

Multi-terminal Josephson junctions as topological materials

Happy birthday!
Dear Quantronics



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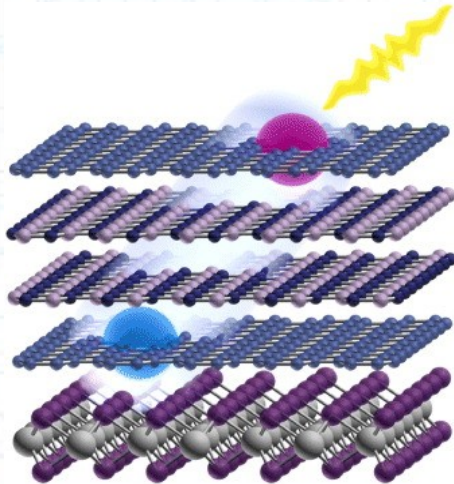
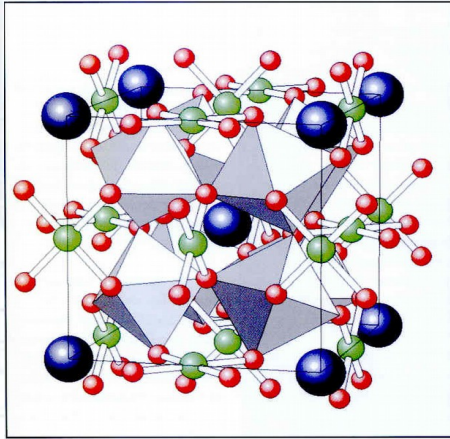
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*nano*SCIENCES
FOUNDATION



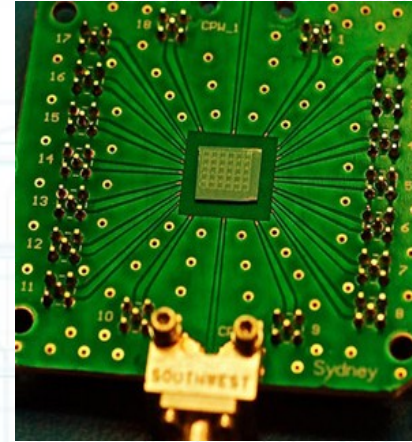
The summary

material



material :
say, 5 d

device



Nanodevice
(multi-terminal
sup.junction)

The back-to-the-future summary

- **1982** Quantum Hall Effect
- A challenge to super-electronics
- 1988 Likharev Zorin: Bloch+Josephson synchro
- 2013 Hriscu Nazarov: synchro!
- **Now: 2015** simplest way to achieve QHE

Outline

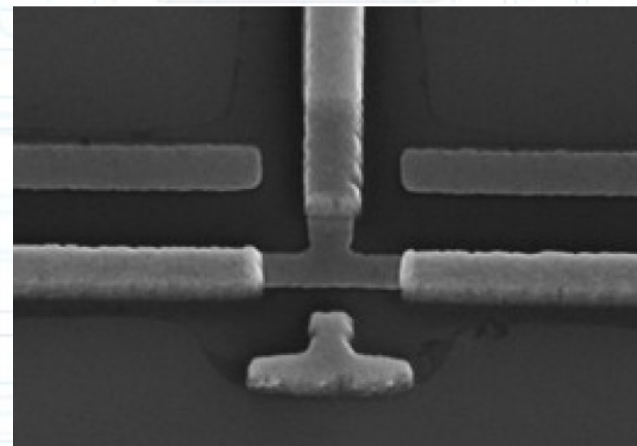
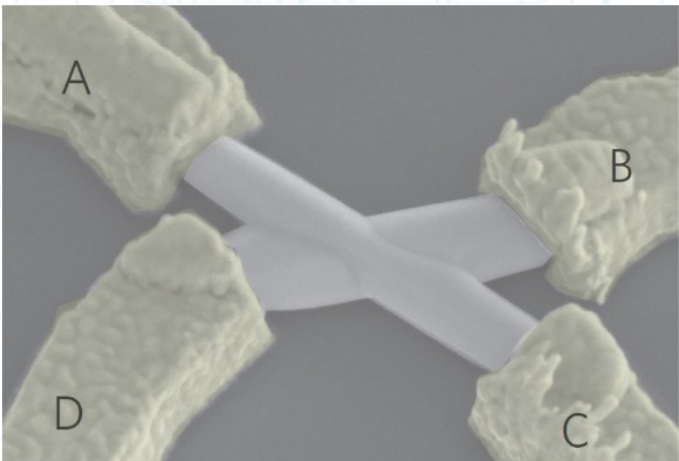
- Topological materials
- Superconducting junctions
- The analogy
- Weyl singularities in 3D
- Tunable 2D from 3D
- Transconductance quantization =>
Quantum Hall effect
- Perspectives

Topological materials

- Bandstructure: Eigenenergies $E(q_x, q_y, q_z)$
- From eigenstates: Berry curvature field
- $$B_z(\vec{q}) = i \left\langle \frac{\partial \Psi}{\partial q_x} \middle| \frac{\partial \Psi}{\partial q_y} \right\rangle - \left\langle \frac{\partial \Psi}{\partial q_y} \middle| \frac{\partial \Psi}{\partial q_x} \right\rangle$$
- 2D topological invariant (Chern number)
- $$C = \frac{1}{2\pi} \int dq_x dq_y B_z(\vec{q})$$
- Manifestation: Quantum Hall effect
→ quantized transconductance
- +much-much more (\mathbb{Z}_2 , spin Hall, surface modes, surface structuring, 1d wires, Majorana)

Superconducting junctions

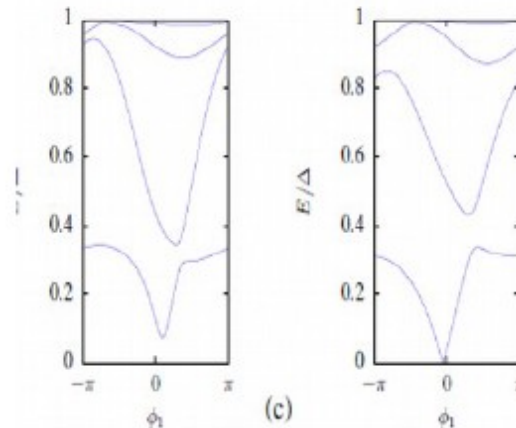
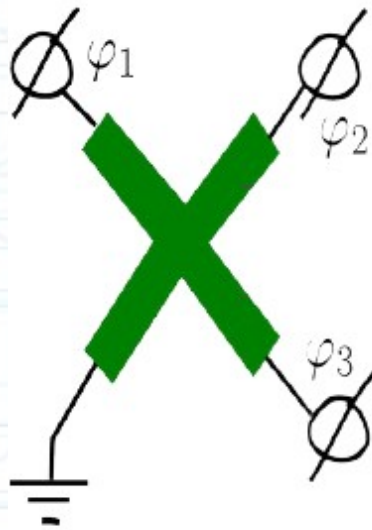
- Josephson junction $E = -E_J \cos \varphi$
- More transparent – Andreev bound states
- $E = -\frac{1}{2} \sum_p E_p$
- $E_p = \Delta \sqrt{1 - T_p \sin(\varphi/2)^2}$
- More terminals – more superconducting phases
- same Andreev states $E_p(\phi_1, \dots, \phi_{n-1})$
- same Andreev states



The analogy

- Bandstructure \Leftrightarrow Andreev spectrum

- $q_x, q_y, q_z \leftrightarrow \varphi_1, \varphi_2, \varphi_3$

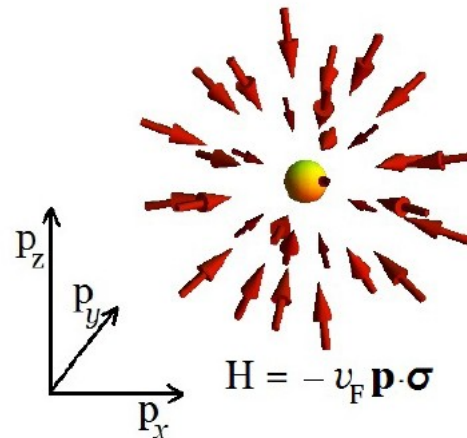
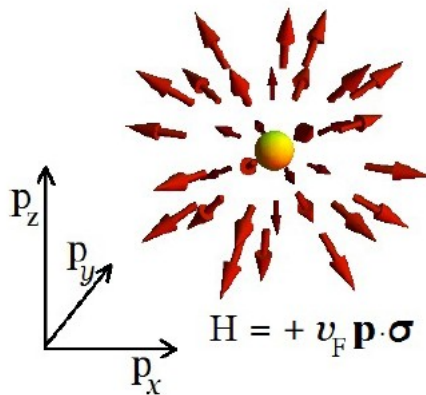
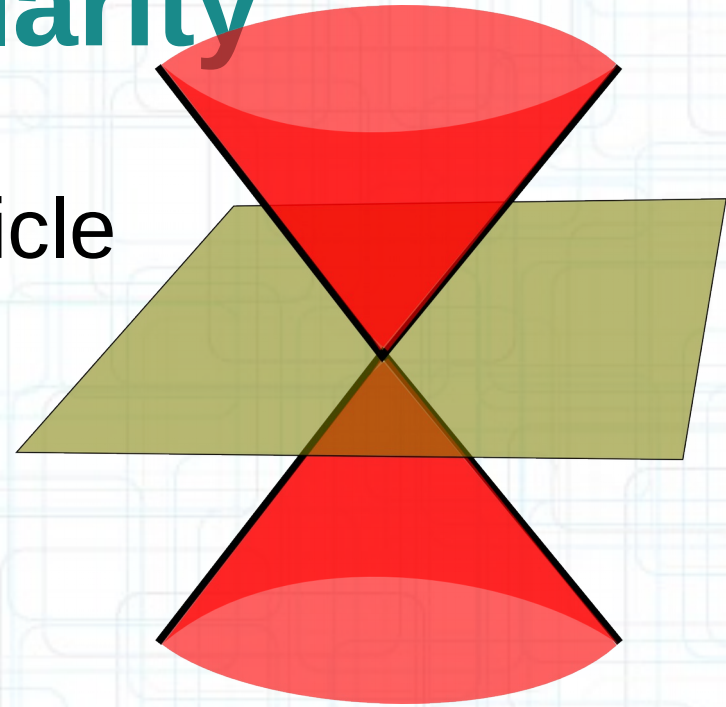


- Incomplete

- Gap edge, continuous spectrum
- Filling: all quasimomenta, phases - one

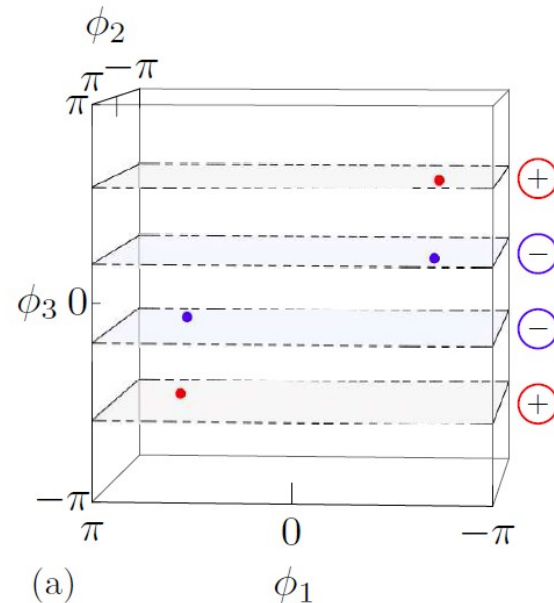
Weyl singularity

- Massless relativistic particle
- Conical spectrum
- In the vicinity of certain q
- $\hat{H} = \vec{\sigma} \cdot (\delta\vec{q})$
- Occurrence in materials – predicted :)
- A monopole of the Berry curvature field



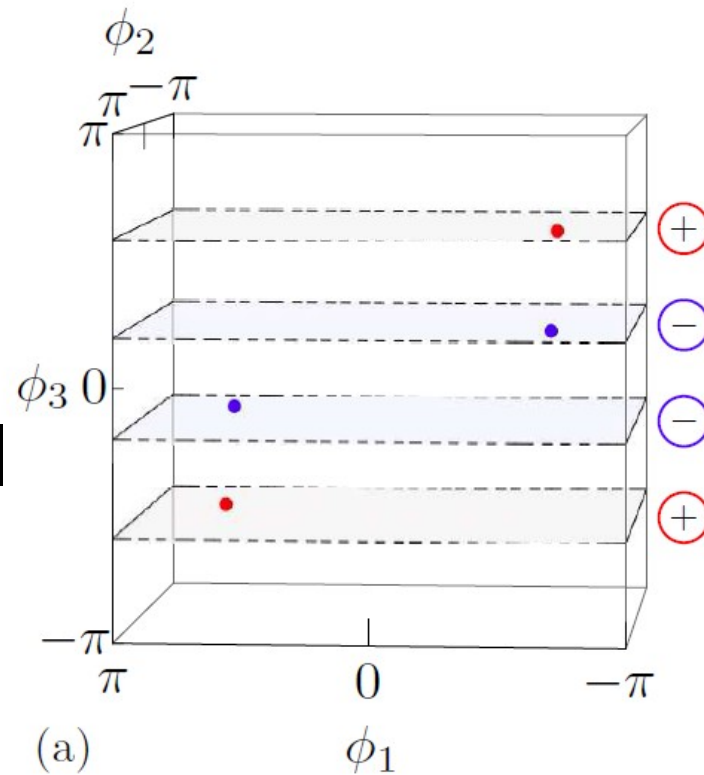
Weyl singularities in 4-terminal junctions

- At zero energy – important
- Three fields – three parameters – three phases – one-goal game
- Beenakker formula – from scatt.matrix
- Come in pairs of 4
- (+- and inversion symmetry)



From 3D to 2D

- Flexibility of dimensions
- 2D bandstructure – a moving plane
- φ_3 – control parameter
- Plane passes W.S =
- Change of Chern number
- Topologically non-trivial
2D material
- Tunable by φ_3



Berry curvature and transport

- Currents – functions of the phases
- Change the phases **adiabatically**

$$I_{\alpha}(t) = \frac{2e}{\hbar} \frac{\partial E}{\partial \phi_{\alpha}} - 2e \dot{\phi}_{\beta} B^{\alpha\beta}$$

Leading order

First correction

- Sensitive to the local Berry curvature

Role of quasiparticles

- Single – level contribution

$$I_{\alpha}(t) = \frac{2e}{\hbar} \frac{\partial E}{\partial \phi_{\alpha}} - 2e \dot{\phi}_{\beta} B^{\alpha\beta}$$

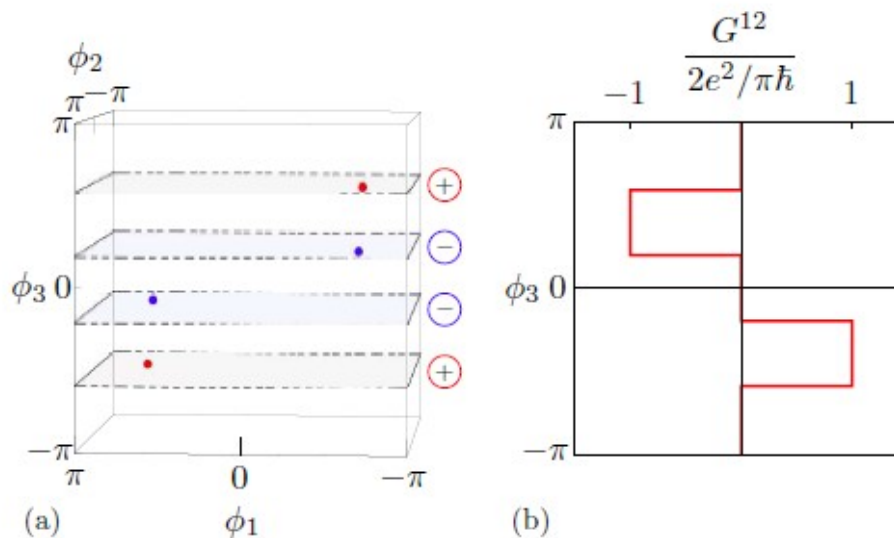
Leading order

First correction

- *No qp: I*
- *1 qp: 0*
- *2qp: $-I$*
- Sensitive to the quasiparticle poisoning
- Let's be optimistic

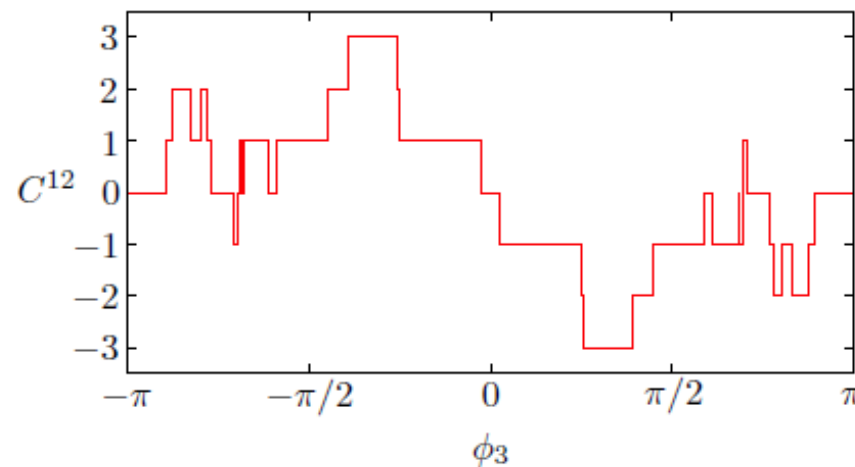
Transconductance quantization

- Apply (incommensurate) voltages
- Phases are swept over BZ
- Sup.current vanishes
- What remains?
- $I_1 = G_{12}V_2; I_2 = -G_{12}V_1; G_{12} \equiv (2e^2/\pi\hbar)C$



Experiment to do

- Instead of Quantum Hall bar
- Make 4 – terminal sup. Junction
- Shake it till Weyl singularities
- Apply voltages to 2 leads
- Measure d.c. currents in the leads
- Tune it by the 3rd phase



Outlook

- Multi -dimensional materials!
- More complex topologies
- Edge? Edge!
 - Structuring in charge space