

Quantum Ising chain, Majorana fermions and tunable square qubits with quantum phase-slip

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Saclay – Delft connection 25+ years

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PHYSICAL REVIEW LETTERS

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Single Cooper-Pair Tunneling in Small-Capacitance Junctions

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(Received 2 April 1990)

We present observations of charging effects for Cooper pairs in short linear arrays of small-capacitance Josephson junctions. Current-voltage characteristics show a Coulomb gap for Cooper-pair tunneling when the charging energy exceeds the Josephson coupling energy. In a double junction a zero-voltage current is observed that is modulated by a gate voltage applied to the metal island between the junctions. For longer arrays a crossover from Coulomb blockade of Cooper-pair tunneling to a supercurrent is observed when the ratio of Josephson coupling to charging energy is increased.

We are much indebted to D. V. Averin, M. H. Devoret, D. Esteve, M. Peters, U. Geigenmüller, G. Schön, B. van Wees, and P. Hadley for enlightening discussions. We thank the Commissariat à l'Énergie Atomique for hospitality during several visits. This work is supported

CONTROLLED TRANSFER OF SINGLE CHARGE CARRIERS

C. Urbina, H. Pothier, P. Lafarge, P.F. Orfila, D. Esteve and M. Devoret
SPSRM, Centre d'Etudes Nucléaires de Saclay, 91191 Gif-sur-Yvette, France

and

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Abstract

We have designed and operated two devices, the turnstile and the pump, that transfer electrons one by one. They are both based on the existence of stable electrostatic configurations in arrays of ultrasmall tunnel junctions. While the turnstile only works in the normal state the pump could in principle achieve the transfer of single Cooper pairs.

cannot be absorbed. There is a finite tunneling rate only if V_j lies close to V_j^c , within an energy interval determined by the Josephson coupling energy and the fluctuation spectrum of the electromagnetic environment⁶.

The turnstile

The turnstile consists of four nominally identical normal

thank you for the collaboration

keep going, all of you
for the next 30 years

arXiv:cond-mat/0108266v2

19 Aug 2001

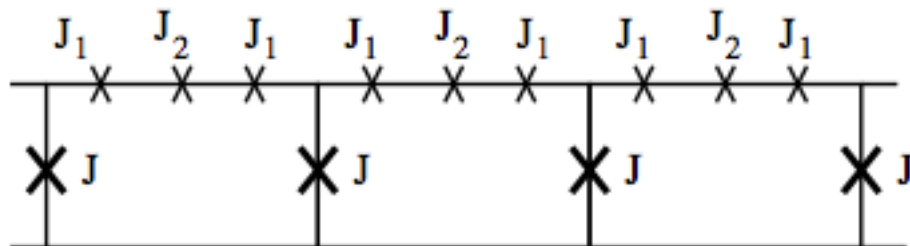
Quantum spin chains and Majorana states in arrays of coupled qubits

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(1) *Physics Department and Center for Materials Science & Engineering, Massachusetts Institute of Technology, 77 Massachusetts Ave, Cambridge, MA 02139*

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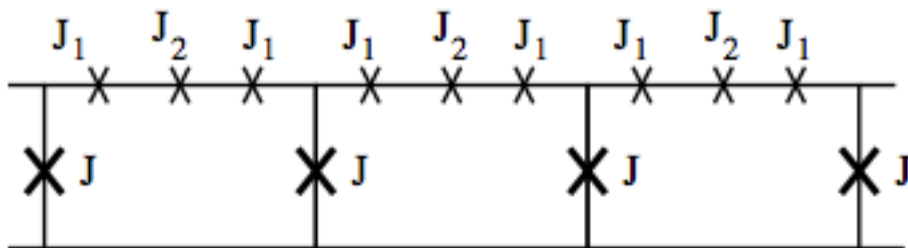
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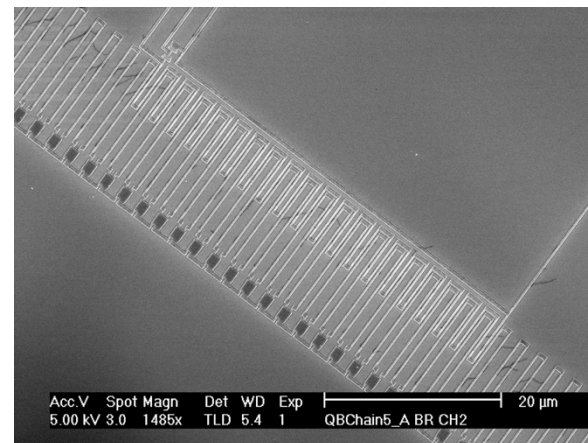
Quantum state transfer in arrays of flux qubits

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Department of Physics and Astronomy, University of Basel,
Klingelbergstr. 82, 4056 Basel, Switzerland

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New Journal of Physics 7 (2005) 181



Floor Paauw 2008

4 (2014) 5535

You, Wang, Zhang, Nori



OPEN

Encoding a qubit with Majorana modes in superconducting circuits

SUBJECT AREAS:

QUBITS

QUANTUM INFORMATION

J. Q. You^{1,2}, Z. D. Wang³, Wenxian Zhang⁴ & Franco Nori^{2,5}

Ising chain

one-dimensional array of $\frac{1}{2}$ spins, one polarization

$$H = -J \sum_{\langle i,j \rangle} \sigma_i^z \sigma_j^z - h \sum_i \sigma_i^z - \Delta \sum_i \sigma_i^x$$

J : exchange constant, interaction between nearest neighbors

$J > 0$ ferromagnetic

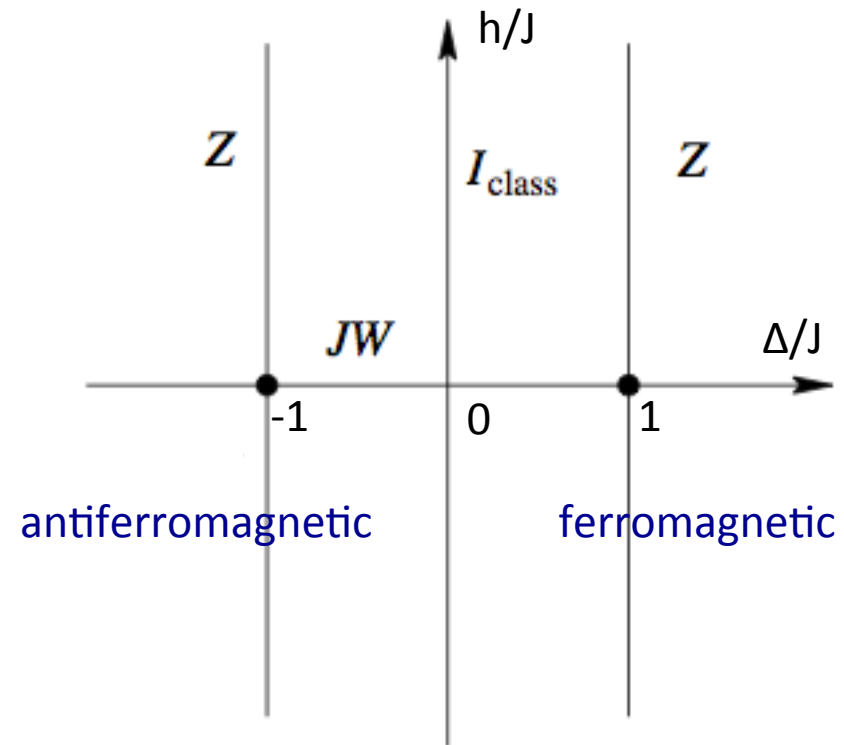
$J < 0$ antiferromagnetic

h : applied magnetic field

Δ : on-site quantum transitions

$h=0$, $|\Delta/J| < 1$:

Jordan-Wigner transformation



$h=0$

$$H = -J \sum_{\langle i,j \rangle} \sigma_i^z \sigma_j^z - \Delta \sum_i \sigma_i^x$$

mapping from $\uparrow\downarrow$ spins at fixed positions to
spinless fermions that can hop along the chain

Jordan-Wigner transformation (1928)

$$\sigma_i^x = 2a_i^\dagger a_i - 1$$

$$\sigma_i^z = (a_i + a_i^\dagger) \prod_{j < i} \sigma_j^x$$

$$H = -\sum_i J (a_i - a_i^\dagger) (a_{i+1} + a_{i+1}^\dagger) - 2\Delta a_i^\dagger a_i$$

$$\{a_i, a_j\} = 0$$

$$\{a_i^\dagger, a_j\} = \delta_{ij}$$

$h=0$

$$H = -J \sum_{\langle i,j \rangle} \sigma_i^z \sigma_j^z - \Delta \sum_i \sigma_i^x$$

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Jordan-Wigner transformation

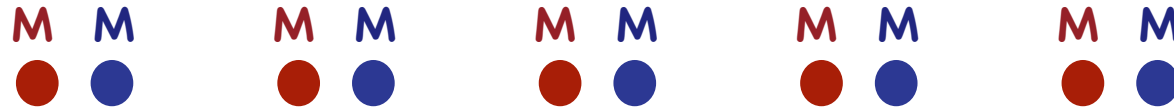
$$\begin{aligned} \sigma_i^x &= 2a_i^\dagger a_i - 1 & \{a_i, a_j\} &= 0 \\ \sigma_i^z &= (a_i + a_i^\dagger) \prod_{j < i} \sigma_j^x & \{a_i^\dagger, a_j\} &= \delta_{ij} \\ H &= -\sum_i J(a_i - a_i^\dagger)(a_{i+1} + a_{i+1}^\dagger) - 2\Delta a_i^\dagger a_i \end{aligned}$$

Majorana fermions:

$$\begin{aligned} \gamma_j^A &= \frac{1}{\sqrt{2}}(a_j + a_j^\dagger) & \{\gamma_j^a, \gamma_{j'}^{a'}\} &= \frac{1}{2} \delta_{aa'} \delta_{jj'} \\ \gamma_j^B &= \frac{1}{i\sqrt{2}}(a_j - a_j^\dagger) & \gamma_j^a \gamma_j^a &= \frac{1}{2} \end{aligned}$$

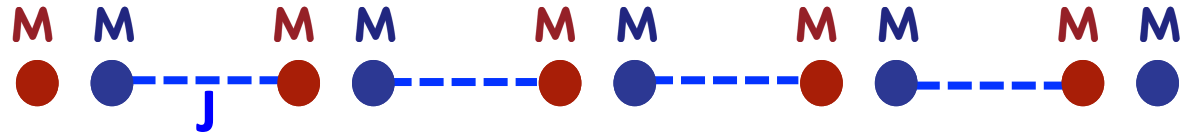
$$H = -2i \sum_{j=1}^n \left(J \gamma_j^B \gamma_{j+1}^A - \Delta \gamma_j^B \gamma_j^A \right)$$

Majorana chain

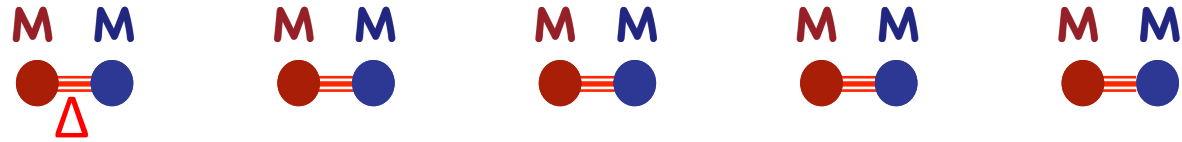


$$H = -2i \sum_{j=1}^n \left(J \gamma_j^B \gamma_{j+1}^A - \Delta \gamma_j^B \gamma_j^A \right)$$

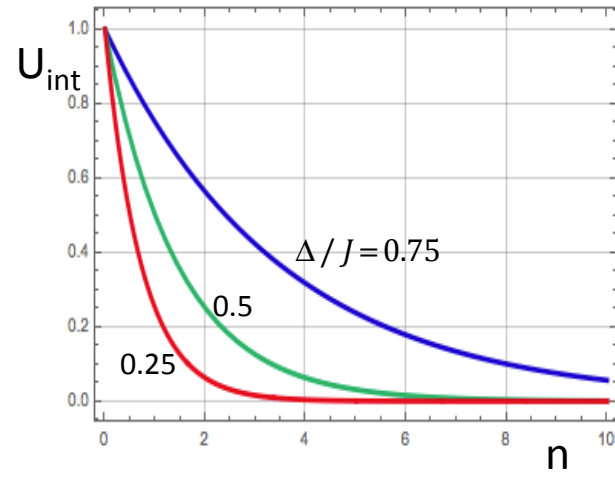
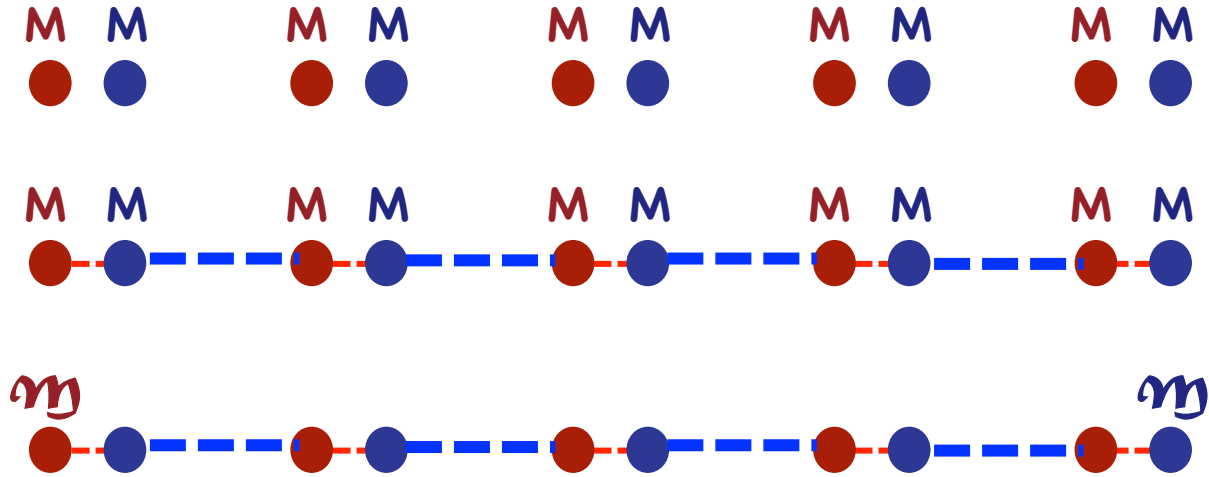
Majorana chain



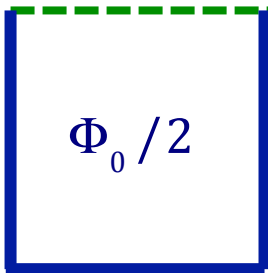
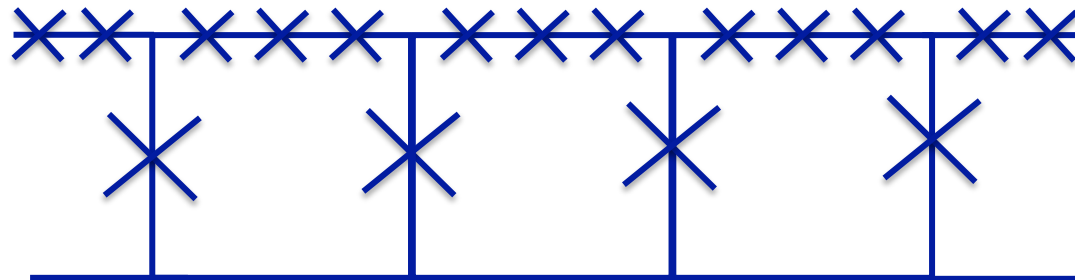
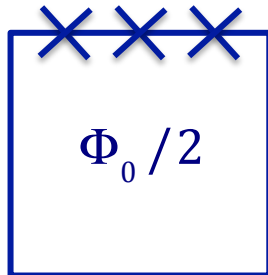
Majorana chain



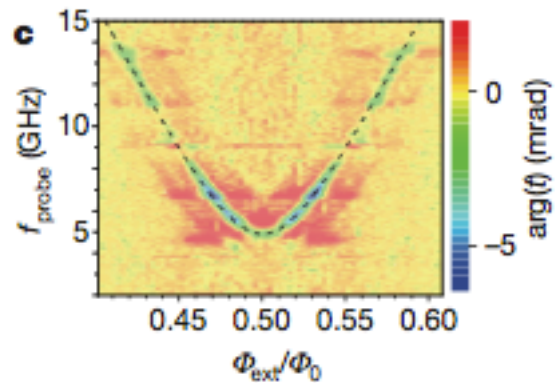
Majorana chain

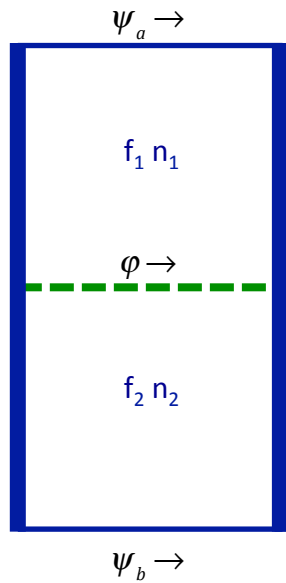


flux qubit with Josephson junctions



flux qubit with quantum phase-slip:
quantum tunneling of a fluxoid across a superconducting nanowire





wire:

$$I = \varphi \Phi_0 / L$$

$$U = \Phi_0 I \varphi = \varphi^2 \Phi_0^2 / L$$

loop:

$$f = \Phi / \Phi_0$$

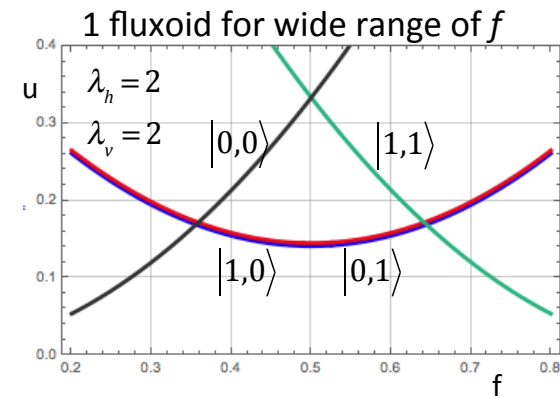
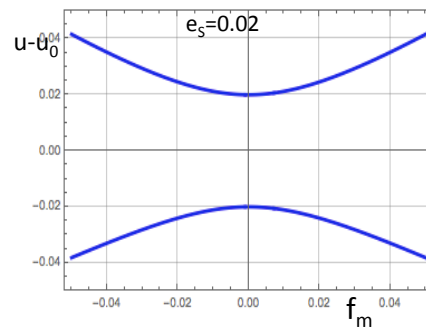
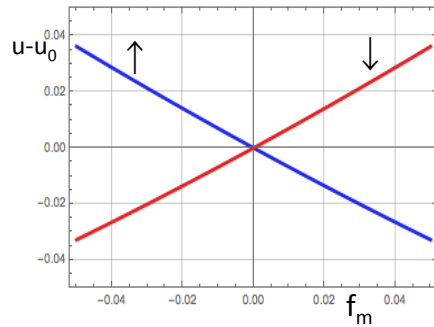
$$\sum \varphi = f - n$$

node:

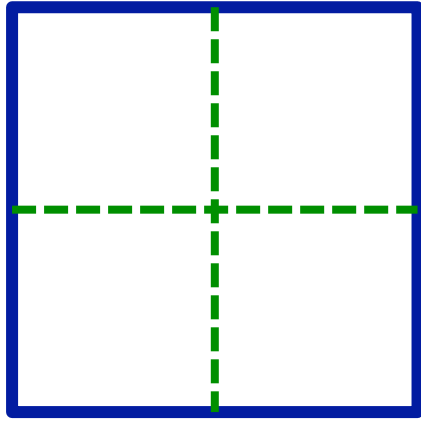
$$\sum I = 0$$

$$f_1 = 0.5 + f_m$$

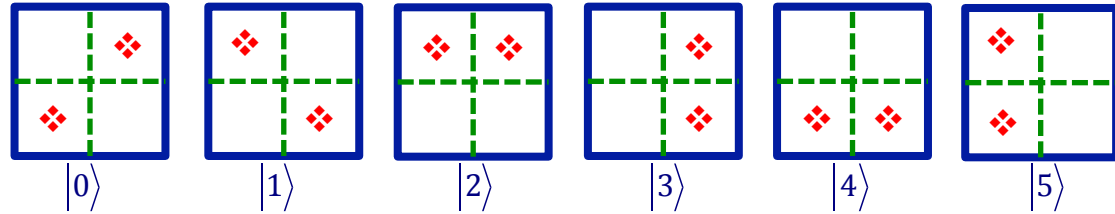
$$f_2 = 0.5 - f_m$$



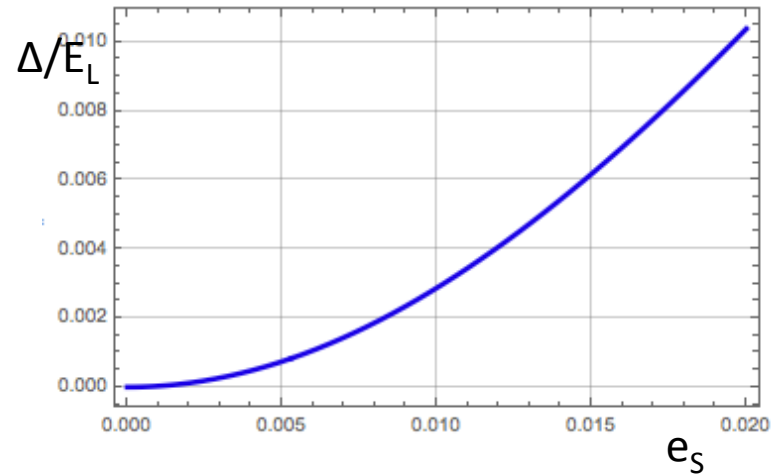
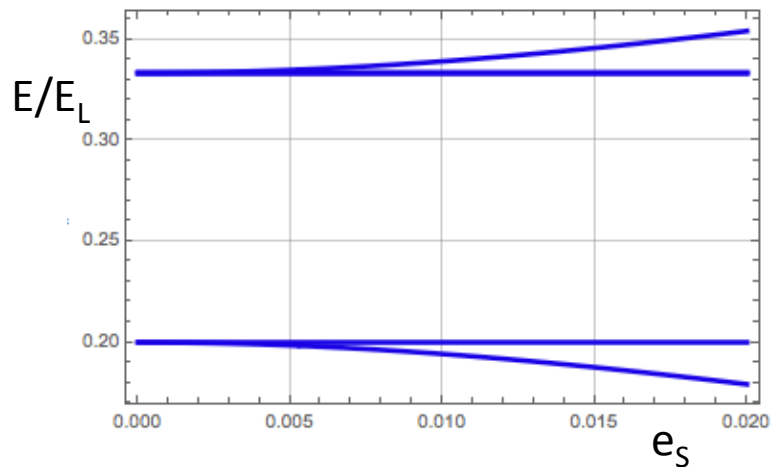
square with 4 loops
 $f=0.5$, 2 fluxoids



inner wires: quantum phase-slip
 amplitude E_S
 outer wires: phase creep

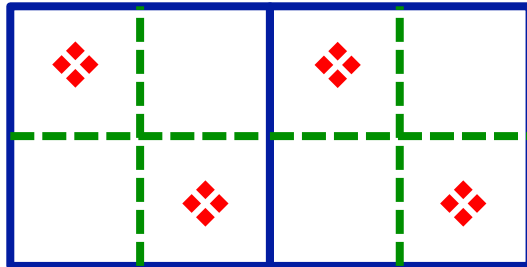


energies normalized to the inductive energy E_L
 of the QPS wire
 $e_S = E_S/E_L$



square QPS flux qubit

ferromagnetic coupling
 $J = 0.020$



energy values
 E_L/h can be 500 GHz

$J = 0.02$ means $J/h = 10$ GHz

$\Delta = 0.2 J$ means $\Delta/h = 2$ GHz

no influence of charges

no islands

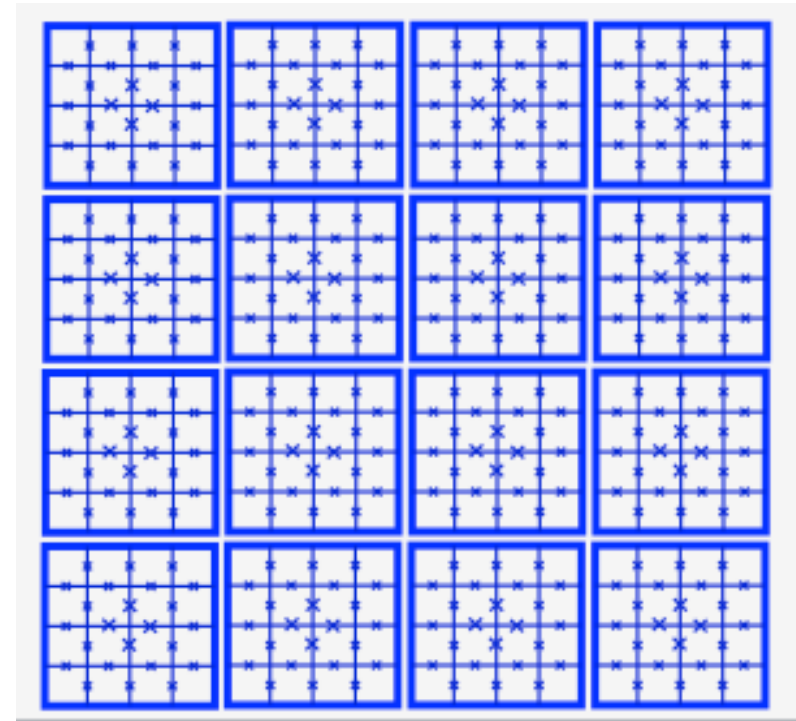
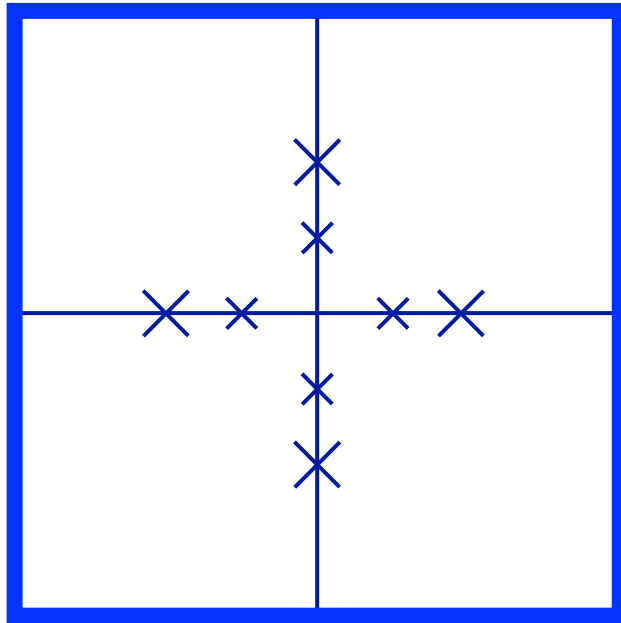
wavelength of plasma oscillations much larger than sample size

no influence of H , dH/dx or dH/dy

but

quantum phase-slip untested

why not with Josephson junctions?



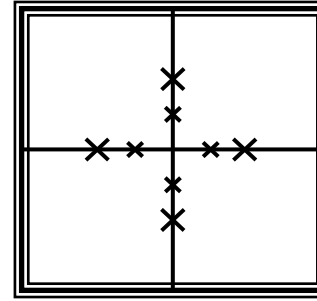
Bilbao 2012
Quantum Simulation conference

Aharonov Casher effect

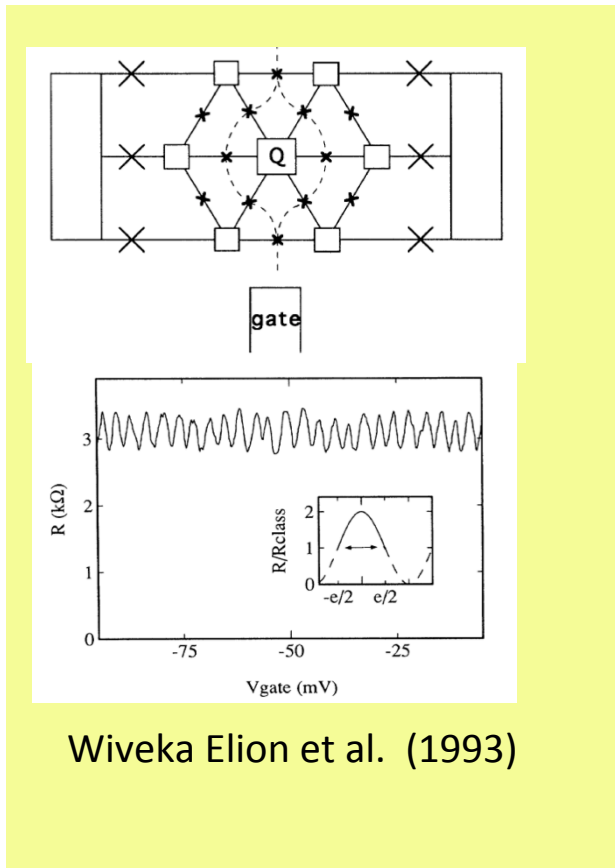
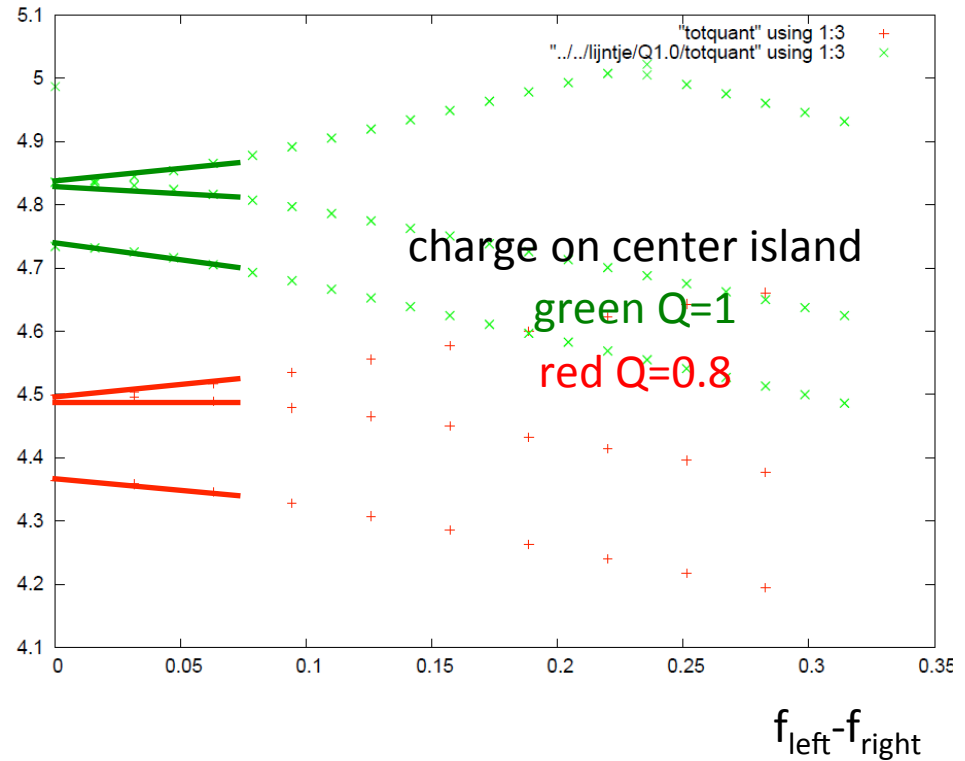
quantum calculation Jos Thijssen

5 degrees of freedom,

9 variables, 9^5 basis vectors

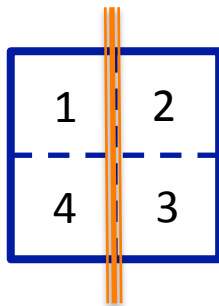
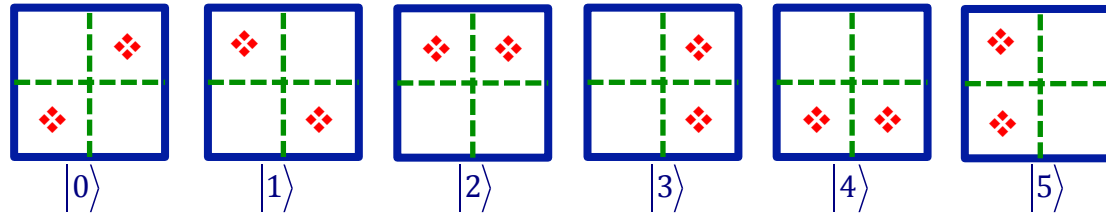


3 lowest energy levels



Wiveka Elion et al. (1993)

fade-out

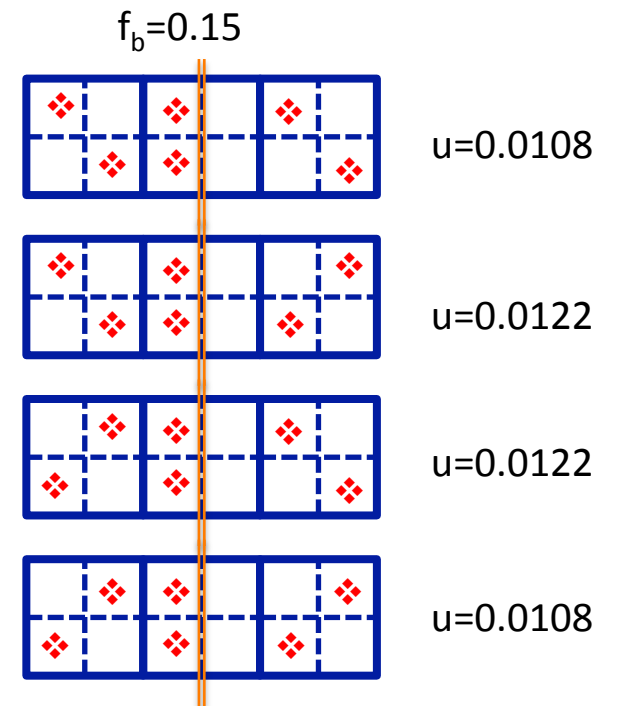
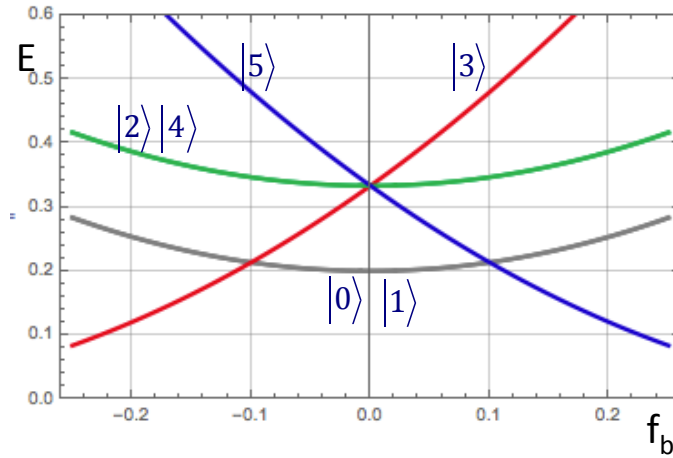


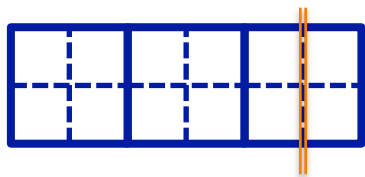
$$f_1 = 0.5 + f_b$$

$$f_2 = 0.5 - f_b$$

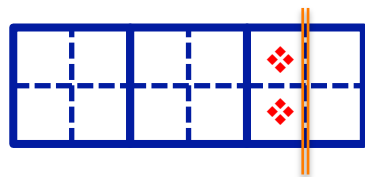
$$f_3 = 0.5 - f_b$$

$$f_4 = 0.5 + f_b$$

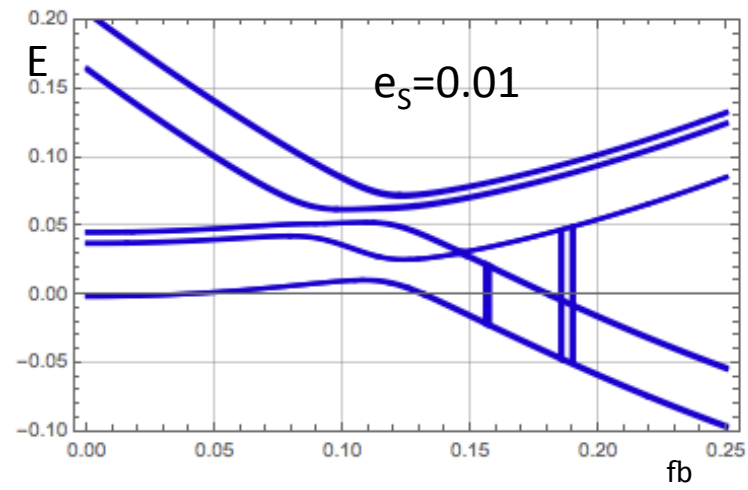
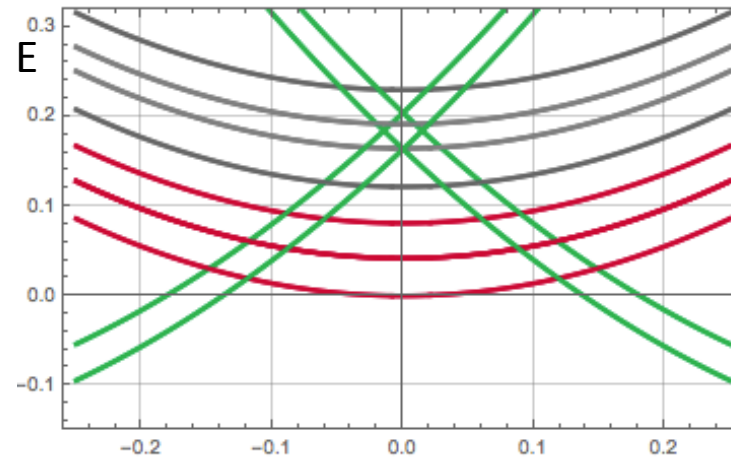




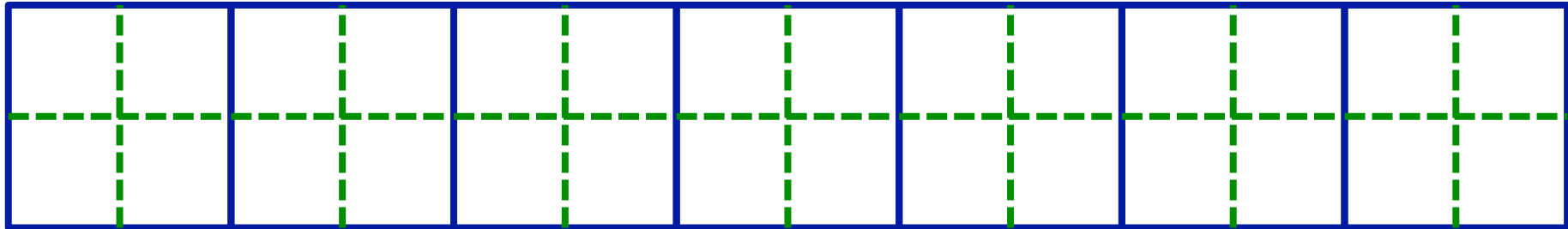
3 qubits



2 qubits



7 qubit chain



7 qubits, 128 classical states

each state:

68 phase values

28 loop equations

40 node equations

lower energy states:

ground states

0000000 1111111

$E=0$, 2 states

$E=0.042$ (2J) 12 states

$E=0.085$ (4J) 32 states

$E=0.126$ (6J) 36 states

$E=0.168$ (8J) 32 states

$E=0.204$ (10J) 12 states

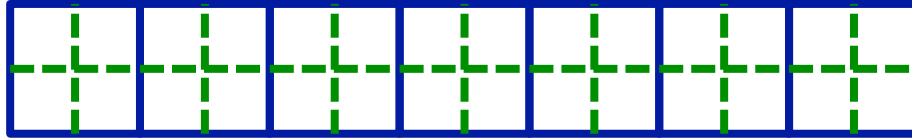
$E=0.245$ (12J) 2 states

energy between 0.0420 and 0.0432

1000000 0000001 0111111 1111110

1100000 0000011 0011111 1111100

1110000 1111000 0001111 1111000



lower energy states:

ground states

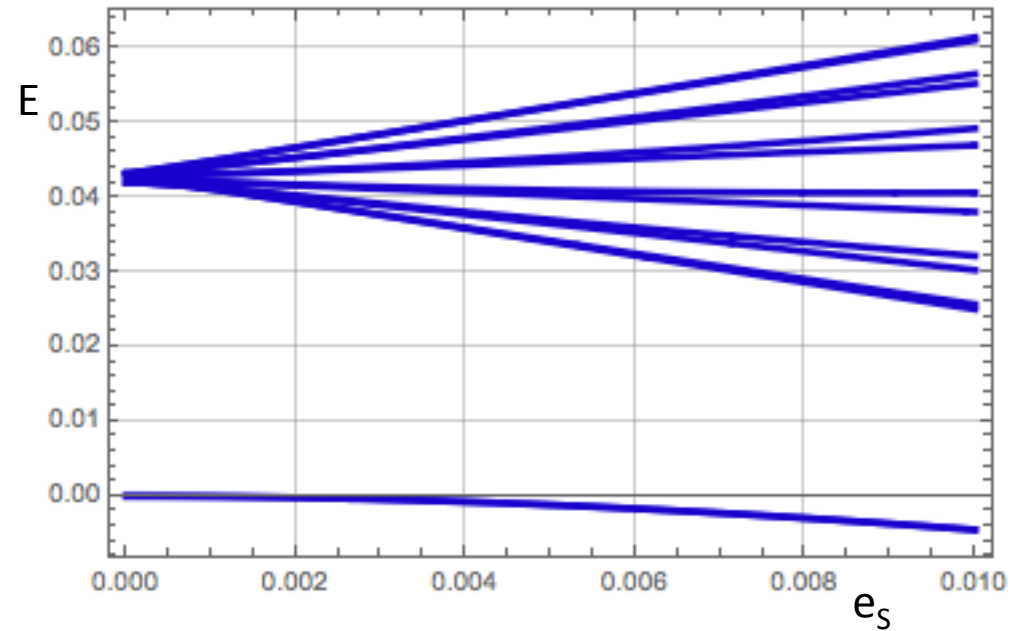
000000 111111

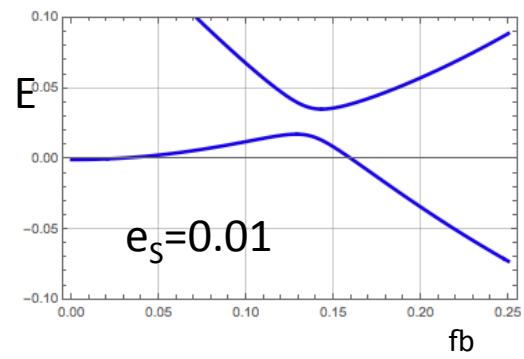
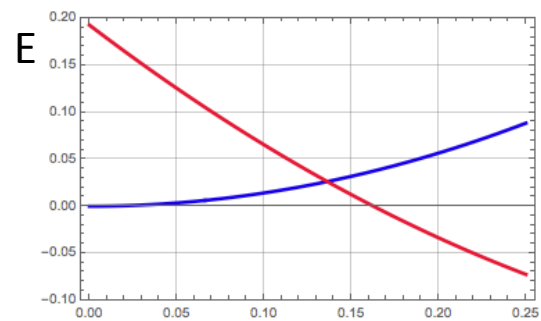
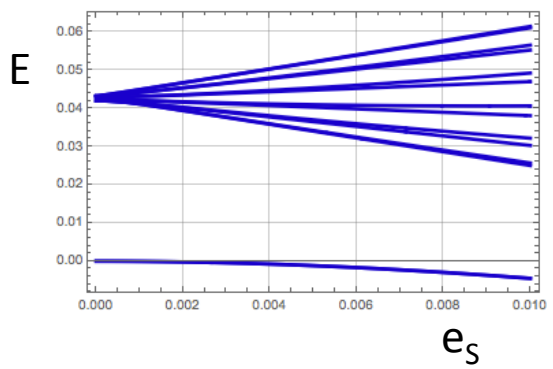
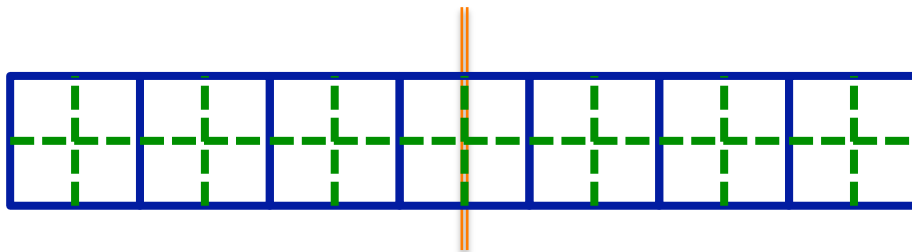
energy between 0.0420 and 0.0432

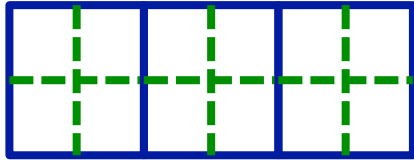
100000 000001 011111 111110

110000 000011 001111 111100

111000 111100 000111 111100







3 squares straight

$$u_{\uparrow\uparrow\uparrow} = 0$$

$$u_{\uparrow\uparrow\downarrow} = 0.0419$$

$$u_{\uparrow\downarrow\uparrow} = 0.0811$$

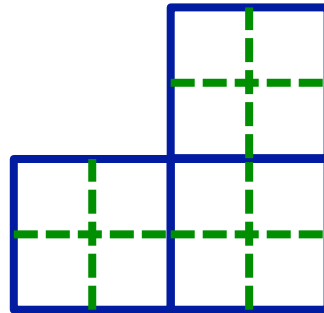
$$u_{\uparrow\downarrow\downarrow} = 0.0419$$

$$u_{\downarrow\uparrow\uparrow} = 0.0419$$

$$u_{\downarrow\uparrow\downarrow} = 0.0811$$

$$u_{\downarrow\downarrow\uparrow} = 0.0419$$

$$u_{\downarrow\downarrow\downarrow} = 0$$



3 squares corner

$$u_{\uparrow\uparrow\uparrow} = 0$$

$$u_{\uparrow\uparrow\downarrow} = 0.0389$$

$$u_{\uparrow\downarrow\uparrow} = 0.0820$$

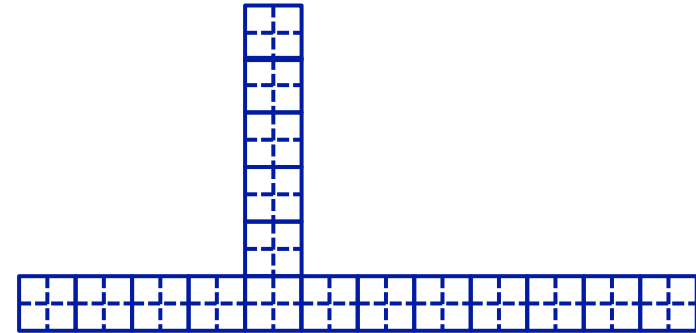
$$u_{\uparrow\downarrow\downarrow} = 0.0389$$

$$u_{\downarrow\uparrow\uparrow} = 0.0389$$

$$u_{\downarrow\uparrow\downarrow} = 0.0820$$

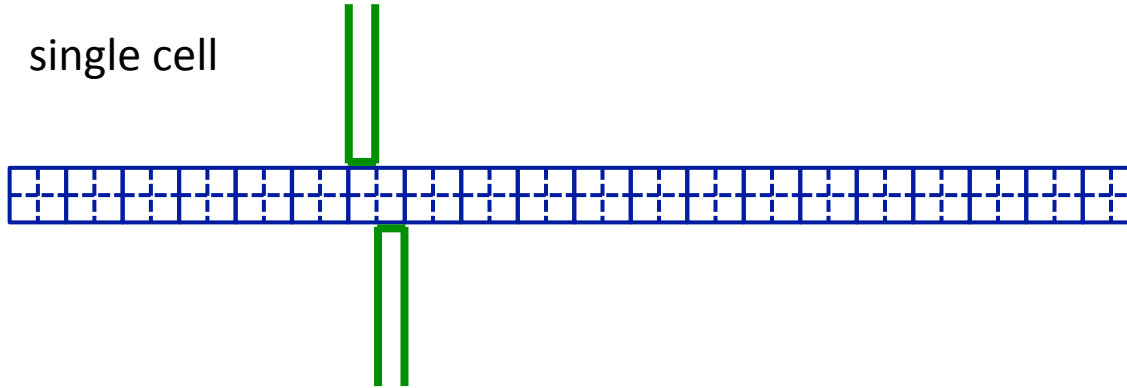
$$u_{\downarrow\downarrow\uparrow} = 0.0389$$

$$u_{\downarrow\downarrow\downarrow} = 0$$

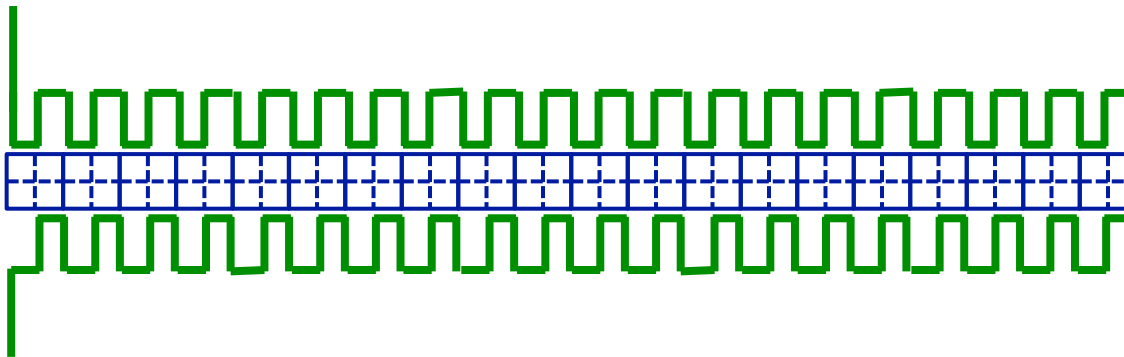


drive, readout

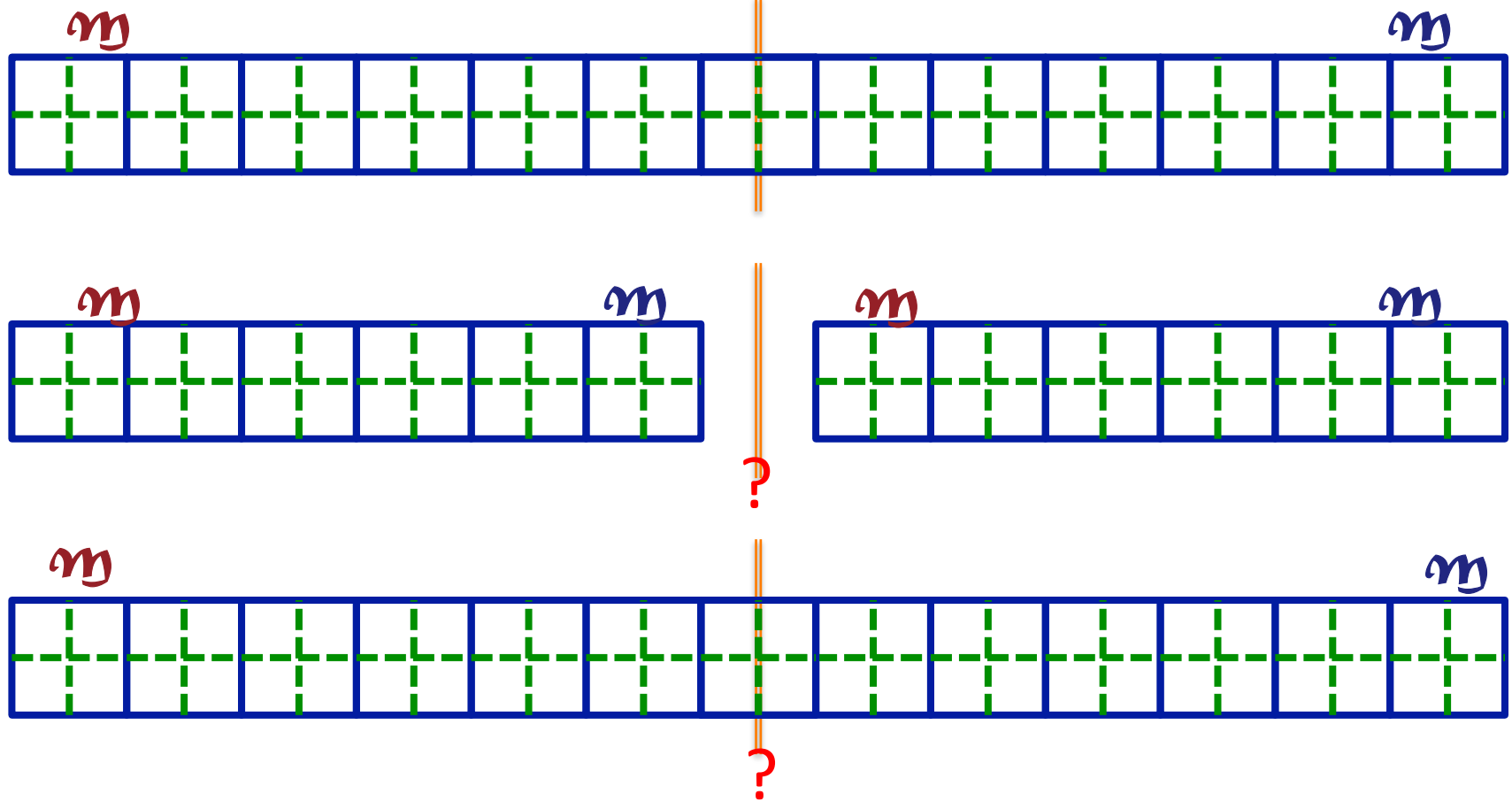
single cell



collective system



fusion



conclusions

aspects of Majorana physics can be studied in quantum Ising chains

Ising chains can be realized with coupled superconducting qubits

square 4-cell qubits with quantum phase-slip nanowires are in principle ideally suited (experimental test needed)

- not sensitive to offset charges or nonlocal magnetic fields
- adiabatic fade-out/in of individual qubits is possible