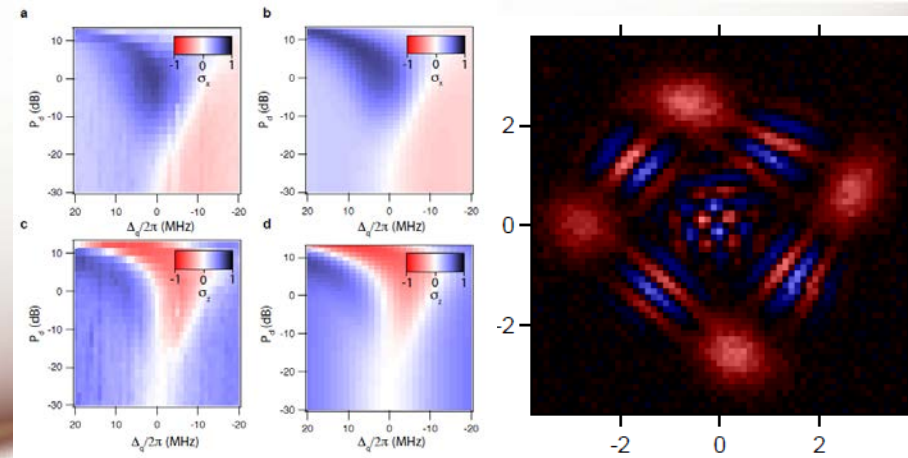




Departments of Physics
and Applied Physics, Yale University



Quantum Reservoir Engineering

Towards Quantum Simulators
with Superconducting Qubits

SMG
Claudia De Grandi
(Yale University)
Siddiqi Group
(Berkeley)



Noise and Dissipation: Friend or Foe?



Clerk et al. *Rev. Mod. Phys.* **82**, 1155 (2010)

Challenge: Quantum state control
in the presence of noise and dissipation

How do we correct non-unitary errors?

$$\rho' = \sum_j E_j \rho E_j^\dagger; \quad \sum_j E_j^\dagger E_j = I$$

Challenge: Quantum state control
in the presence of noise and dissipation

How do we correct non-unitary errors?

$$\rho' = \sum_j E_j \rho E_j^\dagger; \quad \sum_j E_j^\dagger E_j = I$$

‘Active’ feedback: classical measurement result
fed back to controller which applies a unitary that is
conditioned on measurement result.

Challenge: Quantum state control in the presence of noise and dissipation

How do we correct non-unitary errors?

$$\rho' = \sum_j E_j \rho E_j^\dagger; \quad \sum_j E_j^\dagger E_j = I$$

‘Autonomous’ feedback: embedded into the system
as a quantum ‘bath’

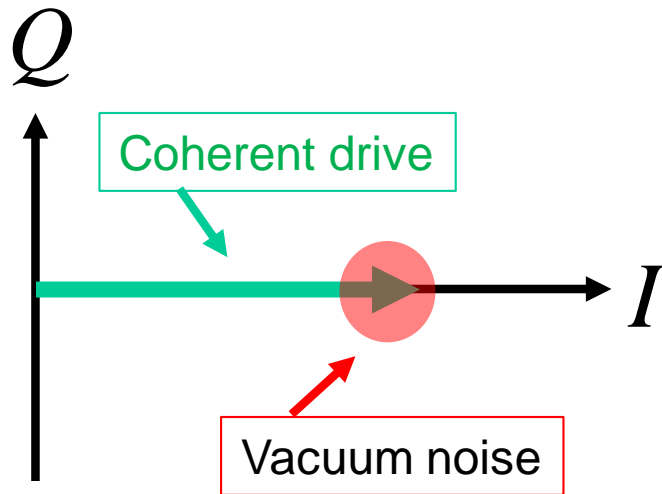
Paradox:

We can use dissipation as a tool to
create and maintain coherence. (Poyatos et al. PRL 1996)

Interference between coherent drives and quantum
noise of bath can be useful.

(Murch et al. PRL 2012)

Example: Photon shot noise in a driven damped cavity



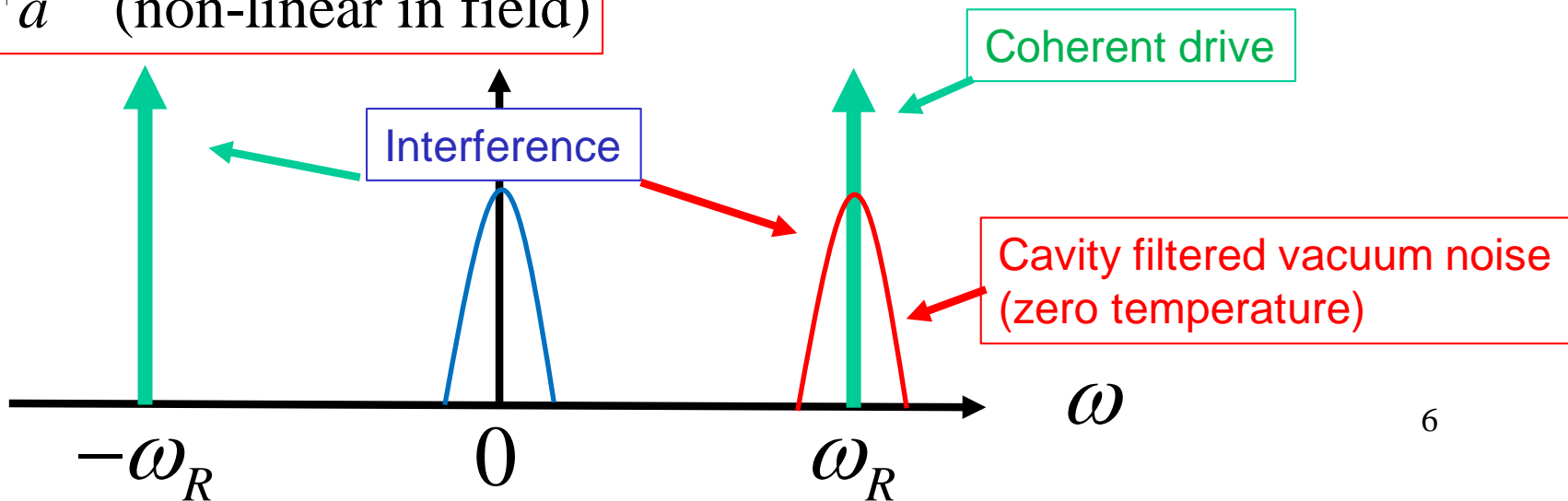
Interference leads to amplitude fluctuations

$$\langle (\hat{n} - \bar{n})^2 \rangle^{1/2} = \bar{n}^{1/2}$$

'Rectification of vacuum noise'

$$\hat{n} = a^\dagger a \quad (\text{non-linear in field})$$

$$\langle \hat{n}(t) \hat{n}(0) \rangle = \bar{n}^2 + \bar{n} e^{-\frac{\kappa}{2}|t|} \quad \text{Nota Bene}$$



Example: Photon shot noise in a driven damped cavity

'Rectification of vacuum noise' when drive is **red detuned** from cavity

Spectral density of shot noise moves up to positive frequencies

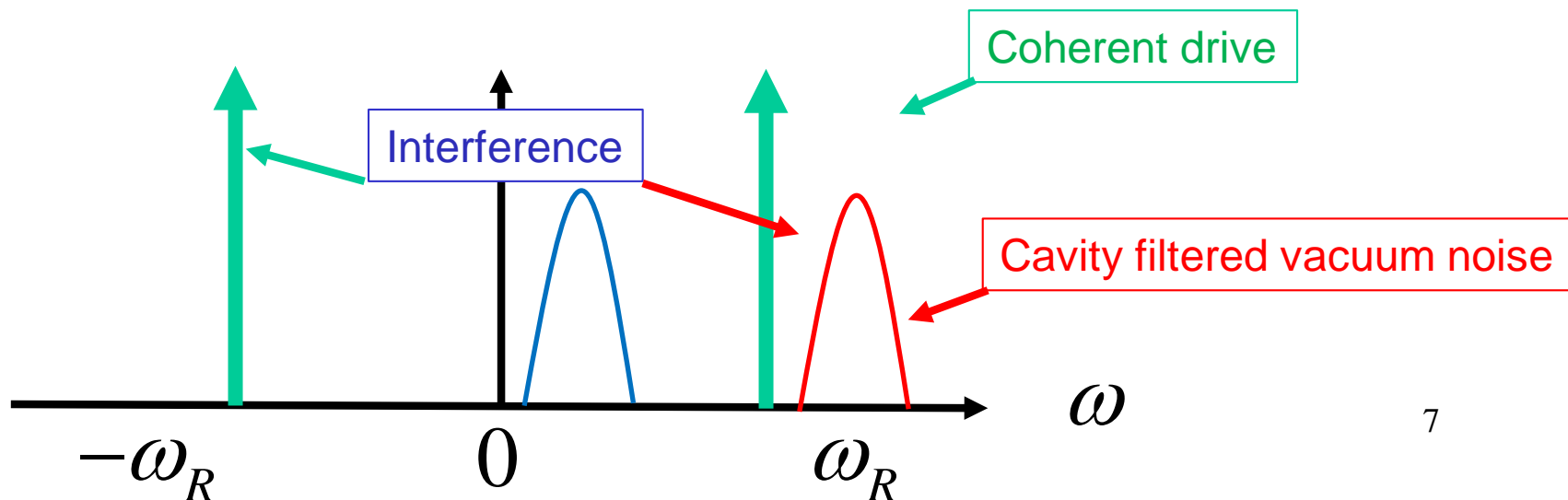
Bath can absorb energy (by Raman scattering of pump photon up to cavity)

Bath cannot emit energy

Cooling!

$$V = \chi O_{\text{system}} (\hat{n} - \bar{n})$$

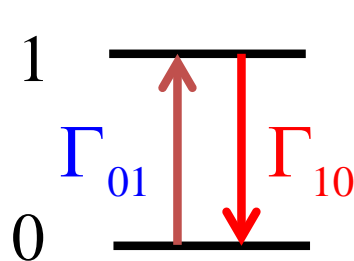
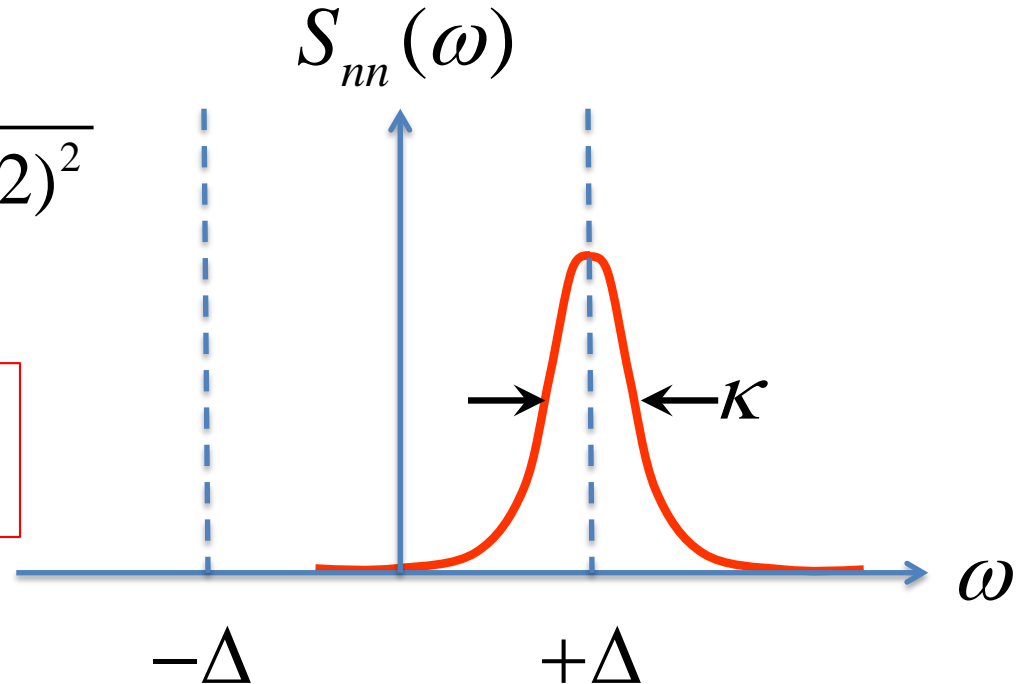
shot noise



Shot noise spectral density for drive red-detuned by distance Δ
 Clerk et al. *Rev. Mod. Phys.* **82**, 1155 (2010)

$$S_{nn}(\omega) = \bar{n} \frac{\kappa}{(\omega - \Delta)^2 + (\kappa/2)^2}$$

Quantum shot noise spectral density is ASYMMETRIC



$$\Gamma_{10} = \chi^2 S_{nn}(+\Delta) \text{ bath absorbs}$$

$$\Gamma_{01} = \chi^2 S_{nn}(-\Delta) \text{ bath emits}$$

Fermi Golden Rule Picture: Clerk et al. *Rev. Mod. Phys.* **82**, 1155 (2010)

Autonomous Feedback: Quantum Reservoir Engineering

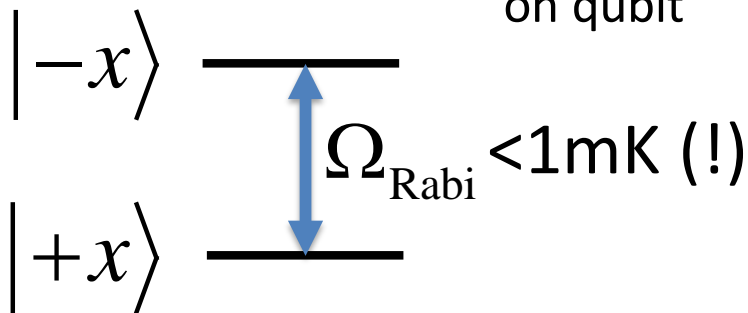
We can use photon shot noise to ‘cool’ a qubit to any point on the Bloch sphere, e.g. $|+x\rangle = \frac{1}{\sqrt{2}}(|g\rangle + |e\rangle)$

$$H = -\Omega_{\text{Rabi}} \sigma^X + \chi \sigma^Z (\hat{n} - \bar{n})$$

Rabi drive
on qubit

dispersive coupling

shot noise of driven cavity



Dissipator ‘cools’ qubit via jump operator:

‘Cavity-assisted quantum bath engineering’
(*Phys. Rev. Lett.* 2012)

$$c = | +x \rangle \langle -x |$$

Berkeley: K. W. Murch, S. J. Weber, I. Siddiqi
Yale: U. Vool, D. Zhou, SMG

Two qubit entanglement:
S. Shankar et al. (Devoret)
Nature (2013).

Quantum Reservoir Engineering to create persistent Ramsey fringes

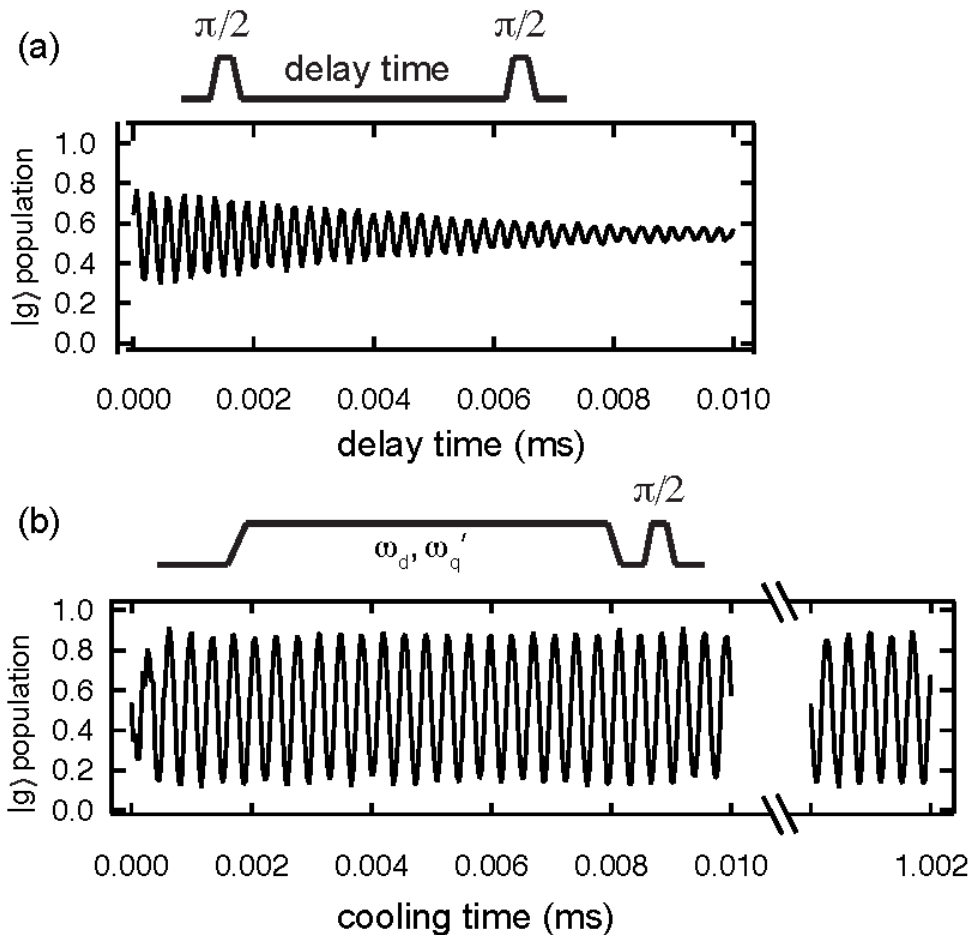
Dissipator ‘cools’ qubit via
jump operator:

$$c = | +x \rangle \langle -x |$$

$$T_{\text{eff}} \sim 150 \mu\text{K}$$

Persistent Ramsey fringes
(with enhanced visibility)

Murch et al. PRL (2012)



Canonical Ensemble Quantum Simulator for 1D Bose-Hubbard Model

Berkeley:

Shay Hacoen-Gourgy

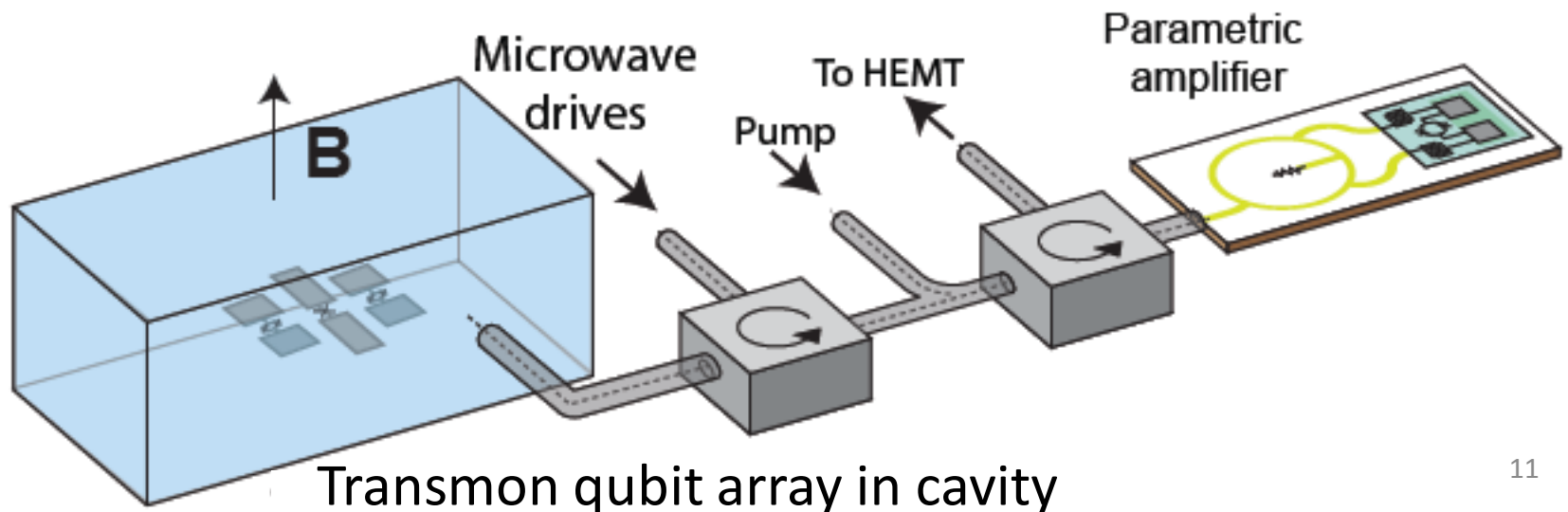
Vinay Ramasesh

Irfan Siddiqi

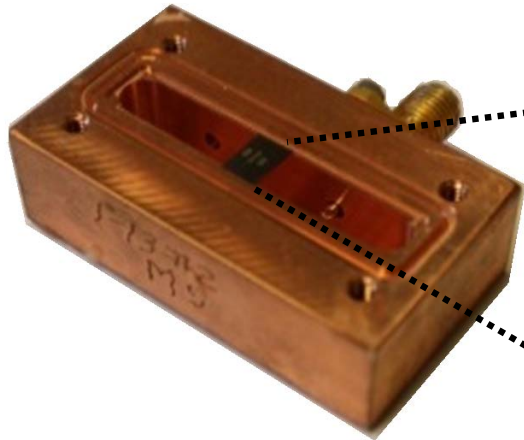
Yale:

Claudia De Grandi

SMG



End qubits flux-tunable (SQUIDs)



Cavity

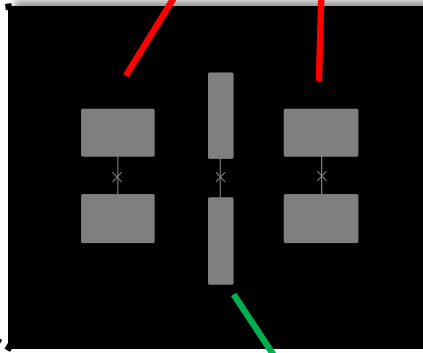
$$\omega_M = 2\pi \times 7.116 \text{ GHz}$$

$$\kappa = 2\pi \times 10 \text{ MHz}$$

$$g_1 = 2\pi \times 0.149 \text{ GHz}$$

$$g_2 = 2\pi \times 0.264 \text{ GHz}$$

$$g_3 = 2\pi \times 0.145 \text{ GHz}$$



Middle qubit fixed (single-junction)

$$\omega_M = 2\pi \times 4.856 \text{ GHz}$$

$$\alpha_M = -2\pi \times 240 \text{ MHz}$$

Couplings

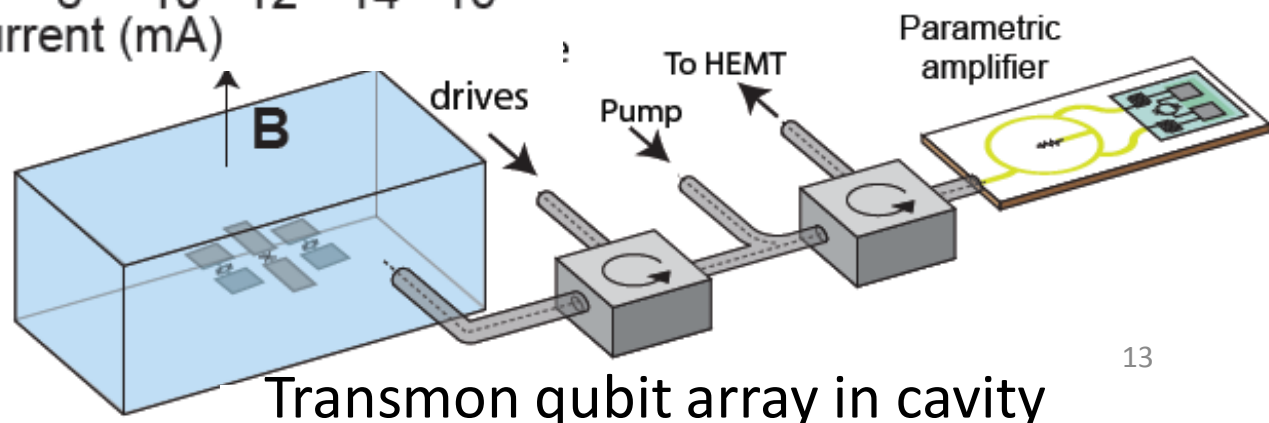
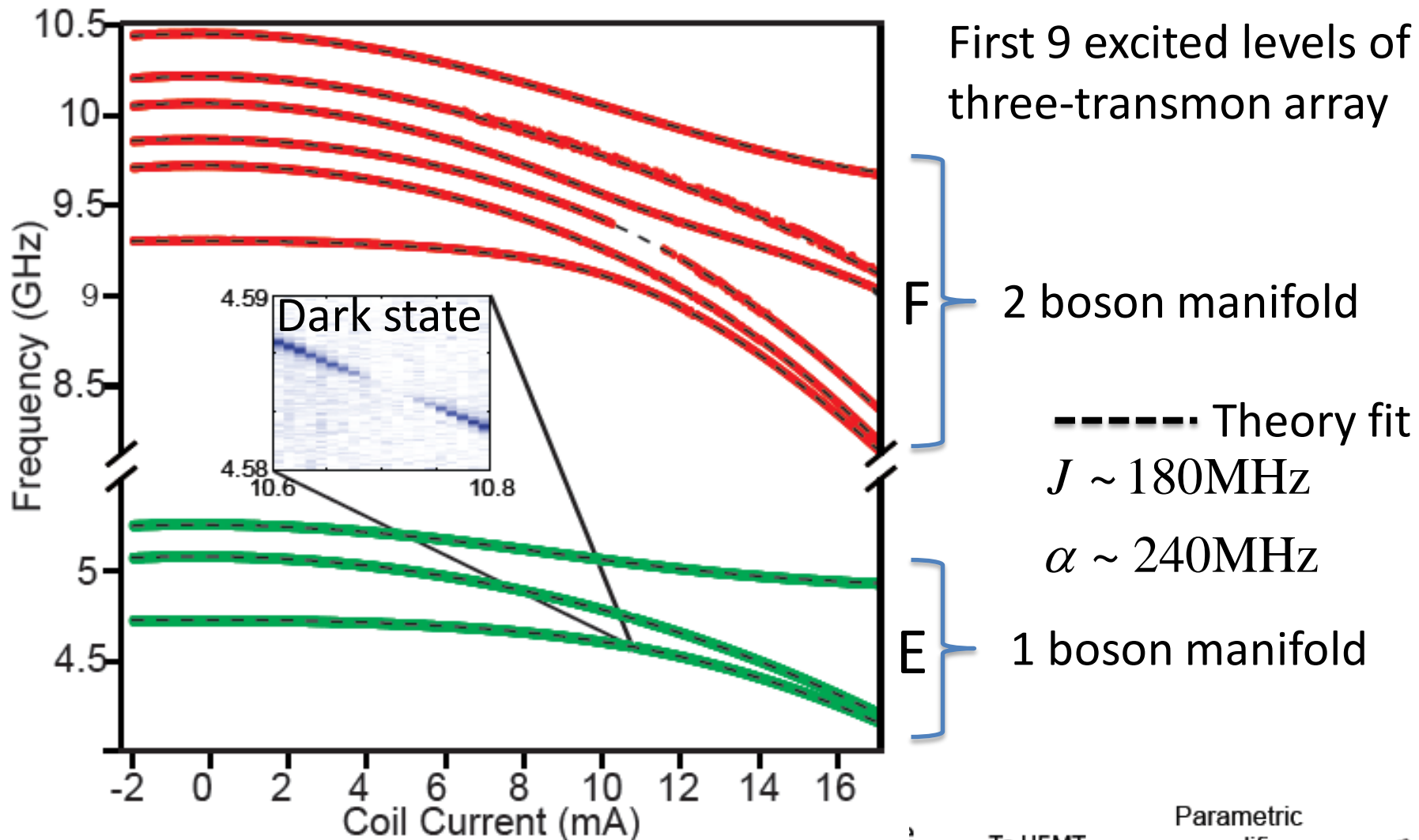
$$J_{12}, J_{23} = 2\pi \times 0.177 \text{ GHz}$$

$$J_{13} = 2\pi \times 0.026 \text{ GHz}$$

$$\omega_L = 2\pi \times 5.058 \text{ GHz}$$

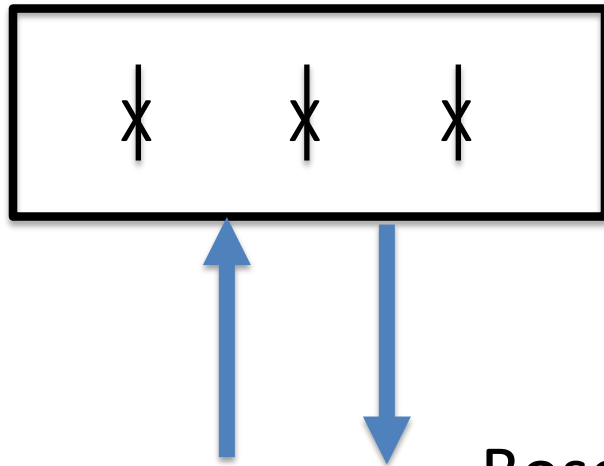
$$\omega_R = 2\pi \times 5.161 \text{ GHz}$$

$$\alpha_L = \alpha_R = -2\pi \times 214 \text{ MHz}$$



Canonical Ensemble Quantum Simulator for 1D Bose-Hubbard Model

Engineer bath to remove entropy and energy but not bosons!
 [cf. Jake Taylor 'Chemical Potential for Light']



$$\left[H_0, \sum_j b_j^\dagger b_j \right] = 0$$

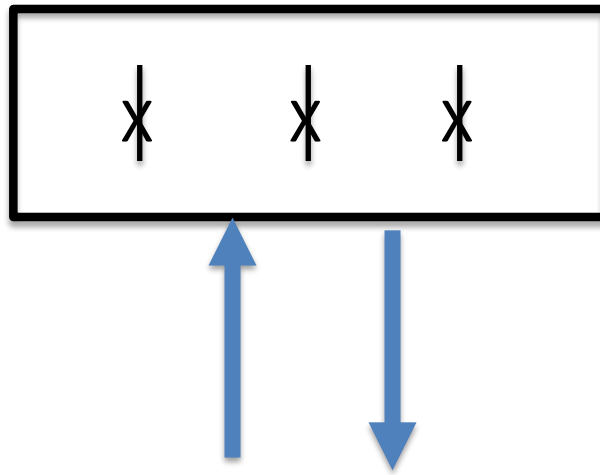
Bose-Hubbard Model

$$H_0 = \sum_j \omega_j b_j^\dagger b_j + \sum_{jk} J_{jk} b_j^\dagger b_k - \frac{1}{2} \sum_j \alpha_j b_j^\dagger b_j^\dagger b_j b_j$$

site energy

boson hopping
 (dipole coupling)

Hubbard $U < 0$



Couple Bose-Hubbard lattice to cavity photons
to create dissipation

$$H_0 = \omega_Q a^\dagger a + \sum_j \omega_j b_j^\dagger b_j + \sum_{jk} J_{jk} b_j^\dagger b_k + g_j (a b_j^\dagger + a^\dagger b_j)$$

$$V = -\frac{1}{2} \sum_j \alpha_j b_j^\dagger b_j^\dagger b_j b_j$$

N.B. $\left[H_0, \sum_j b_j^\dagger b_j \right] \neq 0$

Diagonalize Quadratic H_0

$$H_0 = \omega_Q a^\dagger a + \sum_j \omega_j b_j^\dagger b_j + \sum_{jk} J_{jk} b_j^\dagger b_k + g_j (a b_j^\dagger + a^\dagger b_j)$$

$$a = \Lambda_{00} A + \sum_j \Lambda_{0j} B_j$$

$$b_k = \Lambda_{k0} A + \sum_j \Lambda_{kj} B_j$$

$$H_0 = \tilde{\omega}_Q A^\dagger A + \sum_j \tilde{\omega}_j B_j^\dagger B_j$$

$$H_0 = \tilde{\omega}_Q A^\dagger A + \sum_j \tilde{\omega}_j B_j^\dagger B_j$$

Quartic Term becomes (in RWA):

$$V = -\frac{1}{2} \sum_j \alpha_j b_j^\dagger b_j^\dagger b_j b_j$$

$$V \sim -\sum_{ijkl} \alpha_{ijkl} B_i^\dagger B_j^\dagger B_k B_l + A^\dagger A \sum_{ij} \chi_{ij} B_i^\dagger B_j + \chi_0 A^\dagger A^\dagger A A + \dots$$

Normal modes scatter due to Hubbard interactions

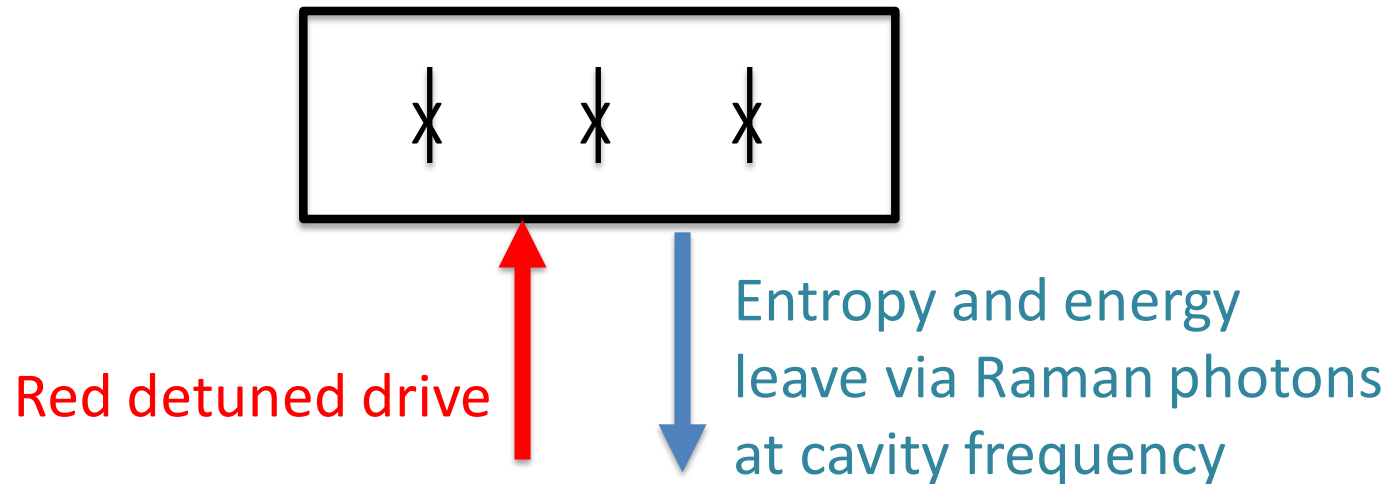
Dispersive coupling of (dressed) cavity to normal modes

Small cavity self-Kerr

$$\left[H_0 + V, \sum_j B_j^\dagger B_j \right] = 0^*$$

(Fine print re. Purcell effect)

Quantum Bath Engineering for 1D Bose-Hubbard Model



$$A^\dagger A \sum_{ij} \chi_{ij} B_i^\dagger B_j = \hat{n} \sum_{ij} \chi_{ij} B_i^\dagger B_j$$

Shot noise fluctuations cause cooling transitions which conserve particle number.

$$A^\dagger A \sum_{ij} \chi_{ij} B_i^\dagger B_j = \hat{n} \sum_{ij} \chi_{ij} B_i^\dagger B_j$$

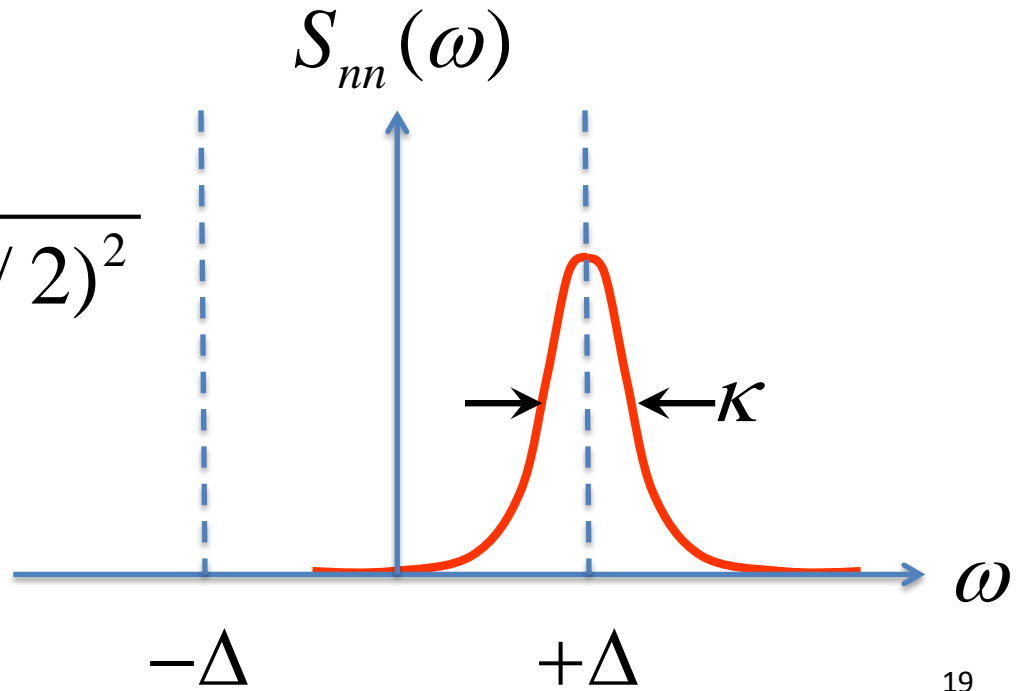
Shot noise fluctuations cause cooling transitions which conserve particle number.

Fermi Golden Rule cooling rate

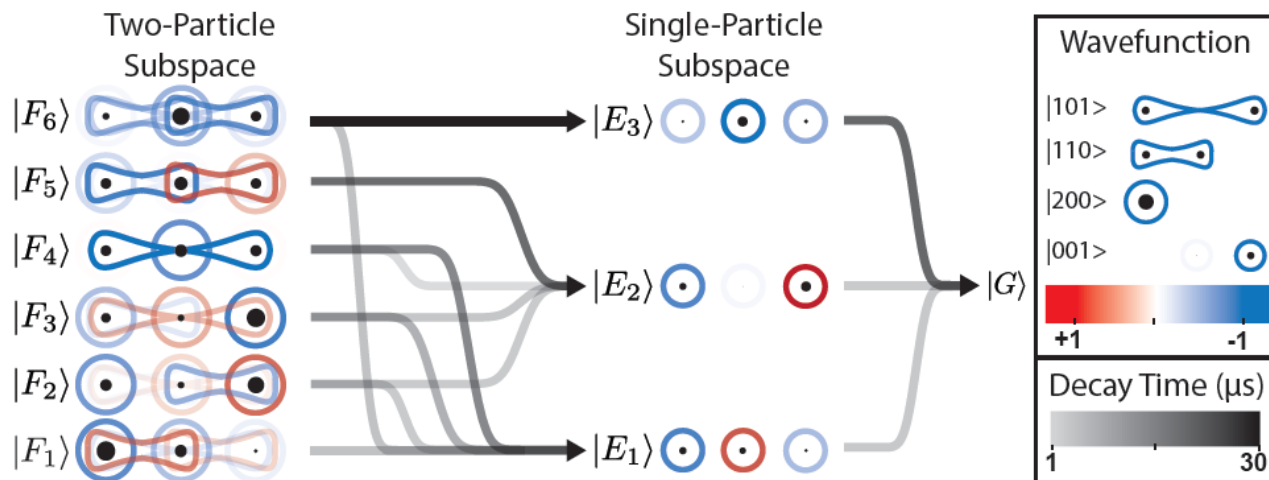
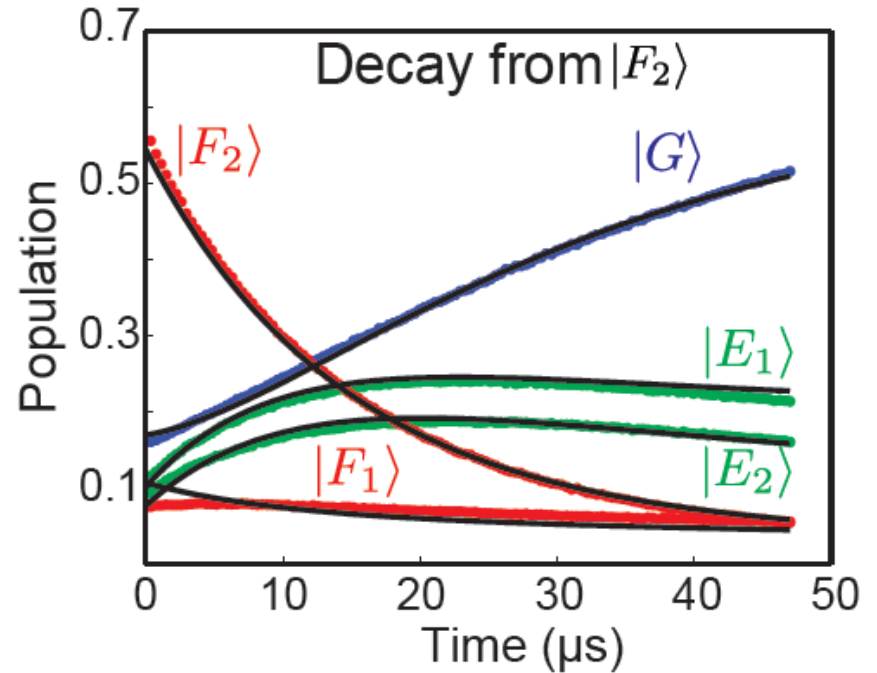
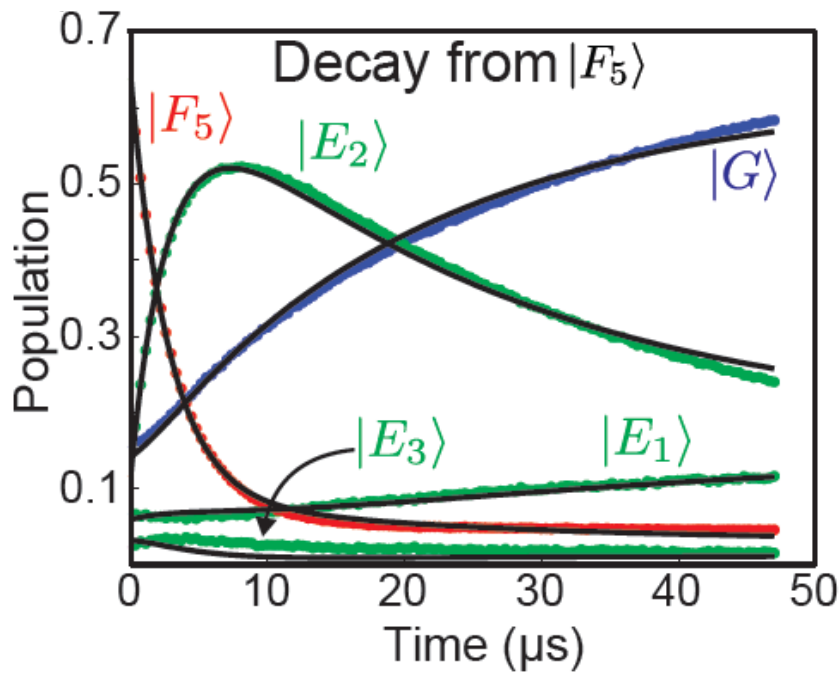
$$\Gamma_{1 \rightarrow 2} = \left| \langle \Psi_2 | \sum_{ij} \chi_{ij} B_i^\dagger B_j | \Psi_1 \rangle \right|^2 S_{nn}(\omega_1 - \omega_2)$$

$$S_{nn}(\omega) = \bar{n} \frac{\kappa}{(\omega - \Delta)^2 + (\kappa / 2)^2}$$

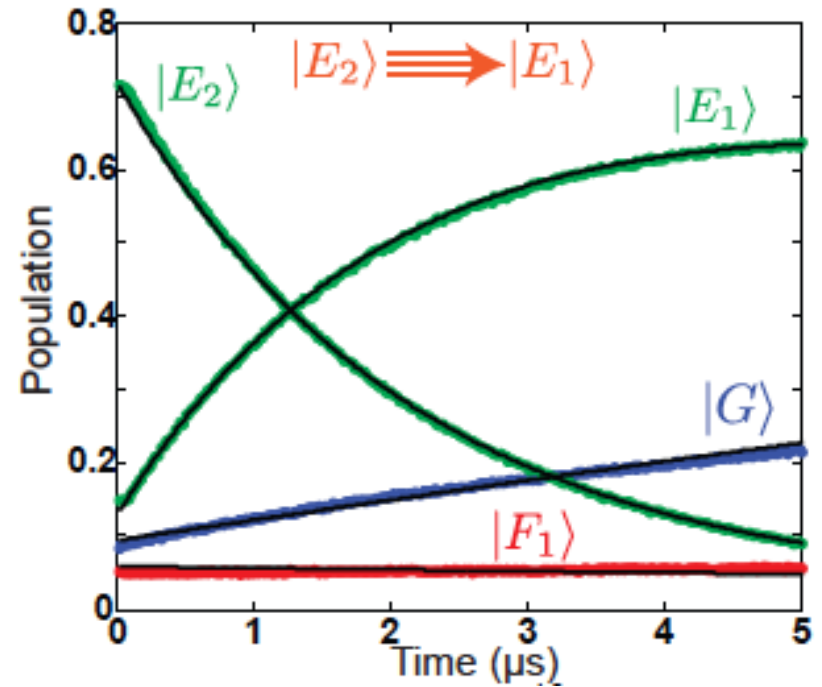
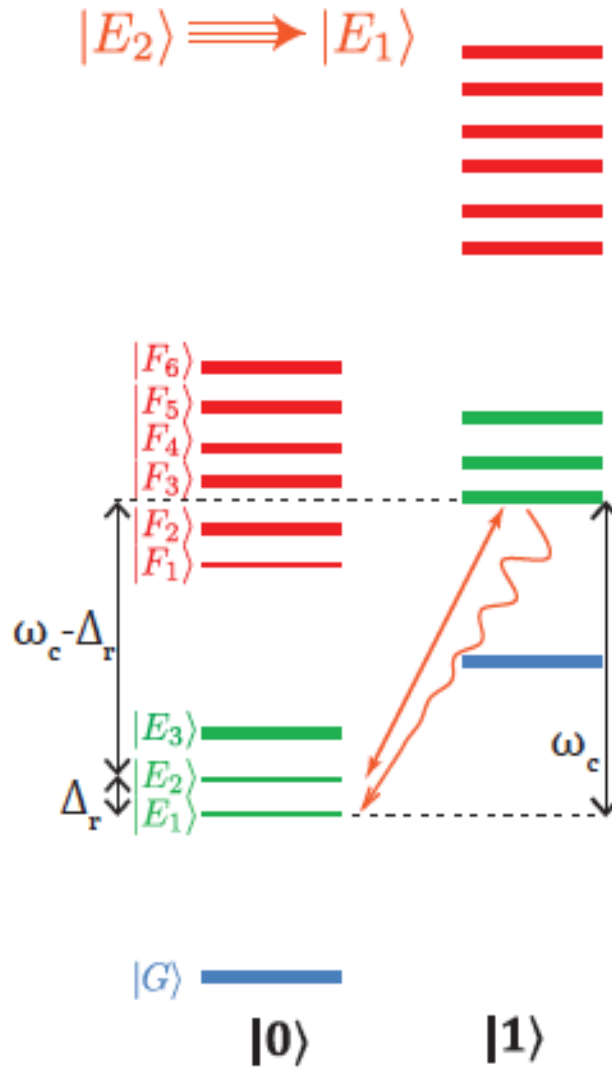
Drive detuning



NATURAL DECAY DYNAMICS

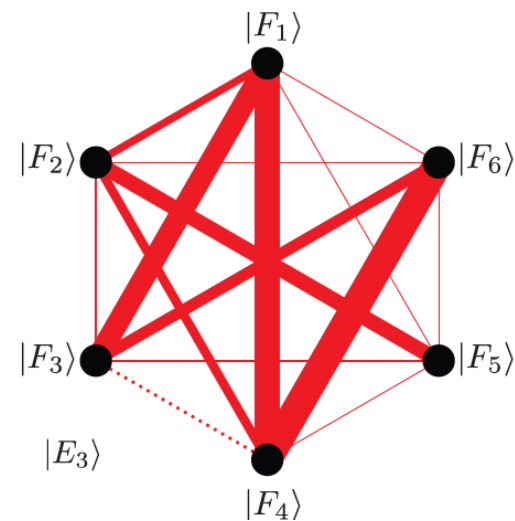
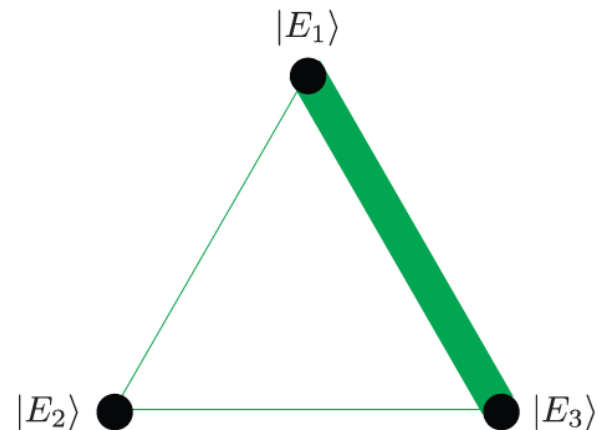
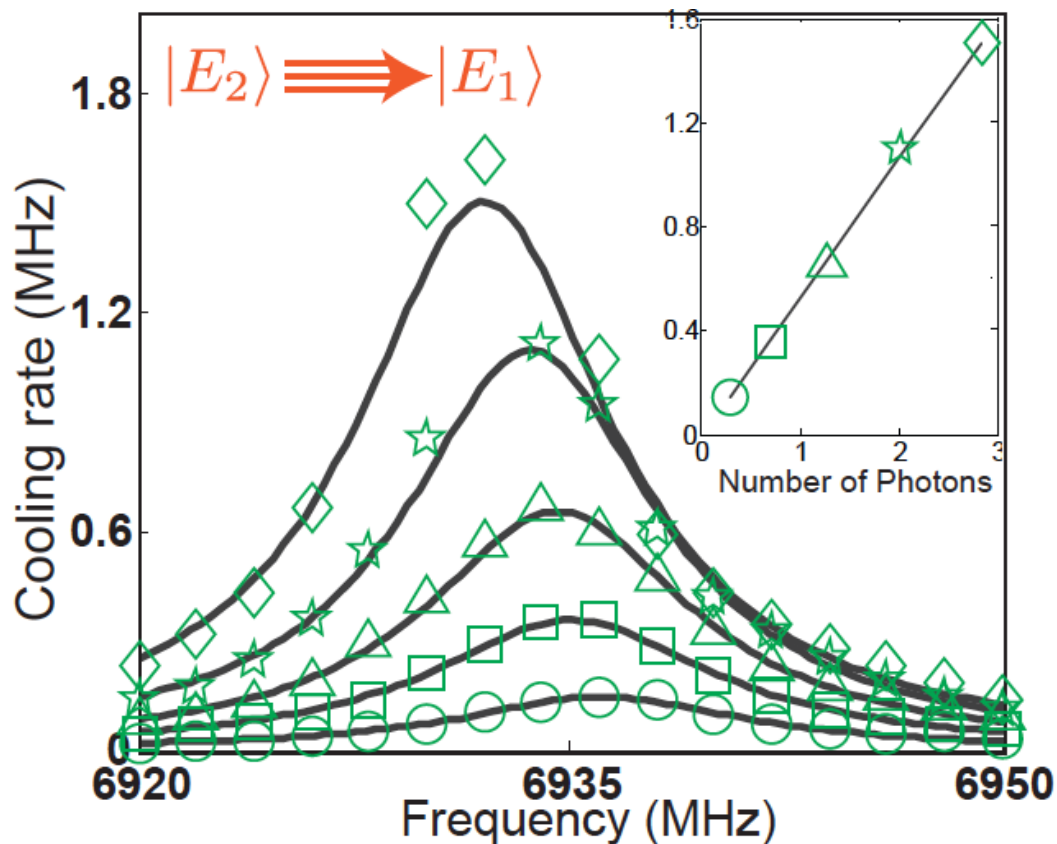


COOLING



- **COHERENT DRIVES TRANSFER EXCITATIONS BETWEEN SUBSPACES**
- **COOLING DRIVES TRANSFER EXCITATIONS WITHIN A SUBSPACE**
- **ACCESS DARK STATES**

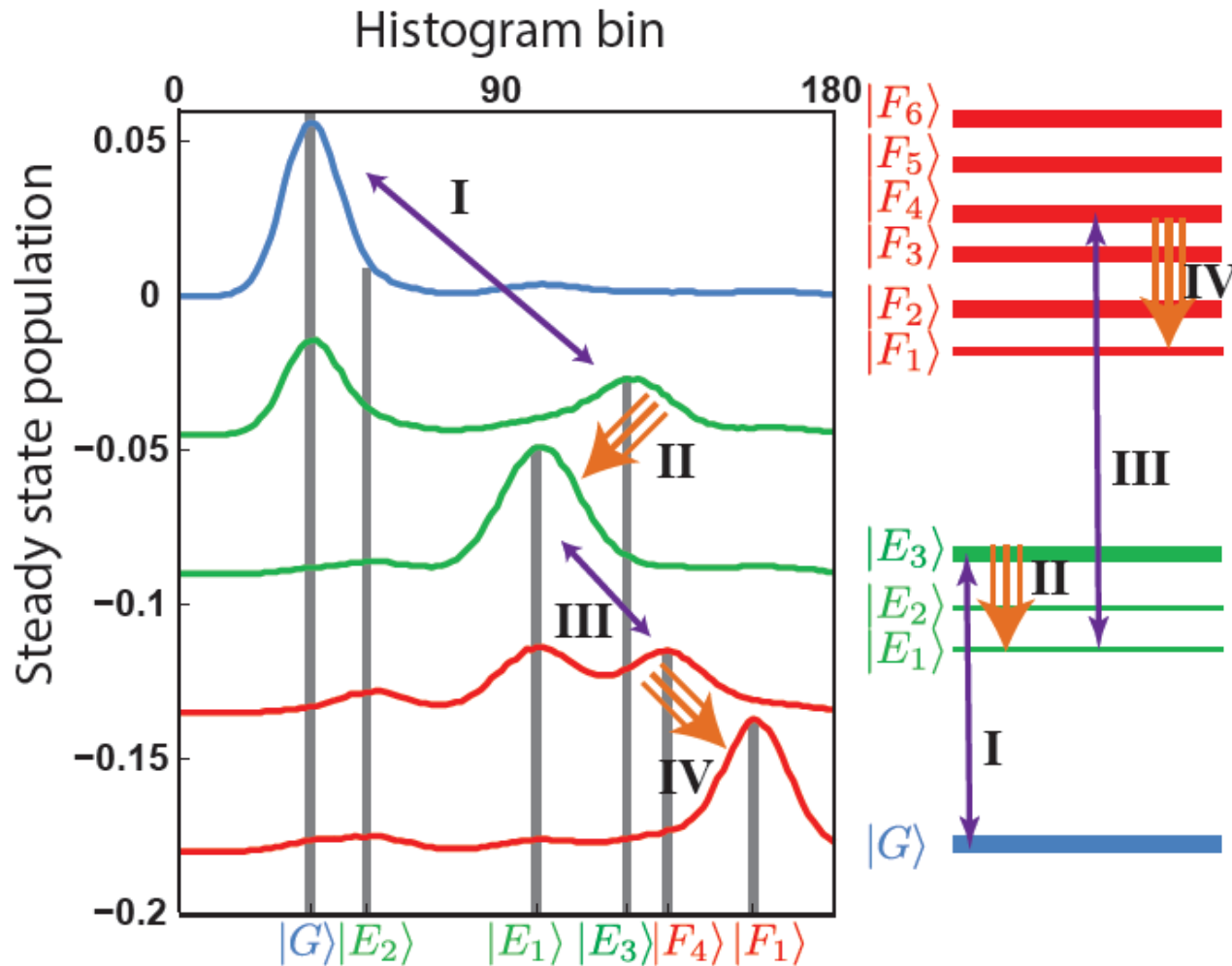
COOLING RATES



$$R_{i \rightarrow f} = 4\alpha^2 \bar{n} |M_{if}|^2 \frac{\kappa}{(\Delta E_{if} - \Delta_r)^2 + (\kappa/2)^2}$$

Cooling rate (MHz/Photon)

STATE STABILIZATION: $|F_1\rangle$



- Can stabilize any many-body eigenstate
- Preserves particle number in Bose-Hubbard Model

PERSPECTIVES

- STATE SYMMETRY CAN FACILITATE COOLING/QBE
- COOLING OF QUBIT CHAINS
 - PREPARE & HOLD N-BODY EIGENSTATE (EG. RESET)
 - DISSIPATIVE BOSE-HUBBARD PHYSICS
 - ACCESS DARK STATES
 - ENERGY FLOW / THERMODYNAMICS
 - QUANTUM EMULATION IN LONG-CHAINS / ARRAYS
- FOR FUTURE QUANTUM SIMULATOR NEED:
 - ABILITY TO MEASURE LOCAL QUBIT CORRELATORS IN MANY-BODY STATE

EXTRA SLIDES BEYOND HERE

Dissipation and decoherence are caused by coupling to quantum and thermal noise of the surrounding reservoir.

Example:
$$H = \frac{\omega_q}{2} \sigma^z + \hat{F}(t) \sigma^x$$

Classically the noise correlator is real:
$$G_{FF}(t) \equiv \langle \hat{F}(t) \hat{F}(0) \rangle$$

and hence its spectral density
$$S_{FF}(\omega) \equiv \int dt e^{i\omega t} \langle \hat{F}(t) \hat{F}(0) \rangle$$

is symmetric in frequency:
$$S_{FF}(\omega) = S_{FF}(-\omega)$$

Not true quantum mechanically because:
$$[\hat{F}(t), \hat{F}(0)] \neq 0$$

Example: $H = \omega_q \sigma^z + \hat{F}(t) \sigma^x$

$[\hat{F}(t), \hat{F}(0)] \neq 0$ Implies $\hat{F}(t)\hat{F}(0)$ is not Hermitian

and so the noise correlator $G_{FF}(t) \equiv \langle \hat{F}(t)\hat{F}(0) \rangle$ is complex

and hence its spectral density $S_{FF}(\omega) \equiv \int dt e^{i\omega t} \langle \hat{F}(t)\hat{F}(0) \rangle$

can be asymmetric in frequency: $S_{FF}(\omega) \neq S_{FF}(-\omega)$

harmonic oscillator example:

$$\hat{F} = F_{\text{ZPF}} \left(\hat{b} + \hat{b}^\dagger \right)$$

$$G_{FF}(t) \equiv \left\langle \hat{F}(t) \hat{F}(0) \right\rangle = F_{\text{ZPF}}^2 \left[\left\langle \hat{b}(t) \hat{b}^\dagger(0) \right\rangle + \left\langle \hat{b}^\dagger(t) \hat{b}(0) \right\rangle \right]$$

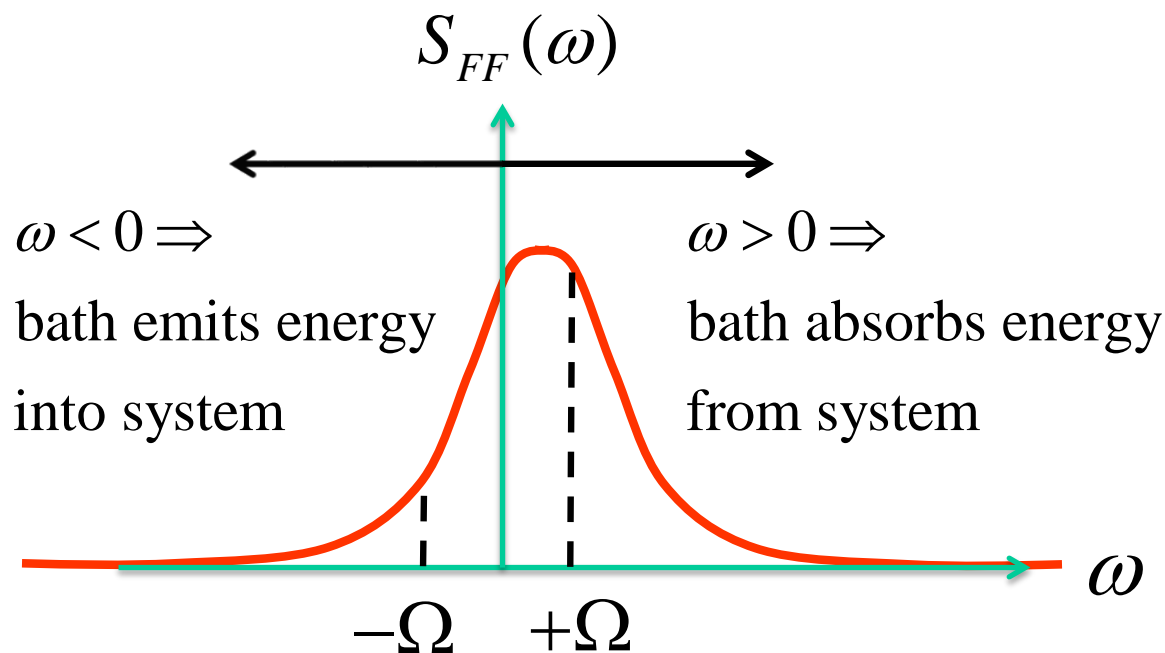
$$G_{FF}(t) = F_{\text{ZPF}}^2 \left[(n_B + 1) e^{-i\omega_0 t} + n_B e^{+i\omega_0 t} \right] \quad (\text{is complex})$$

$$S_{FF}(\omega) = 2\pi \left[(n_B + 1) \delta(\omega - \omega_0) + n_B \delta(\omega + \omega_0) \right] \quad (\text{is asymmetric})$$

Stimulated emission
(bath absorbs)

Absorption
(bath emits)

quantum noise NOT (necessarily) symmetric in frequency:

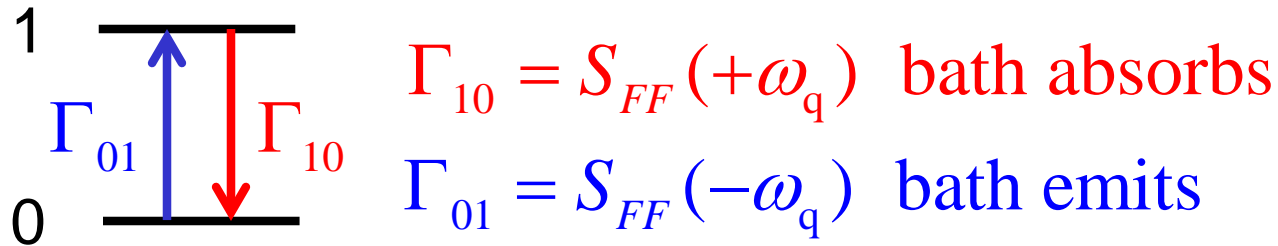


In thermal equilibrium:
(or pseudo-equilibrium)

$$\frac{S_{FF}(+\Omega)}{S_{FF}(-\Omega)} = e^{\beta\hbar\Omega} = 1 \text{ for } \hbar \rightarrow 0$$

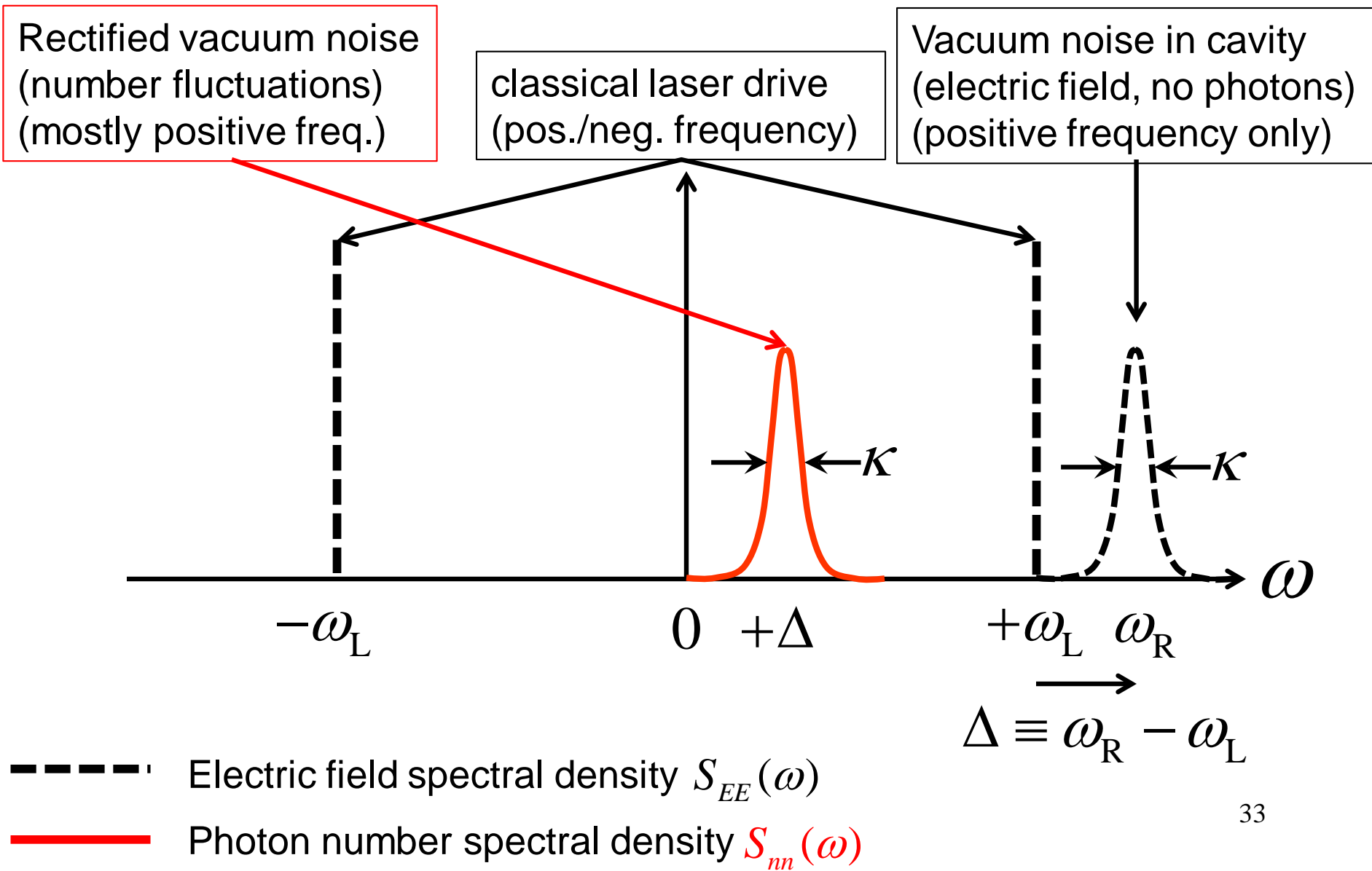
Fermi's Golden Rule in terms of noise spectral density

Example:
$$H = \frac{\omega_q}{2} \sigma^z + \hat{F}(t) \sigma^x$$



For quantum noise picture of FGR see:
Clerk et al. *Rev. Mod. Phys.* **82**, 1155 (2010)

Shot noise as 'rectified vacuum noise'



Rabi experiment: decoherence of driven qubit in rotating frame:

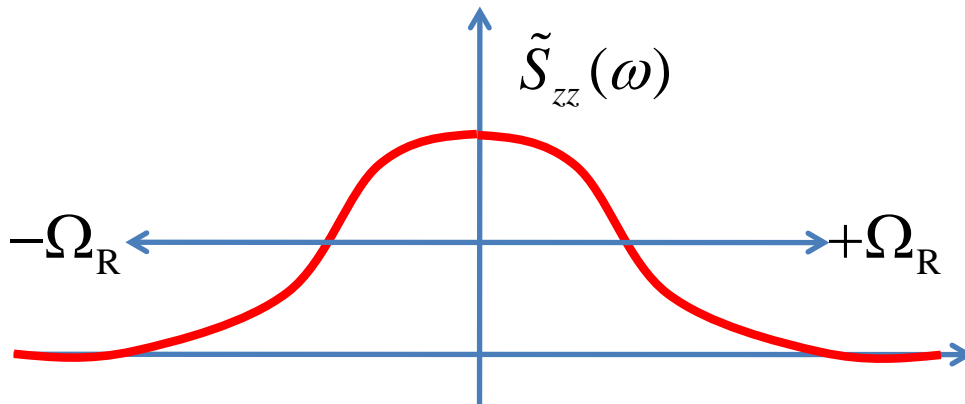
$$H = \frac{\Omega_R}{2} \sigma^x + \frac{1}{2} U(t) \left[\vec{\xi}(t) \cdot \vec{\sigma} \right] U^\dagger(t)$$

G. Ithier, et al., *Phys. Rev. B* **72**, 134519 (2005).

$$\frac{1}{T_{\text{Rabi}}} = \frac{3}{4} \frac{1}{T_1} + \frac{1}{2} \frac{1}{T_\varphi(\Omega_R)}$$

$$\frac{1}{T_\varphi(\Omega_R)} = \frac{1}{4} \left\{ \tilde{S}_{zz}(+\Omega_R) + \tilde{S}_{zz}(-\Omega_R) \right\} \approx \frac{1}{T_\varphi}$$

but can be smaller if Rabi rate is high



$$\hbar\Omega_R \ll k_B T$$

noise is classical so symmetric in frequency (and probably non-eq.)

Choose cavity drive detuning to match mechanical oscillator frequency

$$\Delta \equiv \omega_R - \omega_L = \Omega_M$$

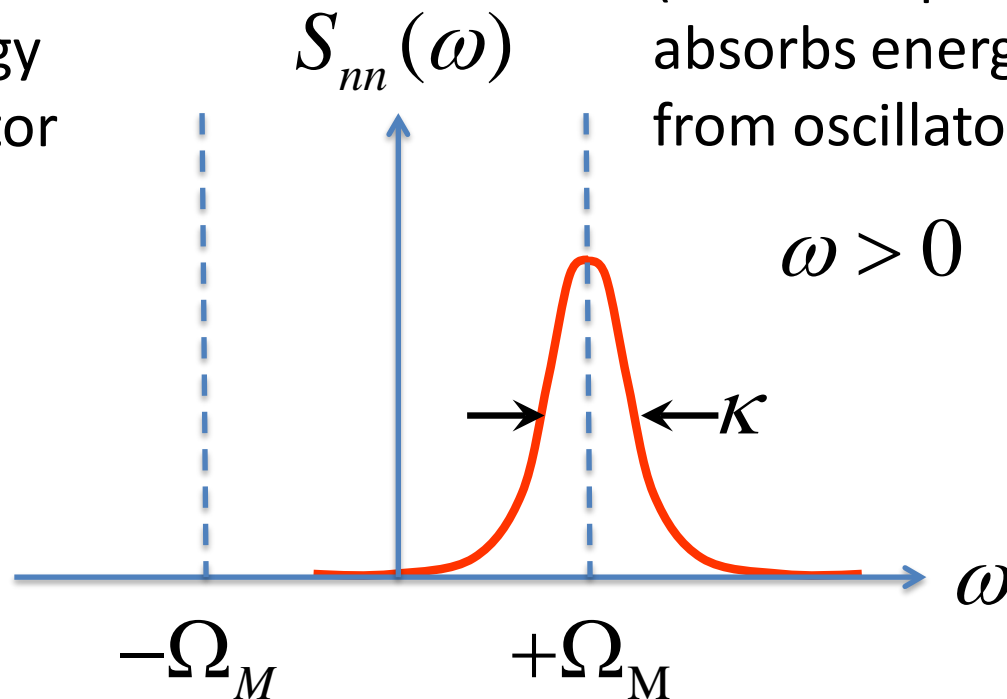
Resolved sideband limit: $+\Omega_M > \kappa \Rightarrow$ noise is cold

Shot noise
emits energy
into oscillator

$$\omega < 0$$

Shot noise
(radiation pressure)
absorbs energy
from oscillator

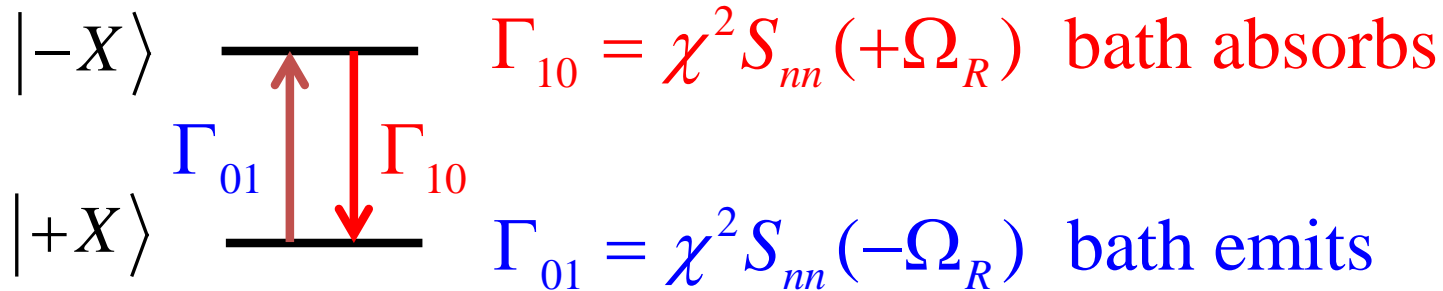
$$\omega > 0$$



Fermi's Golden Rule in terms of noise spectral density

$$H = -\Omega_{\text{Rabi}} \sigma^X + \chi \sigma^Z (\hat{n} - \bar{n})$$

shot noise of driven cavity



For quantum noise picture of FGR see:

Clerk et al. *Rev. Mod. Phys.* **82**, 1155 (2010)