Architecture for Error Correction: Majority Vote vs. Repeated Parity (Surface Code)

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Figure of Merit:
Threshold
Qubit number

(Difference is double-exponential)

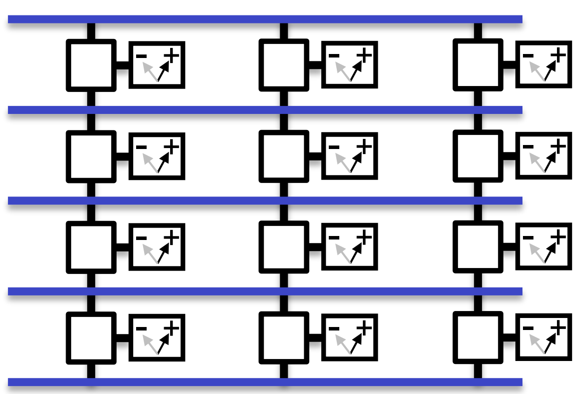
Majority Vote vs. Repeated Parity

Majority Vote

Simple
Data & measure errors same
No threshold

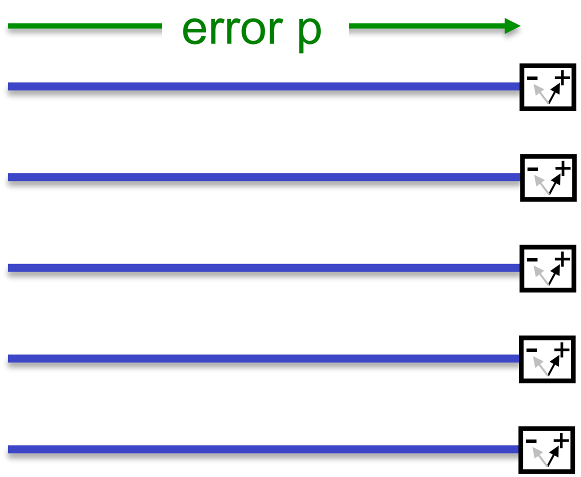
Logical:
00000 or
11111

Repeated Parity



Better resource scaling

Logical Error for Majority Vote



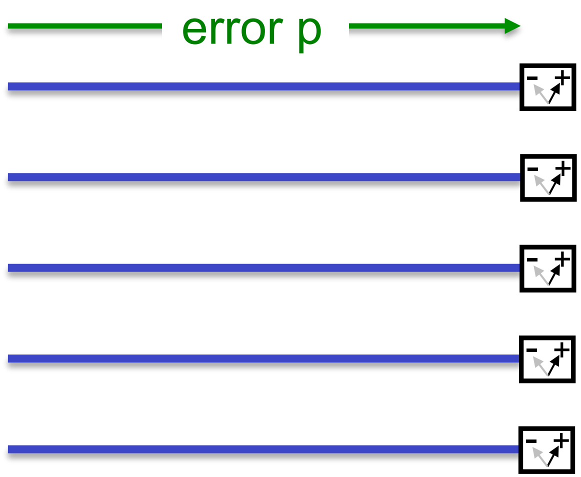
$$P_l = \sum_{j=n/2}^{n} \binom{n}{j} p^j (1-p)^{n-j}$$

$$\cong 0.5\left[4p(1-p)\right]^{n/2}$$

$$\to 0 \quad \text{for} \quad n \to \infty$$

Any information ($p \neq 0.5$) gives subthreshold

Logical Error for Majority Vote



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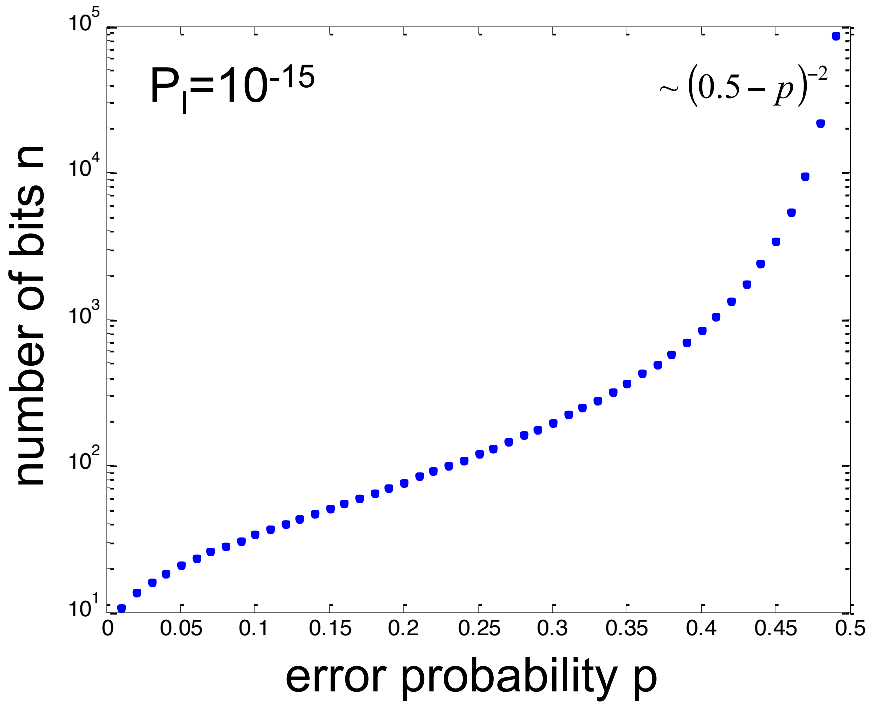
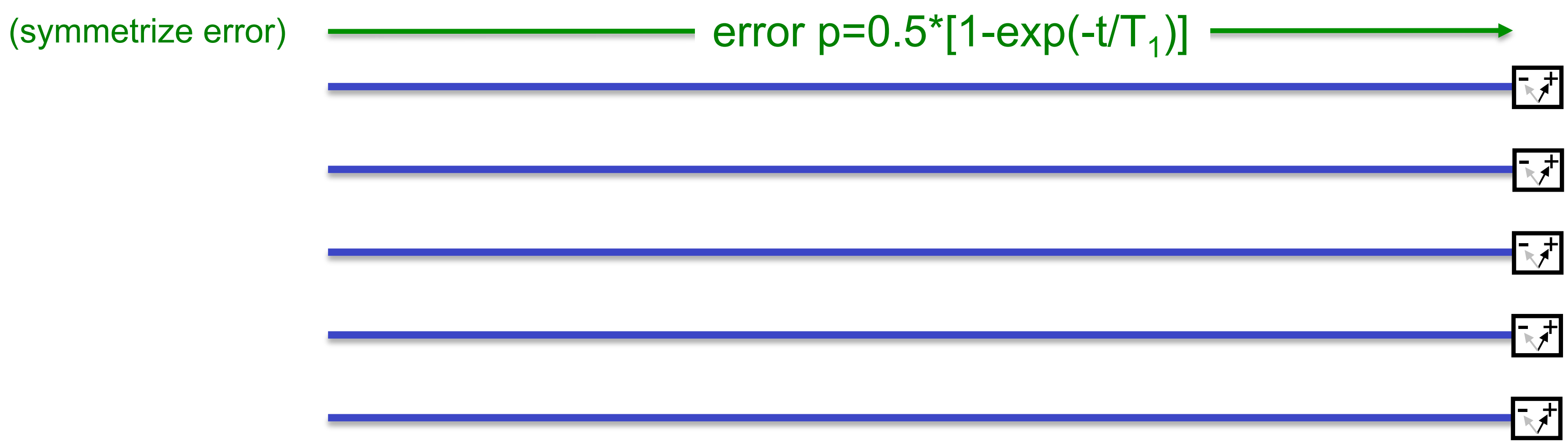


But, number of bits impractical as $p \rightarrow 0.5$

Error of Majority Vote for Long Time



$$n \approx \exp(2t / T_1)$$

Bad to require exponentially large resources

Resource Scaling (CS figure of merit)

x is scaling parameter

examples:
- Google: number of users ~ 10^9
- Error correction: time $x=2t/T_1 \sim 10^9$

resource scaling

	1	hashing
	log x	
	$x^{1/2}$	
	x	
	x log x	FFT, quicksort
	x^2	FT
	x^3	
	…	
majority vote	exp x	

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resource scaling

resource scaling		example
1	↑ scalable	hashing
log x		
$x^{1/2}$		
x		
x log x		FFT, quicksort
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x^3		
…		
exp x		majority vote

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- Google: number of users ~ 10^9
- Error correction: time $x=2t/T_1$ ~ 10^9

resource scaling

resource scaling		examples
1		hashing
log x	repetitive measure	
$x^{1/2}$	scalable ↑	
x		
x log x		FFT, quicksort
x^2		FT
x^3		
…		
exp x	majority vote	

Repeated Parity (Surface Code)

0000… & 1111…
gives same parity

1) Repeated measure: log x scaling

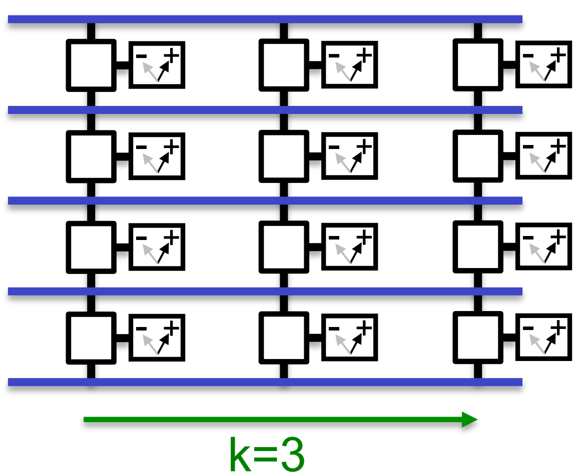
2) Parity measurement is compatible with quantum and surface code

More complex, more error sites

For good scaling, consider only small data errors

$$P_{lk} = 0.5k(4p)^{n/2}$$

Repeated Parity



For 10x each order n/2, want p = 0.025

n/2 scales as log k

Repeated Parity (Surface Code)

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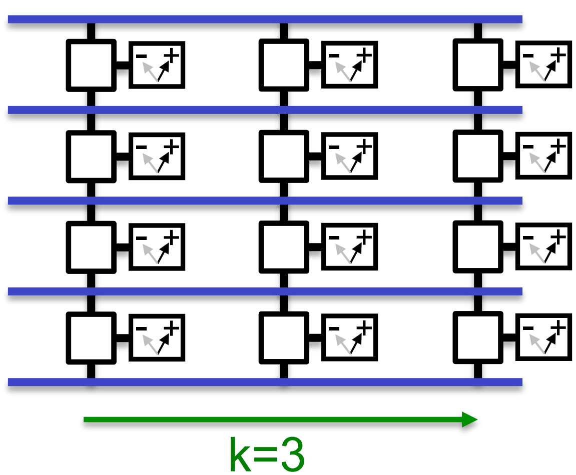
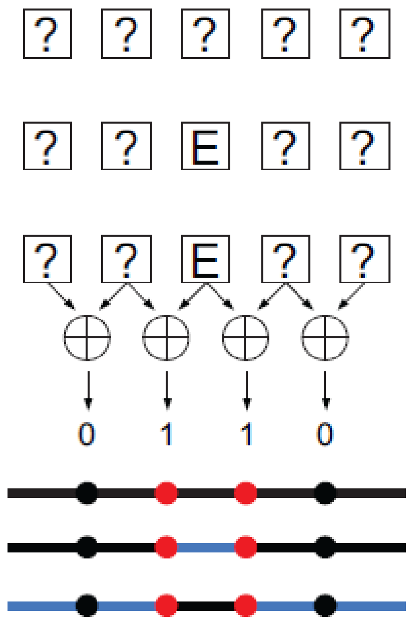
Repeated
Parity



For majority vote, same time

$$P_l = 0.5(4kp)^{n/2}$$

Bit-Flip Error Correction of Data - Decoding



? = 0 or 1

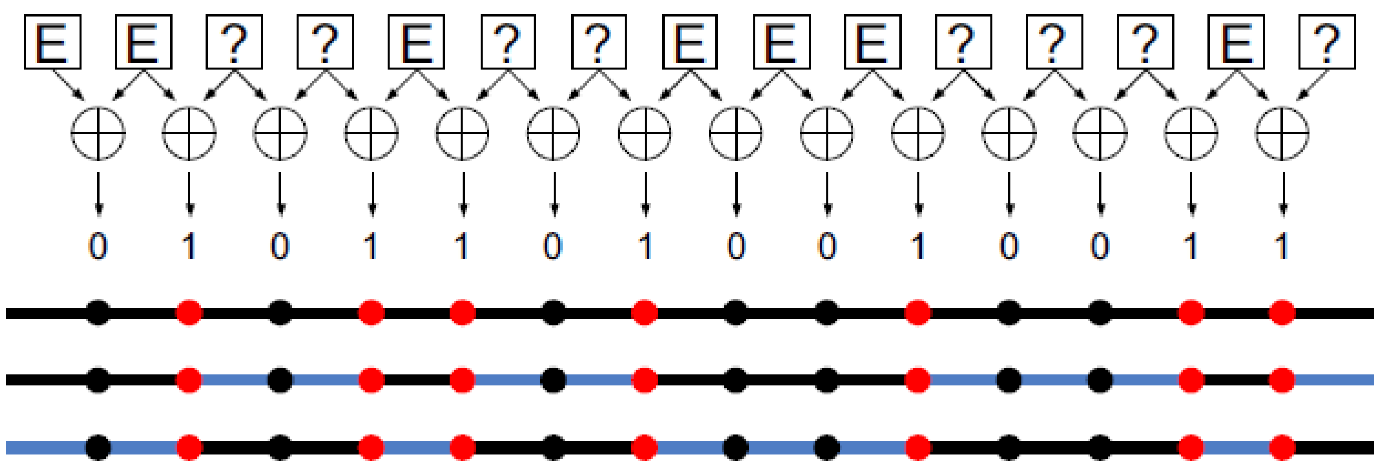
E = bit flip error (with probability p)

XOR measurement of error

Red dot = error detected

Blue line = error decoded into bit flip

Also possible, but less probable (p^4 vs p)



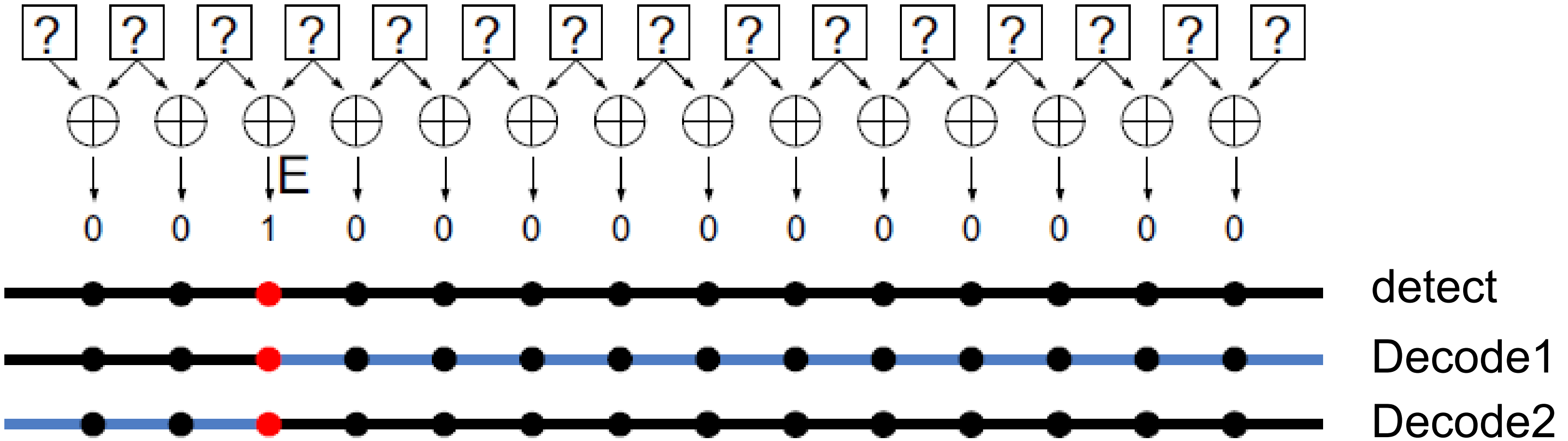
p^8

p^7 more likely

Logical error in decoding with probability

$$P_L \cong \frac{15!}{8!\,7!} p^8$$

Subtlety is for Measurement Errors!

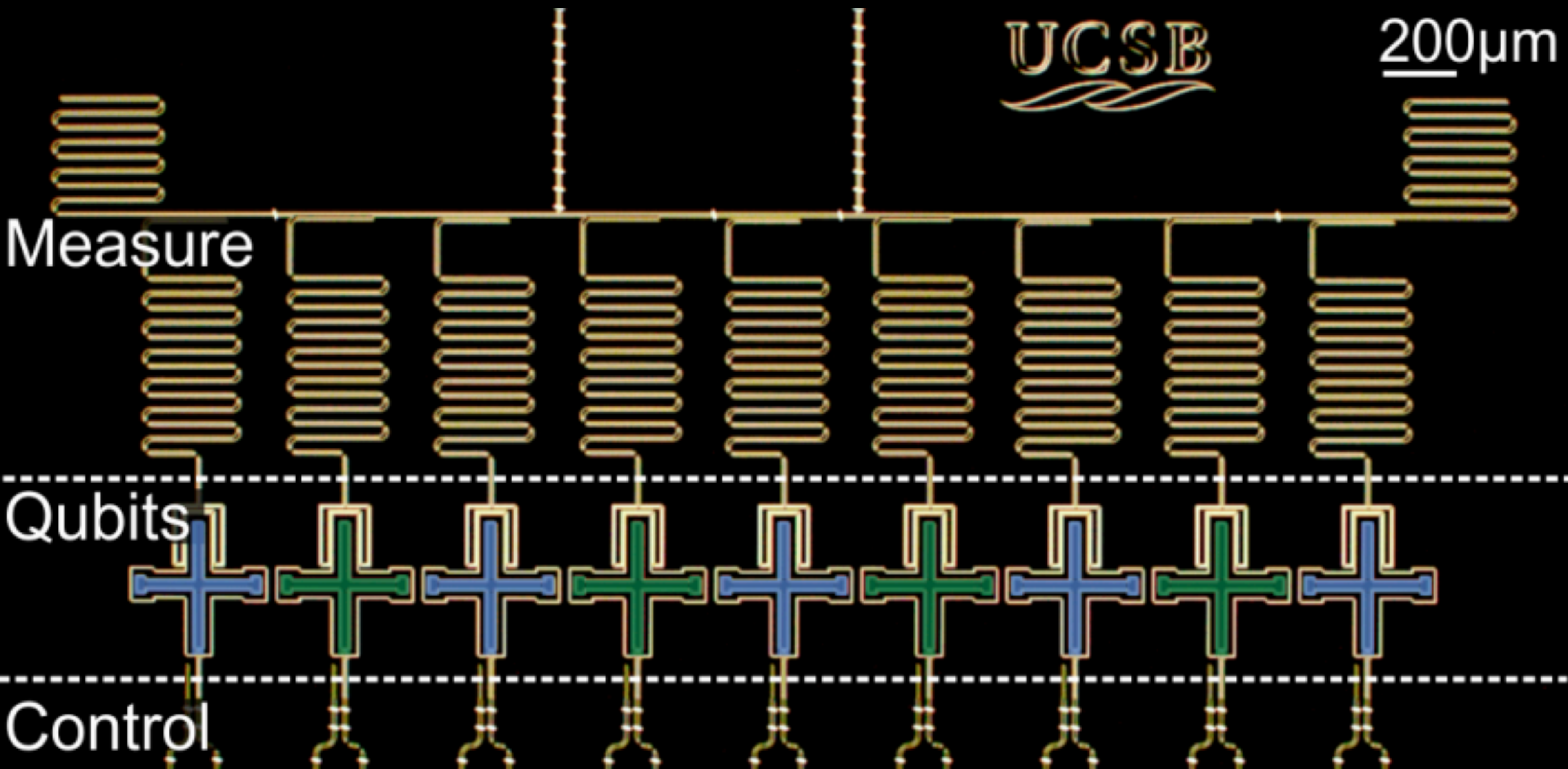


Both decodings are wrong!!

Need to look at measurement vs. time

(The difficultly of quantum hardware)

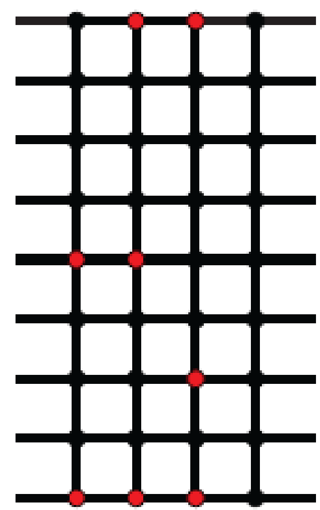
Repetition Code: device



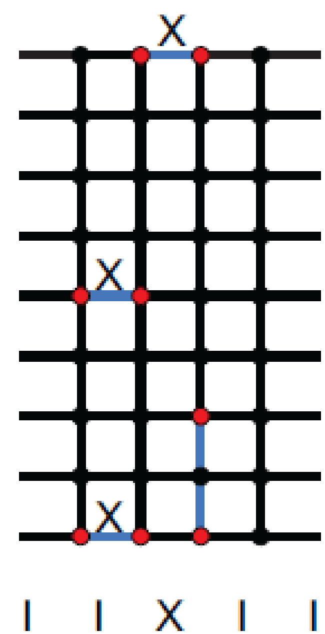
9 Qubit Experiment: Example data

Error detection and decoding:

	raw data	extended	detected errors
in	000000000		
		0 0 0 0	
		0 0 0 0	
1	0 1 1 0	0 1 1 0	. D D .
2	0 0 0 0	0 0 0 0	. . . .
3	0 1 1 0	0 1 1 0	. . . .
4	0 0 0 0	0 0 0 0	. . . .
5	1 0 1 0	1 0 1 0	D D . .
6	0 0 0 0	0 0 0 0	. . . .
7	1 0 0 0	1 0 0 0	. . D .
8	0 0 0 0	0 0 0 0	. . . .
		0 1 1 0	D D D .
fin	0 0 1 0 0		

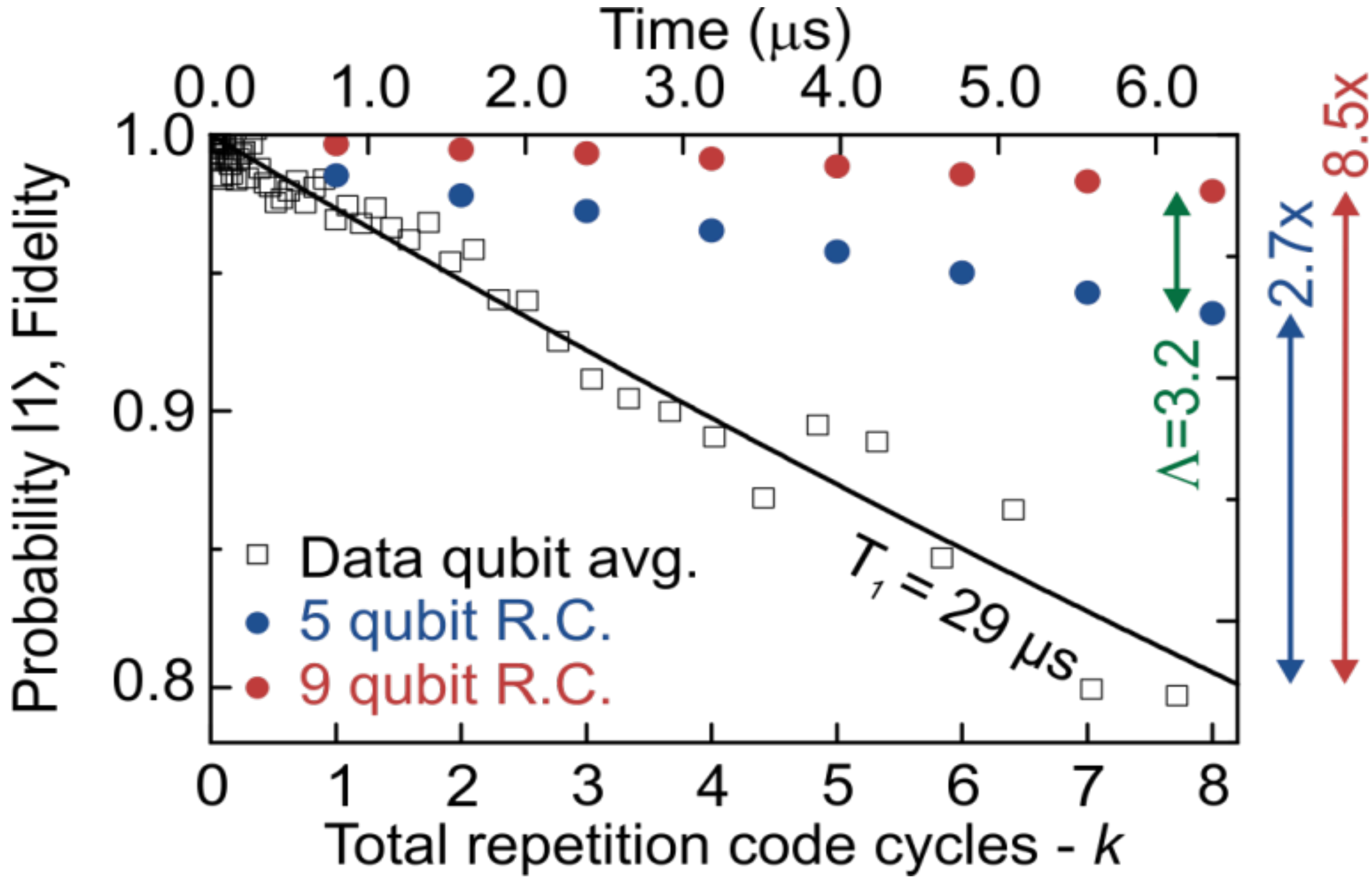


graph



decoding

9 Qubit Data: Bit-Flip Error Correction Works!

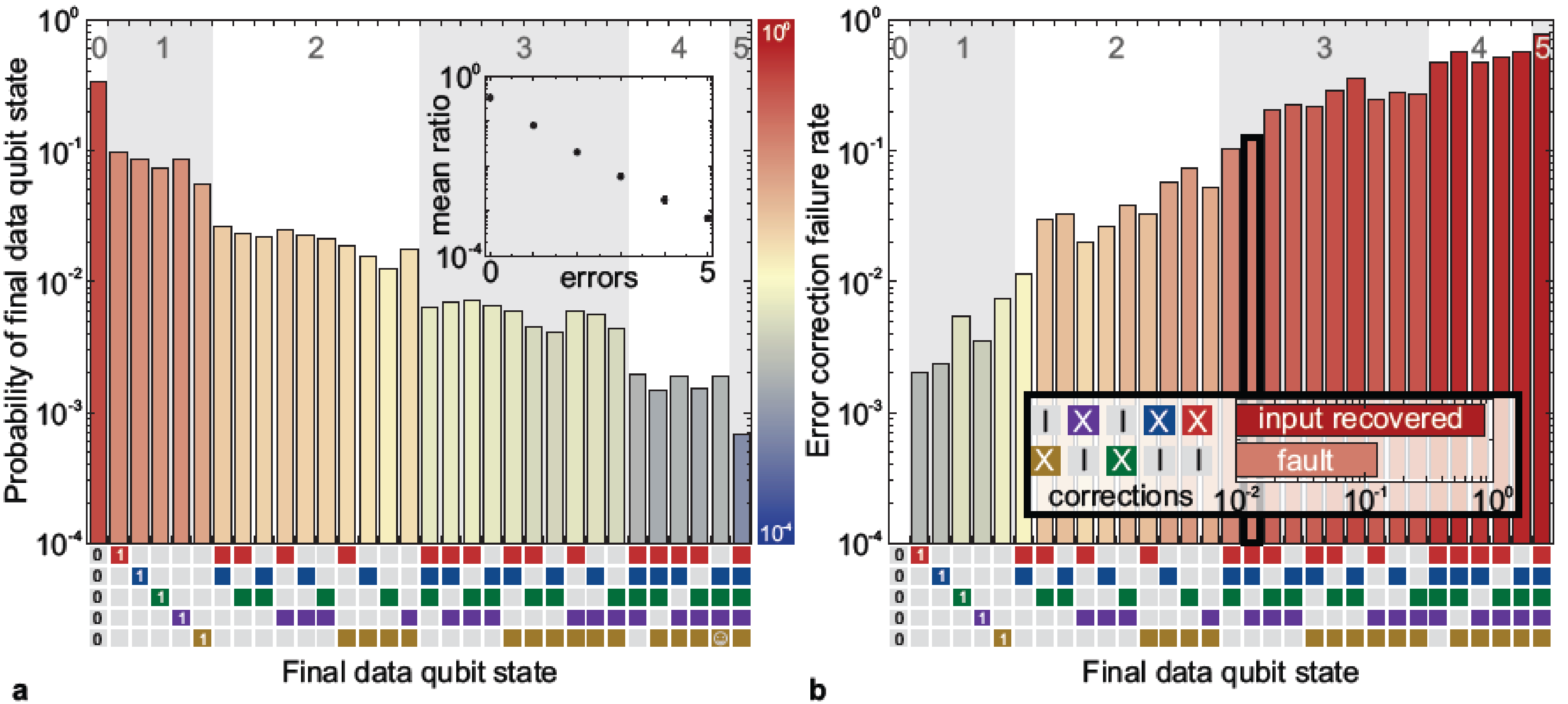


Slower decay of logical qubit state:
1st order: 2.7x
2nd order: 8.5x

Λ = 3.2 > 1, so better memory for higher order (fault tolerant behavior)

Nature, 5 March

Errors for Majority Vote are Corrected



$P_{1,data} \sim 10^{-1}$

$P_3 \sim 8 \times 10^{-3}$

2x data error for each order

Only 20% of 3 errors remain uncorrected

Summary and Conclusions

1) Impact/meaning of “below threshold” depends on resource scaling
   - majority: $\exp(10^9)$
   - repetitive: $\log(10^9)$

2) Need to make gates better