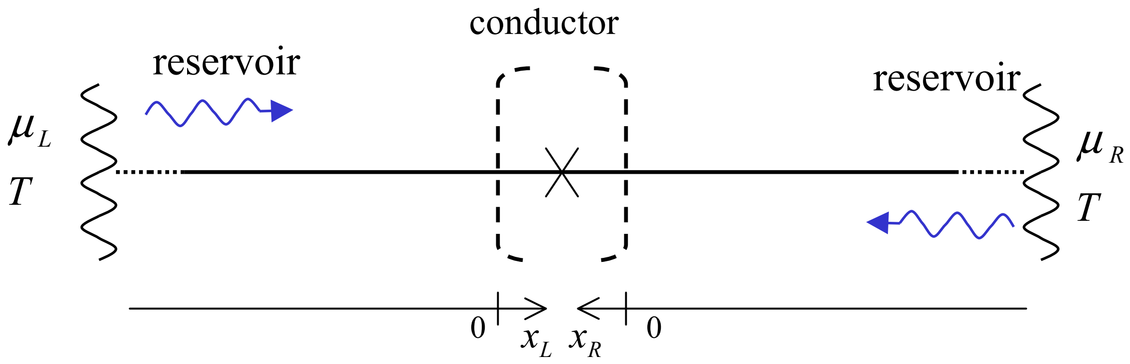
(see more in last part of the talk, if time permits)

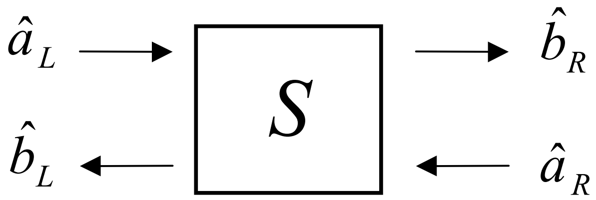
For a detector at finite temperature

$$P\propto S_I(\nu).(N_{Det.}+1)-S_I(-\nu).N_{Det.}$$

Lesovik and Loosen, JETP. Lett. 65, 295 (1997)
Y. Gavish, Y. Imry and Y. Levinson, Phys.Rev.B. 62, 10637 (2000)
see poster Marjorie CREUX

II . 2.b. quantum shot noise for a single mode





$$S = \begin{pmatrix} s_{LL} & s_{LR} \\ s_{RL} & s_{RR} \end{pmatrix} \equiv \begin{pmatrix} r & it \\ it & r \end{pmatrix}$$

$$S\,S^{+} = S^{+}S = 1$$

scattering states in the reservoirs:

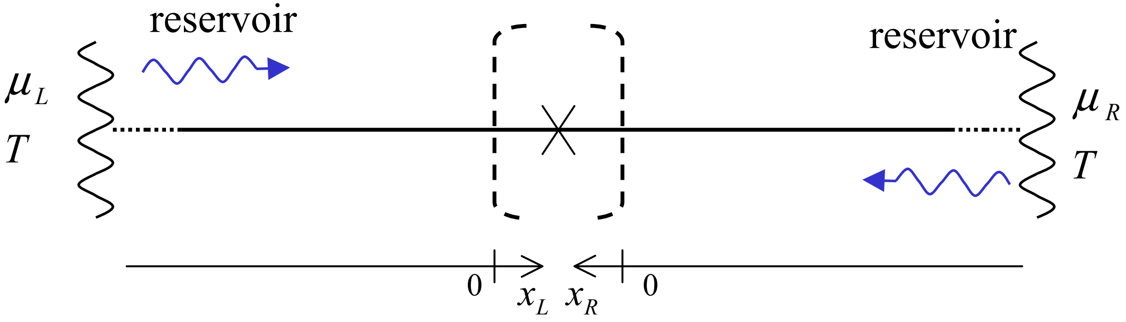
$$\hat{\psi}(x_L) = \frac{e}{h}\int d\varepsilon \frac{1}{\sqrt{2\pi\hbar v_L(\varepsilon)}}\left(\hat{a}_L(\varepsilon)e^{ik_L x_L} + \hat{b}_L(\varepsilon)e^{-ik_L x_L}\right)$$

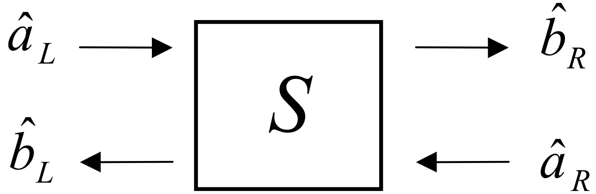
$$\hat{\psi}(x_R) = \frac{e}{h}\int d\varepsilon \frac{1}{\sqrt{2\pi\hbar v_R(\varepsilon)}}\left(\hat{a}_R(\varepsilon)e^{ik_R x_R} + \hat{b}_R(\varepsilon)e^{-ik_R x_R}\right)$$

$$t^2 = D = 1 - R$$

$$r^2 = R$$

to summarize:





$$\boxed{\begin{gathered} S = \begin{pmatrix} s_{LL} & s_{LR} \\ s_{RL} & s_{RR} \end{pmatrix} \\ S\,S^{+} = S^{+}S = 1 \end{gathered}}$$

$$I = \frac{e}{h}\int d\varepsilon \left\{ f_L(\varepsilon) - \left(|s_{LL}|^2 f_L(\varepsilon) + |s_{LR}|^2 f_R(\varepsilon) \right) \right\}$$

Landauer formula

$$\boxed{I = \frac{e}{h}\int d\varepsilon |s_{LR}|^2 \left(f_L(\varepsilon) - f_R(\varepsilon) \right)}$$

$$G = \frac{e^2}{h}\int d\varepsilon \left(-\frac{\partial f}{\partial \varepsilon} \right) D(\varepsilon)$$

$$D = |S_{LR}|^2 = |S_{RL}|^2 = 1 - |S_{LL}|^2 = 1 - |S_{RR}|^2 = 1 - R$$

$$G = \frac{e^2}{h}\; D(\varepsilon_F) \quad @\,\mathrm{T} = 0$$

Current fluctuations will be calculated in the left reservoir lead

$$\hat{\psi}(x_L,t)=\frac{e}{h}\int d\varepsilon \frac{1}{\sqrt{2\pi\hbar v_L(\varepsilon)}}\left(\hat{a}_L(\varepsilon)e^{ik_L x_L}+\hat{b}_L(\varepsilon)e^{-ik_L x_L}\right)e^{-i\varepsilon t/\hbar}$$

$$\hat{b}_L=r\hat{a}_L+it\hat{a}_R$$

$$\hat{I}(x_L,t)=\frac{e}{h}\int d\varepsilon d\varepsilon'\left\{\hat{a}_L^+(\varepsilon')\hat{a}_L(\varepsilon)-\hat{b}_L^+(\varepsilon')\hat{b}_L(\varepsilon)\right\}\frac{v_L(\varepsilon')+v_L(\varepsilon)}{2\sqrt{{}_L(\varepsilon')v_L(\varepsilon)}}e^{i(k_L-k_L')x_L}\;e^{i(\varepsilon'-\varepsilon)t}$$

... plus a factor

$$\propto\quad \frac{(v_L(\varepsilon')-v_L(\varepsilon))}{\left(v_L(\varepsilon')v_L(\varepsilon)\right)^{1/2}}\left(a_L{}^+b_L\ldots b_L{}^+a_L\right)$$

also : ~ 1

negligeable if $\varepsilon'-\varepsilon \sim h\nu \ll E_F$

contribute to fluctuations within reservoirs

$$\hat{I}(x_L,t)=\frac{e}{h}\int d\varepsilon d\varepsilon'\left\{\begin{matrix} t^2\left(\hat{a}_L^+(\varepsilon')\hat{a}_L(\varepsilon)-\hat{a}_R^+(\varepsilon')\hat{a}_R(\varepsilon)\right)-\ldots \\ \ldots irt\left(\hat{a}_L^+(\varepsilon')\hat{a}_R(\varepsilon)-\hat{a}_R^+(\varepsilon')\hat{a}_L(\varepsilon)\right)\end{matrix}\right\}\;e^{i(k_L-k_L')x_L}\;e^{i(\varepsilon'-\varepsilon)t}$$

contribute to fluctuations between reservoirs
(partitionning)

$$\hat{I}(0)\hat{I}(\tau)=\left(\frac{e}{h}\right)^2\int d\varepsilon'''d\varepsilon''d\varepsilon'd\varepsilon\ \left[t^2\left(\hat{a}_L^+(\varepsilon''')\hat{a}_L(\varepsilon'')-\hat{a}_R^+(\varepsilon''')\hat{a}_R(\varepsilon'')\right)-irt\left(\hat{a}_L^+(\varepsilon''')\hat{a}_R(\varepsilon'')-\hat{a}_R^+(\varepsilon''')\hat{a}_L(\varepsilon'')\right)\right]....$$

$$...\times\left[t^2\left(\hat{a}_L^+(\varepsilon')\hat{a}_L(\varepsilon)-\hat{a}_R^+(\varepsilon')\hat{a}_R(\varepsilon)\right)-irt\left(\hat{a}_L^+(\varepsilon')\hat{a}_R(\varepsilon)-\hat{a}_R^+(\varepsilon')\hat{a}_L(\varepsilon)\right)\right]e^{-i(\varepsilon-\varepsilon')\tau}$$

$$\left\langle \hat{I}(x_L,0)\,\hat{I}(x_L,\tau)\right\rangle = ??$$

normal pairing : $\varepsilon'''=\varepsilon''$ and $\varepsilon'=\varepsilon$

gives the contribution: $\left\langle\hat{I}(0)\right\rangle\left\langle\hat{I}(\tau)\right\rangle=\left\langle I_L\right\rangle^2$

→ the *fluctuations* : $\left\langle\Delta\hat{I}(x_L,0).\Delta\hat{I}(x_L,\tau)\right\rangle=\left\langle\hat{I}(x_L,0)\,\hat{I}(x_L,\tau)\right\rangle-\left\langle\hat{I}(x_L,0)\right\rangle\left\langle\hat{I}(x_L,\tau)\right\rangle$

again come from the exchange term : $\varepsilon'''=\varepsilon$ and $\varepsilon'=\varepsilon''$

first part:

$$\begin{cases} D^2\left\langle\hat{a}_L^+(\varepsilon''')\hat{a}_L(\varepsilon'')\,\hat{a}_L^+(\varepsilon')\hat{a}_L(\varepsilon)\right\rangle\equiv f_L(\varepsilon)\,\left(1-f_L(\varepsilon')\right)\delta(\varepsilon'''-\varepsilon)\,\delta(\varepsilon''-\varepsilon') \\ D^2\left\langle\hat{a}_R^+(\varepsilon''')\hat{a}_R(\varepsilon'')\,\hat{a}_R^+(\varepsilon')\hat{a}_R(\varepsilon)\right\rangle\equiv f_R(\varepsilon)\,\left(1-f_R(\varepsilon')\right)\delta(\varepsilon'''-\varepsilon)\,\delta(\varepsilon''-\varepsilon') \end{cases}$$

= emission noise of each reservoir. the D^2 term indicates two-particle emitted by a reservoir and transmitted

$$\hat{I}(0)\hat{I}(\tau)=\left(\frac{e}{h}\right)^2\int d\varepsilon'''d\varepsilon''d\varepsilon'd\varepsilon\ \left[t^2\left(\hat{a}_L^+(\varepsilon''')\hat{a}_L(\varepsilon'')-\hat{a}_R^+(\varepsilon''')\hat{a}_R(\varepsilon'')\right)-irt\left(\hat{a}_L^+(\varepsilon''')\hat{a}_R(\varepsilon'')-\hat{a}_R^+(\varepsilon''')\hat{a}_L(\varepsilon'')\right)\right]\ldots$$

$$\ldots\times\left[t^2\left(\hat{a}_L^+(\varepsilon')\hat{a}_L(\varepsilon)-\hat{a}_R^+(\varepsilon')\hat{a}_R(\varepsilon)\right)-irt\left(\hat{a}_L^+(\varepsilon')\hat{a}_R(\varepsilon)-\hat{a}_R^+(\varepsilon')\hat{a}_L(\varepsilon)\right)\right]e^{-i(\varepsilon-\varepsilon')\tau}$$

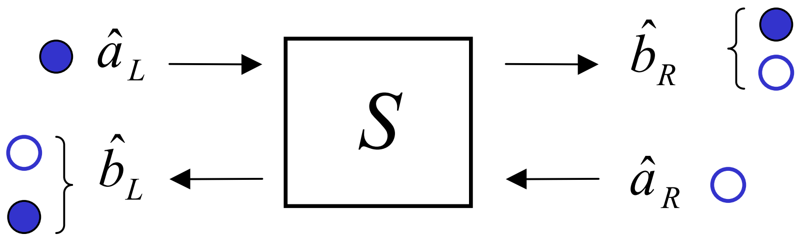
→ the *fluctuations* : $\left\langle\Delta\hat{I}(x_L,0).\Delta\hat{I}(x_L,\tau)\right\rangle=\left\langle\hat{I}(x_L,0)\,\hat{I}(x_L,\tau)\right\rangle-\left\langle\hat{I}(x_L,0)\right\rangle\left\langle\hat{I}(x_L,\tau)\right\rangle$

again come from the exchange term : $\varepsilon'''=\varepsilon$ and $\varepsilon'=\varepsilon''$

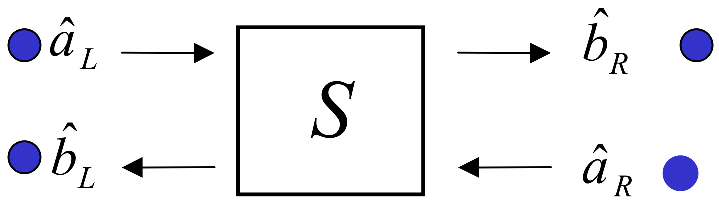
second part:

$$\left\{\begin{array}{ll}(-irt)^2\left\langle\,\hat{a}_L^+(\varepsilon''')\hat{a}_R(\varepsilon'')\left(-\,\hat{a}_R^+(\varepsilon')\hat{a}_L(\varepsilon)\right)+\left(-\,\hat{a}_R^+(\varepsilon''')\hat{a}_L(\varepsilon'')\right)\hat{a}_L^+(\varepsilon')\hat{a}_R(\varepsilon)\,\right\rangle & \\ \Rightarrow\qquad RD\left[f_L(\varepsilon)\,\left(1-f_R(\varepsilon')\right)+f_R(\varepsilon)\,\left(1-f_l(\varepsilon')\right)\right]\delta(\varepsilon'''-\varepsilon)\,\delta(\varepsilon''-\varepsilon') & \end{array}\right.$$

= partition shot noise..



partition noise



'Pauli blocking' of partition noise

$$S_I(\nu) = 2\int_{-\infty}^{\infty} d\tau\, \overline{I(t)I(t+\tau)}\, e^{i2\pi\nu\tau}$$

complete finite frequency, finite temperature and voltage formula:

$$\boxed{S_I(\nu)=2\frac{e^2}{h}\int d\varepsilon\left\{D^2\left[f_L(\varepsilon)\,\left(1-f_L(\varepsilon-h\nu)\right)+f_R(\varepsilon)\,\left(1-f_R(\varepsilon-h\nu)\right)\right]+RD\left[f_L(\varepsilon)\,\left(1-f_R(\varepsilon-h\nu)\right)+f_R(\varepsilon)\,\left(1-f_l(\varepsilon-h\nu)\right)\right]\right\}}$$

reservoir emission noise

shot noise

EQUILIBRIUM : $(f_R = f_L = f)$

$$\boxed{\begin{array}{ll} S_I\,(\nu)=4\,D\dfrac{e^2}{h}\dfrac{\hbar\omega}{e^{\frac{\hbar\omega}{k_BT}}-1} & G = D\dfrac{e^2}{h} \\ S_I\,(\nu)=4\,G\,k_BT & h\nu << k_BT \end{array}}$$

$$D = D^2 + D(1-D)$$

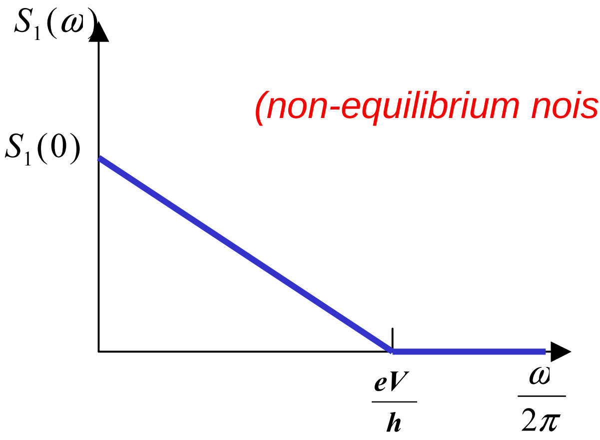
thermal noise
of reservoirs

partition noise of
thermal electrons

equilibrium noise
=
thermal noise
+
partition noise
(equal amount)

NONEQUILIBRIUM, and T=0 : $(\mu_L = \mu_R + eV)$

$$\boxed{\begin{array}{ll} S_I(\nu)=2.D(1-D)\dfrac{e^2}{h}\left(eV-h\nu\right) & eV \geq h\nu \\ \quad =0 & eV < h\nu \end{array}}$$



(non-equilibrium noise)

II. 2. C. zero frequency shot noise and multimode case

shot noise at low frequency and zero temperature

(one mode)

$$S_I = 2eI\,(1-D) = 2eI\,.F \qquad \text{for} \quad \nu \to 0$$

Poisson (Schottky)

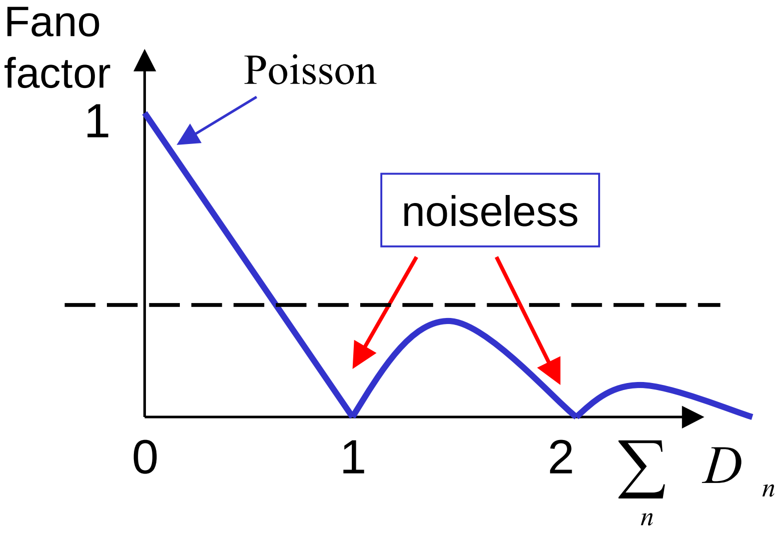
reduction factor (Fano)

generalization to multiple modes:

$$Tr\left[S_{21}S_{21}^{+}(1-S_{21}S_{21}^{+})\right]$$

$$\downarrow$$

$$\boxed{S_I = 2eI\,\frac{\sum_n D_n(1-D_n)}{\sum_n D_n} = 2eI\,.F}$$



G. B. Lesovik, Pis'ma Zh. Eksp. Teor. Fiz. 49 (1989) 513 [JETP Lett. 49 (1989) 592].

Introduction

I. Electronic scattering (a brief introduction)

II. Quantum Shot noise

- 1 - Quantum partition noise
  - - one and two particle partitioning :electrons/ photons
  - - electronic shot noise
- 2- scattering derivation of quantum shot noise
  - a- $S(\omega)$ for an ideal one mode conductor
  - b- quantum shot noise for a single mode
  - c-zero frequency shot noise and multimode case
- → 3- experimental examples
- 4- current noise cross-correlations
  - - scattering derivations
  - - electronic analog of the optical Hanbury-Brown Twiss experiment
  - - electronic quantum exchange

III Shot Noise and Interactions:

IV. Shot noise: ***the*** tool to detect entanglement

V. Shot noise and high frequencies

EXPERIMENTAL EXAMPLES

numbers:

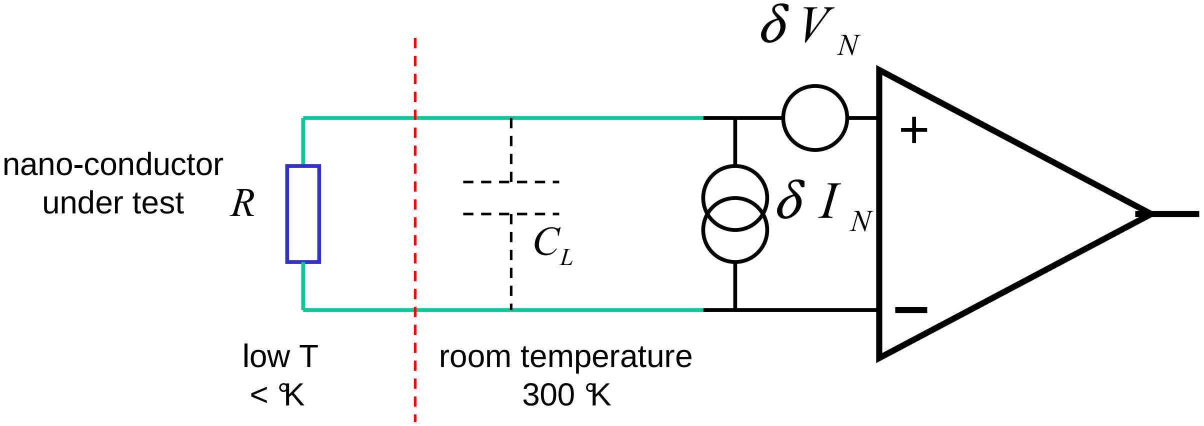
$$100\text{mK} \approx 10\mu\text{V}$$

$$\frac{2e^2}{h}\times 10\mu\text{V} \approx 0.8\,\text{nA}$$

$$S_I = 2eI \approx 2.6\,10^{-38}\,\text{A}^2/\text{Hz} = \left(16\,\text{fA}/\sqrt{\text{Hz}}\right)^2$$

$$S_V = \left(200\text{pV}/\sqrt{\text{Hz}}\right)$$

detection noise:



noise power added by
the amplifier reffered
to resistor R:

$$k_B T_N \Delta f = \frac{1}{4R}\left(\delta V_N\right)^2 + \frac{R}{4}\left(\delta I_N\right)^2$$

optimal resistor:

$$R_{opt.} = \frac{\sqrt{\left(\delta V_N\right)^2}}{\sqrt{\left(\delta I_N\right)^2}}$$

lowest T_N:

$$k_B T_N \Delta f = \frac{1}{2}\sqrt{\left(\delta V_N\right)^2}\sqrt{\left(\delta I_N\right)^2}$$

excellent room temperature commercial LNA (100kHz range) :

$\sqrt{(\delta V_N)^2} \approx 1.3\,\mathrm{nV}/\sqrt{\mathrm{Hz}}$

(LI75A from NF)

$\sqrt{(\delta I_N)^2} \approx 13\,\mathrm{fA}/\sqrt{\mathrm{Hz}}$

$R_{opt.} \approx 100\mathrm{kOhms}$

$\mathrm{T_N}^{opt} \approx 700\mathrm{mK}$

well adapted to quantum point contacts,quantum dots, STM,etc,... provided microphonic noise sources in the audio range are carefully eliminated

(one meter coax limits to *f* < 16 kHz for 100kOhm sample)

good room temperature commercial 80MHz range LNA:

$\sqrt{(\delta V_N)^2} \approx 0.45\,\mathrm{nV}/\sqrt{\mathrm{Hz}}$

(220 FS from NF)

$\sqrt{(\delta I_N)^2} \approx 130\,\mathrm{fA}/\sqrt{\mathrm{Hz}}$

$R_{opt.} \approx \text{few kOhms}$

$\mathrm{T_N}^{opt} \approx 2.5\mathrm{K}$

well adapted to diffusive wire in semi-conductors, superconducting/2DEG hybride junctions, ...
Higher frequency (up to few MHz without appreciable capacitive shunting)

microwave LNAs (GHz range):

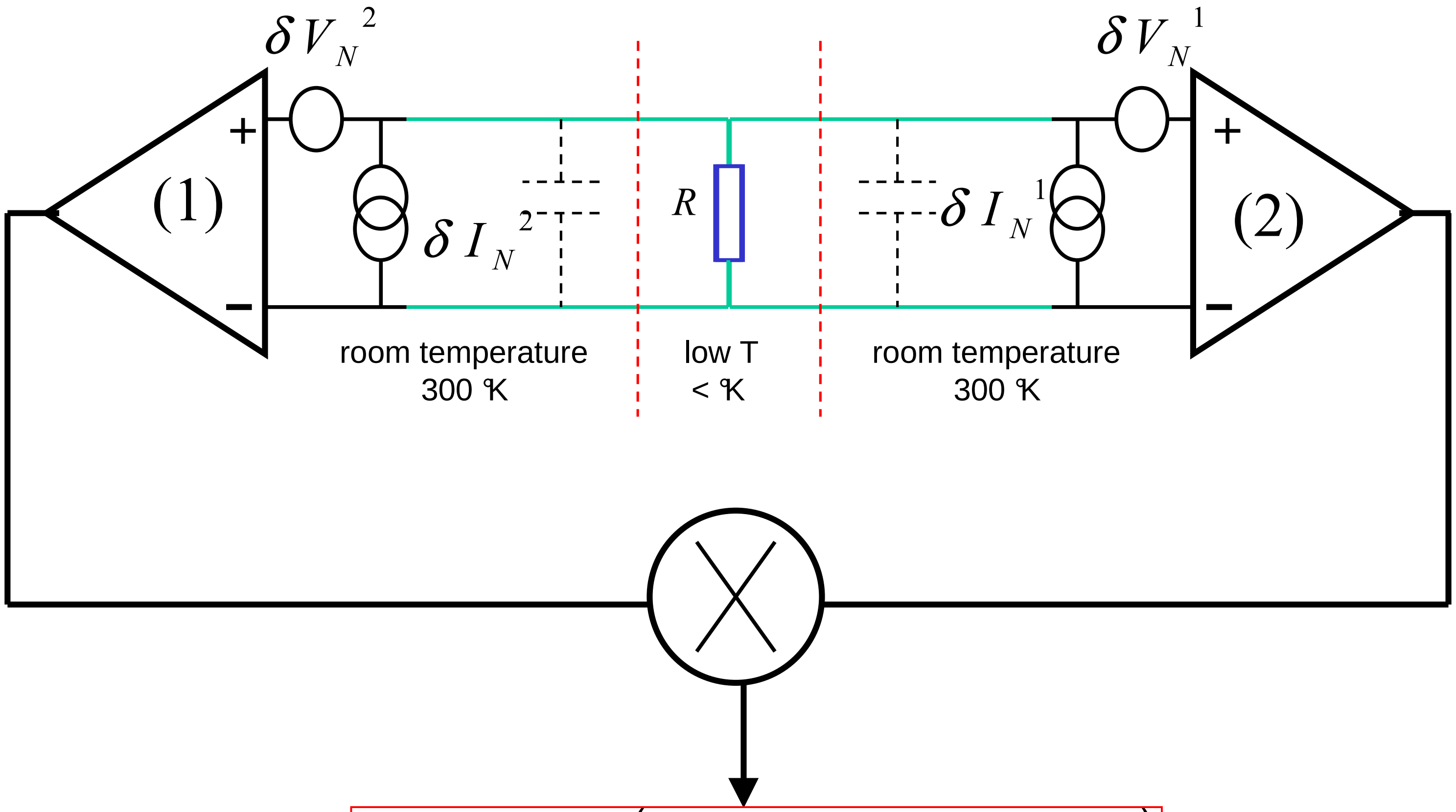
bad coupling : 50 Ohms versus 13kOhms
but fast:

$\sqrt{(\delta V_N)^2} \approx 0.30\,\mathrm{nV}/\sqrt{\mathrm{Hz}}$ (room temperature) $T_N = 30°K$ on $50\,\Omega$

$\sqrt{(\delta V_N)^2} \approx 100-80\,\mathrm{pV}/\sqrt{\mathrm{Hz}}$ (cooled < 20Kelvin) $T_N = 3-2°K$ on $50\,\Omega$

$$\boxed{\delta T_N \equiv \frac{T_N}{\sqrt{\Delta f . \tau}}}$$

cross-correlations



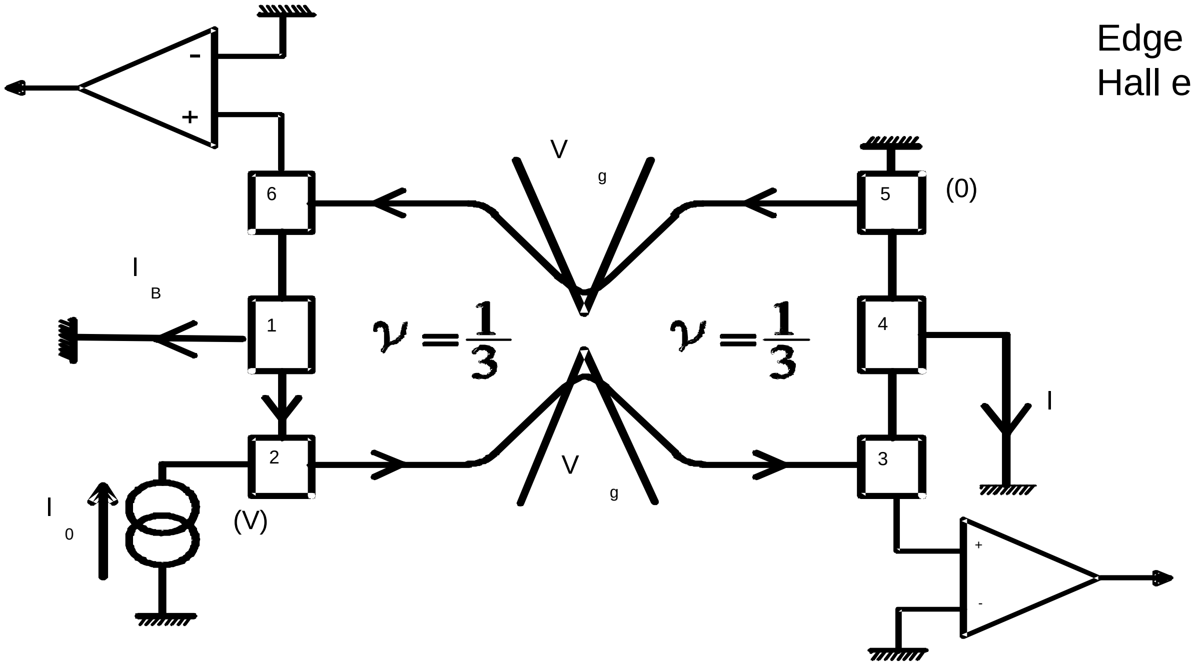
$$S_{V_1V_2}\Delta f = R^2.\left(S_I\,\Delta f + \left(\delta I_N^1\right)^2 + \left(\delta I_N^2\right)^2 \right)$$

physical sample noise
to be measured

(white) current noise of
each amplifier added

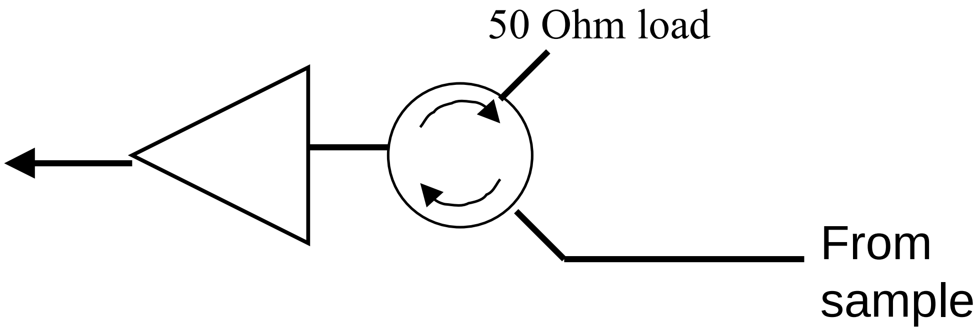
eliminates uncorrelated amplifier voltage noise, thermal noise of leads, reduces microphonic noise.
(like four-point resistance measurements). Improvement in reliability (not noise sensitivity):

trick: make use of chirality, when possible to eliminate current noise of the amplifiers



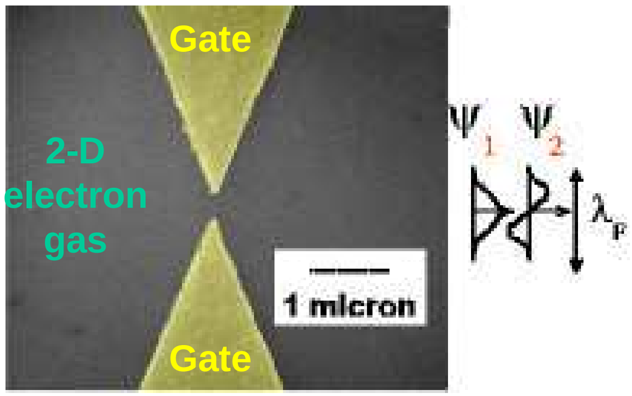
Edge states of the Quantum Hall effect

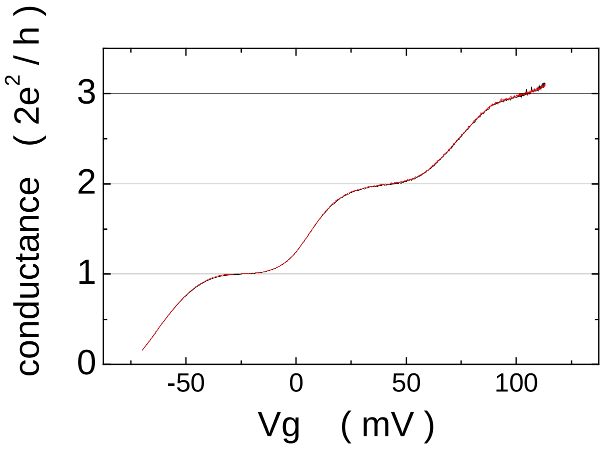
High frequency (microwaves): isolators (also called circulators)



in both cases chirality helps!!

quantum point contact





$$F = \frac{\sum_n D_n (1-D_n)}{\sum_n D_n}$$

$$\lambda_F \approx 70 \text{ nm}$$

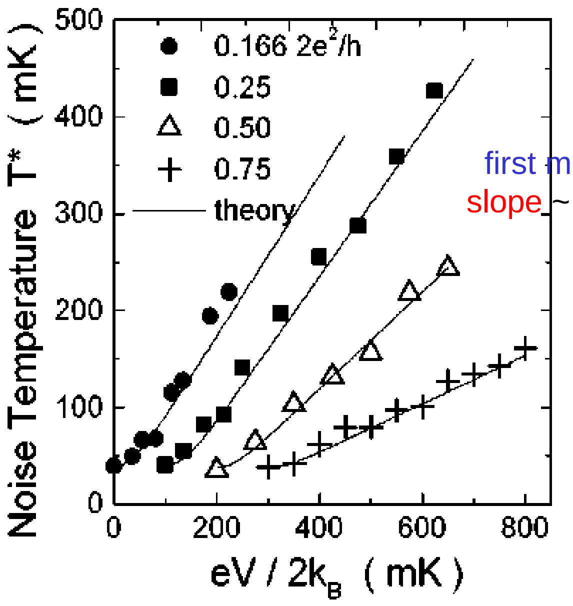
$$l_{elast.} \approx 10-20 \ \mu\text{m}$$

(ballistic conductor)

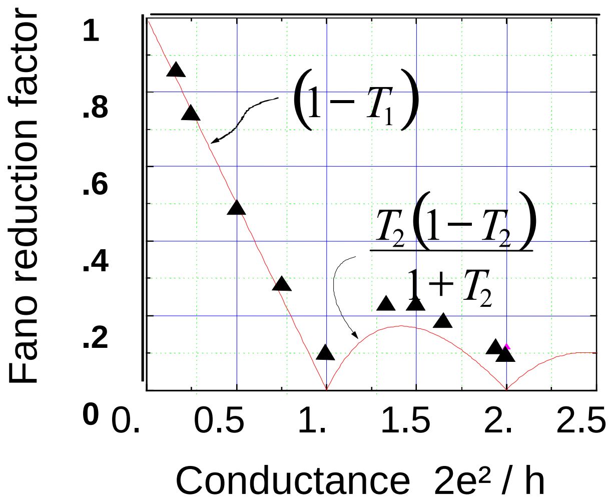
$$G = \frac{2e^2}{h} . \sum_n D_n$$

$$S_I = F.2eI$$

$$T^* = \frac{S_I}{4 G k_B}$$



first mode :
slope ~ (1 - *D*1)



A. Kumar et al. Phys. Rev. Lett. 76 (1996) 2778..
M. I. Reznikov et al., Phys. Rev. Lett. 75 (1995) 3340.

Thermal to shot noise cross-over regime

$$S_I = 2\frac{e^2}{h}k_B T.\sum_n D_n^2 + 2\frac{e^2}{h}eV.\sum_n D_n(1-D_n)\coth\left(\frac{eV}{2k_B T}\right)$$

thermal emission
noise of reservoirs

shot noise

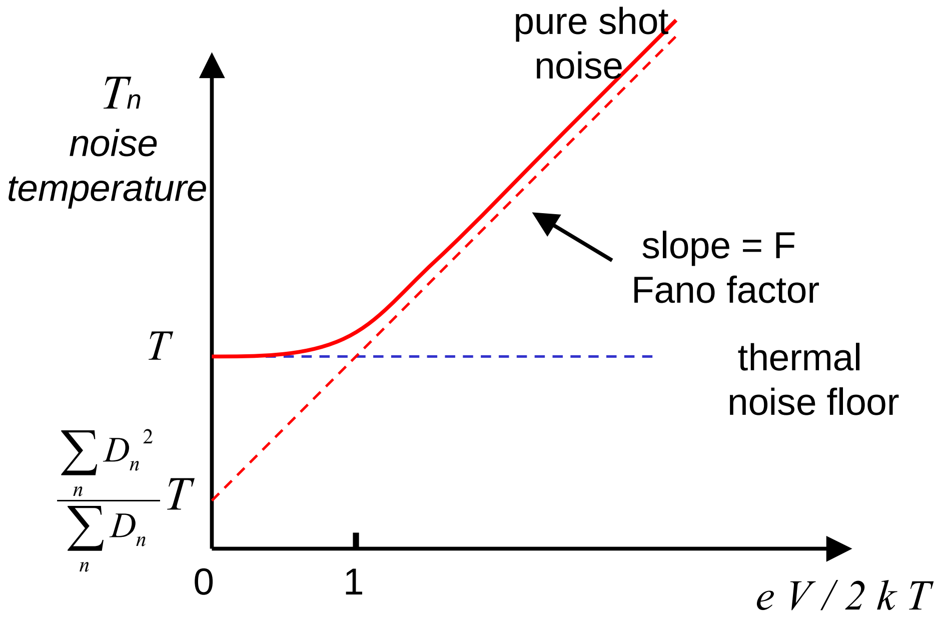
M. Büttiker, Phys. Rev. Lett. 65 (1990) 2901.
R. Landauer and Th. Martin, Physica B 175 (1991)

Johnson Nyquist noise at $V = 0$

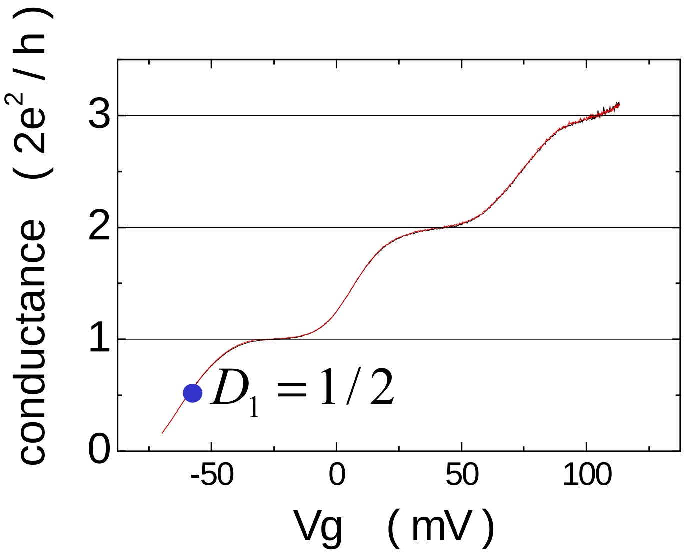
$$S_I = 4\frac{e^2}{h}k_B T.\sum_n D_n = 4Gk_B T$$

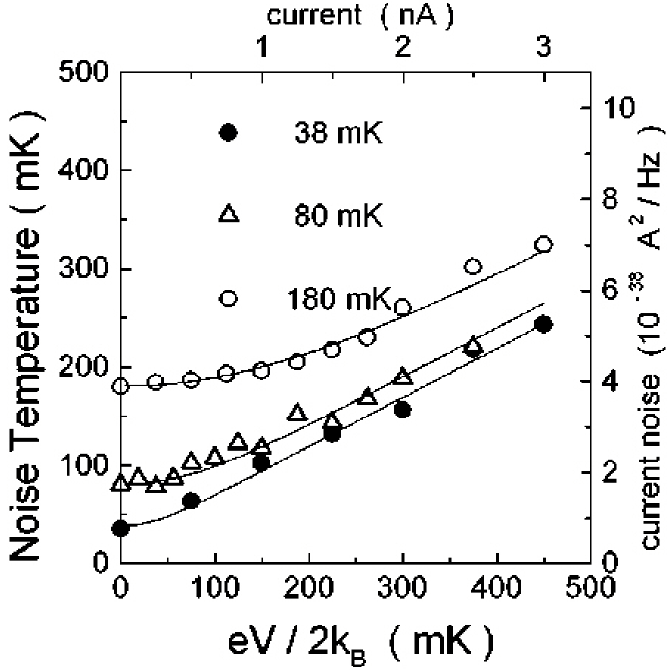
Def. : noise temperature *T_n*

$$T_n = \frac{S_I}{4Gk_B}$$



thermal to shot-noise cross-over checked using a QPC





A. Kumar et al. Phys. Rev. Lett. 76 (1996) 2778..

no adjustable parameter

$$S_I = 2\frac{e^2}{h} k_B T \,.\, D_1^2 + 2\frac{e^2}{h} eV \,.\, D_1 (1 - D_1) \coth\left(\frac{eV}{2 k_B T}\right)$$

- electron shot noise reaches quantum partition noise limit

- in general quantum conductor show sub-poisonian noise

various Fano factor have been observed in agreement with theory

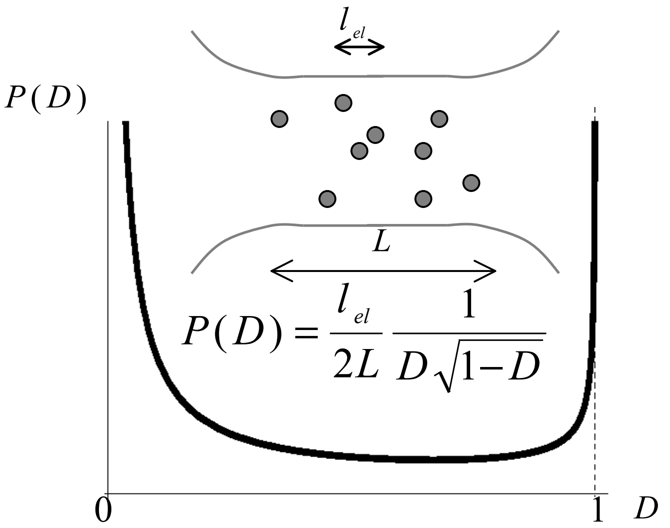
- F =1/3 : diffusive conductors
- F =1/4 : electron billards (quantum chaos)
- F =1/2 : quantum dots
- ….

$$F = \frac{\sum_\lambda \boldsymbol{D}_\lambda (1-\boldsymbol{D}_\lambda)}{\sum_\lambda \boldsymbol{D}_\lambda} \equiv \frac{\langle \boldsymbol{D}(1-\boldsymbol{D})\rangle}{\langle \boldsymbol{D} \rangle}$$

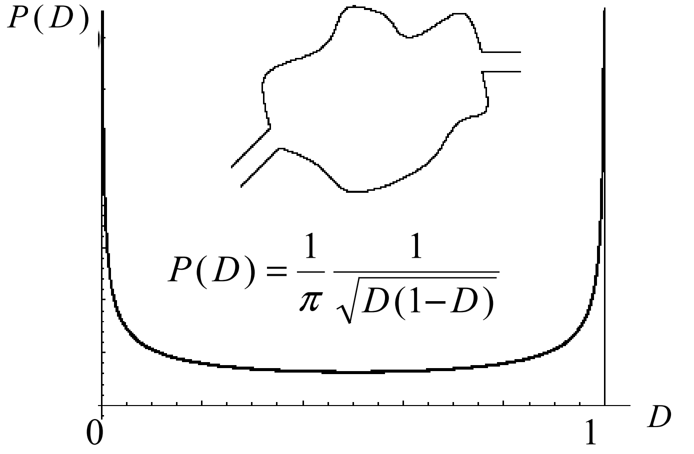
where : $\langle\ \rangle$ is the average over the probability distribution $\boldsymbol{P}(\{\boldsymbol{D}\})$ of transmissions $\boldsymbol{D}_\lambda$

diffusive : $\langle D \rangle = \frac{l_{el}}{L} << 1$

chaotic :



$$P(D) = \frac{l_{el}}{2L} \frac{1}{D\sqrt{1-D}}$$



$$P(D) = \frac{1}{\pi} \frac{1}{\sqrt{D(1-D)}}$$

(question : what is quantum in the 1/3 Fano factor of diffusive electronic systems ?)

D.C. Glattli, NTT-BRL School, 03 november 05

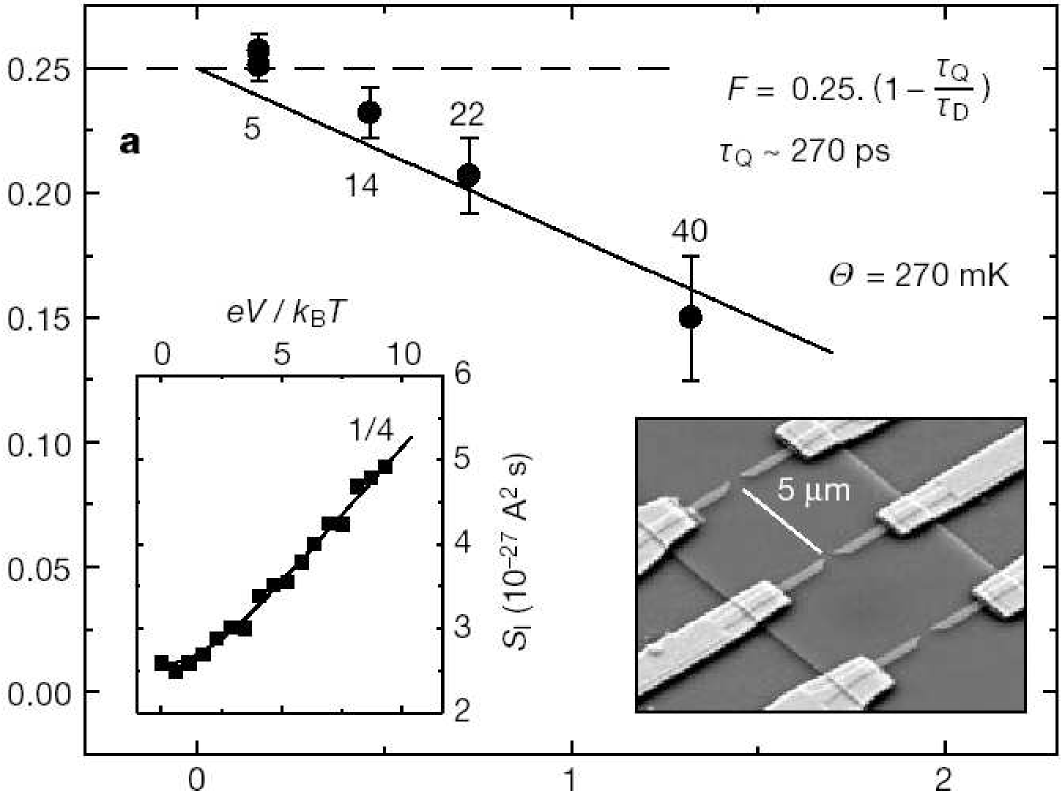
Chaotic cavity (electron billard)

cross-over from quantum to classical (no noise) regime.

noise is quantum !

FANO Factor

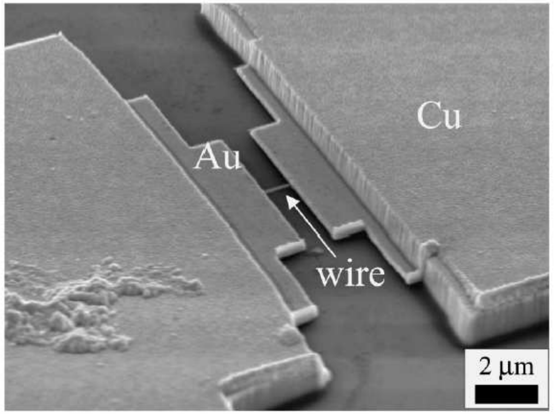
$S_I / 2e|I|$

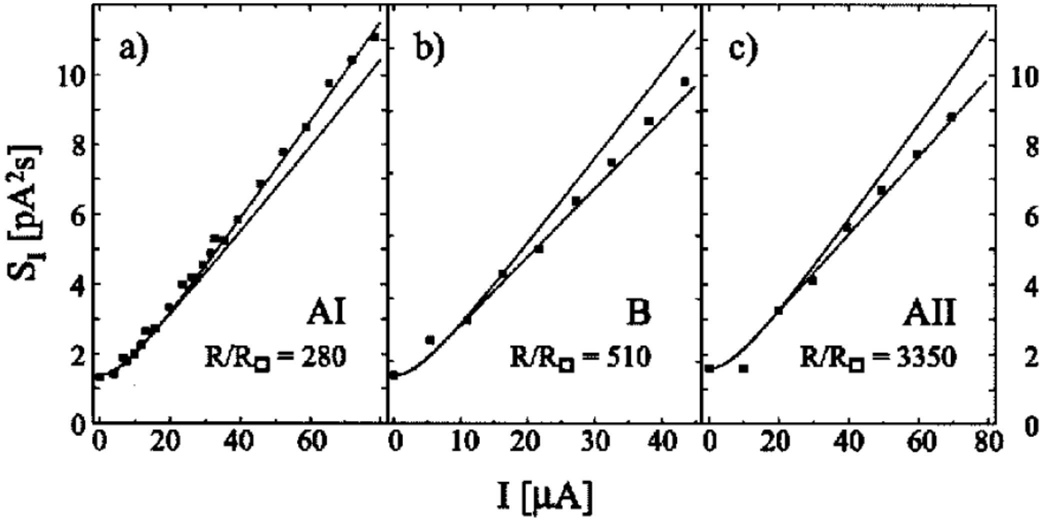


$1/\tau_D$ (GHz)

Oberholzer et al. Nature (2002)

$$S_I = F\,.2eI$$





lower slope : diffusive regime

$$F = 1/3$$

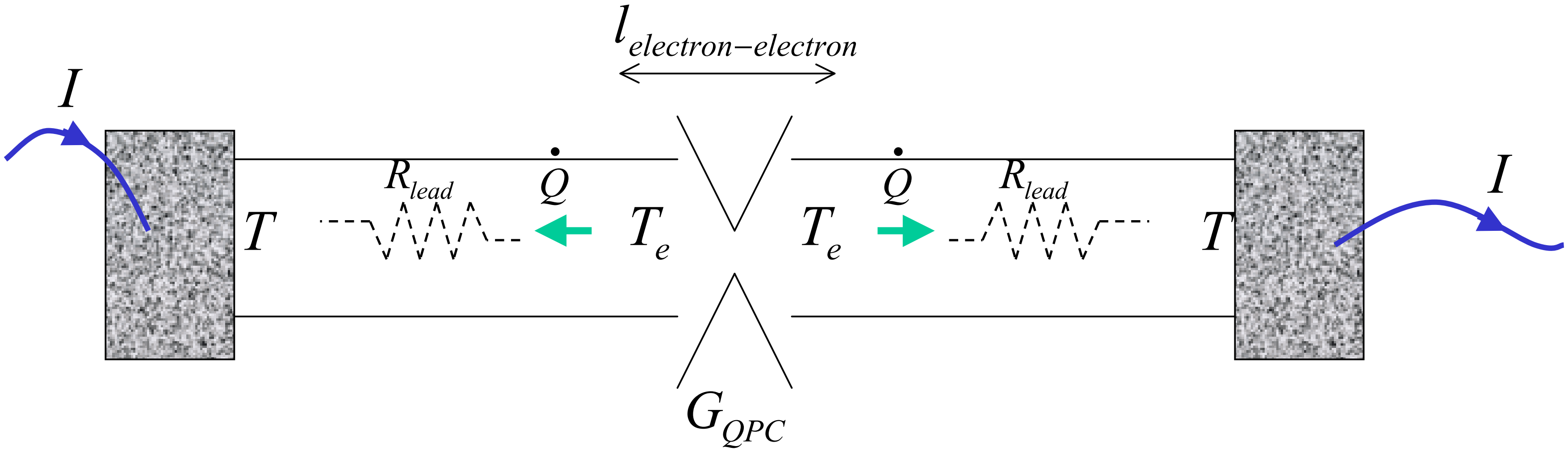
upper slope : hot electron regime
(see later)

$$"F" = \sqrt{3}/4$$

M. Henny, S. Oberholzer, C. Strunk, and C. Schonenberger,
Phys. Rev. B 59 (1999) 2871.

(question : what is quantum in the 1/3 Fano factor of diffusive electronic systems ?)

heating effect : apparent shot noise



$$\dot{Q} = \frac{1}{2} G_{QPC} V^2$$

heat produced

$$\dot{Q} = \frac{\pi^2}{3}\left(\frac{k_B}{e^2}\right)^2 \frac{T_e^{\,2} - T^2}{R_{lead}}$$

heat flow
Wiedeman-Franz

$$T_e^{\,2} = T^2 + \frac{3}{2\pi^2}\left(\frac{e^2}{k_B}\right)^2 G_{QPC} R_{lead} V^2$$

$$S_I = 4\, G_{QPC}\, k_B T_e$$

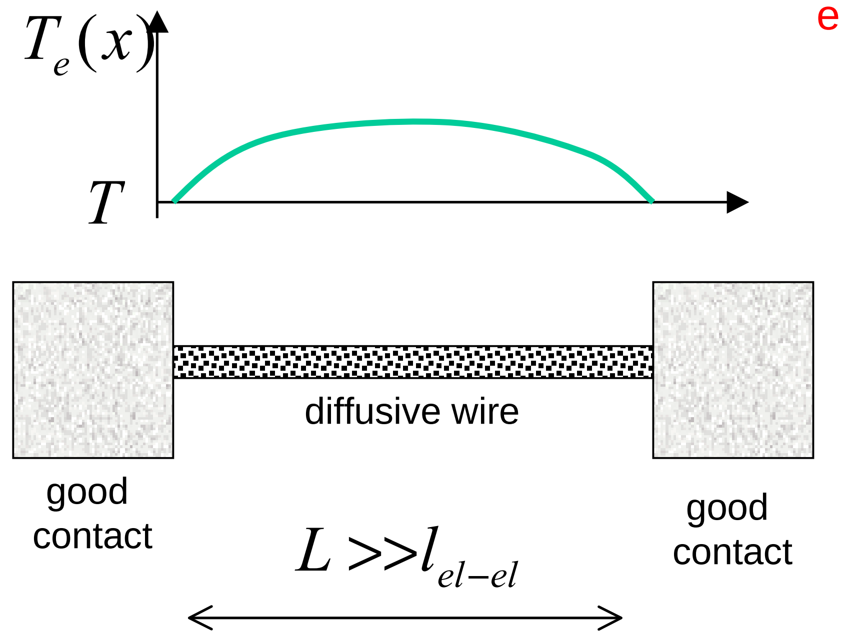
$$\boxed{S_I = 2eI\sqrt{\frac{6}{\pi^2} G_{QPC}\, R_{lead}} \quad \text{for } eV > k_B T}$$

(note : no heating effect if chiral system)

not shot noise, just **heating**,
apparent fano factor F

Important : good, low resistive contacts

electron heating effect in a diffusive wire



$T_e(x)$

T

diffusive wire

good contact

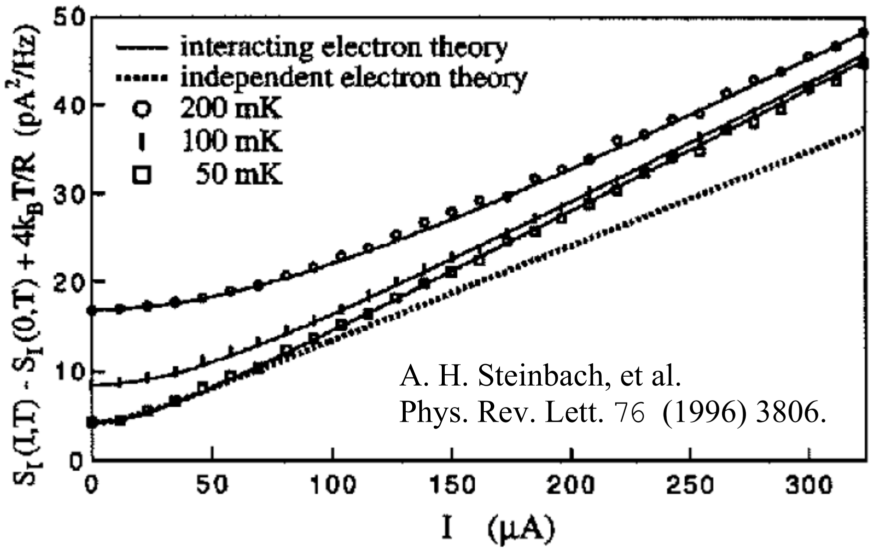
good contact

$L >> l_{el-el}$

$$S_I = 2eI \times "F"$$

$$"F" = \sqrt{3}/4$$

(just electron heating, not transport shot noise)



A. H. Steinbach, et al.
Phys. Rev. Lett. 76 (1996) 3806.

Also :
M. Henny, S. Oberholzer, C. Strunk, and C. Schonenberger,
Phys. Rev. B 59 (1999) 2871.

Introduction

I. Electronic scattering (a brief introduction)

II. Quantum Shot noise

- 1 - Quantum partition noise
  - - one and two particle partitioning :electrons/ photons
  - - electronic shot noise
- 2- scattering derivation of quantum shot noise
  - a- S(ω) for an ideal one mode conductor
  - b- quantum shot noise for a single mode
  - c-zero frequency shot noise and multimode case
- 3- experimental examples
- → 4- current noise cross-correlations
  - - scattering derivations
  - - electronic analog of the optical Hanbury-Brown Twiss experiment
  - - electronic quantum exchange

III Shot Noise and Interactions:

IV. Shot noise: ***the*** tool to detect entanglement

V. Shot noise and high frequencies

II. 4 . current noise correlations.

Zero temperature expression:

$$S_{\alpha\beta} = 2\frac{e^2}{h}\sum_{\gamma\neq\delta}\int d\varepsilon\ \left[s^*_{\alpha\gamma}s_{\alpha\delta}s^*_{\beta\delta}s_{\beta\gamma}\right] \times \{f_\gamma(\varepsilon)(1-f_\delta(\varepsilon)) + f_\delta(\varepsilon)(1-f_\gamma(\varepsilon))\}$$

use the property $\sum_\delta s_{\alpha\delta}s^*_{\beta\delta} = 0$

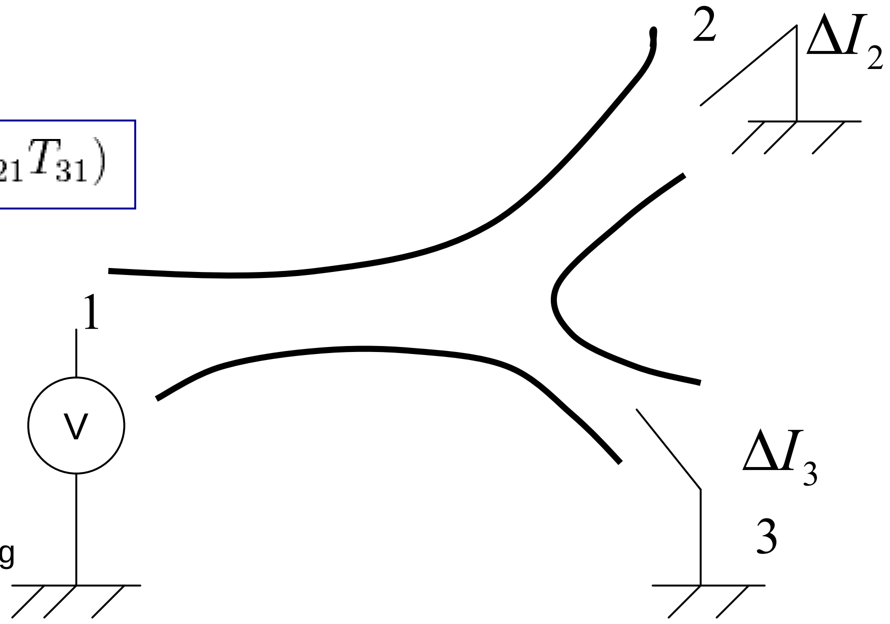
$$S_{\alpha\beta} = -2\frac{e^2}{h}\int d\varepsilon\ \left(\sum_\gamma s_{\alpha\gamma}s^*_{\beta\gamma}f_\gamma(\varepsilon)\right)\left(\sum_\delta s_{\alpha\delta}s^*_{\beta\delta}f_\delta(\varepsilon)\right)$$

cross-correlation between two different leads are always *negative*

Special case of a three terminal branch :
one finds :

$$\boxed{S_{23} = -2\frac{e^2}{h}eV\left(s_{21}s^*_{21}s_{31}s^*_{31}\right) = -2\frac{e^2}{h}eV.(T_{21}T_{31})}$$

$$\Delta I_2\,\Delta I_3 = S_{23}\Delta f < 0$$



binomial partitioning is here replaced by multinomial partitioning
(just ‘gambling’ law!)

Hanbury Brown & Twiss experiment with electrons

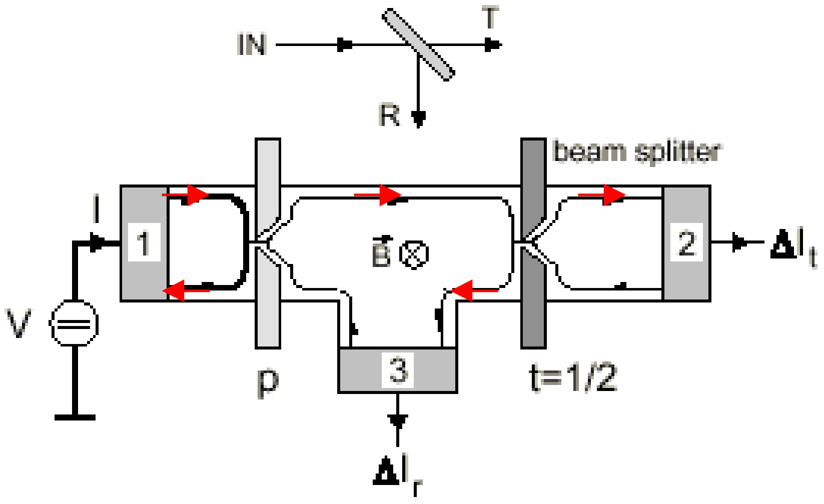
Oberholzer et al. ' 00

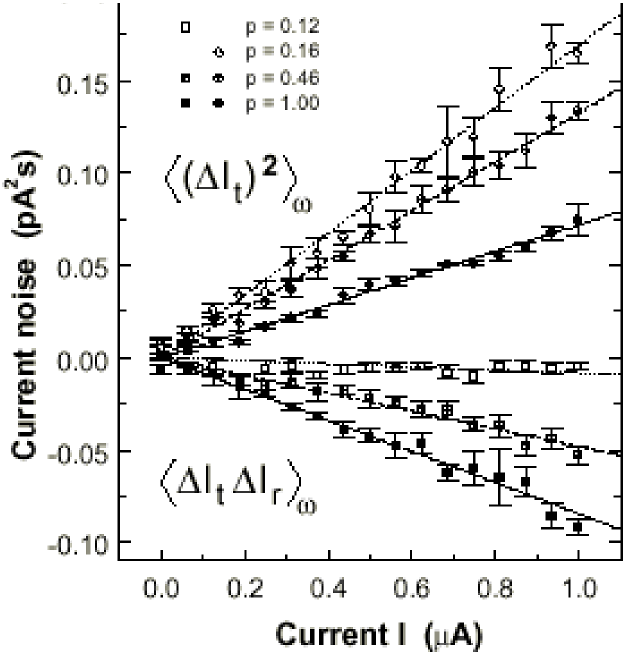
from sub-poissonian statistics

$$\Delta I_t \Delta I_r < 0 \quad = -\Delta I_t^{\,2} = -\Delta I_r^{\,2}$$

to poissonian statistics

$$\Delta I_t \Delta I_r = 0$$





Four terminal lead :

(A) $V_1 = V$; $V_3 = 0$

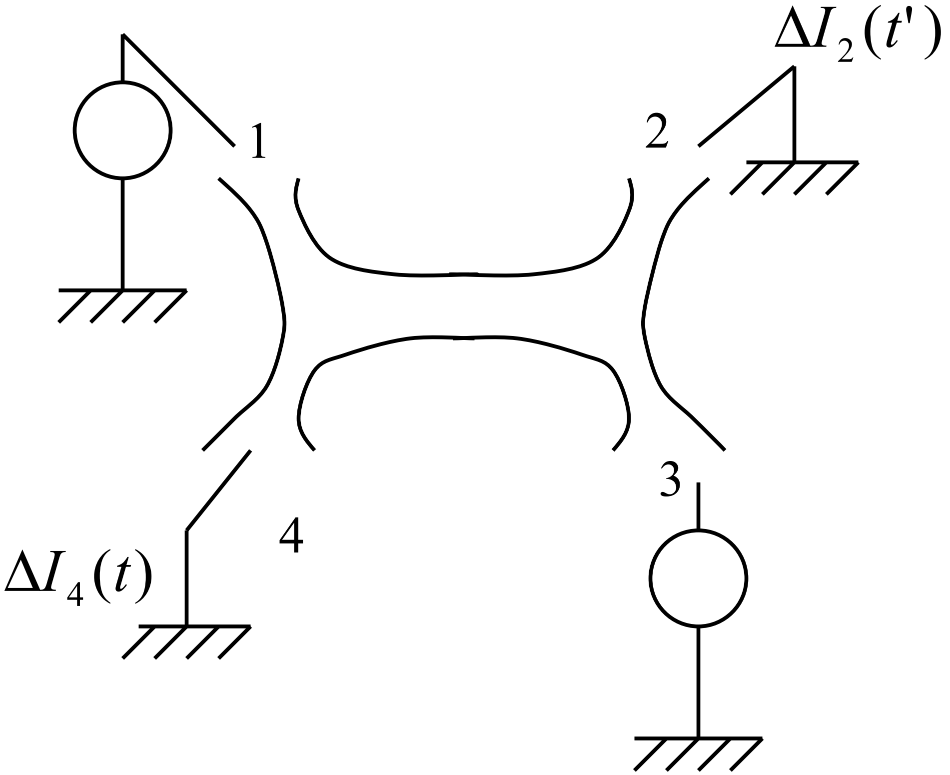
(B) $V_1 = 0$; $V_3 = V$

(A + B) $V_1 = V$; $V_3 = V$

$$S_{(A)} = -2\frac{e^2}{h}eV\left(s_{21}s_{21}^{*}s_{41}s_{41}^{*}\right)$$

$$S_{(B)} = -2\frac{e^2}{h}eV\left(s_{23}s_{23}^{*}s_{43}s_{43}^{*}\right)$$

$$S_{(A+B)} \neq S_{(A)} + S_{(B)}$$



(M. Büttiker, Phys. Rev. B 46 (1992) 12485.

$$\boxed{S_{(A+B)} - \left(S_{(A)} + S_{(B)}\right) = -2\frac{e^2}{h}eV\left[\left(s_{21}s_{23}{}^{*}s_{43}s_{41}{}^{*}\right) + \left(s_{23}s_{21}{}^{*}s_{41}s_{43}{}^{*}\right)\right]}$$

exchange terms : non classical

Introduction

I. Electronic scattering (a brief introduction)

II. Quantum Shot noise

III Shot Noise and Interactions:

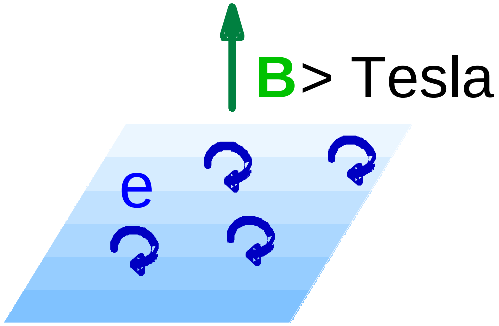
→ 1. Fractional Quantum Hall effect
2.. Superconducting-Normal mesoscopic interfaces
3. Interactions in a QPC : 0.7 structure

IV. Shot noise: ***the*** tool to detect entanglement

V. Combining electrons and photons

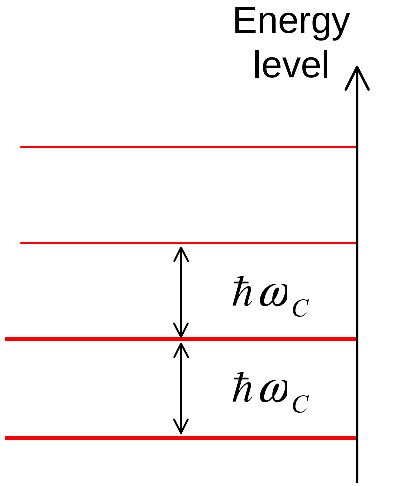
III 1. Quantum Hall effect

Integer Q.H.E. (von Klitzing 80)

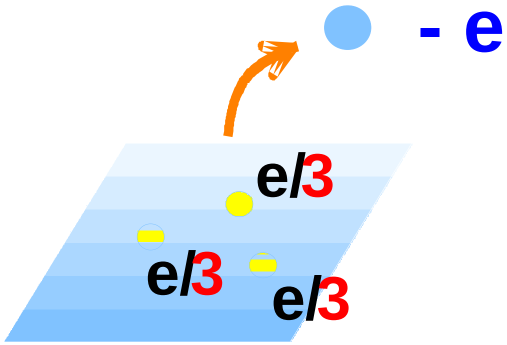


$$R_{\mathrm{Hall}} = \frac{h}{e^2} \cdot \frac{1}{\text{integer}}$$

$$\omega_C = \frac{eB}{m}$$ cyclotron frequency



Fractional Q.H.E. (Tsui, Störmer,Gossard 1982)
(Laughlin 1983)



$$R_{\mathrm{Hall}} = \frac{h}{e^2} \cdot \frac{1}{\text{fraction}}$$

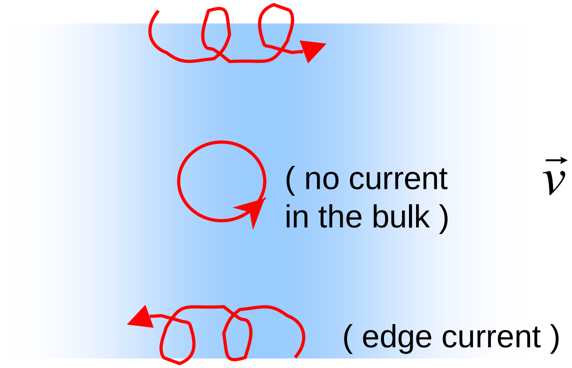
$$\nu = \frac{\text{number of electrons}}{\text{number of quantum states}}$$

$\nu = 1/3$:

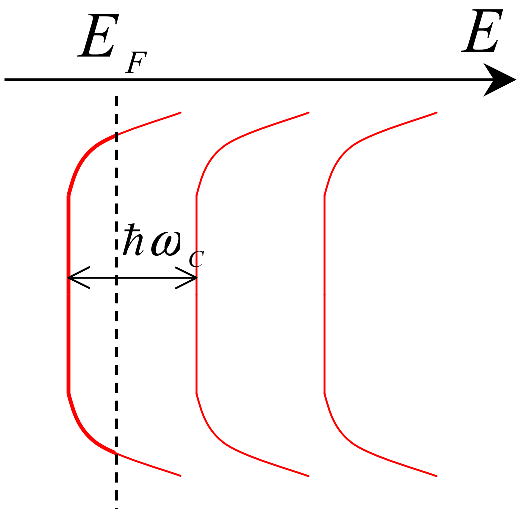
1 electron for 3 quantum states

elementary excitation
≡ empty a quantum state
≡ carry fractional charge e/3

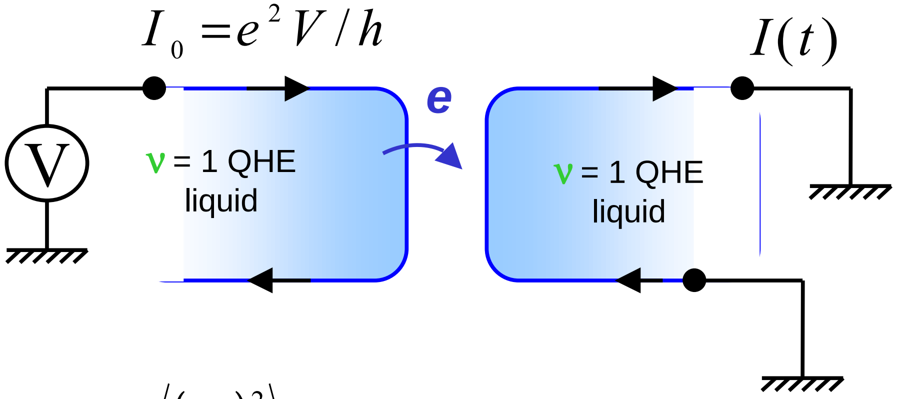
available current is at the edges of the sample



$$\vec{v}_{drift} = \vec{E}_{Confinement} \times B\,\hat{z}$$

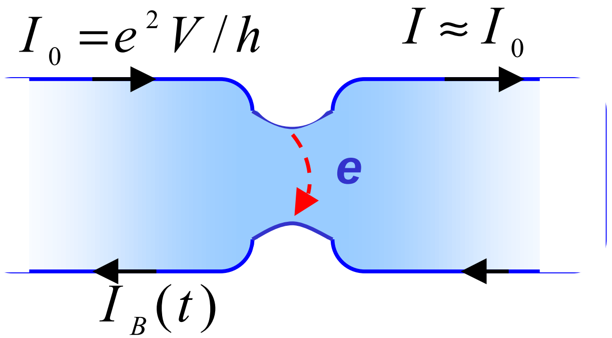


Tunneling trough barrier :

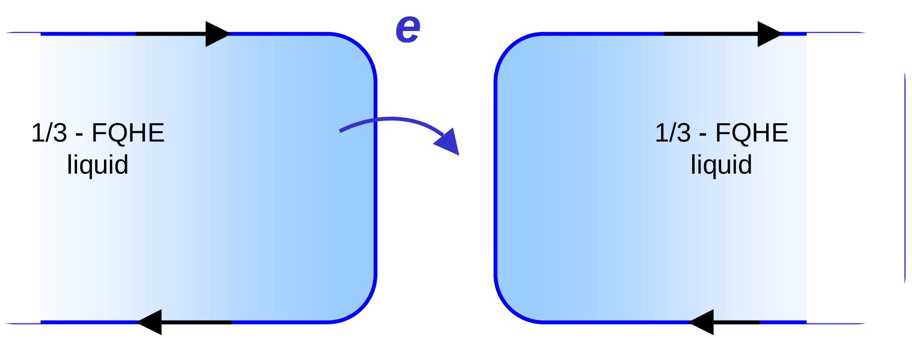


$$\left\langle (\Delta I)^2 \right\rangle = 2eI\Delta f \qquad D << 1$$

Transfer of hole trough Q.H.E. fluid:



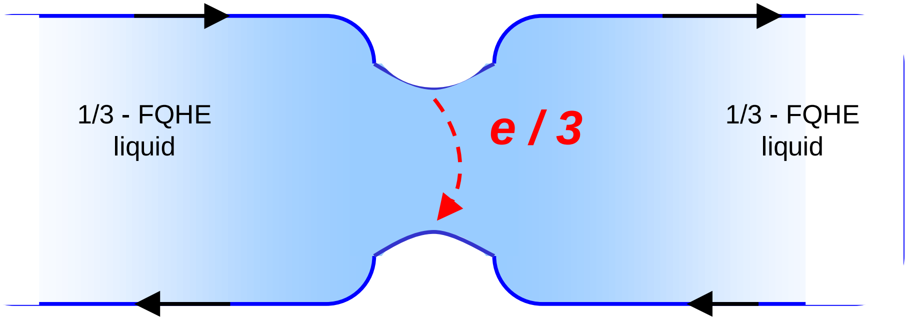
$$\left\langle (\Delta I)^2 \right\rangle = 2eI_B\Delta f \qquad D \approx 1$$



Tunneling trough
barrier :

q = **e**

$$\left\langle (\Delta I)^2 \right\rangle = 2eI\,\Delta f \qquad D << 1$$



Transfer trough
1/3 FQHE fluid:

q = **e/3**

$$\boxed{\left\langle (\Delta I)^2 \right\rangle = 2(e/3)I_B\,\Delta f \qquad D \approx 1}$$

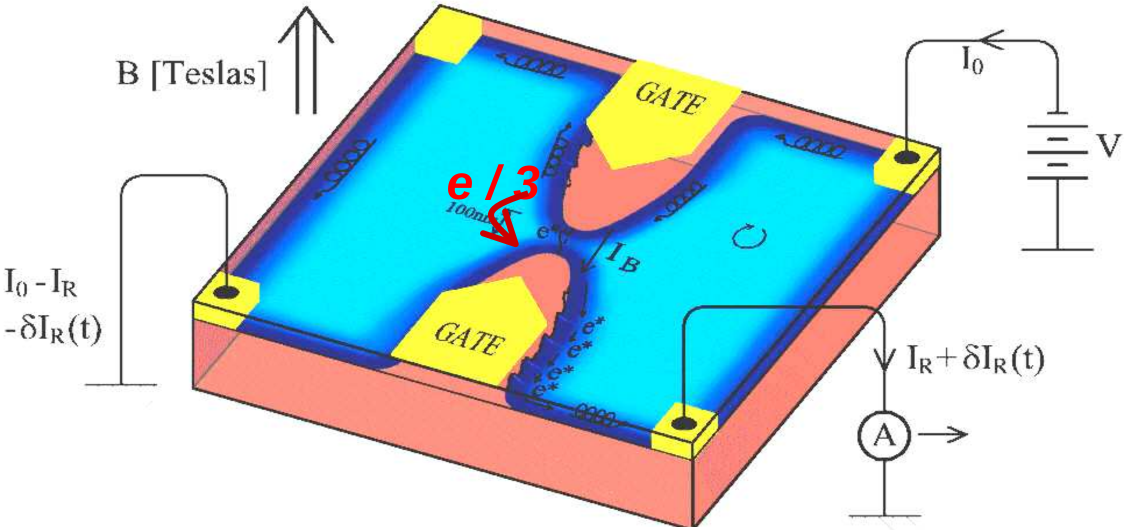
$$\boxed{\frac{\left\langle \Delta I^2 \right\rangle}{\Delta f} = S_I = 2qI}$$

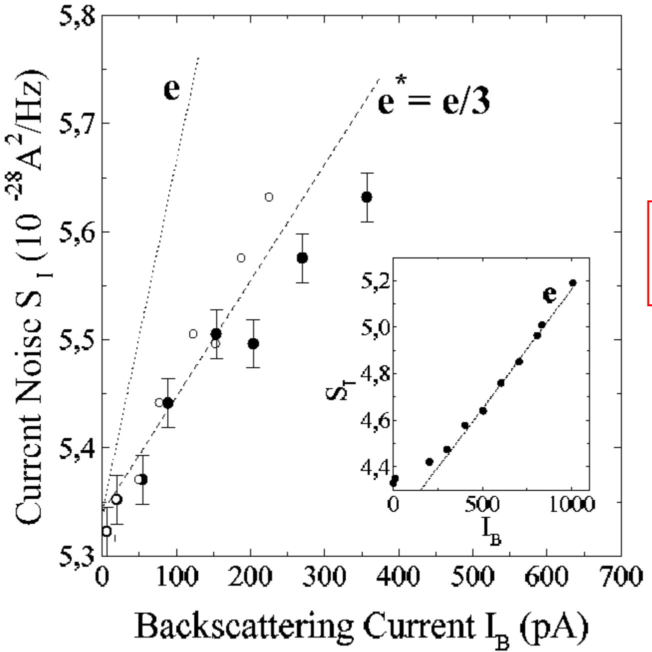
measuring both quasiparticle shot noise (Poissonian regime!)
+ mean current gives the charge with no adjustable parameters.

current is carried by fractional charges

(direct evidence, no unknown parameters)

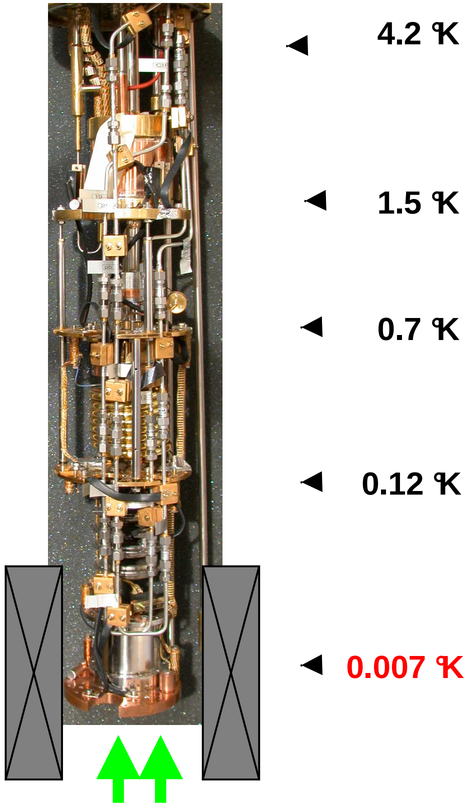
L. Saminadayar, D. C. Glattli, Y. Jin, and B. Etienne,
Phys. Rev. Lett. 79, 2526 (1997).



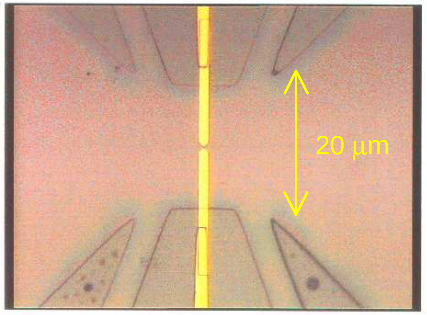


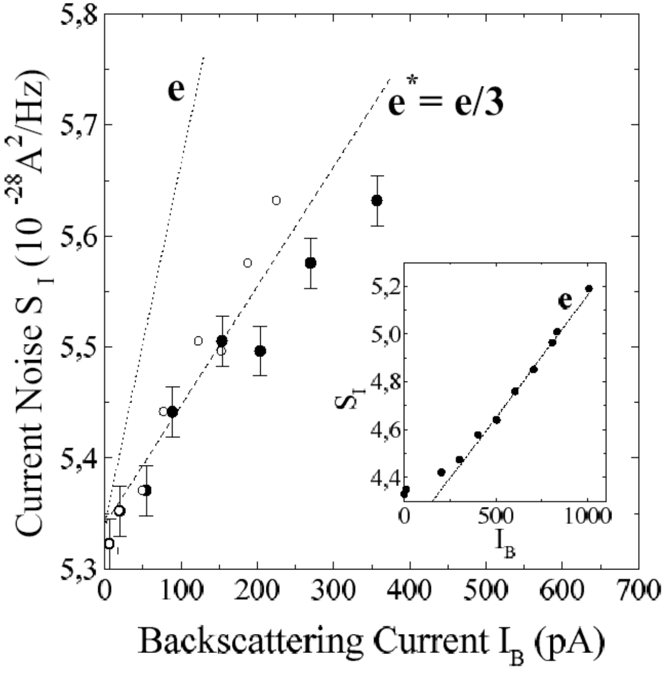
$$\left\langle (\Delta I)^2 \right\rangle = 2(e/3) I_R \Delta f$$

charge q=e/3

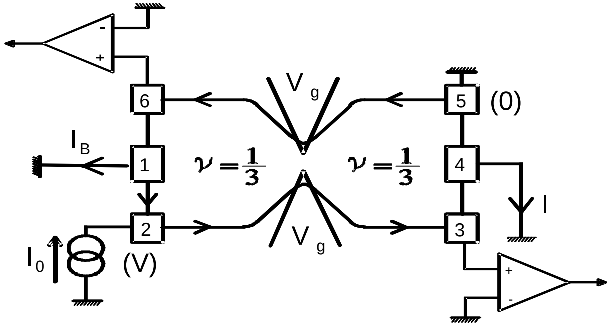


15 Tesla
superconducting coil





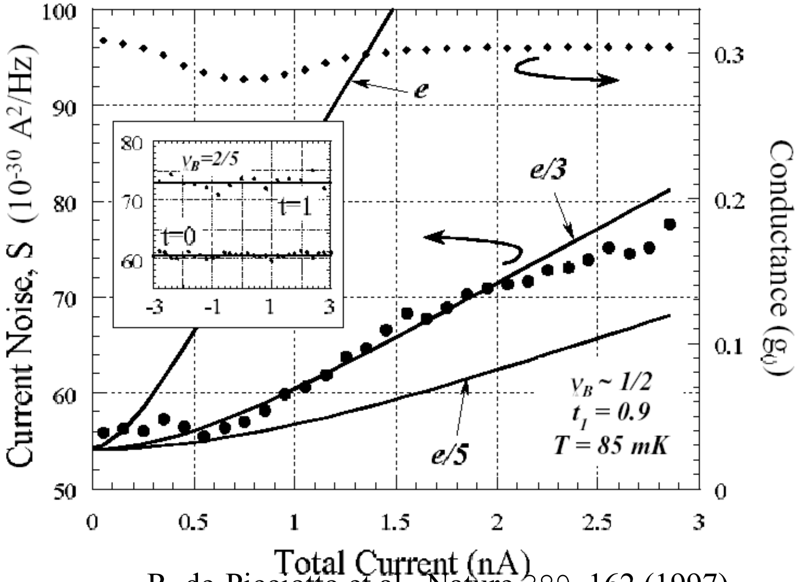
L. Saminadayar, D. C. Glattli, Y. Jin, and B. Etienne,
Phys. Rev. Lett. 79, 2526 (1997).



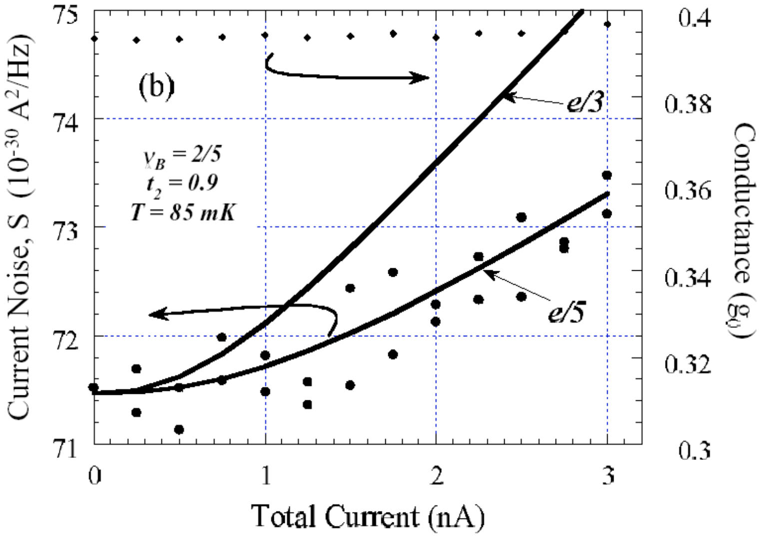
***e / 3** fractional charges are observed*

at filling factor 1/3 (Weizmann and Saclay 97) and

***e / 5** charges at filling 2/5 (Weizmann 99).*

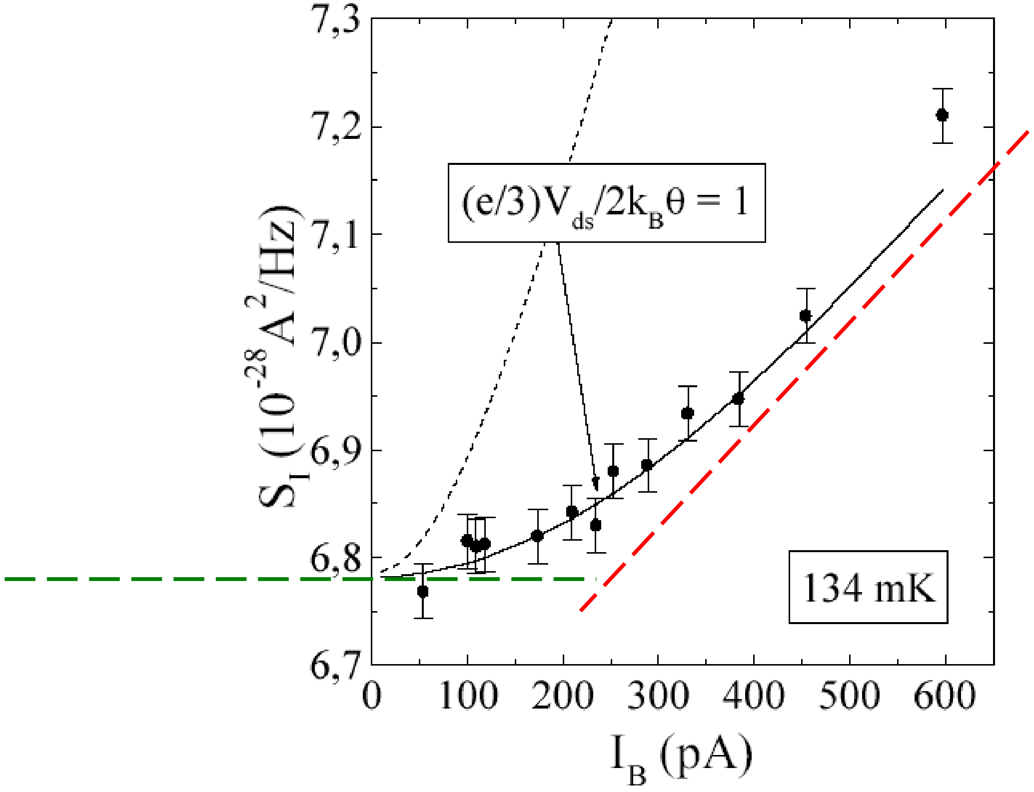


R. de-Picciotto et al., Nature 389, 162 (1997).

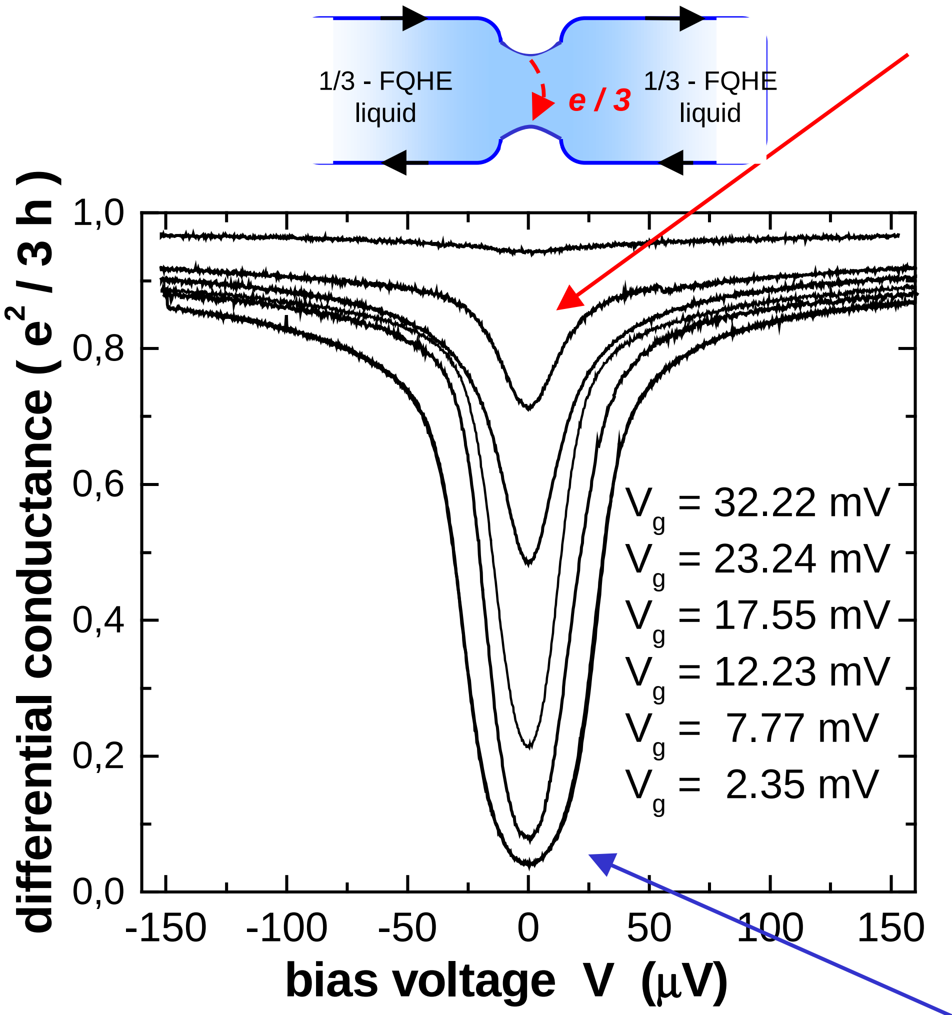


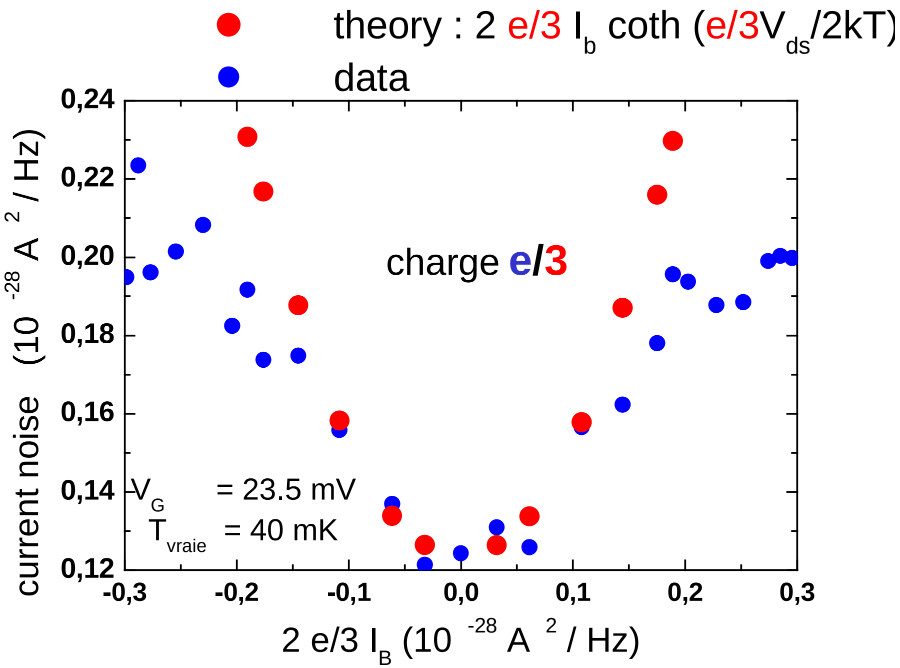
M. Reznikov, R. de-Picciotto, T. G. Griths,
M. Heiblum, and V. Umansky, Nature 399, 238 (1999).

cross-over from thermal noise to fractional charge shot noise

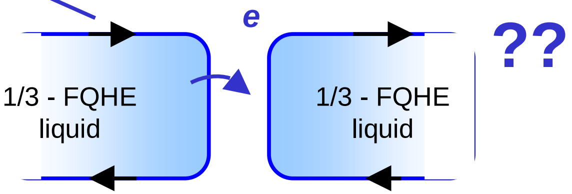


$$S_I = 2\,\frac{e}{3}\,I_B\ \coth\ \frac{eV_{ds}}{3k_BT}$$



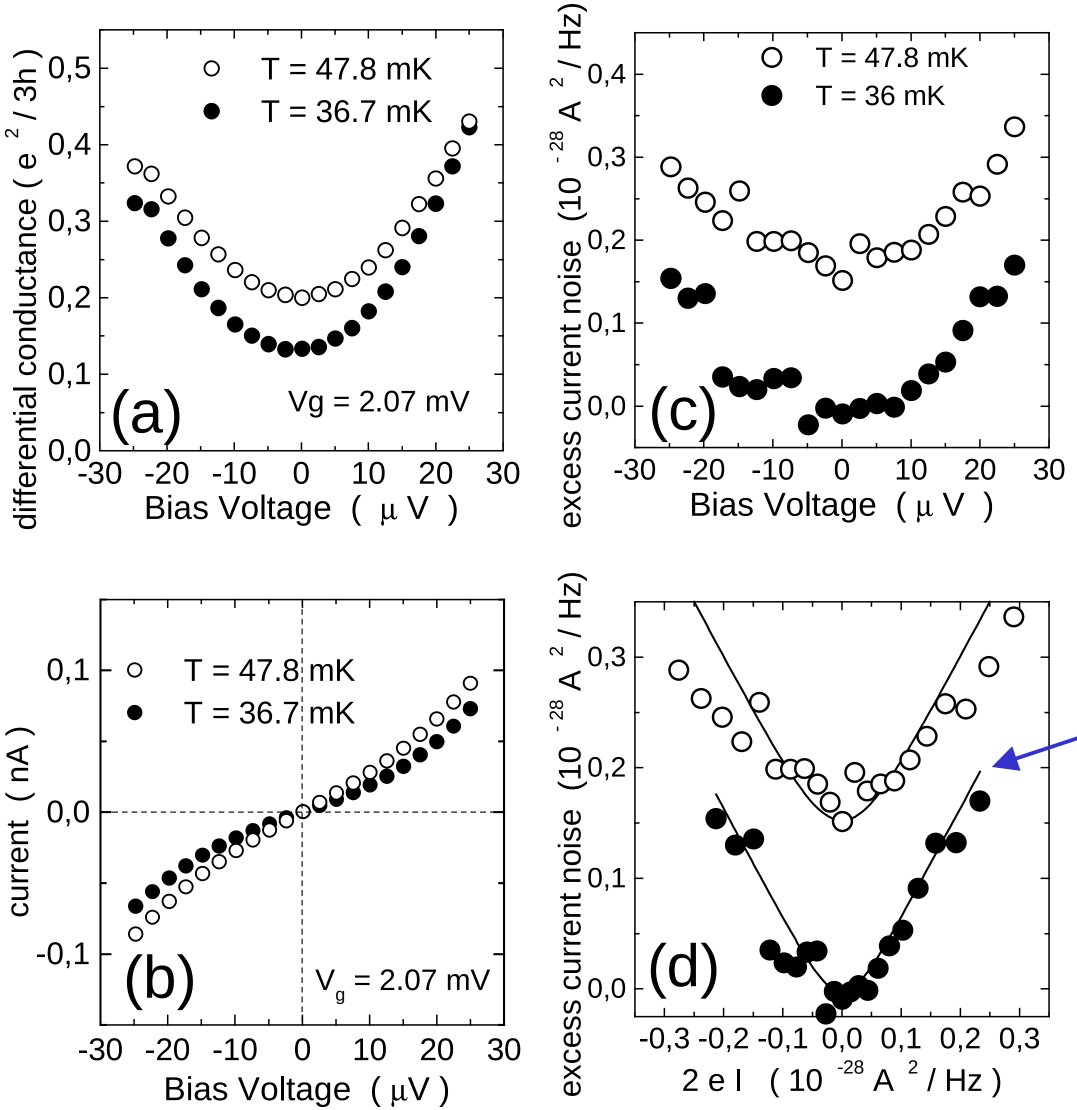


charge **e**



V. Rodriguez et al (2000)

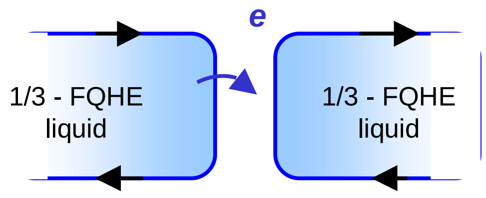
integer charge in the strong backscattering regime



$$S_I = 2\,e\,|I|$$

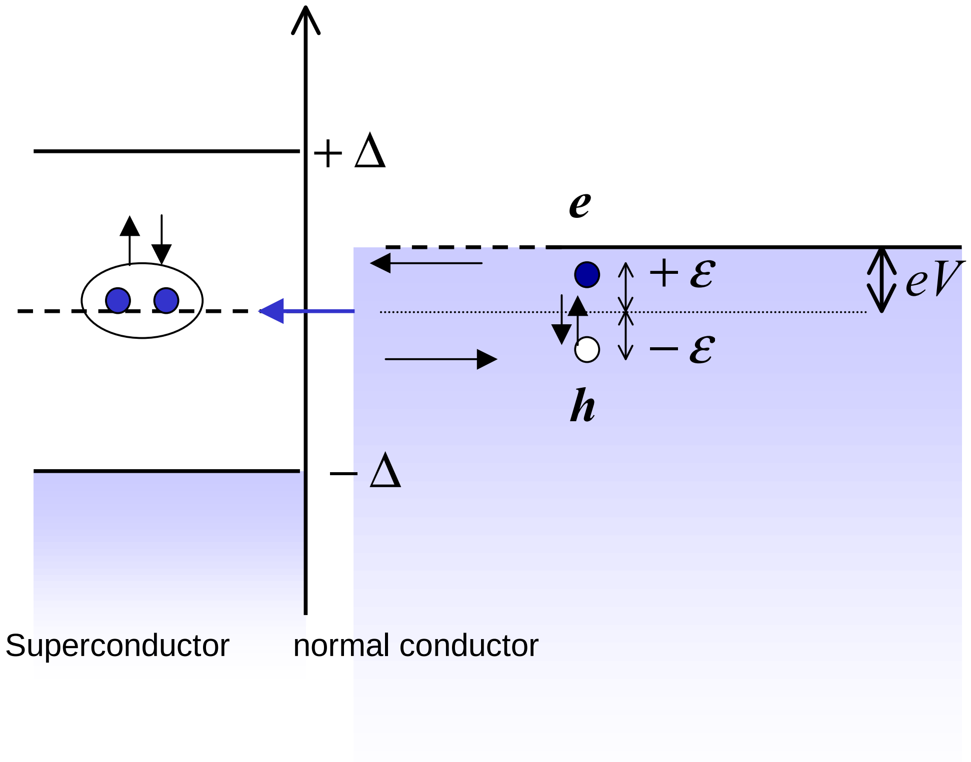
Schottky noise

for charge e



In the same sample, same quantum point contact, one can go from the e/3 regime to the e regime just by changing coupling

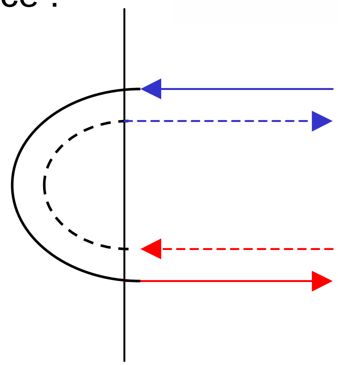
III . 2. normal / superconductor interface:



no single particle current between superconducting and normal metal expected for sub-gap energies.

second order process involving two quasi-particle allows for finite current: Andreev reflection.

ideal interface :

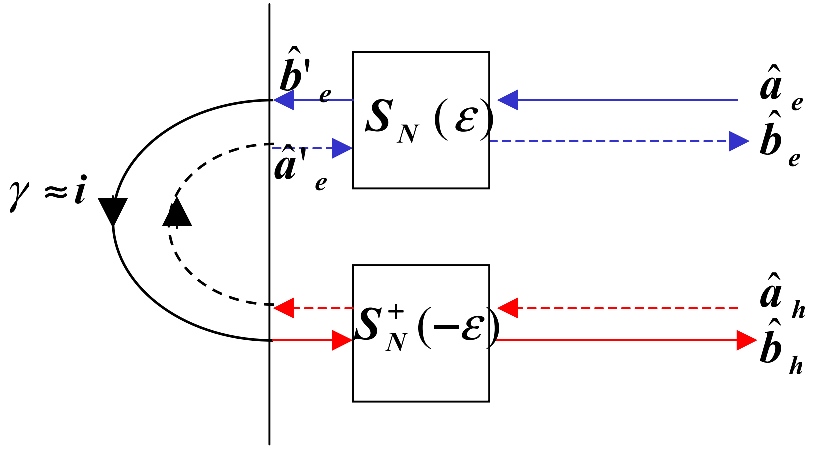


incoming electron (energy $+\varepsilon$) $\rightarrow$ outgoing hole (energy - ε)

plus phase $\gamma \equiv \exp(-i\cos^{-1}(\varepsilon/\Delta))$

incoming hole (energy $-\varepsilon$) $\rightarrow$ outgoing electron (energy $+\varepsilon$)

plus phase $\gamma \equiv \exp(-i\cos^{-1}(\varepsilon/\Delta))$



$$S_N(\varepsilon) = \begin{pmatrix} r_{11} & t_{12} \\ t_{21} & r_{22} \end{pmatrix}$$

scattering matrix in the normal lead

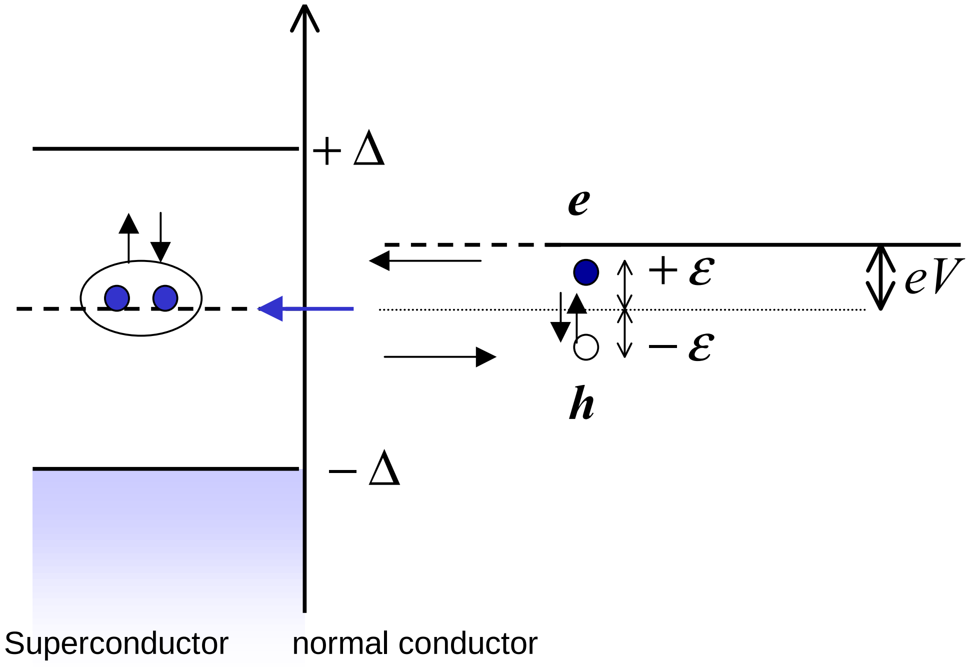
the complete scattering matrix including Andreev reflection and normal scattering is :

$$\begin{pmatrix} \hat{b}_e \\ \hat{b}_h \end{pmatrix} = S \begin{pmatrix} \hat{a}_e \\ \hat{a}_h \end{pmatrix} \qquad S = \begin{pmatrix} s_{ee} & s_{eh} \\ s_{he} & s_{hh} \end{pmatrix}$$

$$s_{he}(\varepsilon) = t_{21}(\varepsilon)\,\gamma\, t_{12}{}^{*}(-\varepsilon) \;+\; t_{21}(\varepsilon)\,\gamma\, r_{22}{}^{*}(-\varepsilon)\;\gamma\, r_{22}(\varepsilon)\, t_{12}{}^{*}(-\varepsilon) \;+\; t_{21}(\varepsilon)\Big(\gamma\, r_{22}{}^{*}(-\varepsilon)\;\gamma\, r_{22}(\varepsilon)\Big)^{2} t_{12}{}^{*}(-\varepsilon) \;+ \dots$$

$$= \frac{t_{21}(\varepsilon)\,\gamma\, t_{12}{}^{*}(-\varepsilon)}{1-\gamma\, r_{22}{}^{*}(-\varepsilon)\;\gamma\, r_{22}(\varepsilon)}$$

(Fabry-Pérot like multiple interferences)



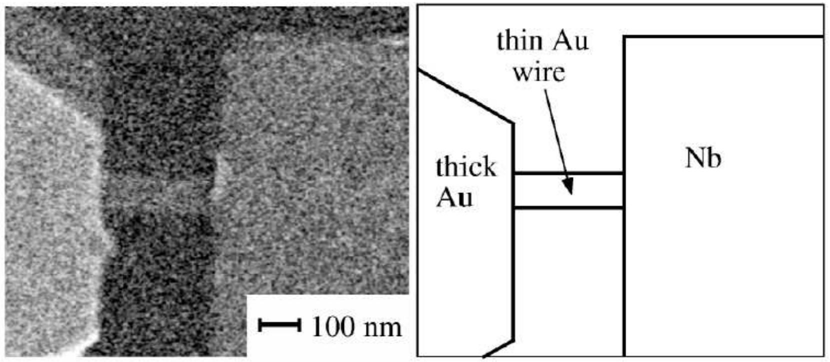
$$G = \frac{(2e)^2}{h}\left|s_{he}\right|^2 = \frac{(2e)^2}{h}\frac{D^2}{(1+R)^2} \qquad R = 1 - D$$

‘doubled’ shot noise :

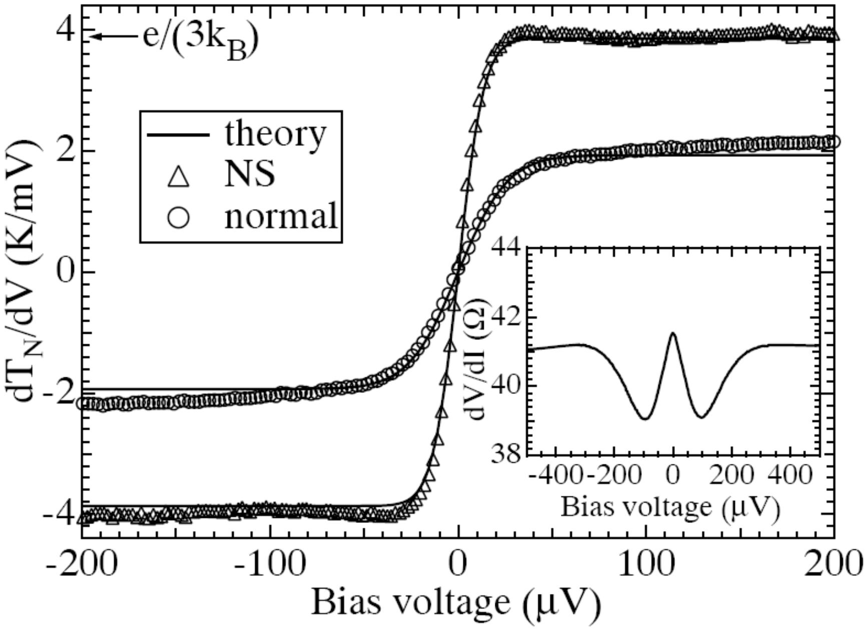
$$S_I = 2\,(2e)\,\frac{e^2}{h}V\left|s_{he}\right|^2\,(1-\left|s_{he}\right|^2\,)$$

- twice the electron charge
- binomial law of quantum partitioning
- noiseless property of the Fermi sea

doubling of the shot-noise for a diffusive S-N junction



A. A. Kozhevnikov, R. J. Schoelkopf, and D. E. Prober,
Phys.Rev.Lett. 84, 3398 (2000)



$$S_I = \frac{1}{3} 2.(2e) I$$

Fano for diffusive regime

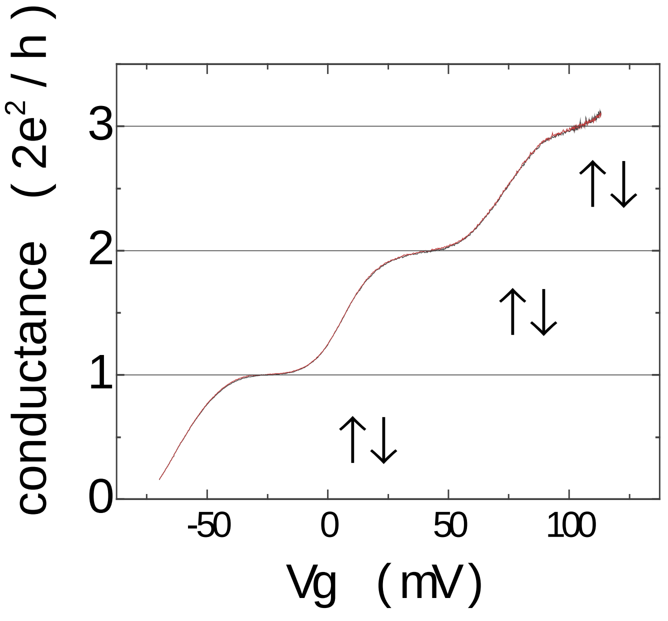
Doubled charge

$$\mathrm{T}_{\mathrm{Noise}} = \frac{S_I}{4G\, k_B}$$

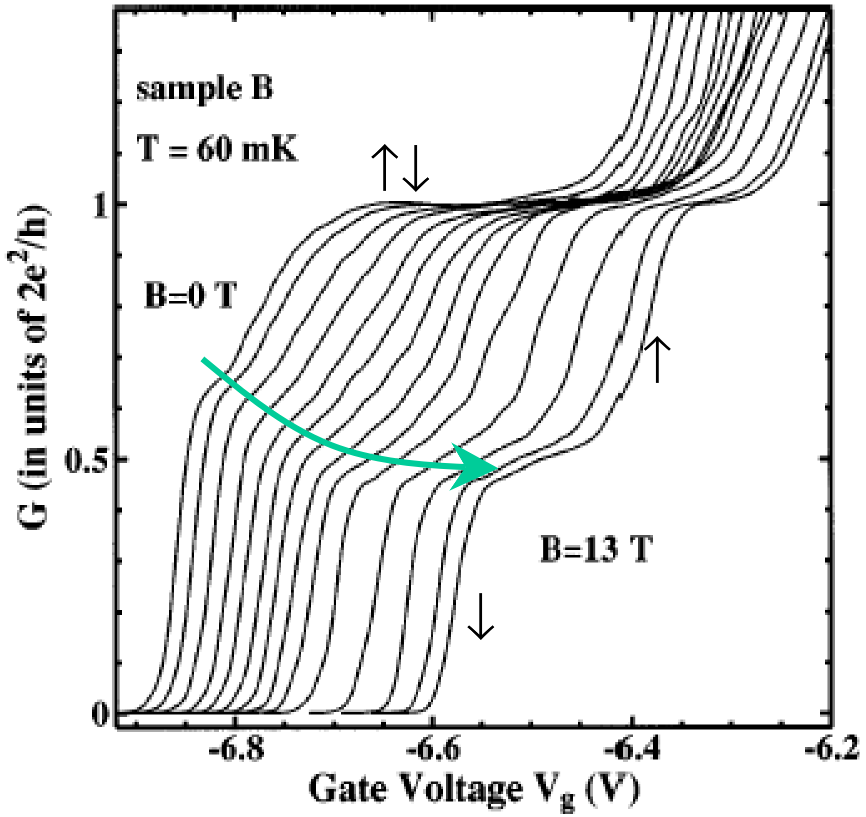
(also M. Sanquer et al. 2001)

III. 3. interaction effects in a Quantum Point Contact : the '0.7x2e²/h' structure

ultra-short QPC: no '0.7'



long QPC: conductance plateau around 0.7X2e²/h



K.J. Thomas *et al.*, Phys.Rev.Lett. **77**, 135 (1996)

plateaus at:

$$G = (\text{integer}) \times 2\frac{e^2}{h}$$

$$(1+1)$$

$$\uparrow + \downarrow$$

- resonance in transmission? (**single** spin degenerate mode)

- or spin degeneracy lifted by interaction? (**two** distinct modes)

↑↓

↑↑

III. 3. interaction effects in a Quantum Point Contact :
the ‘0.7x2e²/h’ structure

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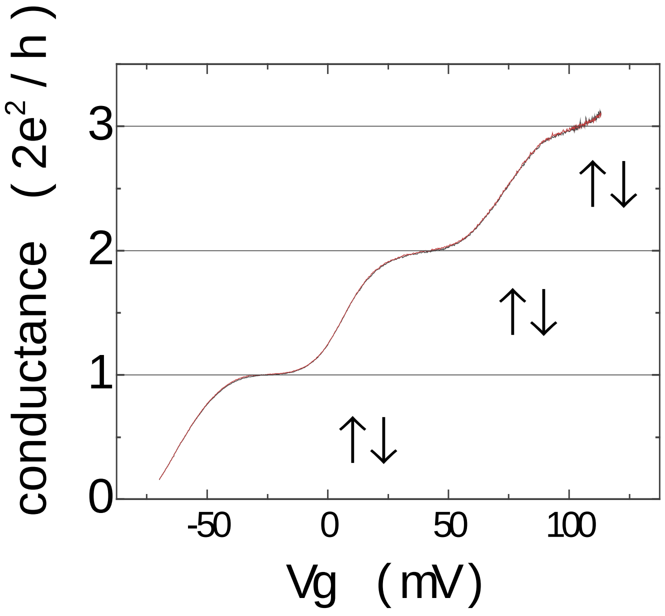

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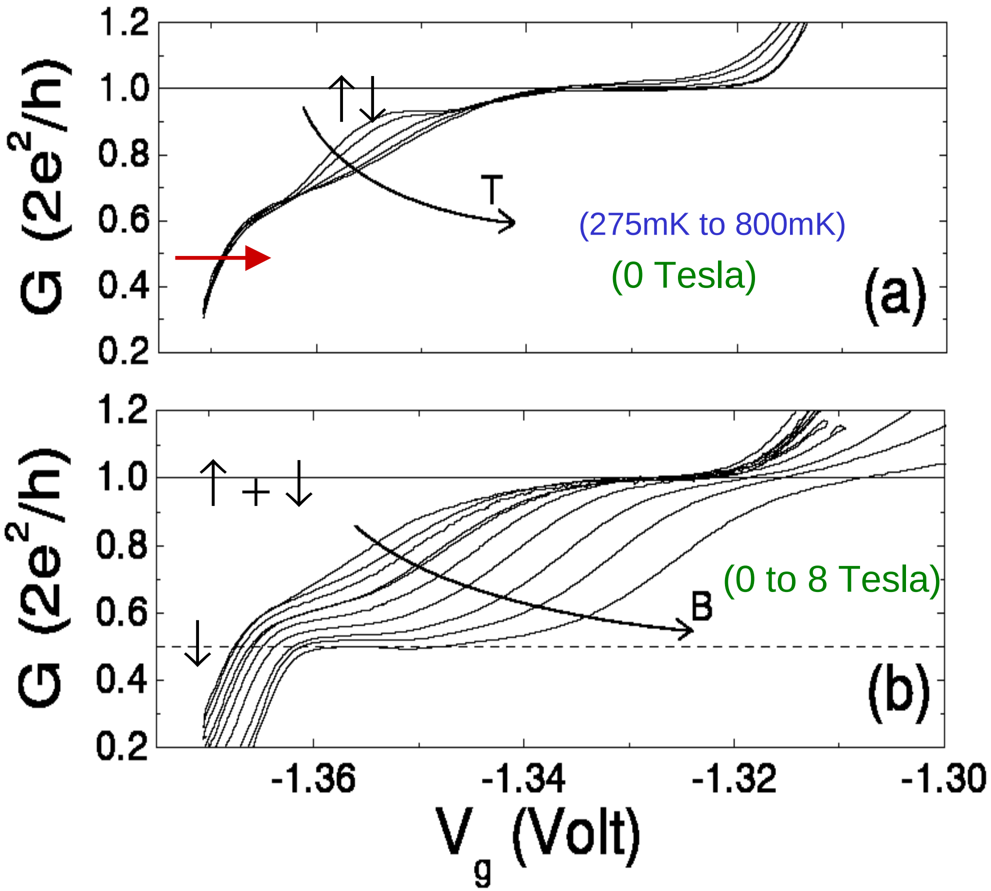

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↑ + ↓

- resonance in transmission? (**single** spin degenerate mode)

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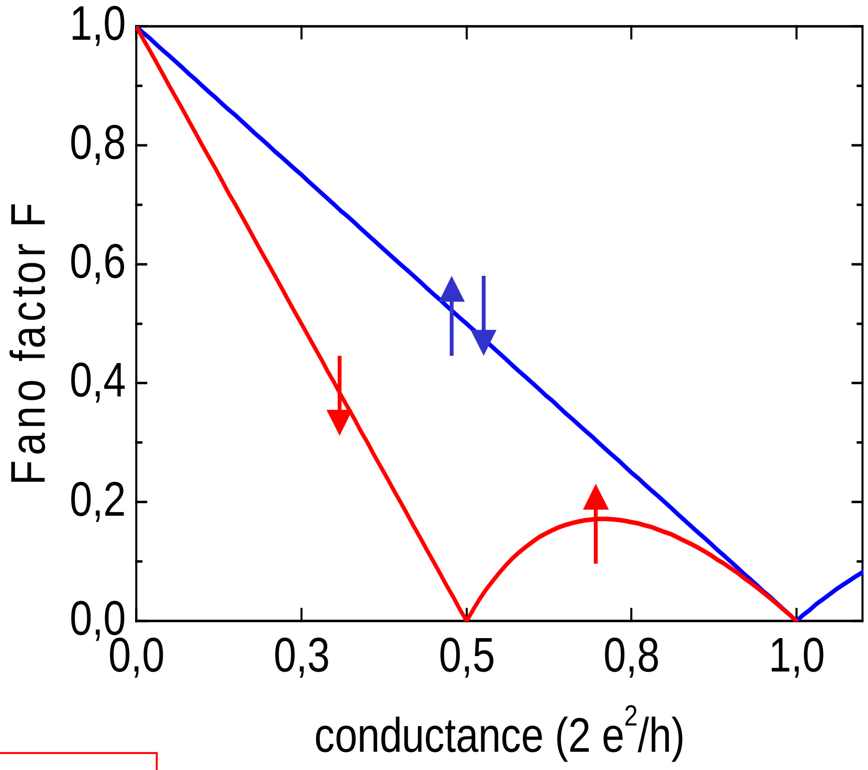
↑↓

↑↑

spin degenerate case

$$\boxed{S_I = 2eI\,(1 - D_{\uparrow\downarrow}) = 2eI\,.\,F_{\uparrow\downarrow}}$$



spin degeneracy fully lifted:

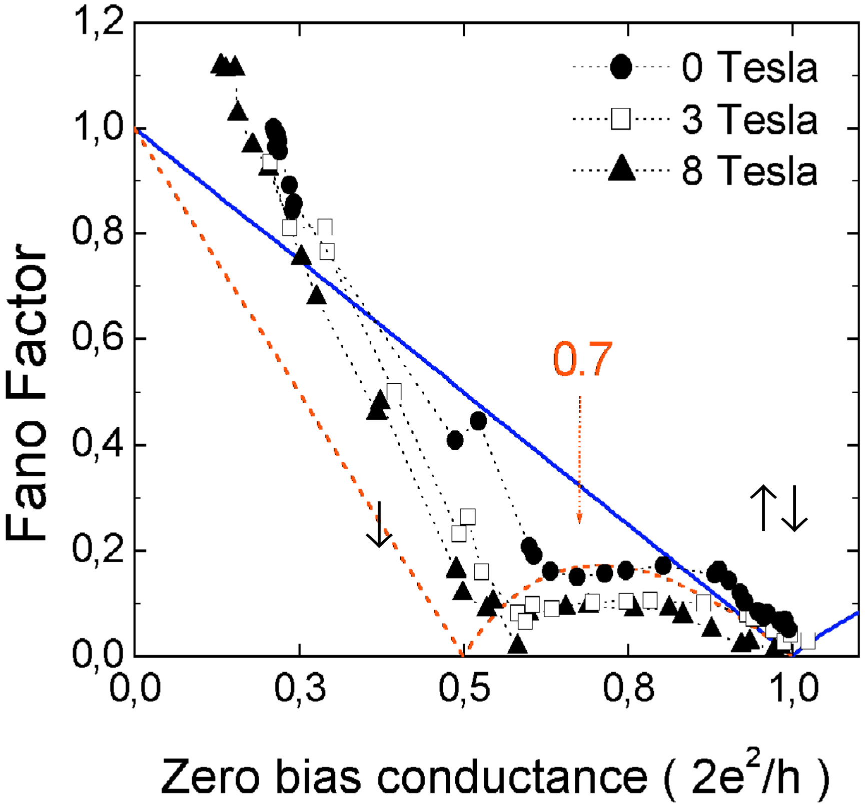
$$\boxed{S_I = 2eI\,\frac{D_\downarrow(1 - D_\downarrow) + D_\uparrow(1 - D_\uparrow)}{D_\downarrow + D_\uparrow} = 2eI\,.\,F_{\uparrow+\downarrow}}$$

conductance can not distinguish between one or two modes

but shot noise can

Here: the Fano factor shows direct signature of **two** modes

→ lifted spin degeneracy scenario.



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