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Comparison of energy and phase relaxation in metallic wires

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We have determined the phase coherence time, τ_ϕ, from magnetoresistance measurements of long, narrow wires of Au, Ag, and Cu, over the temperature range 40 mK-6 K. In the Cu and Au wires, τ_ϕ saturates at low temperature. In the Ag wire, τ_ϕ continues to increase down to the lowest temperatures measured; moreover, its temperature dependence below about 1 K is consistent with theoretical predictions of Altshuler, Aronov and Khmelnitskii published in 1982. These results cast doubt on recent assertions that saturation of τ_ϕ at low temperature is a universal phenomenon in metal wires. We compare these results with those of recent experiments on energy relaxation in similar metallic wires. The results of the two experiments are strongly correlated, suggesting that a single (unknown) mechanism is the cause of the enhanced phase and energy relaxation observed in some samples.

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1. INTRODUCTION

One of the fundamental properties of disordered conductors that has been studied extensively the past 20 years is the phase coherence time of the conduction electrons. At temperatures below about 1 K, the dominant phase-breaking mechanism in nonmagnetic disordered metals is predicted to be electron-electron scattering.[1,2] The corresponding phase-breaking rate decreases as the temperature is lowered as a power law, $\tau_\phi^{-1} \propto T^p$, where p depends on the effective dimensionality of the system. Recent experiments

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and analysis of several older experiments[3] show that the phase coherence time in some samples tends to saturate at a finite temperature, in contrast with theoretical expectations. A second consequence of electron-electron scattering is energy exchange between quasiparticles. Recently, experiments have been performed which provide information on the energy dependence, as well as the overall rate, of energy exchange in mesoscopic wires driven out of equilibrium by a current.[4,5] Those experiments showed that the energy relaxation rate in Cu wires was larger than predicted by theory, and the energy dependence of the scattering rate was different from that predicted.

The purpose of the present work is twofold. First, we aim to shed light on the issue of saturation of the phase coherence time, by measuring the temperature dependence of τ_ϕ in wires of several different metals. Second, we compare our results with those from the energy relaxation experiments, to see if energy and phase relaxation are related in similarly prepared samples. Energy relaxation experiments have now been performed on wires of Ag[6] and Au[7] in addition to the original work on Cu. For purposes of comparison, we have therefore studied phase coherence in Cu, Ag, and Au wires.

2. EXPERIMENT

The wires used in our experiment were deposited on similar substrates, and in the same electron-gun evaporator used to fabricate samples for the energy relaxation experiments.[5,6] The wires were patterned using e-beam lithography and the lift-off process. Sample dimensions are given in Table I. The three samples have very similar resistivity ρ, as deduced from the resistance R, length L, width w and thickness t. The diffusion constants D were deduced from ρ using Einstein's relation. The only difference in the fabrication procedure for these three samples is that the Au sample was deposited on top of a thin (1 nm) layer of Al to improve adhesion to the substrate. Sample widths vary from 65 to 110 nm, hence all the samples are quasi one-dimensional with respect to the phase-breaking length, $L_\phi = \sqrt{D\tau_\phi}$ and the thermal length, $L_T = \sqrt{\hbar D/k_B T}$ over the temperature range studied. The samples were immersed in the dilute phase of the ^{3}He-^{4}He mixture of a dilution refrigerator. Electrical lines to the sample were filtered at the top of the cryostat and again near the sample. Resistance measurements were performed using a standard ac four-terminal technique with a lock-in amplifier. A ratio transformer was used in a bridge circuit to enhance the measurement sensitivity to small changes in sample resistance.

Figure 1 shows the magnetoresistance of the Ag sample at several temperatures. At low temperature, the magnetoresistance is positive, indicating that the spin-orbit scattering length $L_{so} = \sqrt{D\tau_{so}}$ is shorter than L_ϕ. Fig-

TABLE I

Sample material	L (μm)	w (nm)	t (nm)	R (kΩ)	ρ (Ω·nm)	D ($m^2 \cdot s^{-1}$)	L_{so} (nm)
Au	271	100	45	2.0	33	0.010	58.5
Ag	136	65	45	1.4	30	0.012	75
Cu	271	110	45	1.9	35	0.007	520

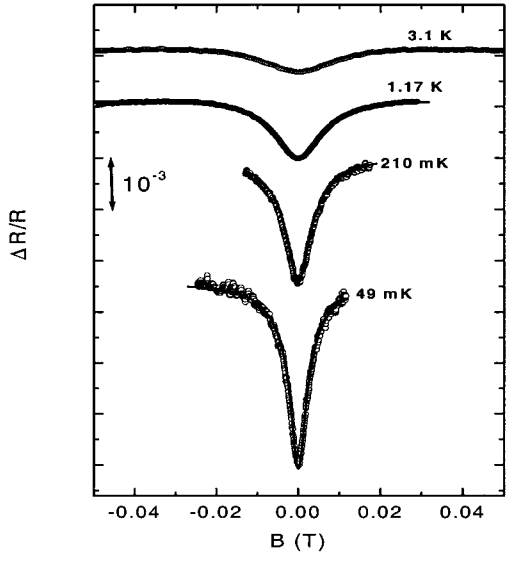

Fig. 1. Magnetoresistance data (circles) and fits to Eq. 1 (solid lines) for Ag sample at different temperatures. The curves are offset vertically for clarity.

ure 1 shows that the magnitude of the weak localization contribution to the magnetoresistance continues to increase down to the lowest temperatures measured. Figure 2 shows similar raw data for the Cu sample. In contrast to the Ag data, the magnitude of the magnetoresistance does not continue to grow at low temperature, but saturates at about 1 K in the Cu sample.

The magnetoresistance is very small in the Au sample, indicating that L_ϕ is very short. In fact, the poor signal-to-noise ratio of the Au data prevented us from obtaining reliable data at temperatures below 1 K. The data we were able to obtain indicate that L_ϕ had already saturated at 6 K.

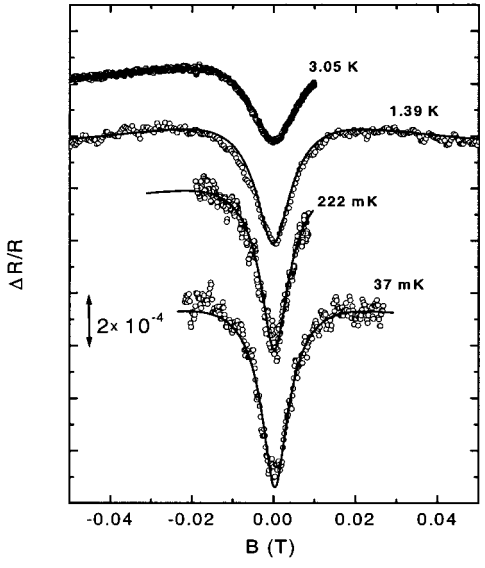

Fig. 2. Magnetoresistance data (circles) and fits to Eq. 1 (solid lines) for Cu sample at different temperatures. The curves are offset vertically for clarity. Note the different vertical scales in Figs. 1 and 2.

3. ANALYSIS AND DISCUSSION

To obtain estimates of L_ϕ and hence τ_ϕ, we fit our data to the weak localization expression for the magnetoresistance in 1D:[8]

$$\frac{\Delta R}{R} = \frac{e^2 R}{\pi\hbar L}\left\{\frac{3}{2}\left[\frac{1}{L_\phi^2}+\frac{4}{3L_{so}^2}+\frac{1}{3}\left(\frac{wBe}{\hbar}\right)^2\right]^{-1/2} - \frac{1}{2}\left[\frac{1}{L_\phi^2}+\frac{1}{3}\left(\frac{wBe}{\hbar}\right)^2\right]^{-1/2}\right\} \quad (1)$$

where we have omitted terms that describe spin-flip scattering by magnetic impurities. Reliable determination of L_{so} from the fit is only possible at the higher temperatures shown, where the magnetoresistance changes slope at high fields. At lower temperatures, we fix the value of L_{so} and fit to the single free parameter L_ϕ.

Echternach et al.[9] have emphasized that Eq. (1) is not strictly valid at temperatures where the dominant phase-breaking mechanism is electron-electron scattering with small energy transfer, also called Nyquist scattering.[1] In that low-temperature limit, phase differences accumulate slowly over many collisions, hence dephasing is not an exponential process. We refer the reader to Ref. 9 for a complete discussion of this issue. Here it suffices to say that experimental data does not allow one to distinguish between the correct magnetoresistance formula in the Nyquist regime and Eq. (1). The reason is that the function that enters in the Nyquist regime, $f(x) = Ai(x)/Ai'(x)$, with Ai the Airy function, is well approximated for all x by $-(0.5+x)^{-1/2}$, with corrections of at most 4%. Use of this approximation in the correct magnetoresistance formula[10] results in Eq. (1) with $\tau_\phi^{-1} = 0.5\tau_N^{-1} + \tau_{\phi 0}^{-1}$, where τ_N^{-1} is the Nyquist dephasing rate and $\tau_{\phi 0}^{-1}$ is the dephasing rate due to electron-phonon scattering and any other mechanisms. Realistically, τ_ϕ defined in this way is the only time accessible to experiment.

Figure 3 shows τ_ϕ versus temperature for our three samples. The differences, already apparent in the raw data, are striking. In the Ag sample, τ_ϕ continues to increase down to the lowest temperatures measured. In the Cu sample, τ_ϕ saturates below 1 K at a value of 2 ns, while in the Au sample, τ_ϕ saturates already at 6 K at a value of 10 ps. The theory of electron-electron scattering in disordered conductors predicts that τ_ϕ should increase as a power law as the temperature is lowered. For the 1D case, the predicted behavior in the Nyquist regime[1,10] is $\tau_\phi = 2\tau_N = 2\left(\frac{e^2R}{\hbar L}\frac{k_B T\sqrt{D}}{\hbar}\right)^{-2/3}$. To see if our Ag data is consistent with this prediction, we have fit the temperature dependence of τ_ϕ to the functional form:

$$\tau_\phi^{-1} = AT^p + BT^3 \tag{2}$$

where the second term is meant to describe electron-phonon scattering at higher temperatures.[11] The best values of the three fit parameters are p=0.61, A=0.7 $\mathrm{ns^{-1}K^{-0.61}}$, and B=55 $\mu\mathrm{s^{-1}K^{-3}}$, compared to the theoretical predictions p=2/3, A=0.6 $\mathrm{ns^{-1}K^{-2/3}}$. Thus the Ag data are in agreement with the theory of Nyquist dephasing in both the overall magnitude and temperature dependence.[12] These results are consistent with those of Wind et al.,[13] who measured both the magnitude and width dependence of the Nyquist dephasing rate in Ag and Al wires above 2 K, and with those of Echternach and al.,[9] who found agreement with theory for the dephasing rate of Au samples down to 100 mK. Our results thus extend by a factor of two lower temperature the experimental confirmation of the Nyquist scattering theory.

What conclusions can we draw from these results? First, our Ag data contradict the recent experimental claim that saturation of τ_ϕ is universal in

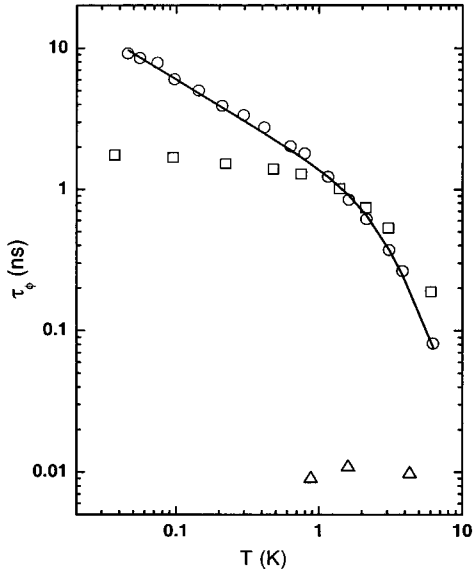

Fig. 3. Phase coherence time for the Ag(○), Cu(□) and Au(△) samples as a function of temperature. The solid line is a fit of the Ag data to Eq. (2).

disordered metal wires.[3] The proposed theoretical expression for the maximum value of τ_ϕ presented in that paper gives a result $\tau_{\phi max} = 2$ ns for our Ag sample, about 5 times shorter than what we observe at our lowest temperature. Second, since the macroscopic parameters (physical dimensions and resistivity) of our three samples are nearly identical, any theoretical model that predicts a maximum value of τ_ϕ based on those parameters alone must be incorrect.[14] Our work thus lends experimental support to recent theoretical papers[15,16] refuting the claim that saturation of τ_ϕ is universal. On the other hand, our Cu and Au data show that some samples do show saturation of τ_ϕ. Since all three of our samples were measured in the same cryostat under the same conditions, it is unlikely that the saturation is due to interference from external electromagnetic radiation, as was recently proposed.[15] The cause of the saturation is not yet known, but we will mention some recent proposals at the conclusion of this paper. The small value of τ_ϕ in our Au sample is curious, when compared with data from other workers.[3,9] Fur-

ther measurements on other samples will reveal whether the Al underlayer has any influence on the dephasing rate.

4. COMPARISON WITH ENERGY RELAXATION

We now compare these results with those from the energy relaxation experiments.[4–7] The analysis of those experiments was performed within the framework of the quantum Boltzmann equation.[17,18] The results of the analysis were expressed in terms of a kernel function, $K(\varepsilon)$, which describes the scattering rate between quasiparticles as a function of the energy ε exchanged in the interaction. In the Appendix, we present a heuristic argument relating $K(\varepsilon)$ to the dephasing rate ${\tau_\phi}^{-1}$. The main result of that calculation is that if $K(\varepsilon) \propto \varepsilon^{-\alpha}$ with $\alpha > 1$, then the dephasing rate should have the temperature dependence $\tau_\phi^{-1} \propto T^{1/\alpha}$. Starting with Ag, Pierre et al.[6] have found that with $\alpha \approx 1.5$ in some samples, and $\alpha = 1.2$ in others. The former result is in agreement with the theory of electron-electron interactions,[2] and is consistent with our observation that ${\tau_\phi}^{-1} \propto T^{0.61}$ at low temperatures. Furthermore, the prefactor in $K(\varepsilon)$ is close to the theoretical prediction, consistent with our similar observation for the dephasing rate. In Cu and Au, the situation is more subtle. The original energy relaxation experiments[5] in Cu found $K(\varepsilon) = \tau_0^{-1}\varepsilon^{-2}$, with $\tau_0 \approx 1$ ns. The characteristic scattering time of 1 ns is very close to the dephasing time we observe at low temperature, but the energy dependence of $K(\varepsilon)$ is not consistent with our observation of τ_ϕ saturation. According to the derivation given in the Appendix, $K(\varepsilon) \propto \varepsilon^{-2}$ should lead to ${\tau_\phi}^{-1} \propto T^{1/2}$. In Au, the situation is similar. The energy relaxation experiments[7] find $K(\varepsilon) = \tau_0^{-1}\varepsilon^{-2}$, with $\tau_0 = 100$ ps, whereas we observe saturation of τ_ϕ at 10 ps. We note that the Au samples used in the energy relaxation experiment did not have the thin Al underlayer.

The fact that the energy dependence of $K(\varepsilon)$ deduced from the energy relaxation experiments in Cu and Au is not consistent with the observed temperature dependence of τ_ϕ may simply be telling us that electron-electron collisions are not responsible for one or both of those observations. The analysis of the energy relaxation experiments assumes that two-body electron-electron collisions are the dominant energy exchange process in the sample. The relation between $K(\varepsilon)$ and τ_ϕ presented in the Appendix assumes the same for dephasing. Since the results of neither experiment agree with the theoretical prediction for electron-electron scattering, it is perhaps not surprising that our comparison of the results from within that framework leads to a contradiction.

5. CONCLUSIONS

What then is the cause of the saturation of τ_ϕ we observe in our Cu and Au samples? It was already argued by Mohanty et al. that magnetic impurities are unlikely to be the cause of the saturation, since samples with magnetic impurities deliberately added do not exhibit a saturation of the dephasing rate.[3] Our results support that conclusion. Although it is well known that magnetic impurities can lead to dephasing via spin-flip scattering, it seems unlikely that they could provide an efficient mechanism of energy exchange in the absence of an external magnetic field. There have been recent suggestions that two-level systems may be responsible.[16,19] One of those proposals[16] relates the dephasing rate to the level of $1/f$ noise in the sample at frequencies close to τ_ϕ^{-1}. That proposal relies on assumptions about the distribution of tunneling centers in disordered metals that have not been directly tested by experiment. The second proposal invokes two-level systems with nearly degenerate ground states, which act as two-channel Kondo impurities.[19] It is argued that such systems can lead to a temperature-independent dephasing rate over a limited temperature range, below which the rate must tend to zero. It remains to be seen if either of these ideas can explain the intriguing results regarding both phase and energy relaxation in metal wires.

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APPENDIX

To facilitate comparison of the energy relaxation and phase relaxation experiments, we present a heuristic derivation of the dephasing rate one obtains from a given $K(\varepsilon)$.[20] The out-scattering term in the Boltzmann equation takes the form:

$$I^{out}(E,\{f\}) = \int d\varepsilon dE' K(\varepsilon) f(E)[1-f(E-\varepsilon)]f(E')[1-f(\varepsilon+E')] \quad (3)$$

where we have suppressed all reference to spatial variables. We assume that the dephasing rate at temperature T is equal to the out-scattering rate with the initial state occupied and with equilibrium Fermi-Dirac distribution functions f_{FD}. Within the framework of Eq. (3), it makes no sense to include

scattering events with energy transfer less than the dephasing rate itself, hence we set the lower limit of the integral to $\hbar/\tau_\phi$.[20] The upper limit is cut off at $\approx k_B T$ by the availability of unoccupied final states. We then have:

$$\tau_\phi^{-1} \approx \int_{\hbar/\tau_\phi}^{k_B T} d\varepsilon K(\varepsilon) g(\varepsilon) \tag{4}$$

where

$$g(\varepsilon) = \int dE' f_{FD}(E')[1 - f_{FD}(E' + \varepsilon)] = \varepsilon[1 - e^{-\varepsilon/k_B T}]^{-1} \tag{5}$$

For $0 \leq \varepsilon \leq k_B T$, $g(\varepsilon) \approx k_B T$, so the final result is:

$$\tau_\phi^{-1} \approx k_B T \int_{\hbar/\tau_\phi}^{k_B T} d\varepsilon K(\varepsilon) \tag{6}$$

If $K(\varepsilon) \propto \varepsilon^{-\alpha}$ with $\alpha > 1$, then the integral is dominated by the low-energy limit, and the dephasing rate has the temperature dependence ${\tau_\phi}^{-1} \propto T^{1/\alpha}$. For the case of disordered metals of dimension d, the theoretical prediction[2] is $K(\varepsilon) \propto \varepsilon^{(d-4)/2}$. In 1D, this leads to the well-known result[1] ${\tau_\phi}^{-1} \propto T^{2/3}$.

We note here some possible limitations of the approach taken above. Most notably, Eq. (4) indicates that dephasing occurs by single scattering events with energy transfer greater than $\hbar/\tau_\phi$. This is in contrast with the discussion given in the original theoretical papers,[1] where dephasing is described by a process of gradual accumulation of phase during many collisions with energy transfer $< \hbar/\tau_\phi$. Second, the approach shown here does not address subtle differences between dephasing rates measured in different experiments. Recently it has been shown both theoretically[21] and experimentally[22] that the dephasing rate measured in weak localization experiments is different from that measured in universal conductance fluctuation experiments. Nevertheless, we find it remarkable that the simple argument outlined above gives the correct temperature dependence of the dephasing rate from the energy dependence of $K(\varepsilon)$.

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12. During the course of the Ag experiment, we slightly underestimated the effect of heating due to the drive current. Since electron cooling at the lowest temperatures occurs almost entirely by thermal conduction through the leads, one can calculate the effect of heating using the Weidemann-Franz relation and the known sample geometry. The drive current at the lowest three temperatures shown was 5 nA, which at 50 mK causes a temperature rise of about 5 mK at the center of the wire. Use of a lower drive current might result in a slightly larger value of p from the fit. This error was corrected in the Cu experiment, where electron heating due to the drive current was kept below a few percent of the lattice temperature.
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