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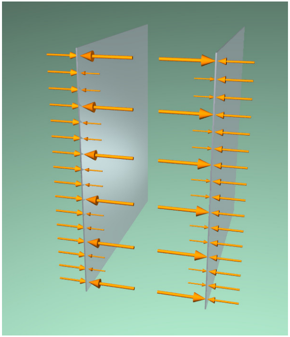
Thermodynamics of the Casimir effect on a transmission line

Gert-Ludwig Ingold
Universität Augsburg

in collaboration with: Astrid Lambrecht (LKB Paris)
Serge Reynaud (LKB Paris)
Marc-Thierry Jaekel (ENS Paris)

Casimir effect

Magdeburg hemispheres (1656)



attractive net pressure due to vacuum fluctuations of the electromagnetic field

→ Casimir effect (1948)

Casimir effect and quantronics

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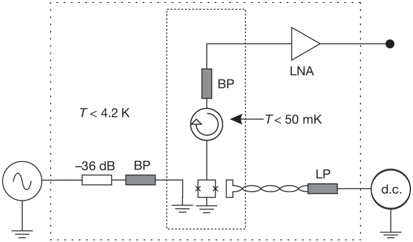
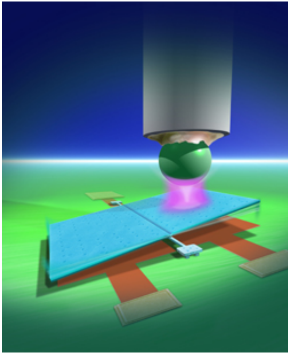
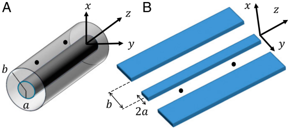
from video *Everything about nothing* ©Thomas Draschan (2007)



C. M. Wilson et al, Nature **479**, 376 (2011)



H. B. Chan et al, Science **291**, 1941 (2001)

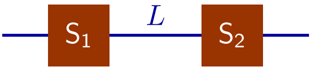


E. Shahmoon et al, Proc. Nat. Acad. Sci. **111**, 10485 (2014)

Scattering approach

$$\mathsf{S} = \begin{pmatrix} r & \bar{t} \\ t & \bar{r} \end{pmatrix}$$

energy shift $\Delta E_{\mathrm{vac}} = \frac{\mathrm{i}\hbar c}{4\pi} \int_0^\infty \mathrm{d}k \ln[\det(\mathsf{S})]$



$$\det(\mathsf{S}) = \det(\mathsf{S}_1)\det(\mathsf{S}_2)\frac{1-[\bar{r}_1 r_2 \exp(2\mathrm{i}kL)]^*}{1-\bar{r}_1 r_2 \exp(2\mathrm{i}kL)}$$

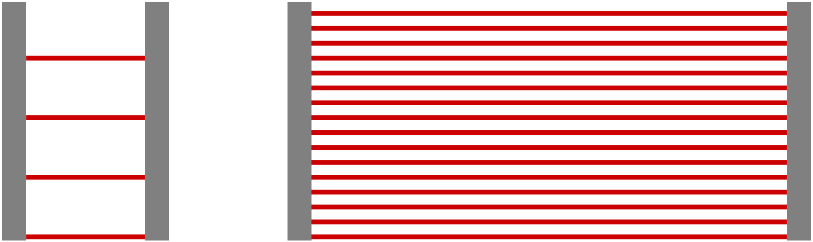
Casimir energy $\Delta E_{\mathrm{vac}}(L) = \Delta E_{\mathrm{vac}} - \Delta E_{\mathrm{vac}}^{(1)} - \Delta E_{\mathrm{vac}}^{(2)}$

M. T. Jaekel, S. Reynaud, J. Phys, I France **1**, 1395 (1991)
GLI, A. Lambrecht, Am. J. Phys. **83**, 156 (2015)

- corresponding differences hold for the Casimir free energy and entropy
- Casimir entropy can be negative

What is the temperature scale?

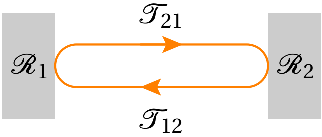
thermal photons also contribute to the Casimir effect



- thermal photons become increasingly important as the distance between the scatterers increases
- dimensionless temperature $\dfrac{k_\mathrm{B}TL}{\hbar c}$
- room temperature corresponds to 7.6µm

Casimir free energy

$$\mathscr{F} = \frac{k_B T}{2} \sum_{n=-\infty}^{+\infty} \ln[\det(1 - \mathscr{R}_1(\xi_n)\mathscr{T}_{12}(\xi_n)\mathscr{R}_2(\xi_n)\mathscr{T}_{21}(\xi_n))] \qquad \xi_n = \frac{2\pi k_B T}{\hbar} n$$



Casimir force

$$F = -\frac{d\mathscr{F}}{dL}$$

- vacuum + thermal photons

Casimir entropy

$$S = -\frac{d\mathscr{F}}{dT}$$

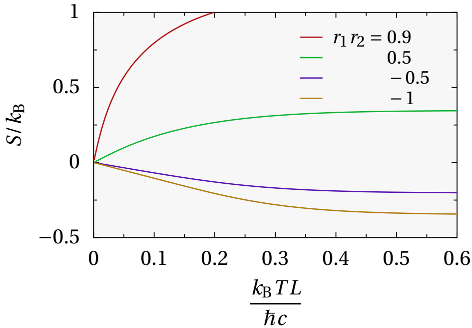
- only thermal photons

Casimir force for perfect mirrors

	$T = 0$	$T \to \infty$
1d, $r_1 = r_2$	$F = \frac{\hbar c \pi}{24 L^2}$	$F = \frac{4\pi (k_B T)^2}{\hbar c} \exp\left(-\frac{4\pi k_B T L}{\hbar c}\right)$
1d, $r_1 = -r_2$	$F = -\frac{\hbar c \pi}{48 L^2}$	$F = -\frac{4\pi (k_B T)^2}{\hbar c} \exp\left(-\frac{4\pi k_B T L}{\hbar c}\right)$
3d, $r_1 = r_2$	$F = \frac{\hbar c \pi^2 A}{240 L^4}$	$F = \frac{k_B T A}{4\pi L^3} \zeta(3)$

- in one dimension, thermal photons suppress the Casimir force
- in three dimensions, an entropic Casimir force occurs in the high-temperature limit

Casimir entropy in 1d

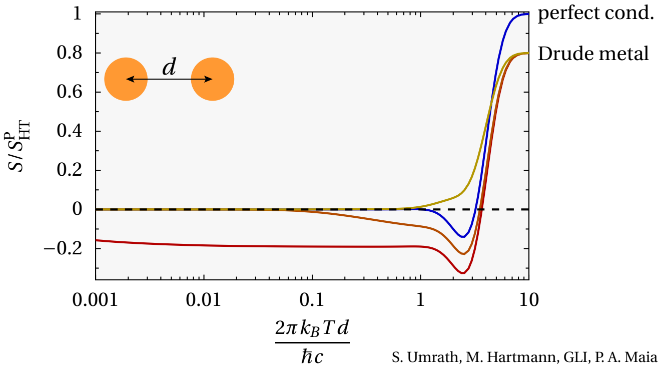


high-temperature limit

$$S = -\frac{k_B}{2}\ln(1 - r_1 r_2)$$

- the Casimir entropy diverges for identical perfectly reflecting scatterers
- in contrast to the 3d case, the Casimir entropy can remain negative in the high-temperature limit
- the Casimir entropy at high temperatures is very sensitive to the reflection properties

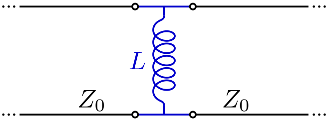
An example in 3d



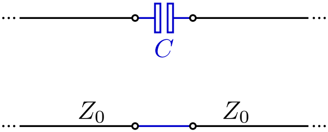
S. Umrath, M. Hartmann, GLI, P. A. Maia Neto (2015)

- positive Casimir entropy at high temperatures
- geometric contribution due to polarization mixing
- dissipative contribution due to TE modes

Tuning the reflection coefficient



$$r(\mathrm{i}\xi) = -\frac{1}{1+\dfrac{2\xi L}{Z_0}} \qquad r(0) = -1$$



$$r(\mathrm{i}\xi) = \frac{1}{1+2\xi C Z_0} \qquad r(0) = 1$$

- scatterers with different phase are relatively easy to realize
- a dissipative element reduces the zero-frequency reflectivity
- dissipation can be described consistently within the scattering formalism

Conclusions

- The Casimir effect in one dimension differs significantly from the three-dimensional case.
- The Casimir entropy in one dimension depends strongly on the reflectivity of the two scatterers. In contrast to the three-dimensional case, it can remain negative even in the high-temperature limit.
- Transmission-line setups allow to tune the reflectivity and might be an interesting tool to study the Casimir effect in one dimension.