

# Quantum interferences in Josephson junctions with large spin orbit coupling

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K. Napolskii, D. Koshkodaev, G. Tsirlina, Y. Kasumov, I. Khodos (Moscow and Chernogolovka)

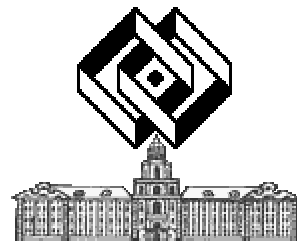
## Proximity effect in material with high spin orbit coupling



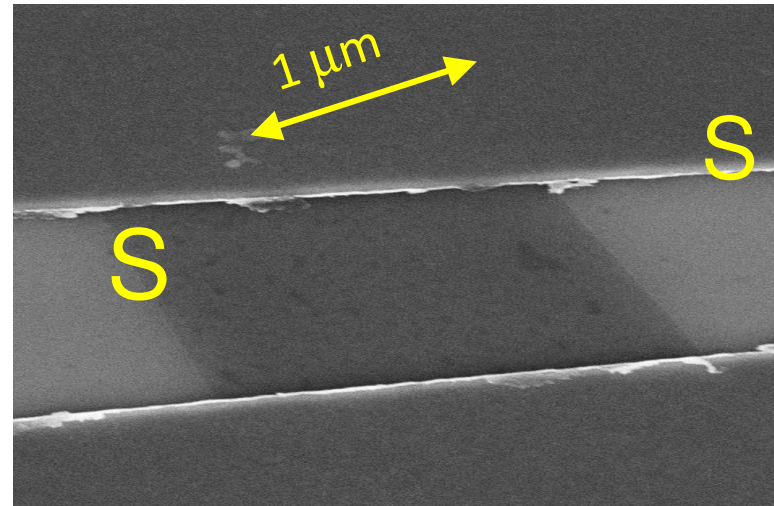
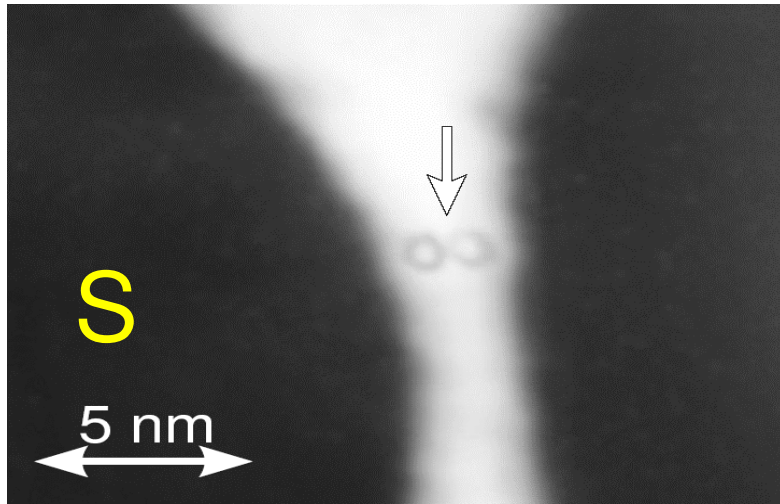
Faculty of Materials  
Science

Faculty of Chemistry

Institute of microelectron  
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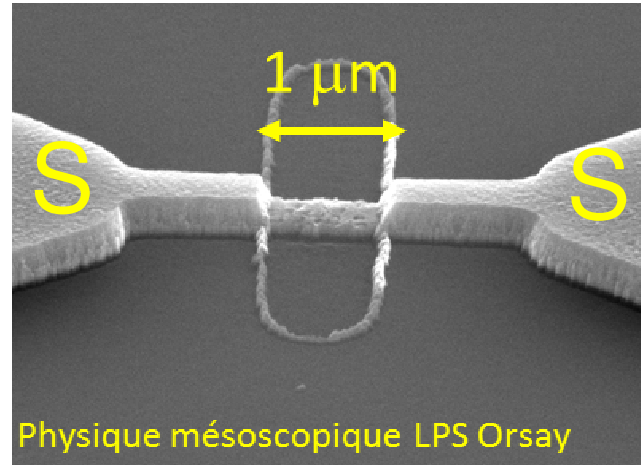
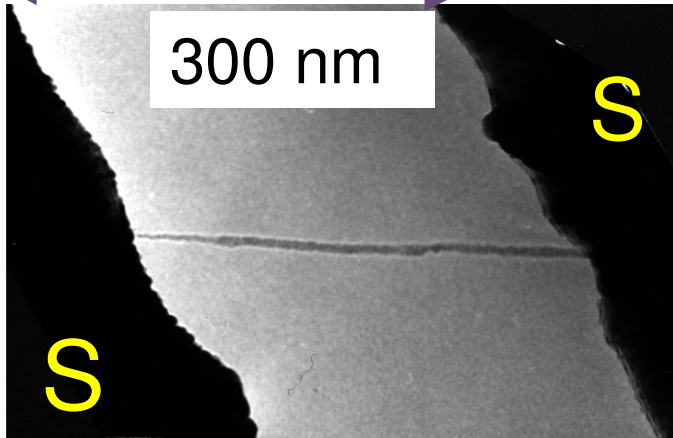


# Guessing game... What's what?



Physique mésoscopique LPS Orsay

Physique mésoscopique LPS Orsay



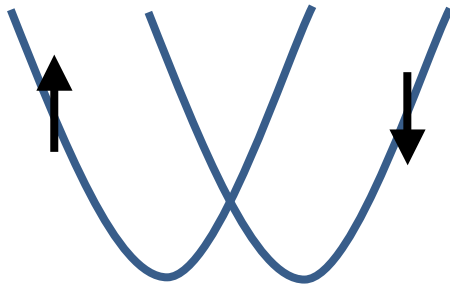
Physique mésoscopique LPS Orsay

Proximity effect reveals spin dynamics, intrinsic pairing, atomic orbitals, dephasing, interference, band structure

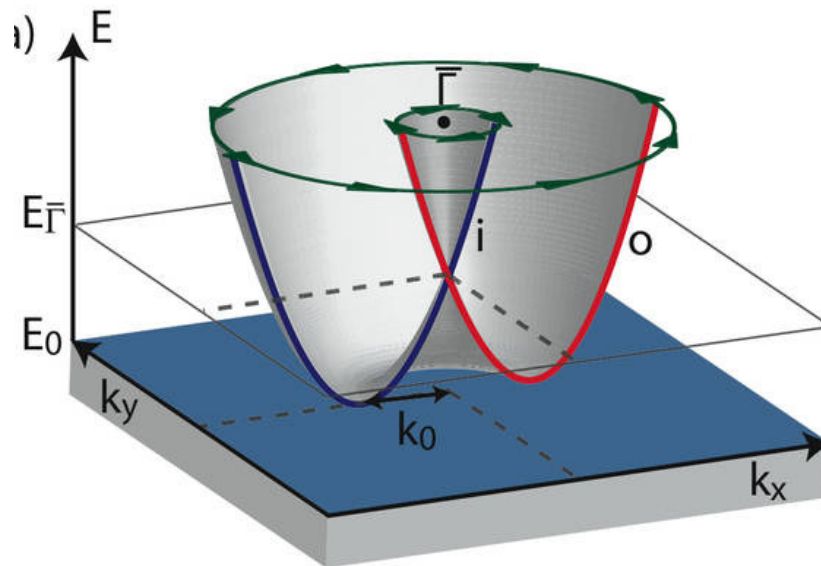
# Spin orbit coupling

$$= \frac{\hbar}{4m^2c^2} \mathbf{s} \cdot (\nabla V \times \mathbf{p}) \quad \text{Spin orbit interactions couple spin and spatial degrees of freedom}$$

$$H_{so} = \underbrace{\gamma_D(k_x\sigma_y + k_y\sigma_x)}_{D_{2d} \text{ Dresselhaus}} + \underbrace{\alpha_{BR}(k_x\sigma_y - k_y\sigma_x)}_{C_{4v} \text{ Bychkov-Rashba}}$$



Spin Split bands



# Spin orbit coupling

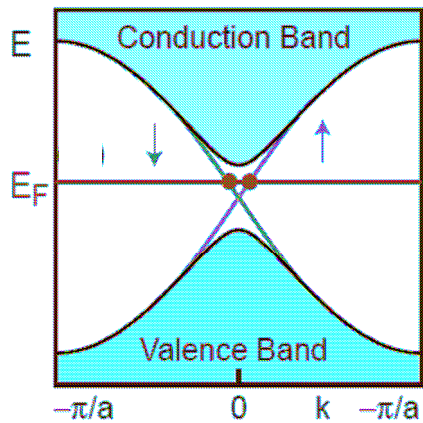
$$= \frac{\hbar}{4m^2c^2} \mathbf{s} \cdot (\nabla V \times \mathbf{p}) \quad \text{Spin orbit interactions couple spin and spatial degrees of freedom}$$

$$H_{so} = \underbrace{\gamma_D(k_x\sigma_y + k_y\sigma_x)}_{D_{2d} \text{ Dresselhaus}} + \underbrace{\alpha_{BR}(k_x\sigma_y - k_y\sigma_x)}_{C_{4v} \text{ Bychkov-Rashba}}$$

depends on the Crystal symmetry:

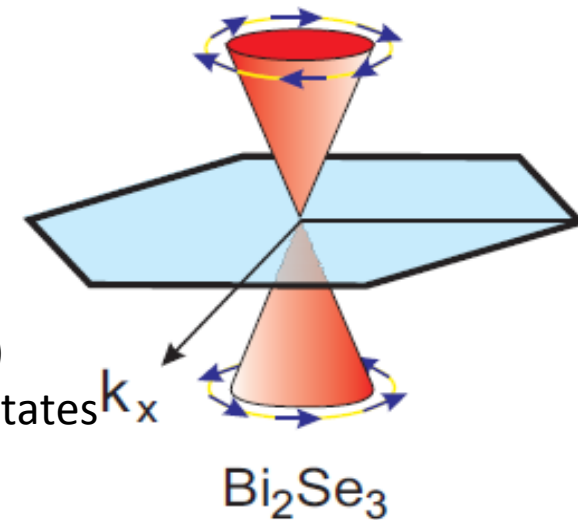
ability to create a topological insulator

2D:

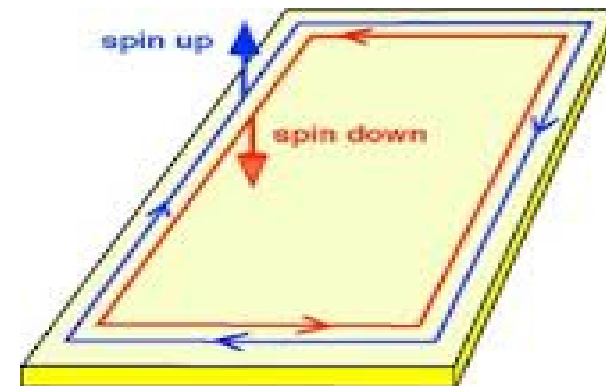


Formation of 1D  
counter propagating  
spin polarised edge states  
Protected from disorder

Topological Insulator

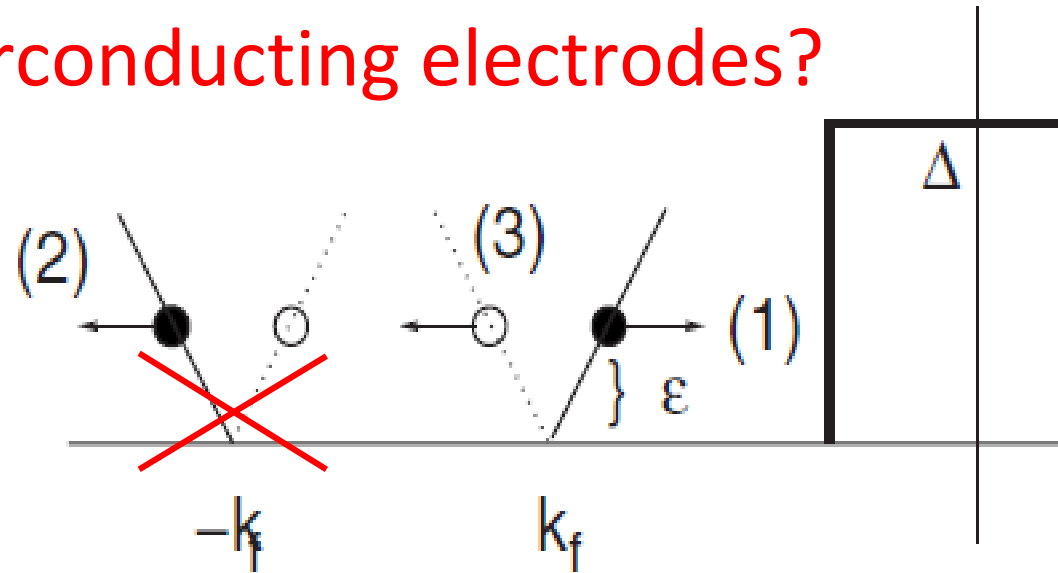


Hasan Kane RMP (2011)  
2D conducting surface states  $k_x$



# What happens with superconducting electrodes?

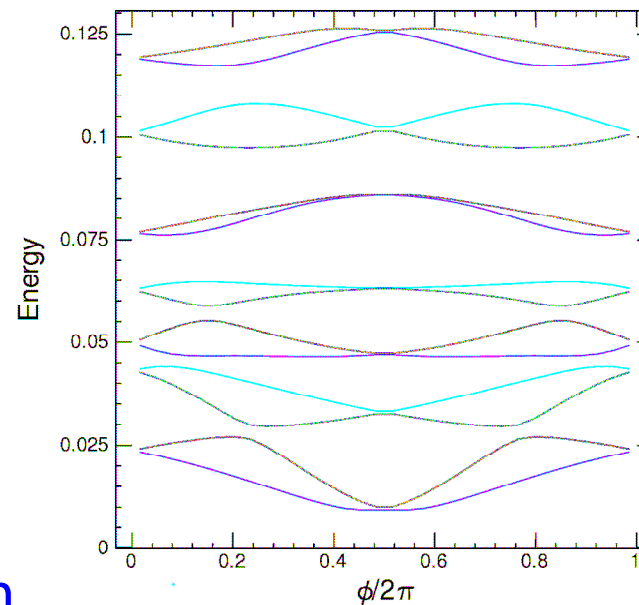
Enhanced Andreev reflection



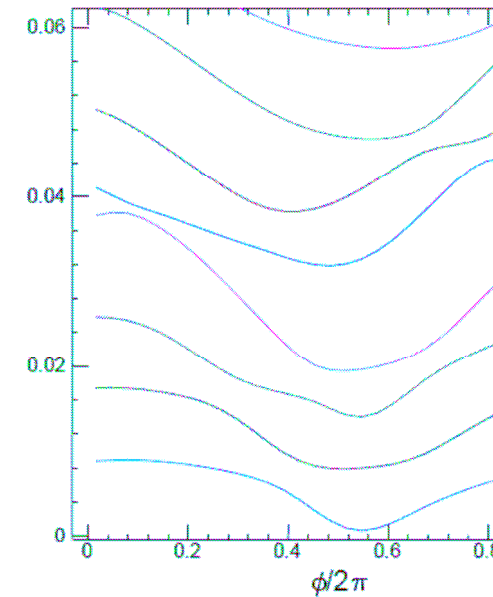
Backward scattering (2) forbidden  
Perfect Andreev reflection (3)

Spin split Andreev states

Josephson junctions in Zeeman field



$$E_{SO} = E_F / 2$$



$$E_z = E_{SO} / 2$$

Possibility of mixed singlet triplet correlation....

Edge states: inhomogeneous current distribution

# Bismuth nanowires

Bulk, surface, edge states

## Josephson effect :

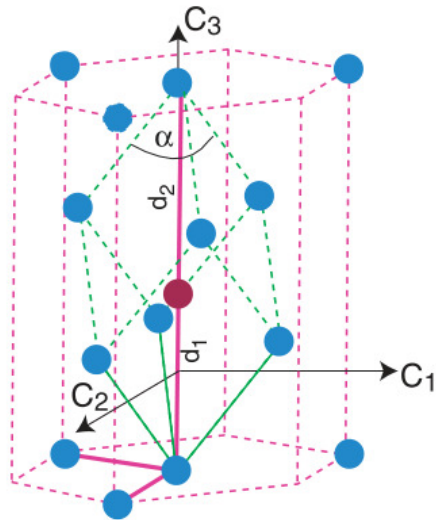
- High critical current: ballistic transport ?
  - Field resistant induced superconductivity:
  - Explanation via orbital and spin effects, and topological edge channels?
  - Investigation of the Andreev spectrum
-

# Bulk Bi

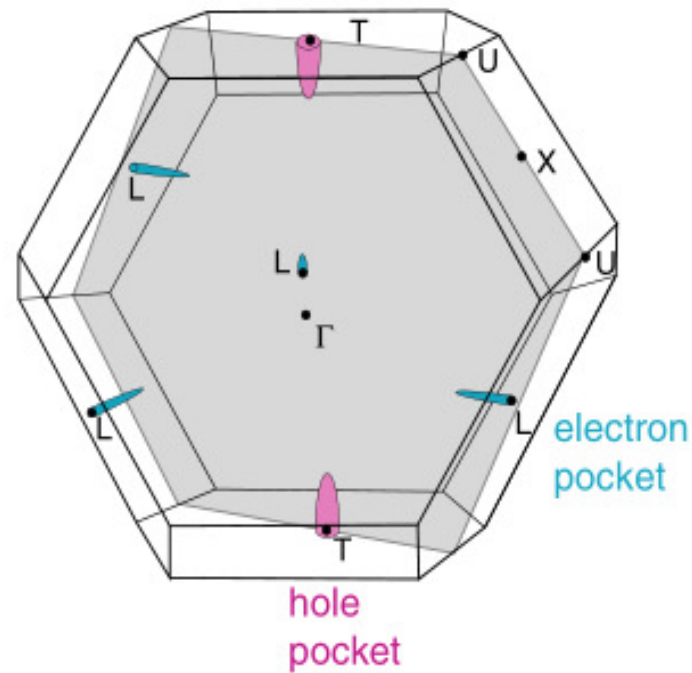
Hofmann 2006 review



rhomb.



Bulk Brillouin zone

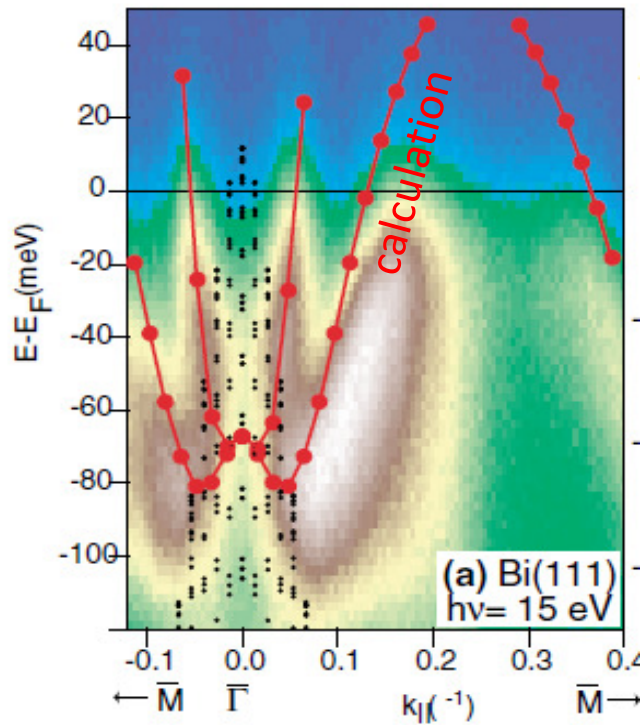
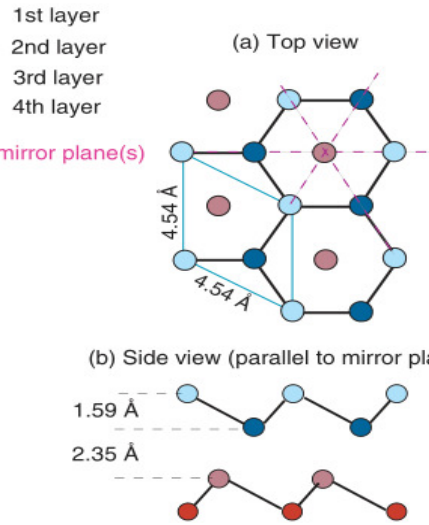


A semi-metal, with  $n \approx 3 \times 10^{17} \text{ cm}^{-3}$ ,  $m^* \approx 0.03 m_e$  and  $\lambda_F \approx 50 \text{ nm}$

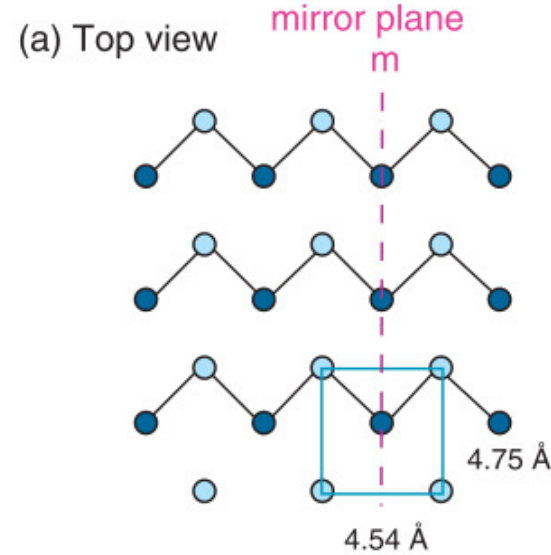
Centrosymmetric: Bulk SO averages to 0

# Spin split states at Bi surfaces

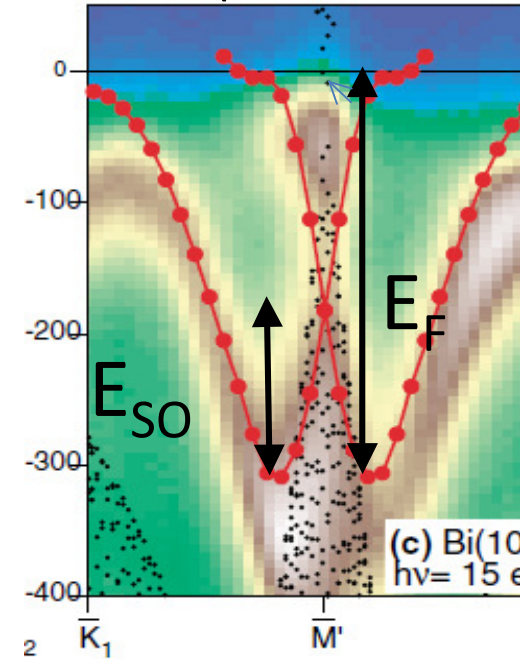
(111)



(100)



ARPES (Hoffman 2004)



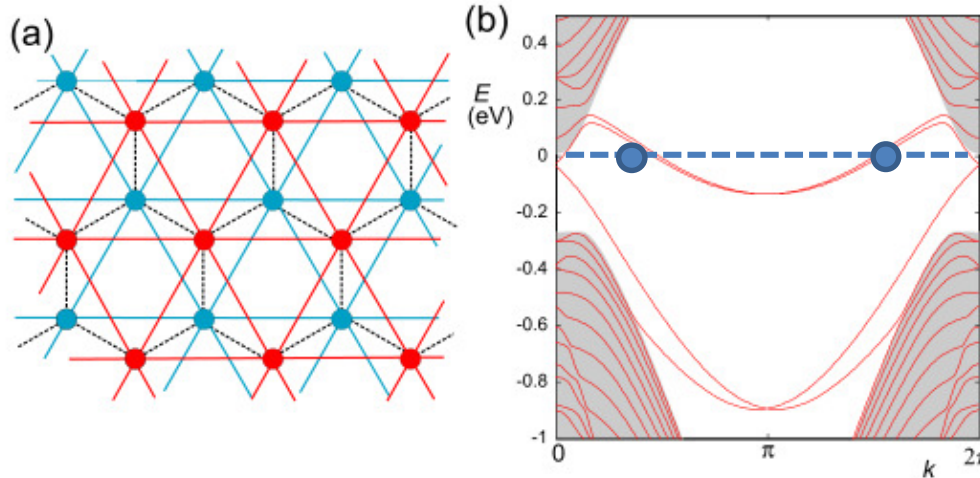
	Carrier density	$\lambda_F$	$m^*$
Bulk	$3 \times 10^{17} \text{ cm}^{-3}$	$\sim 50 \text{ nm}$	0.03
(111) surface	$3 \times 10^{13} \text{ cm}^{-2}$	$\sim \text{nm}$	0.3

$g_{\text{eff}}: 1 \sim 100$

All surfaces are different, but  $E_{SO} \sim E_F \sim 100 \text{ meV}$   
 Dominate transport for this layers or wires  $d < 90 \text{ nm}$



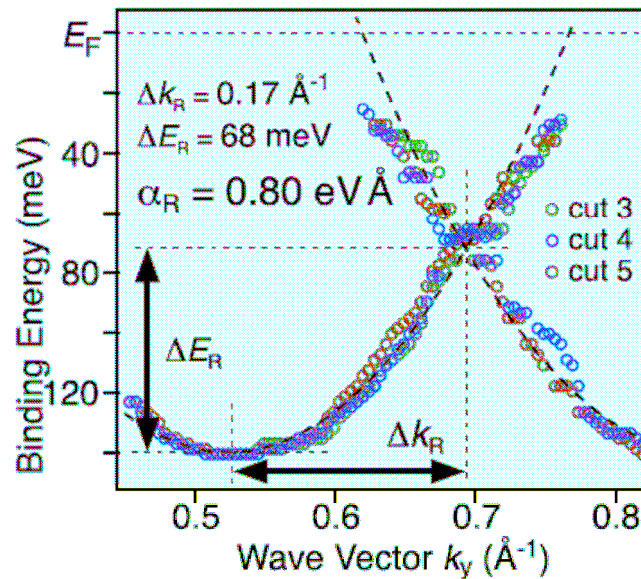
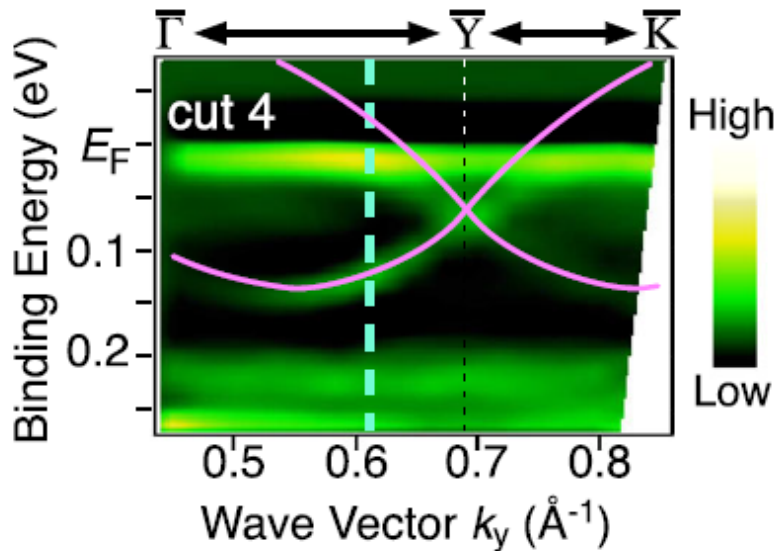
# Edge states on certain surfaces



Murakami 2006

Bi(111) bilayer  
Topological 2D  
insulator

What about thicker layers?

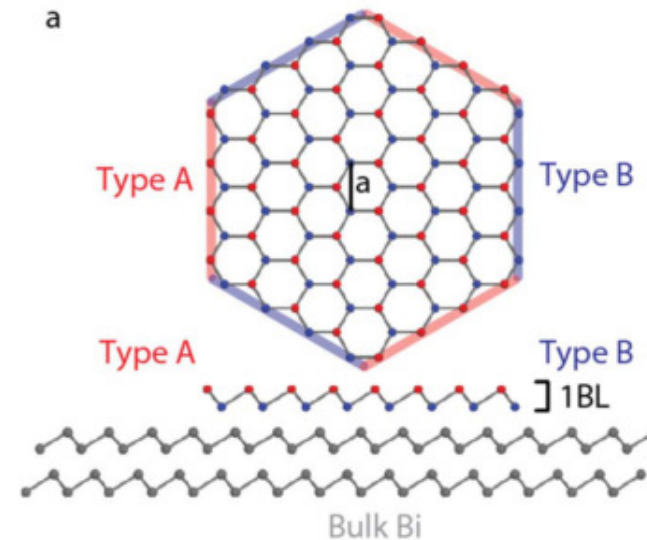
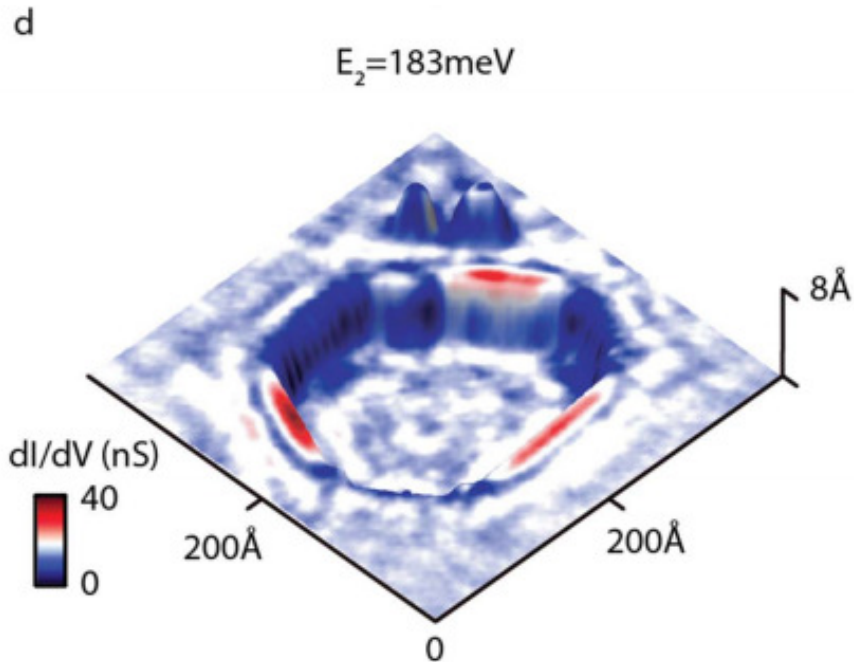


ARPES Takayama et al PRL

15 nm

Edge states but possibly  
not topological

# Bismuth edge states on (111) surface



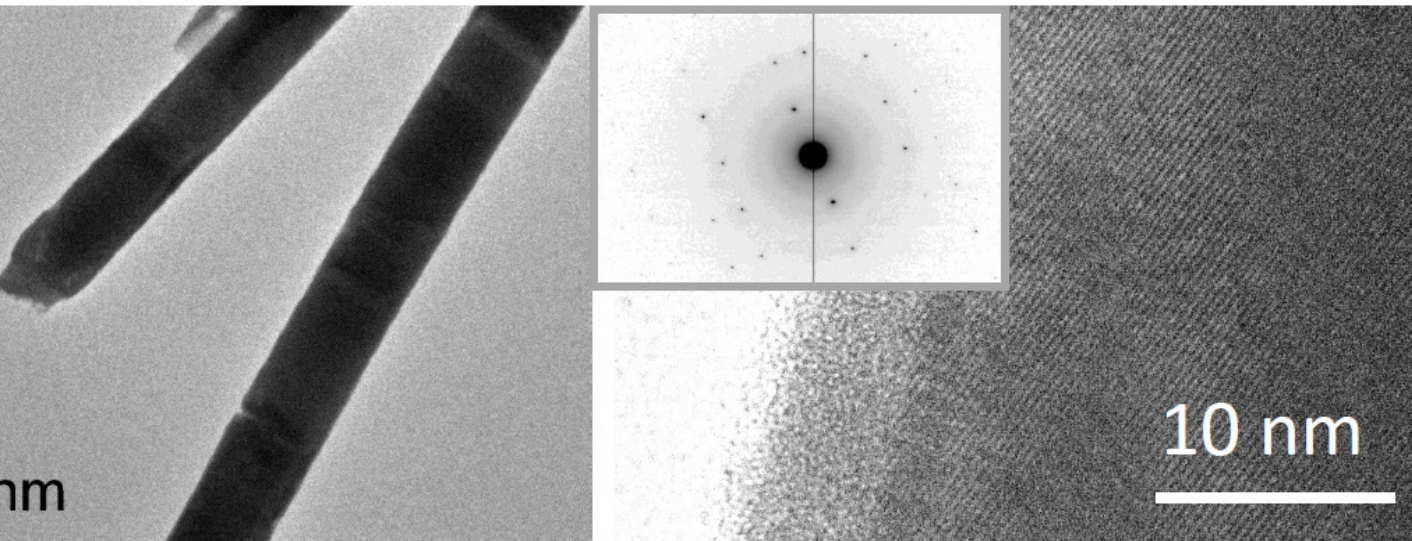
STM on Bi(111) bilayer small pit: **1D edge states at some edges**  
(Drozdov Yazdani, 2014)

**Conclusion:** confined Bi 3D semi metal  $\longrightarrow$  2D metal  $\longrightarrow$  1D edges

# Bi nanowires

Electrochemically grown in 90 nm-wide pores of polycarbonate membrane.

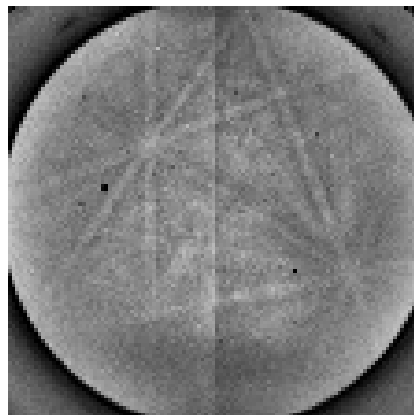
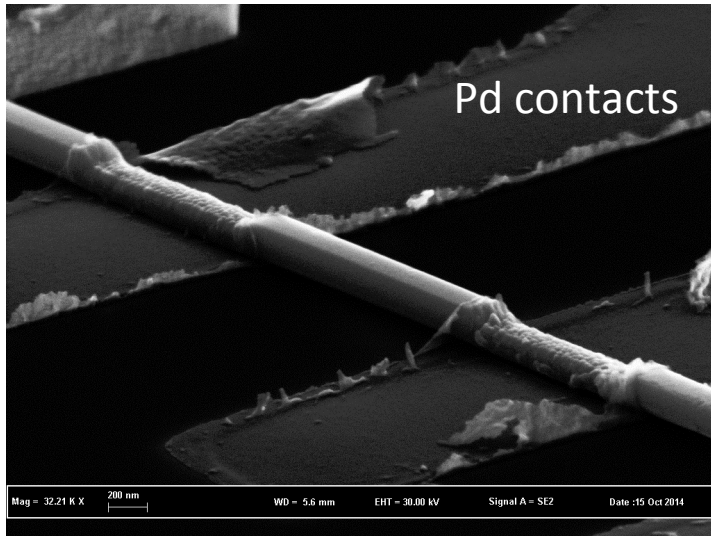
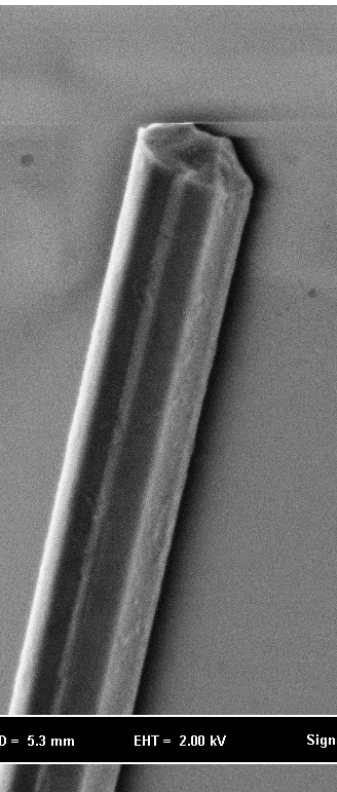
TEM image of similar wires grown in nanopores (G. Tsirlina)



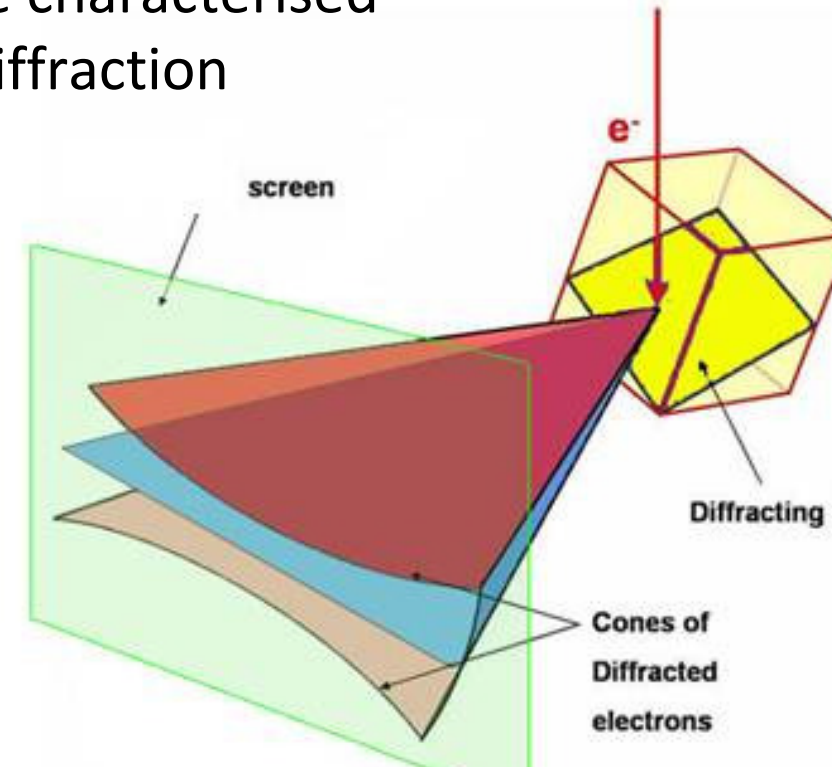
- probably faceted, with several orientations
- Practically monocrystalline (no high angle boundary)
- Protected by membrane residues

No possible characterisation of a selected wire for transport measurements

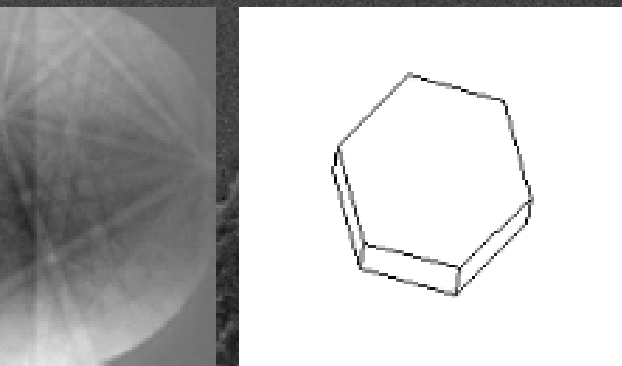
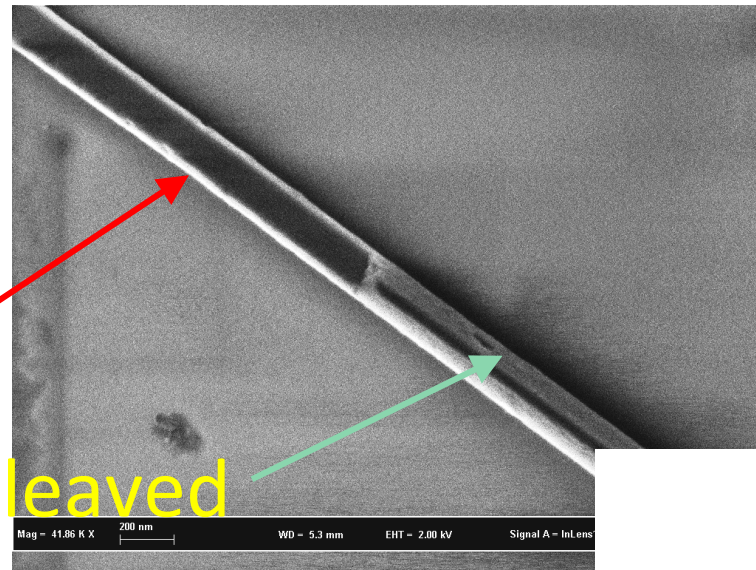
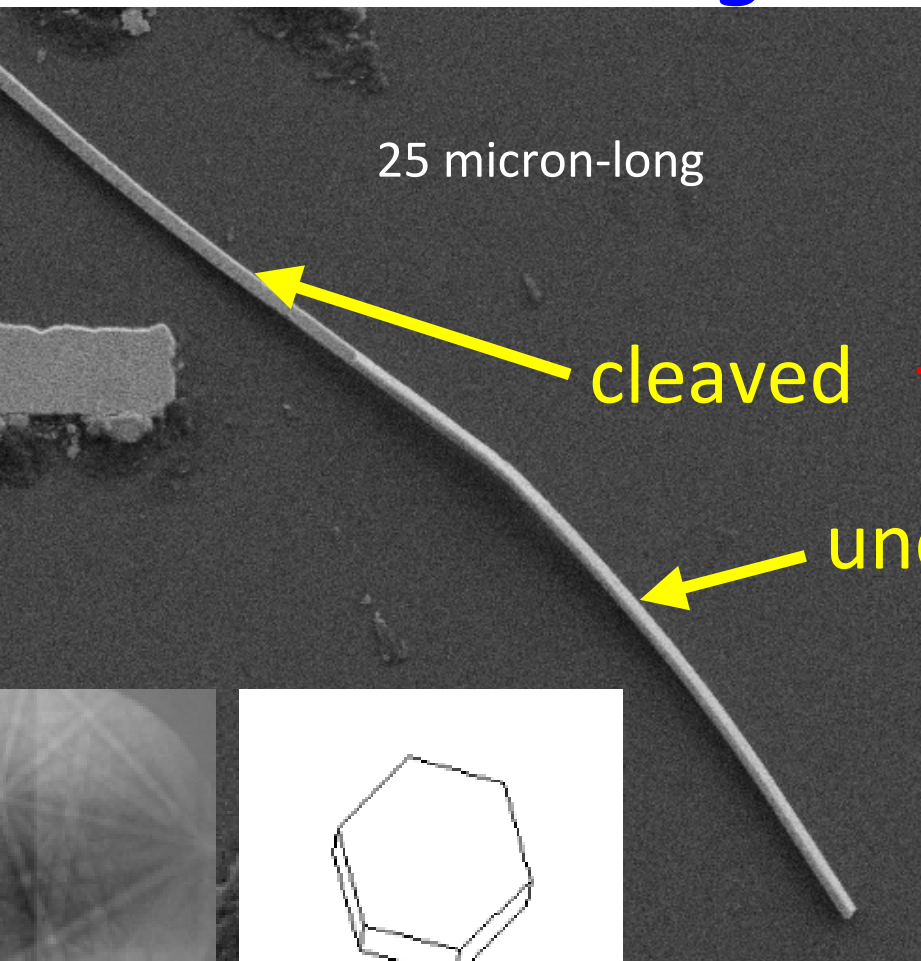
# Bi nanowires grown with sputtering



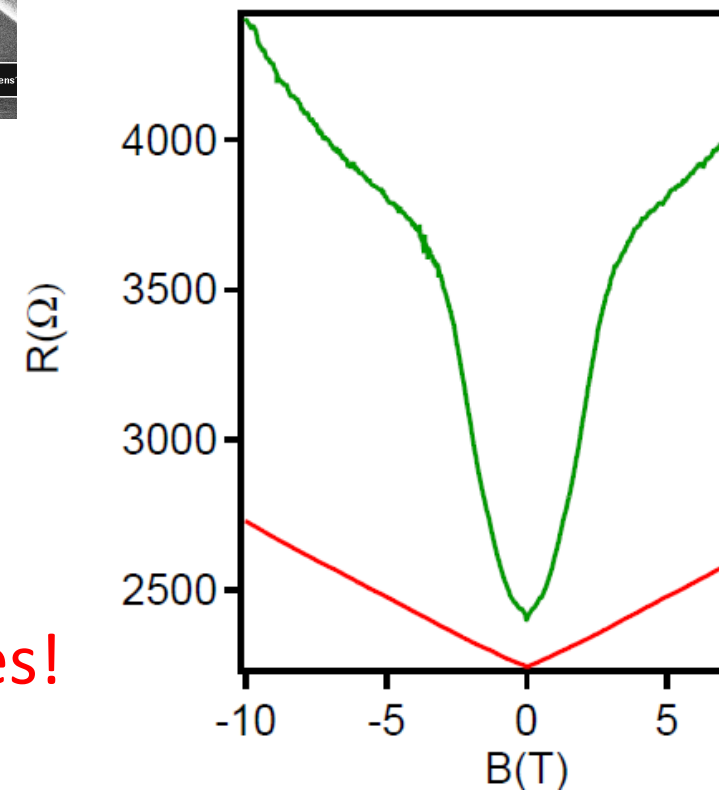
Orientation of Bi nanowire facets  
Can be characterised  
By e diffraction



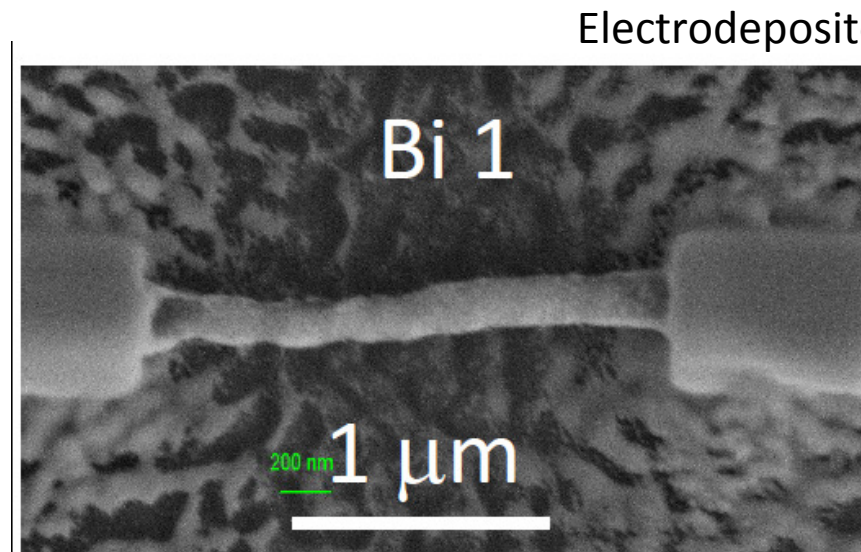
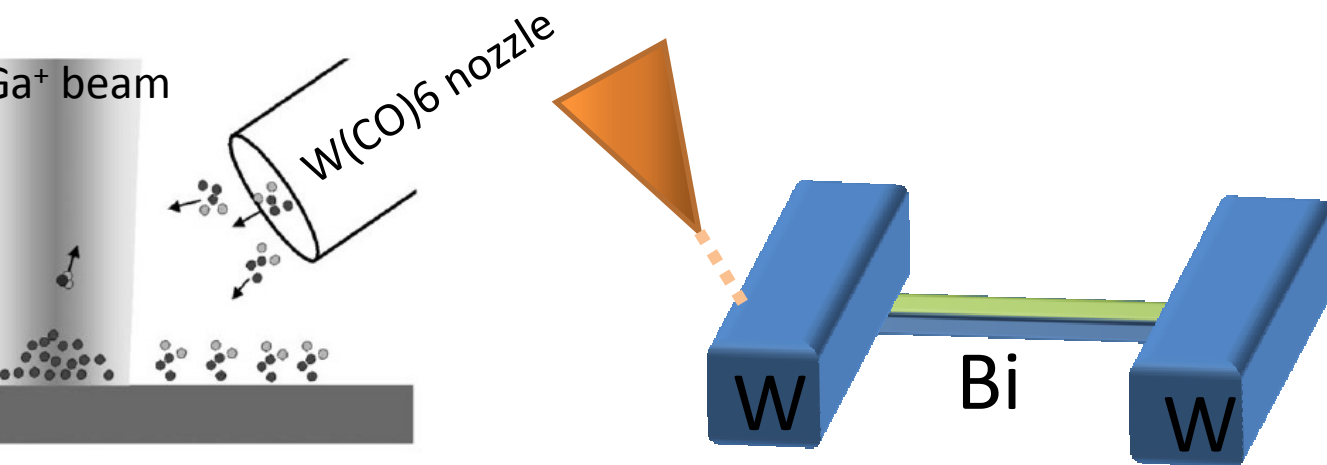
# Two segments with (111) facets



Give different magnetoresistances!



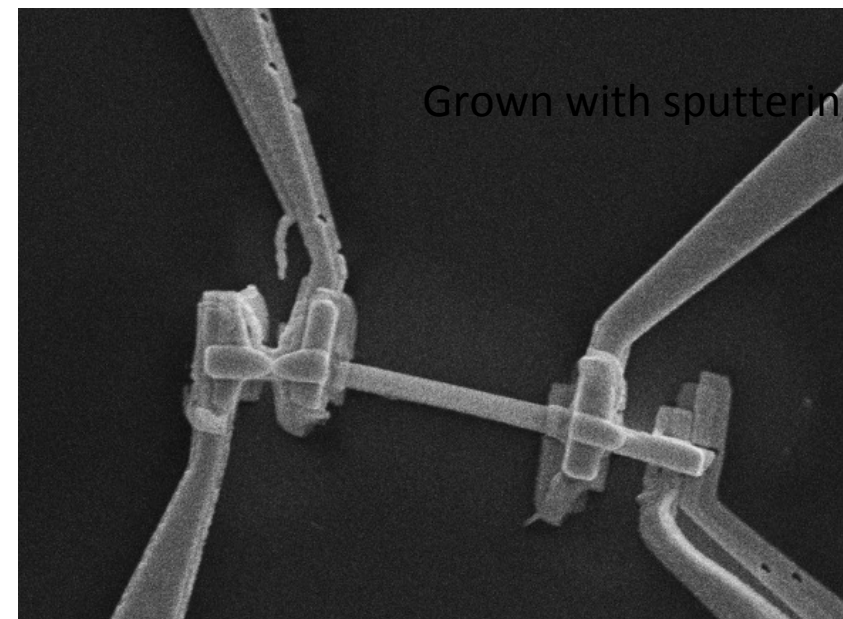
# Superconducting contacts by focused ion beam-assisted deposition



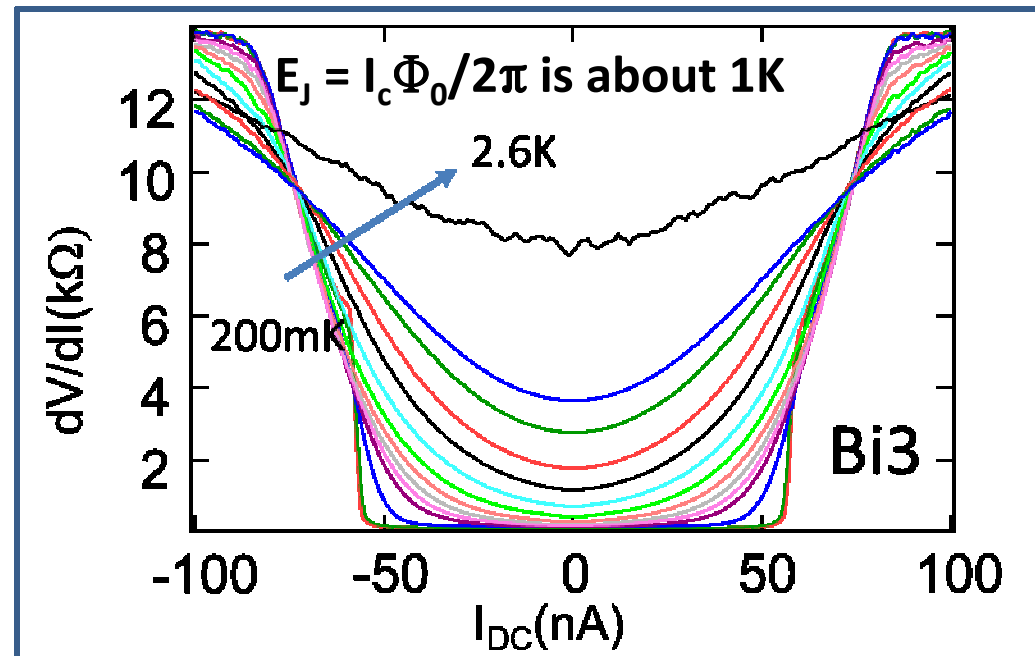
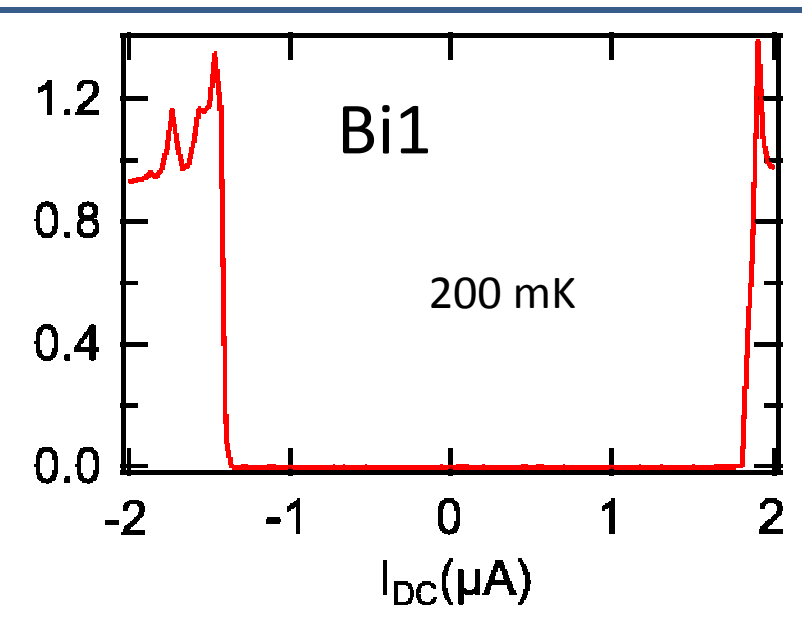
and Ga-doped amorphous W wire,  
roughly 200 nm thick and wide, with great  
superconducting properties:

$T_c \sim 4\text{K}$ ,  $\Delta \sim 0.8\text{ meV}$ ,  $H_c \sim 12\text{ Tesla!}$

Gasumov 2005, Guillamon 2008



# Supercurrent in W/Bi/W junctions



	Bi1	Bi2	Bi3	Bi4
	1 kΩ	10 kΩ	16 kΩ	1,2 kΩ
	1,8 μA	140 nA	70 nA	750 nA
length	1.9 μm	2 μm	1.6 μm	1.4 μm

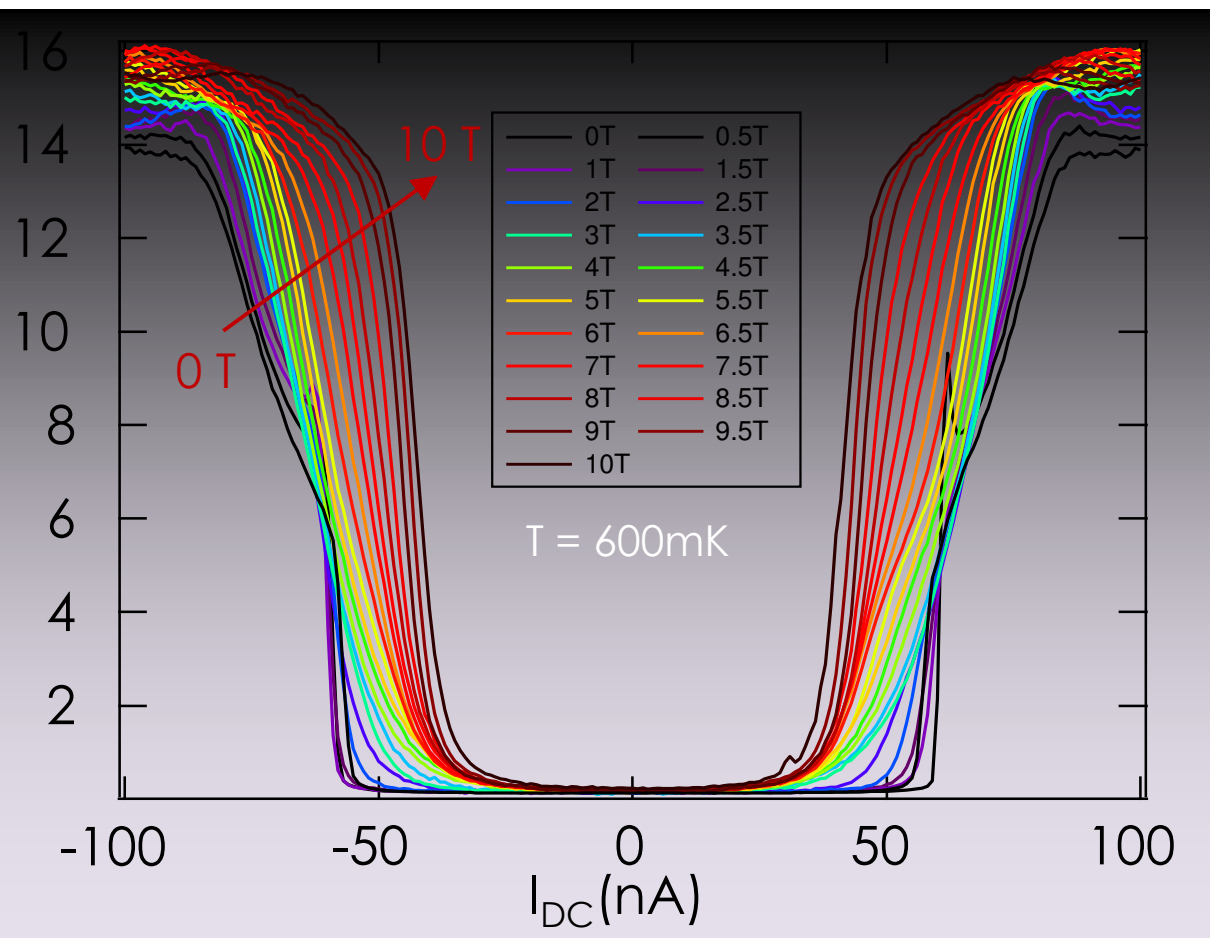
$$R_N I_C \sim 1-2 \text{ meV} \sim \Delta_W:$$

maximum critical current possible for SNS junction !

High critical current at zero field , much higher than for Ag nanowires

Nearly perfect Andreev reflection in spite of interface barriers

# Back to S/Bi/S: Supercurrent persists to huge $\perp$ field!

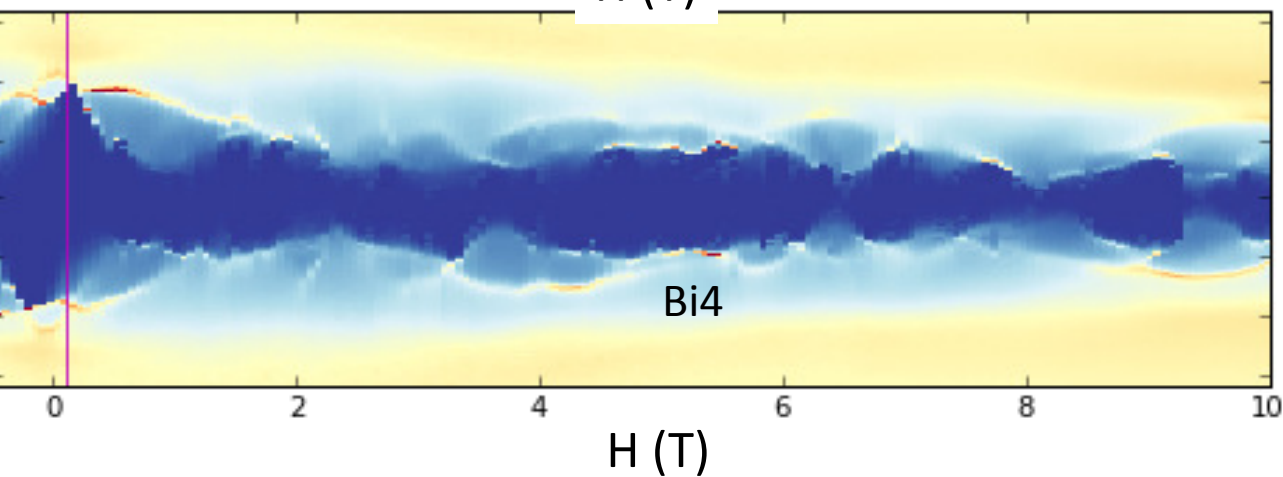
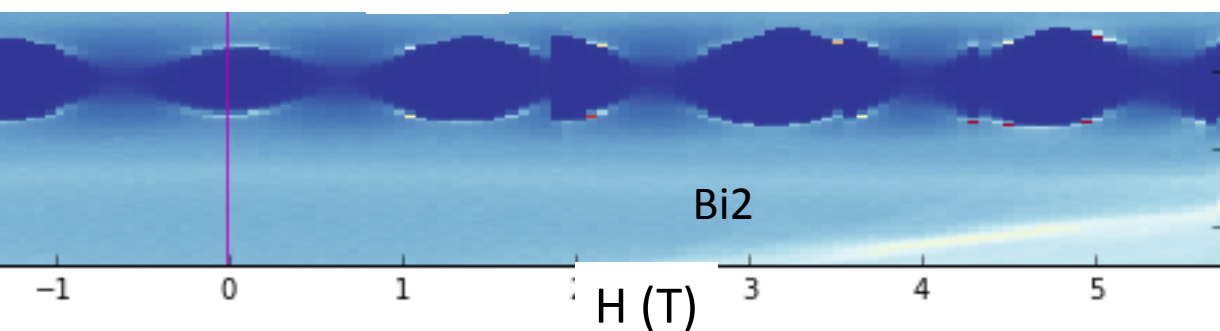
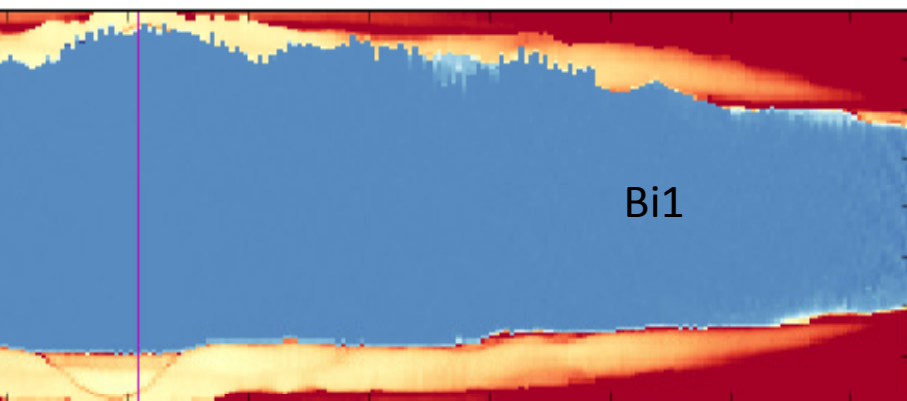


10 T corresponds to  $1000 \Phi_0$  in the  $100\text{ nm} \times 2\text{ }\mu\text{m}$  wire!

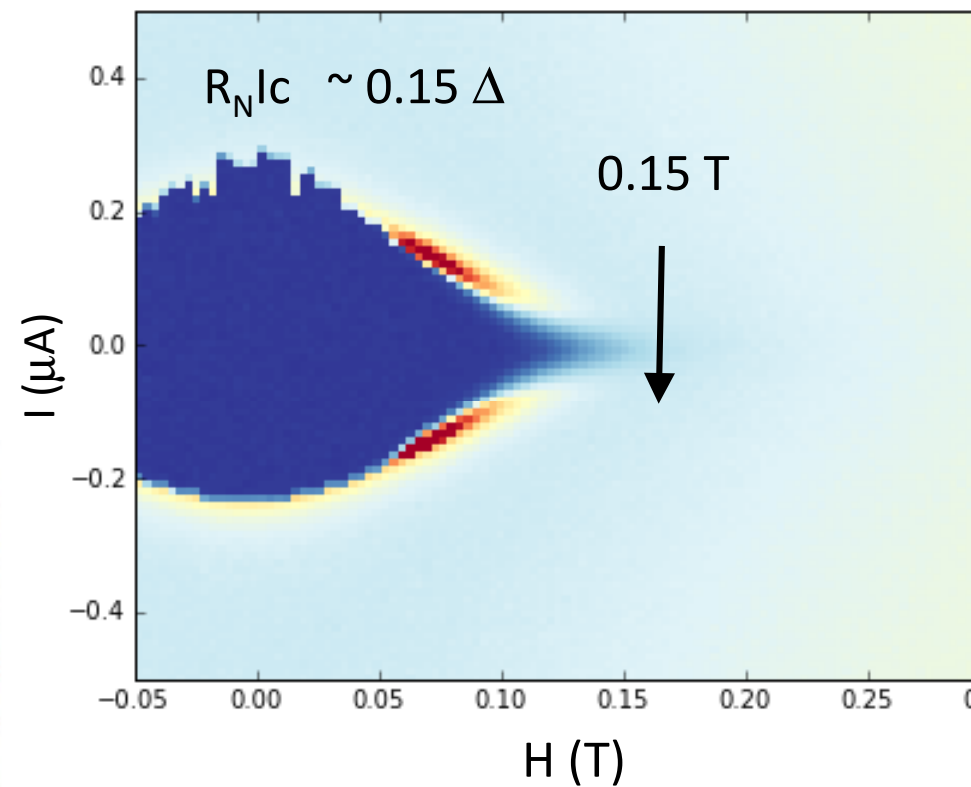
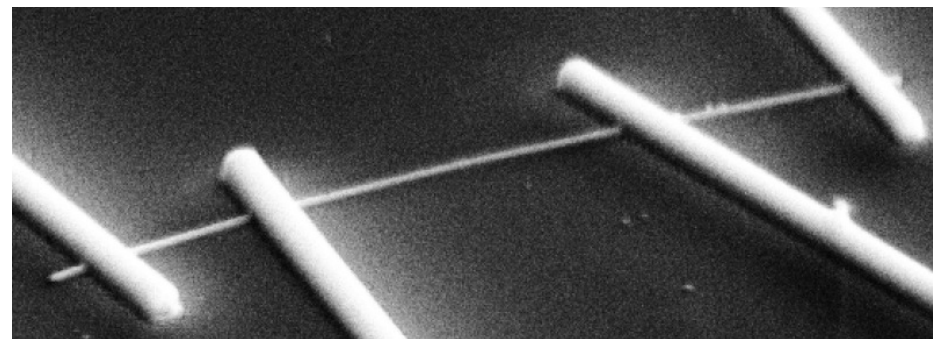
Diffusive multichannel model doesn't work.  
Field dependence implies: few ballistic channels!



# Bi nanowires

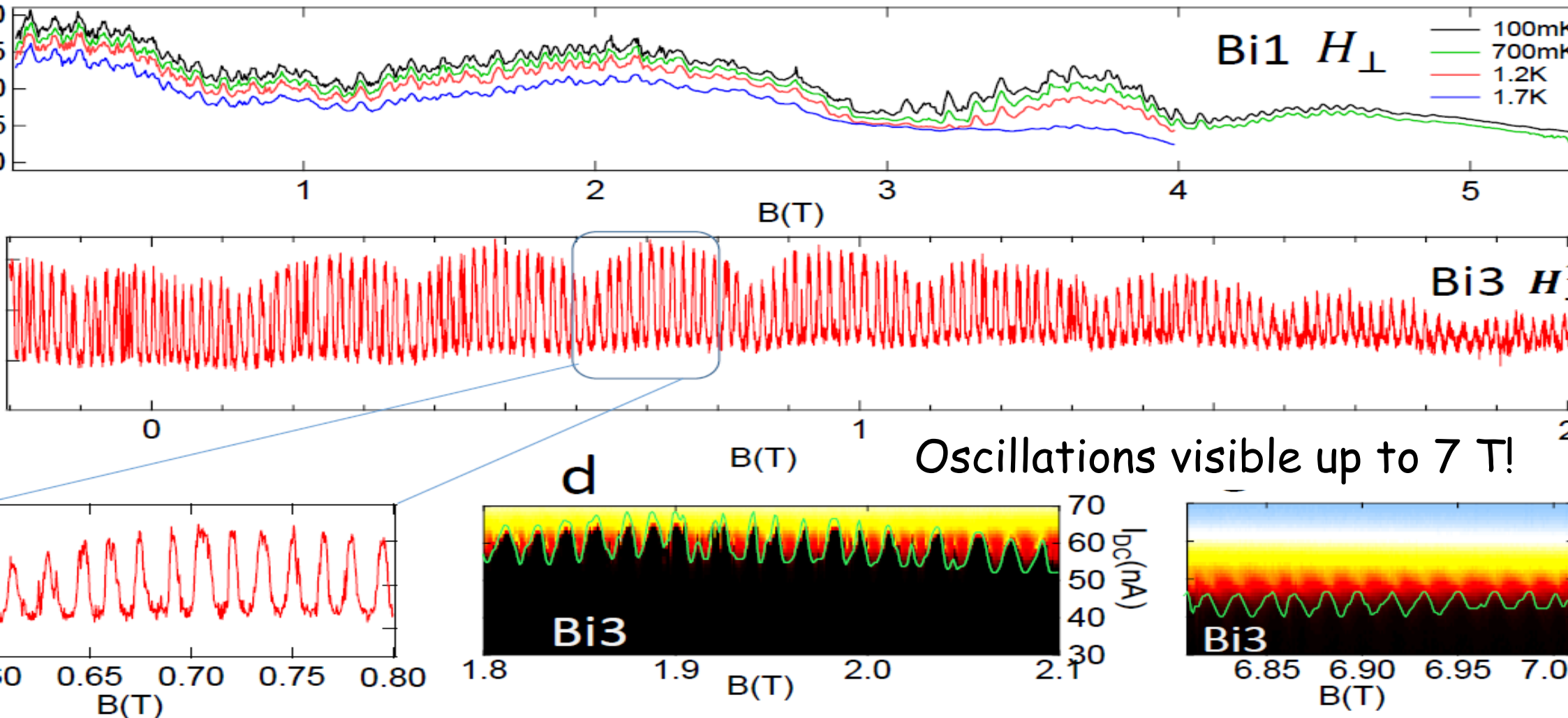


# Ag nanowire



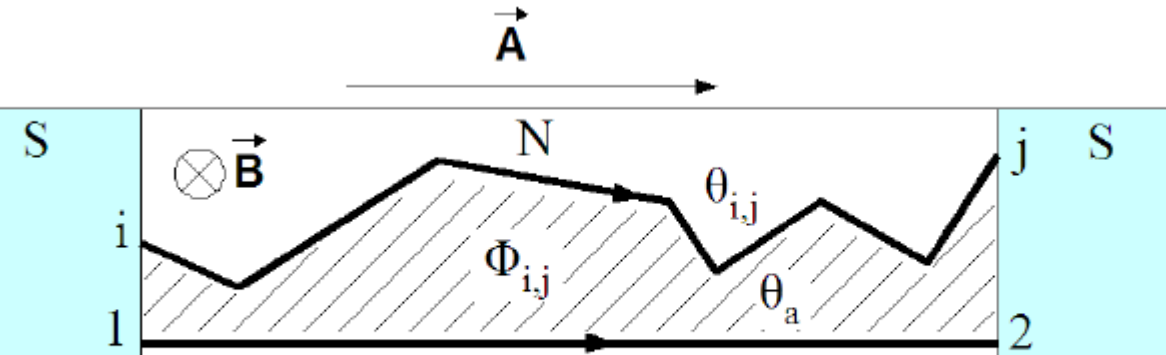
Very different behavior

# Small period field oscillations of the switching current



usive multichannel model doesn't work. Field dependence  
es: few ballistic channels!

# Narrow diffusive sample with many channels: Flux dependent phase variation in sample



Cuevas, Montambaux

$$I_c \propto \left| \langle e^{i\Delta\theta_{i,j}} \rangle_{c_{i,j}} \right| \quad I_c \propto \left| e^{-\langle (\Delta\theta_{i,j})^2 \rangle_{c_{i,j}}/2} \right|$$

Diffusive trajectories encircle different fluxes  
so pick up different phases

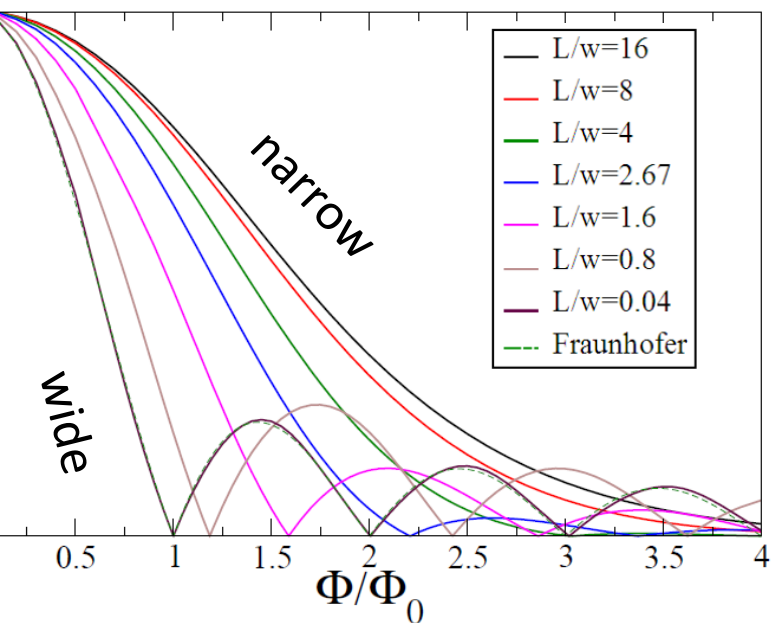
$$\Delta\theta_{i,j} = \frac{2e}{\hbar} \left[ \int_i^j A_x dx - \int_1^2 A_x dx \right] = \frac{2e}{\hbar} \oint A_x dx = \frac{2\pi}{\Phi_0} H S_{i,j} = 2\pi \frac{\Phi_{i,j}}{\Phi_0} \quad (2.5)$$

$$I_c \propto \left| e^{-2\pi^2 H^2 \alpha^2 / \Phi_0^2} \right|$$

~ Gaussian decay of  $I_c$  on scale of  $\Phi_0$   
because dephasing by field

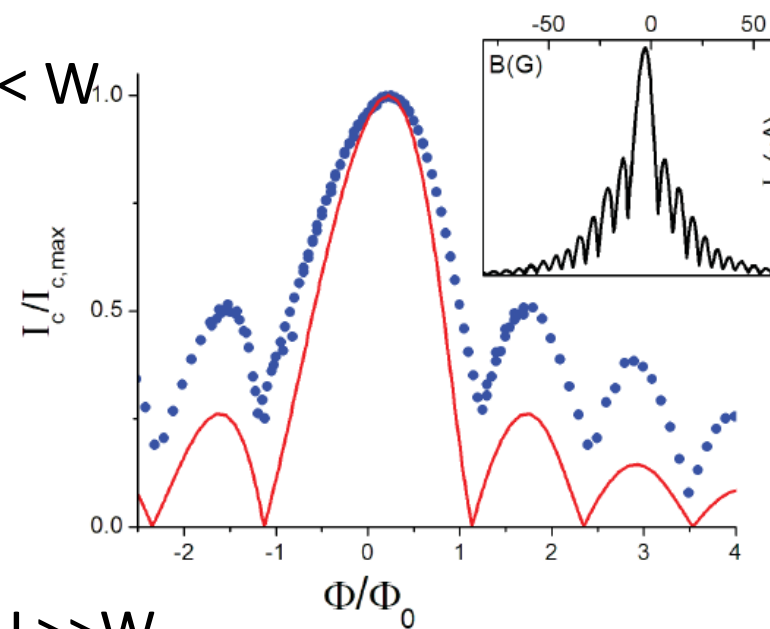
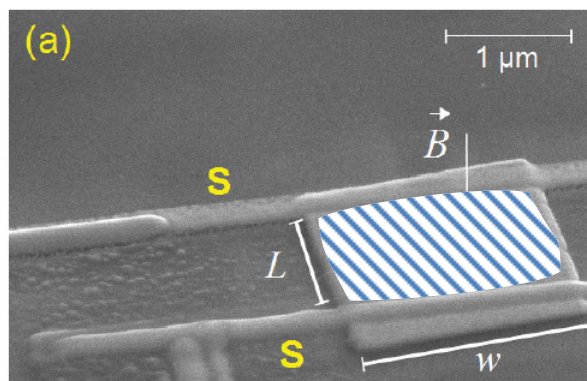
# Role of geometry demonstrated in SNS junctions

theory: Bergeret Cuevas 2008

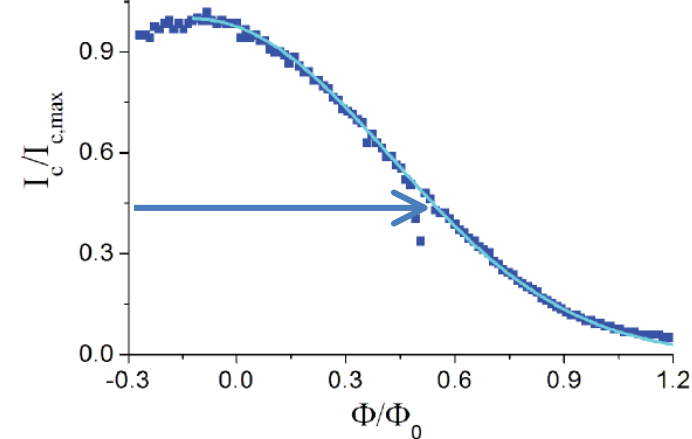
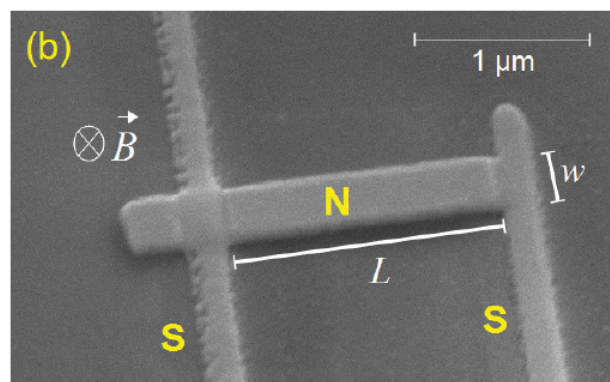


Exp: Chiodi 2012

wide:  $L < W$

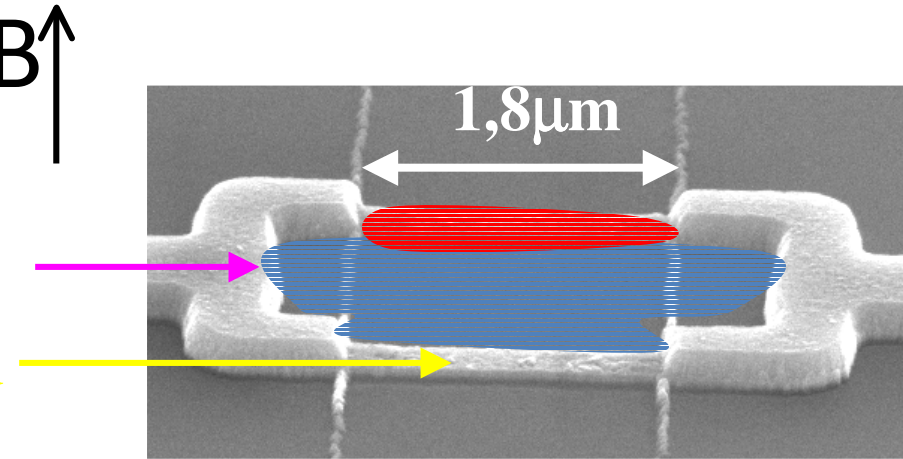


Narrow:  $L \gg W$

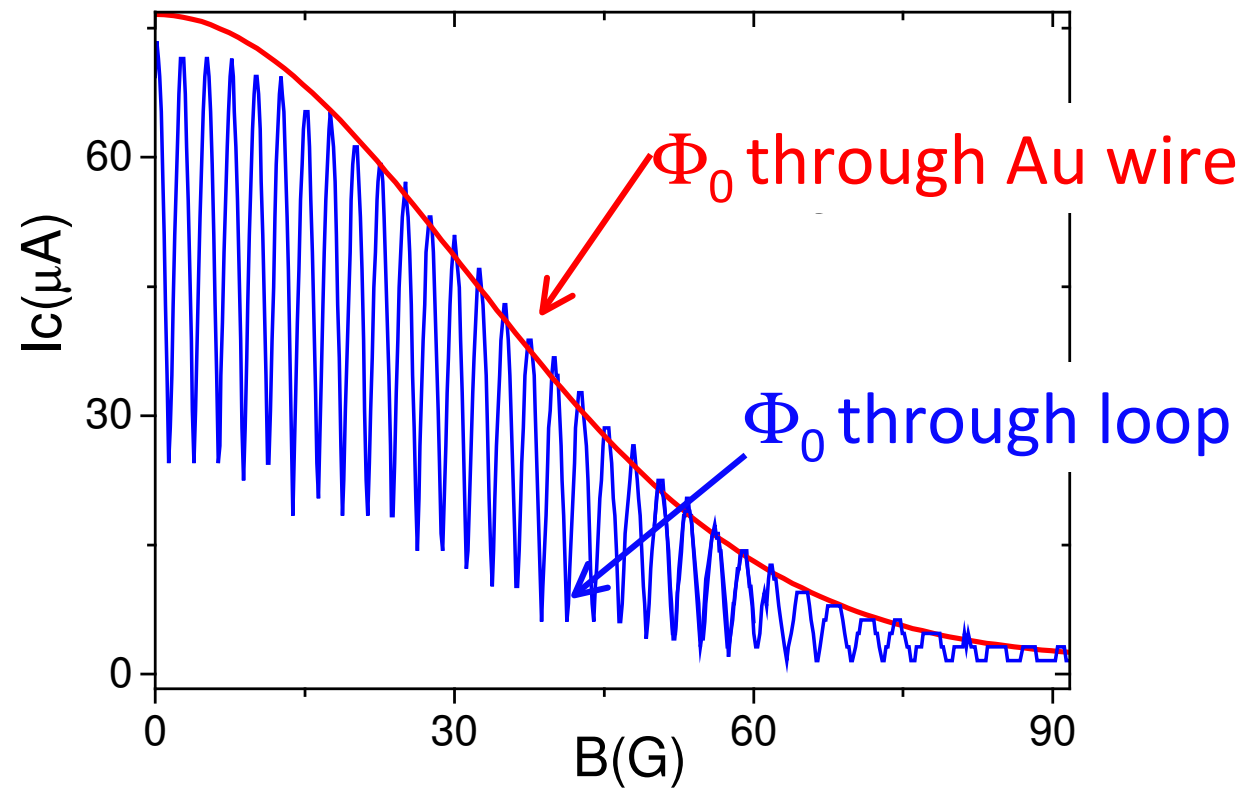


$I_c$  decays on scale of  $\Phi_0$  through sample surface (100 G)

# SNS squid junction



Angers 2008

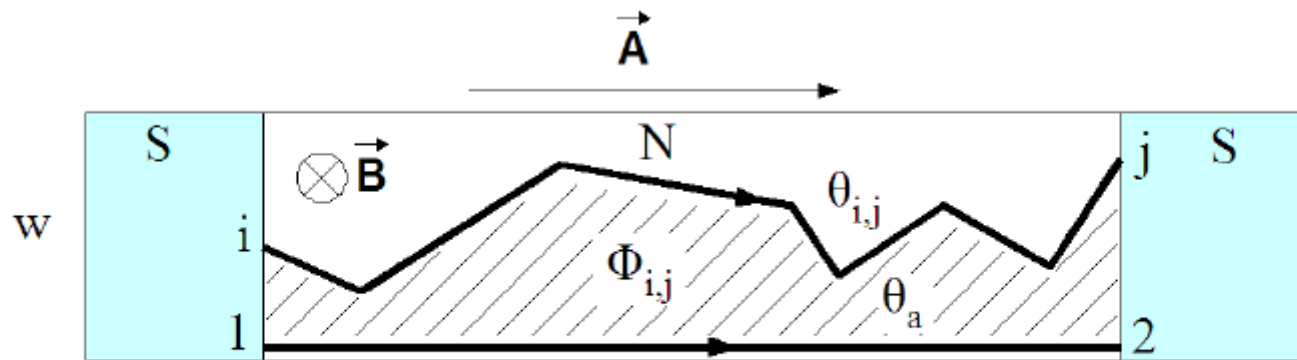


Modulation period a few G, decay scale  $\sim 50$  G

# Back to S/Bi/S: Supercurrent persists up to huge field!

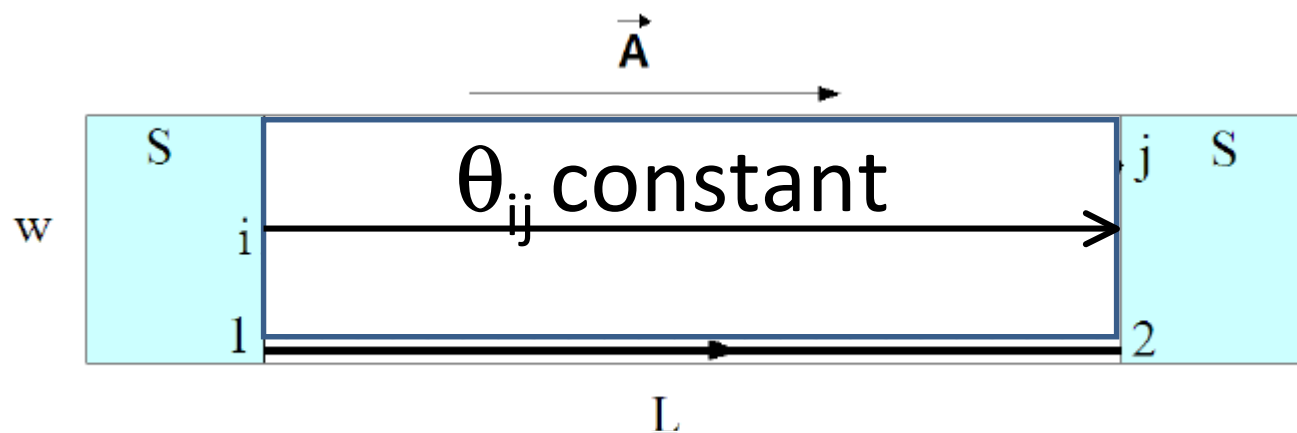
$T_c$  corresponds to  $1000 \Phi_0$  in  
100 nm x 2  $\mu\text{m}$  wire!

Compatible with many diffusive  
channels.



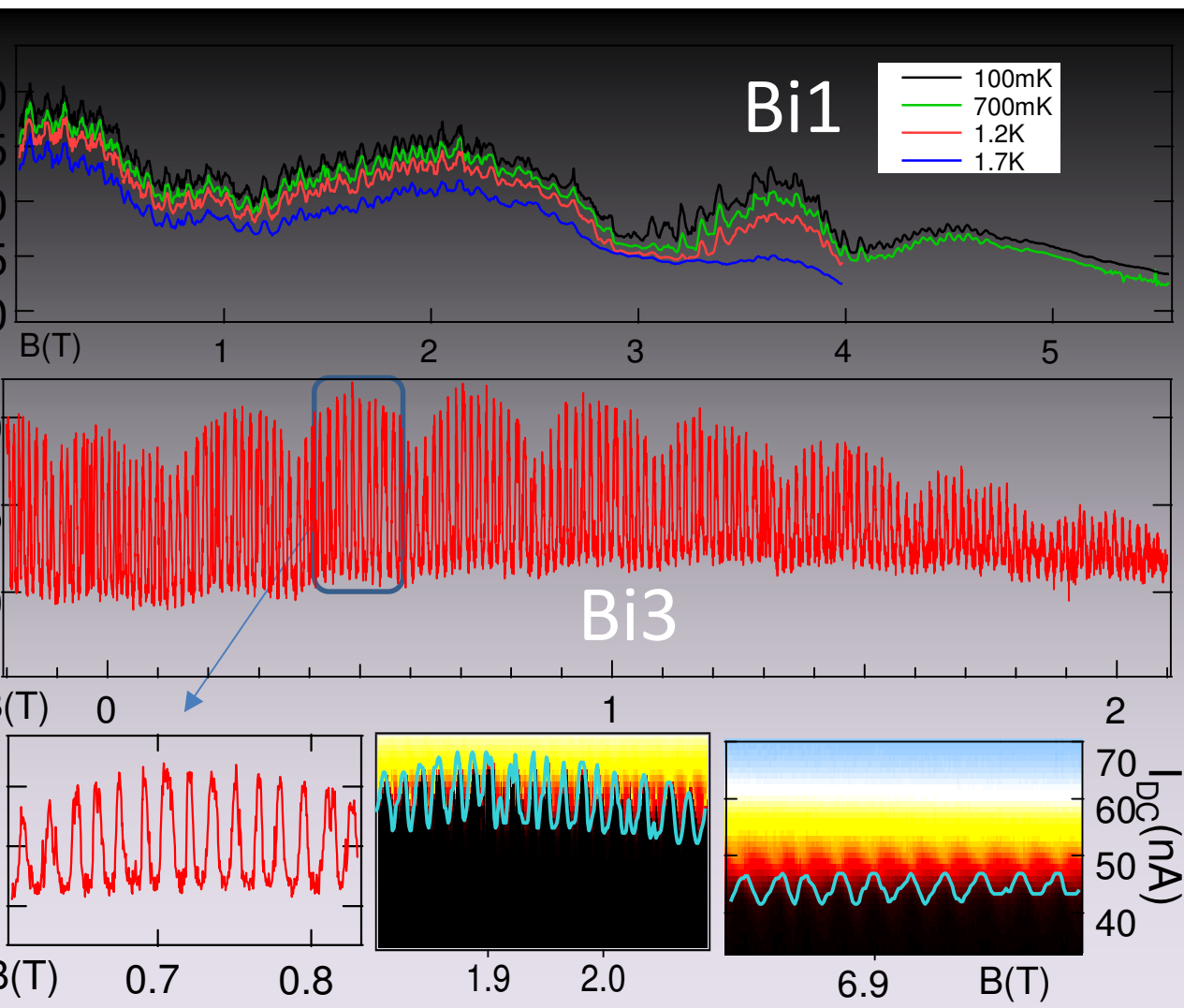
$$I_c \propto \left| \langle e^{i\Delta\theta_{i,j}} \rangle_{c_{i,j}} \right| \propto \left| e^{-\langle (\Delta\theta_{i,j})^2 \rangle_{c_{i,j}} / 2} \right|$$

Field dependence implies: very few ballistic 1D channels!

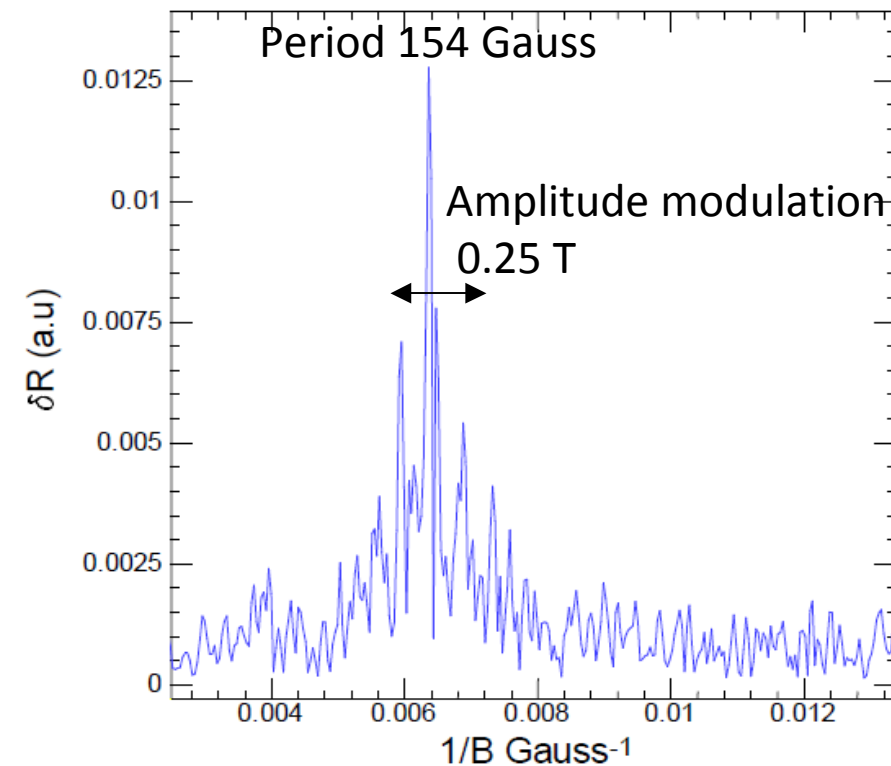


Then no decay of  $I_c$

# SQUID-like $I_c$ oscillations, up to 10 T !



Fourier transform Bi3

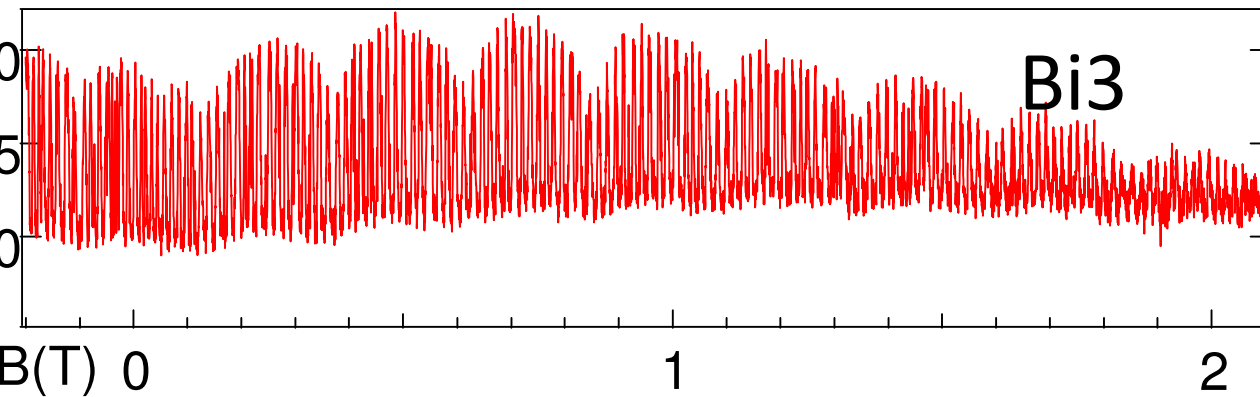


Where is the loop?

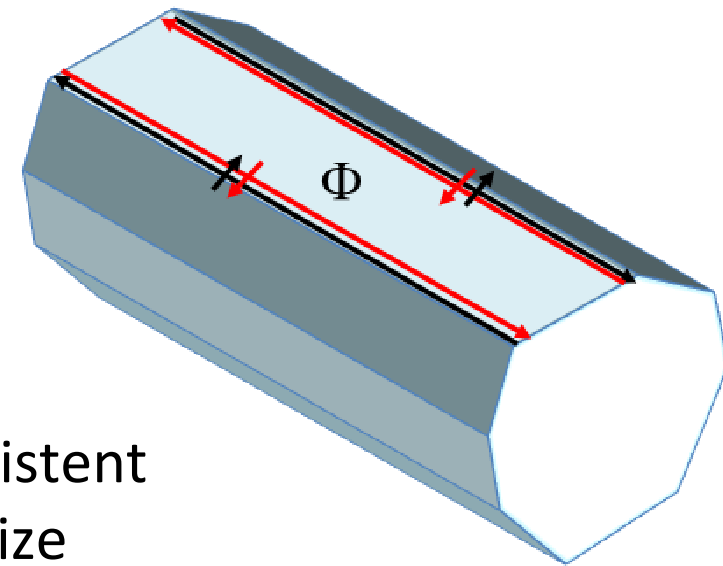
Why no extinction

on the scale of  $1 \Phi_0 / S \sim 400$  Gauss

# SQUID-like $I_c$ oscillations, up to 10 T: Very few ballistic narrow 1D channels



Bi1		Bi3	
$\Delta B =$ 754G	$S = 0.027 \mu\text{m}^2$ =13nm x 2 $\mu\text{m}$	$\Delta B =$ 154G	$S = 0.13 \mu\text{m}^2$ =65nm x 2 $\mu\text{m}$

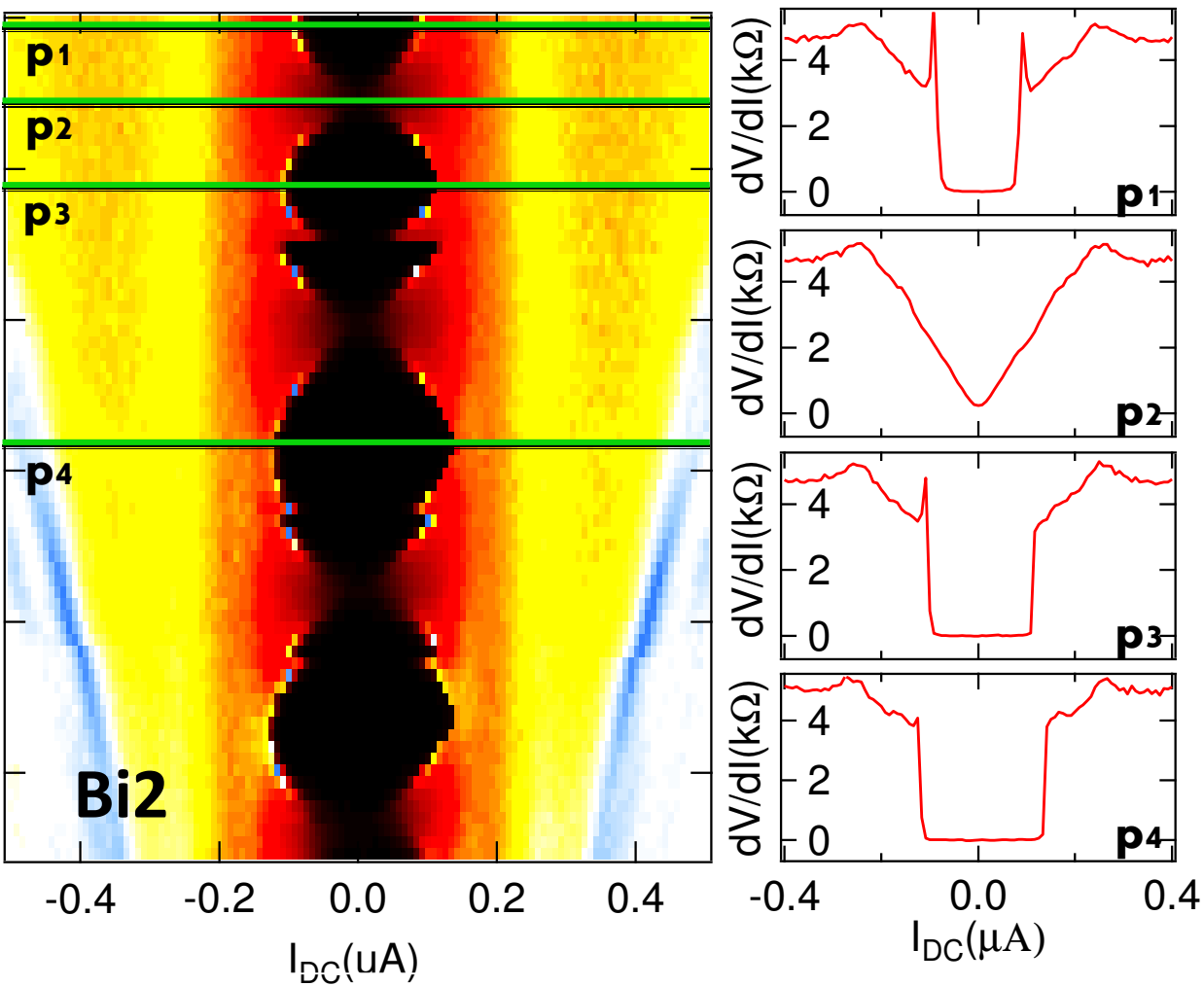


Period consistent with facet size

- 1D edge states on (111) facet (or other topological facet).
- Similar to observations in SC top. Insulators HgTe /HgCd Te, InAs/GaSb
- Decay scale gives extension of edge state (nm!)
- Other surface or bulk states will not contribute at such high fields



# High field range modulation seen for all wires

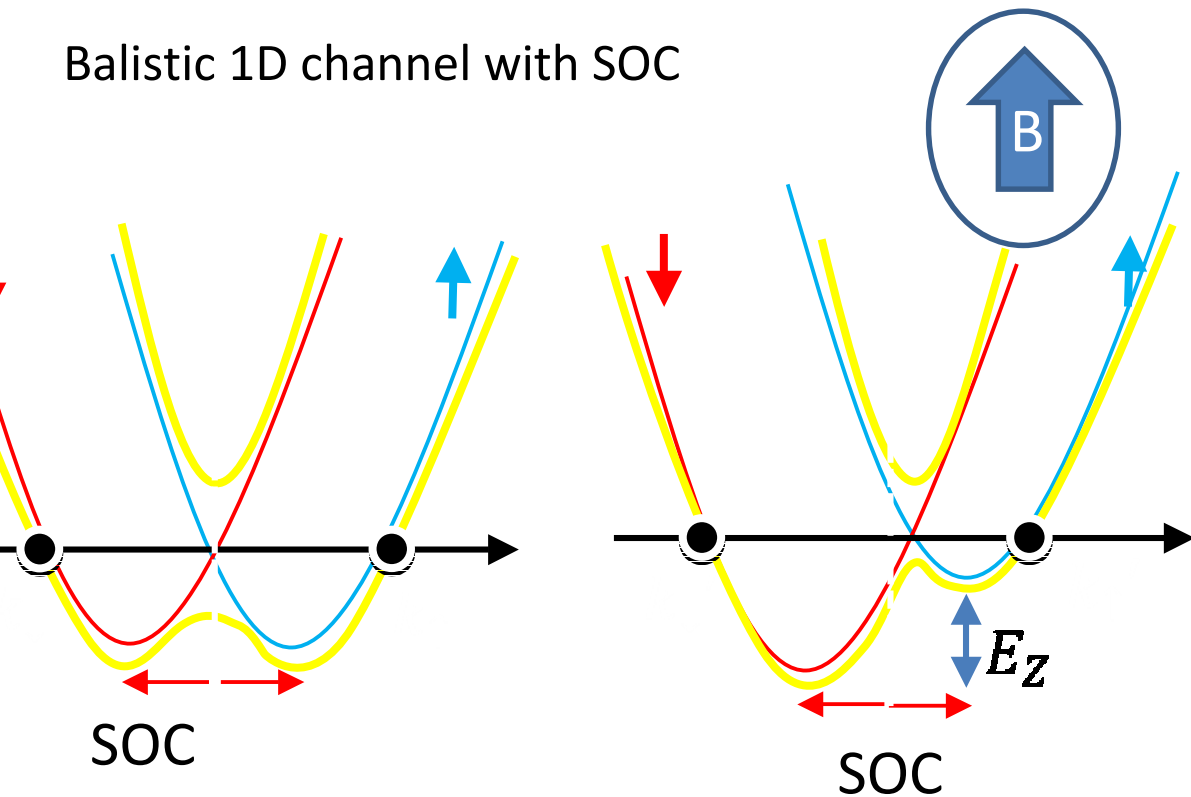


Critical current is larger in 5T than in zero field!

Slow phase modulation by Zeeman energy

# High field scale critical current modulation due to Zeeman dephasing

Balistic 1D channel with SOC



$$\Delta k = E_Z / h v_F = g_{\text{eff}} \mu_B B / h v_F$$

$$\delta\phi = 2\pi L \Delta k = 2 g_{\text{eff}} \mu_B B L / v_F$$

$$k_{\uparrow} = -k_{\downarrow}$$

$$\Delta\phi = 0$$

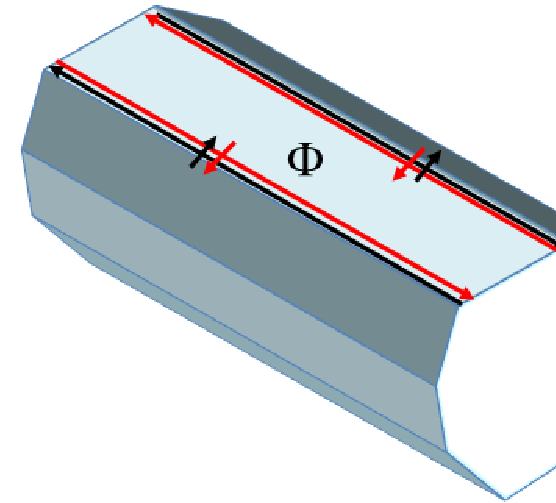
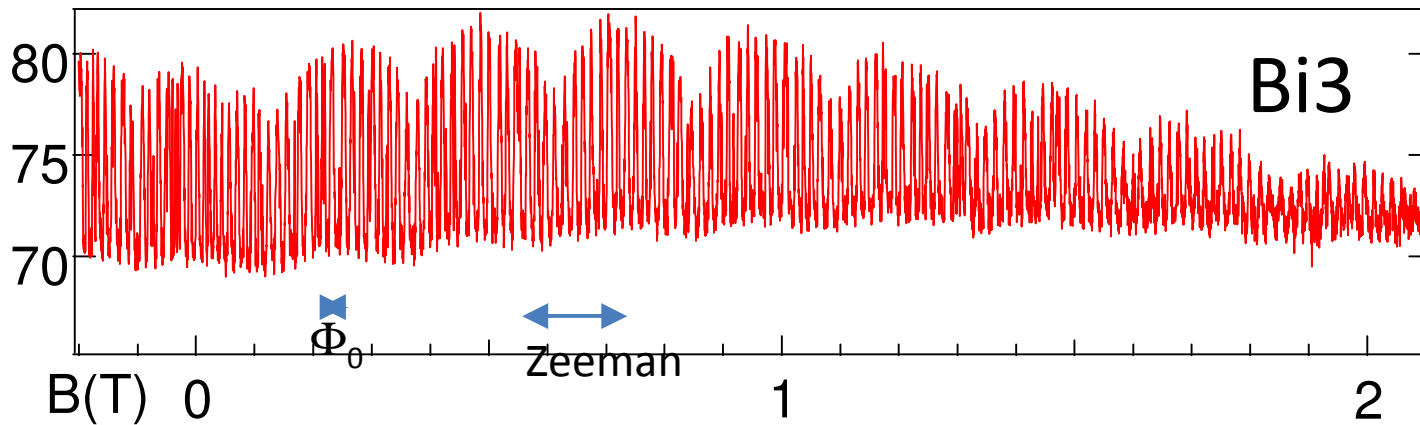
$$k_{\uparrow} = -k_{\downarrow} + \Delta k$$

$$\Delta\phi \neq 0$$

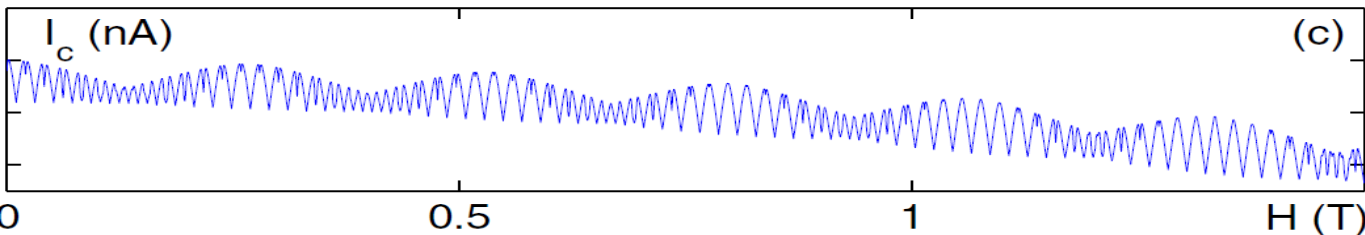
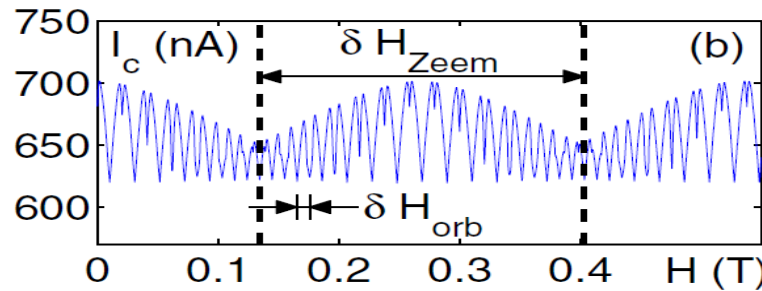
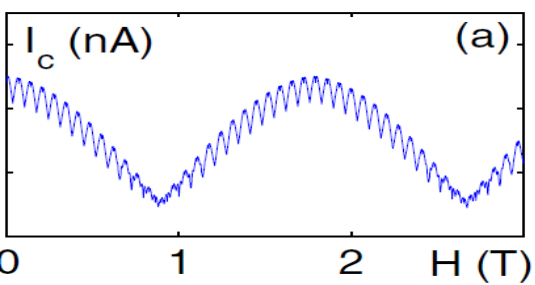
with  $v_F = 3 \times 10^5$  and  $g_{\text{eff}} = 30$

yields period in Tesla range

# SQUID-like $I_c$ oscillations, and high field modulation reproduced with 2 channels



SQUID formula with field modulated  $I_{c_i}$ :  $I_c^2 = I_{c_1}(H)^2 + I_{c_2}(H)^2 + 2 \cdot I_{c_1} I_{c_2} \cos 2\pi \Phi / \Phi_0$



$$I_{c_i}(H) = I_{c_i} (1 - \alpha_i \cos g_{\text{eff}i} \cdot H)$$

$$I_{c_1} = 30 I_{c_2}, \quad g_{\text{eff}2} = 20 g_{\text{eff}1}$$

## Conclusion: what we found in S/Bi nanowire/S junctions

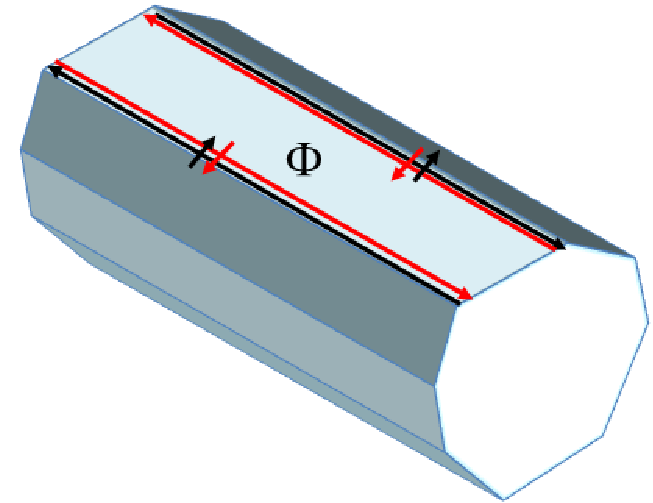
Rapid squid like oscillations (due to orbital phase modulation), and absence of decay with field:

Small number of narrow (<1nm) 1D ballistic (helical) edge states.

~ Tesla range modulation explained by Zeeman (spin) dephasing

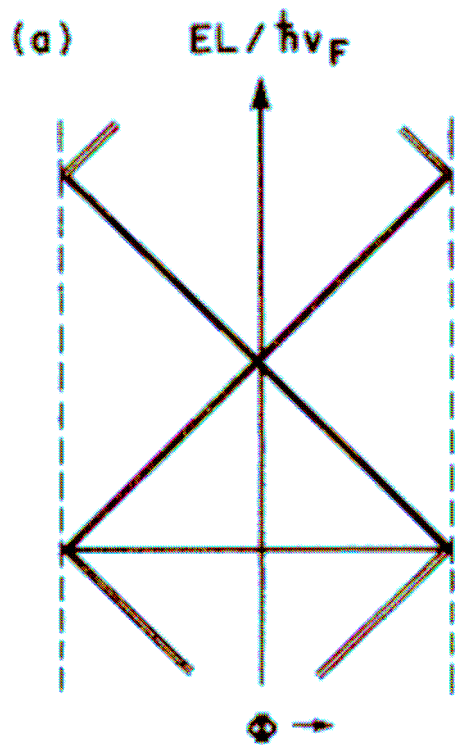
High critical current due to suppressed normal backscattering at helical edges?

Next: Investigation of Andreev states

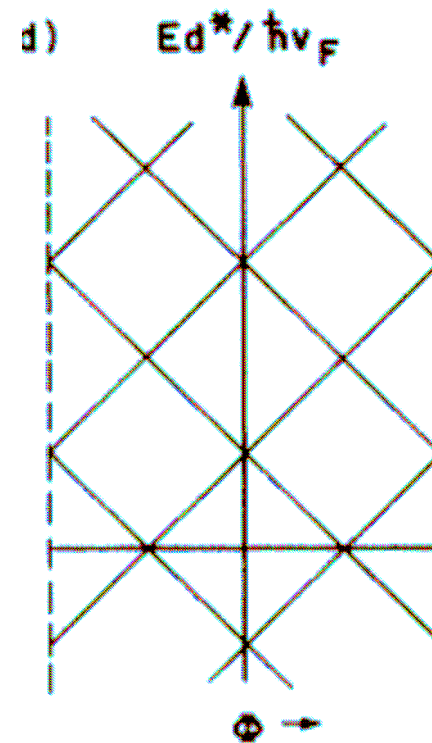


From single electron spectrum in a ring to Andreev states in a long SNS junction  $L \gg \xi_S$

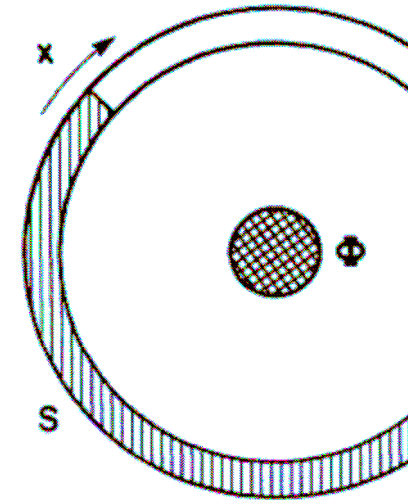
Büttiker Klapwijck 1985



N Ring periodicity  $h/e$

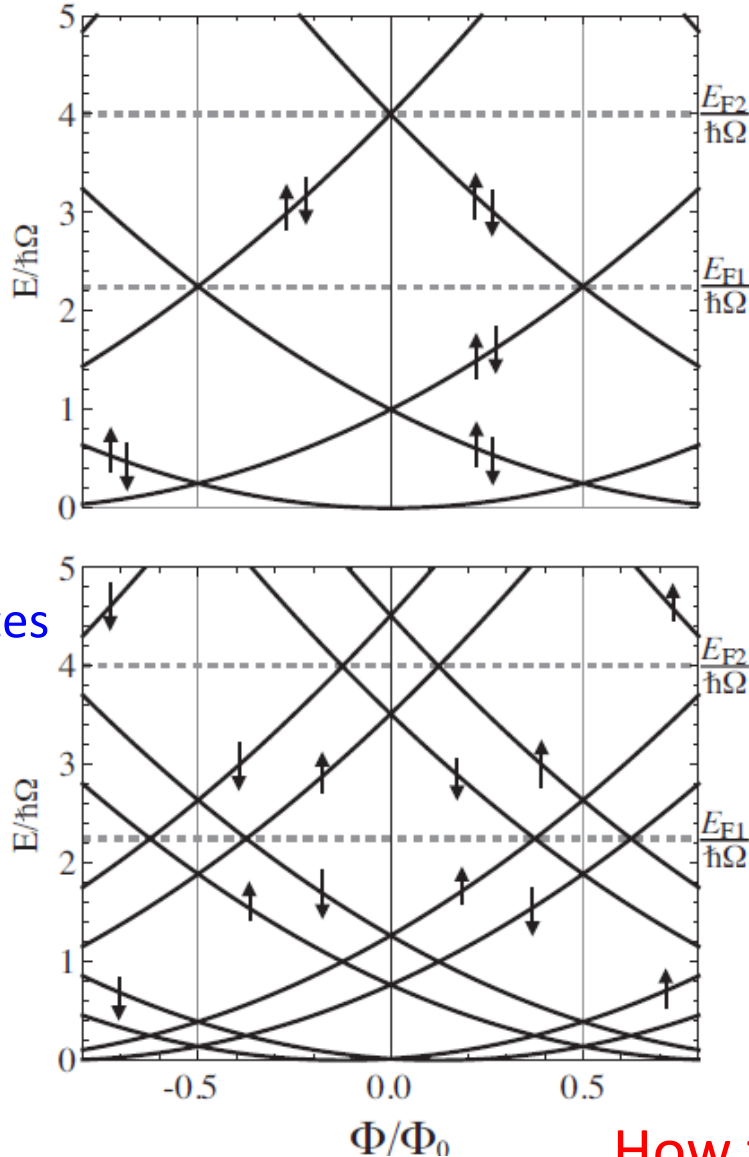


NS Ring periodicity  $h/2e$



# Phase dependent spectrum of a ring in the presence of spin orbit coupling

1D N ring



↑ shifted and ↓ States

$\pm k_{so}L$

No SO coupling

With Rashba SO coupling  
 $\lambda = E_{SO}/E_F = 0.3$

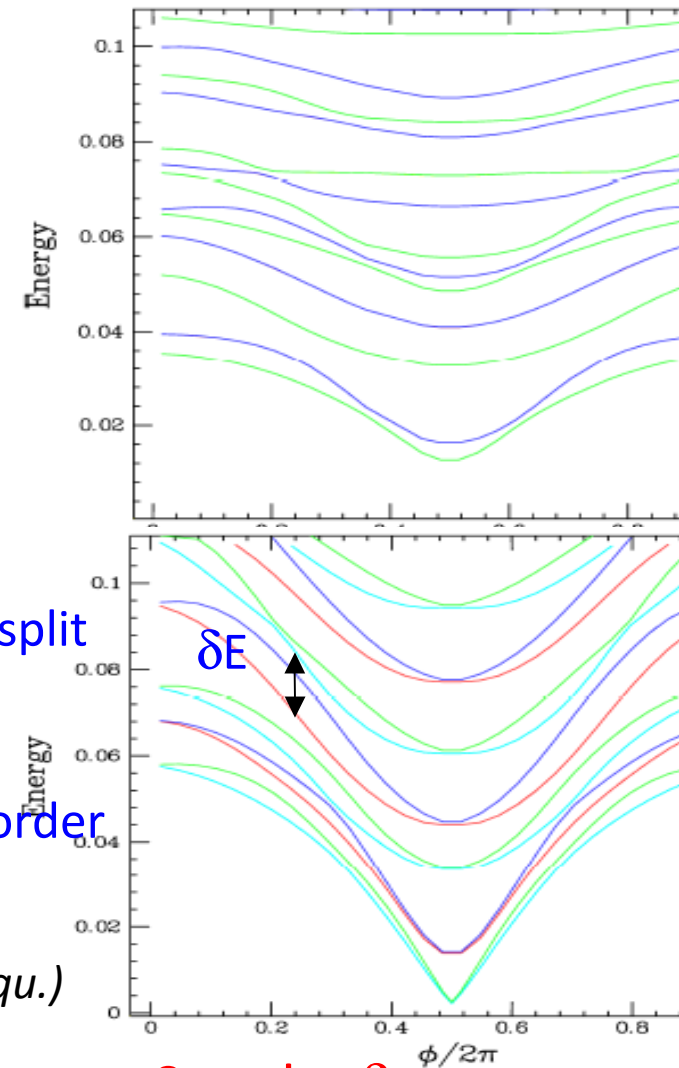
Andreev states are spin split for  $\phi \neq 0, \pi$   
 TRS breaking  $\delta E = \lambda \Delta^2 \tau_D$   
 decreased effective disorder

(A. Murani, A. Chepelianskii, H.B. Simulations of BdG equ.)

How to detect level crossings at 0 and  $\pi$  ?

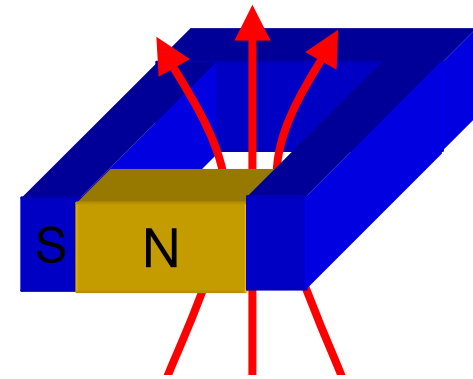
Multichannel

NS ring Andreev states invariant by



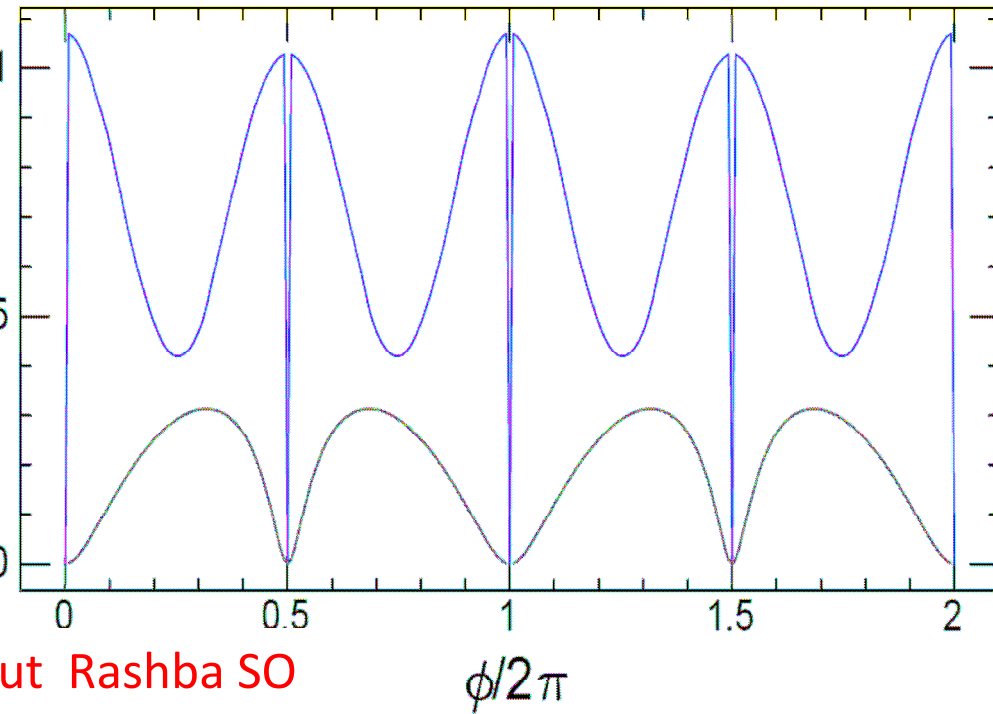
# Finite frequency response

$$\Phi_{\text{ext}} = \Phi_{\text{dc}} + \Phi_{\text{ac}} \cos$$



Au diffusive wire

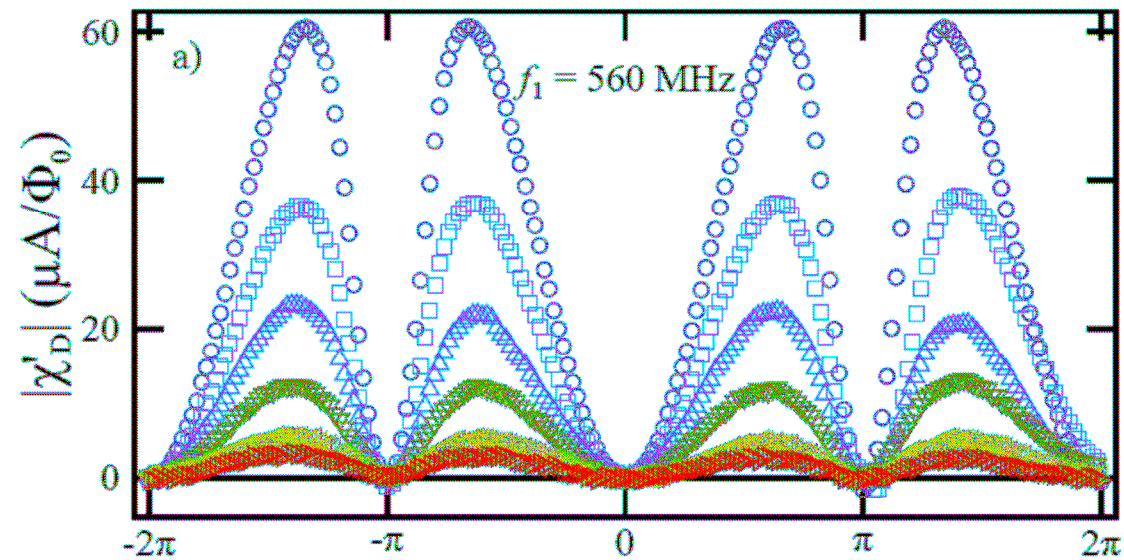
with Rashba SO



without Rashba SO

Josephson current

$$I(\varphi) = \sum f_n(\varphi) i_n(\varphi) \quad i_n(\varphi) = \partial \epsilon_n(\varphi) / \partial \varphi$$



Finite frequency

$$\chi_D(\omega) = - \sum_n i_n^2 \frac{\partial f_n}{\partial \epsilon_n}$$

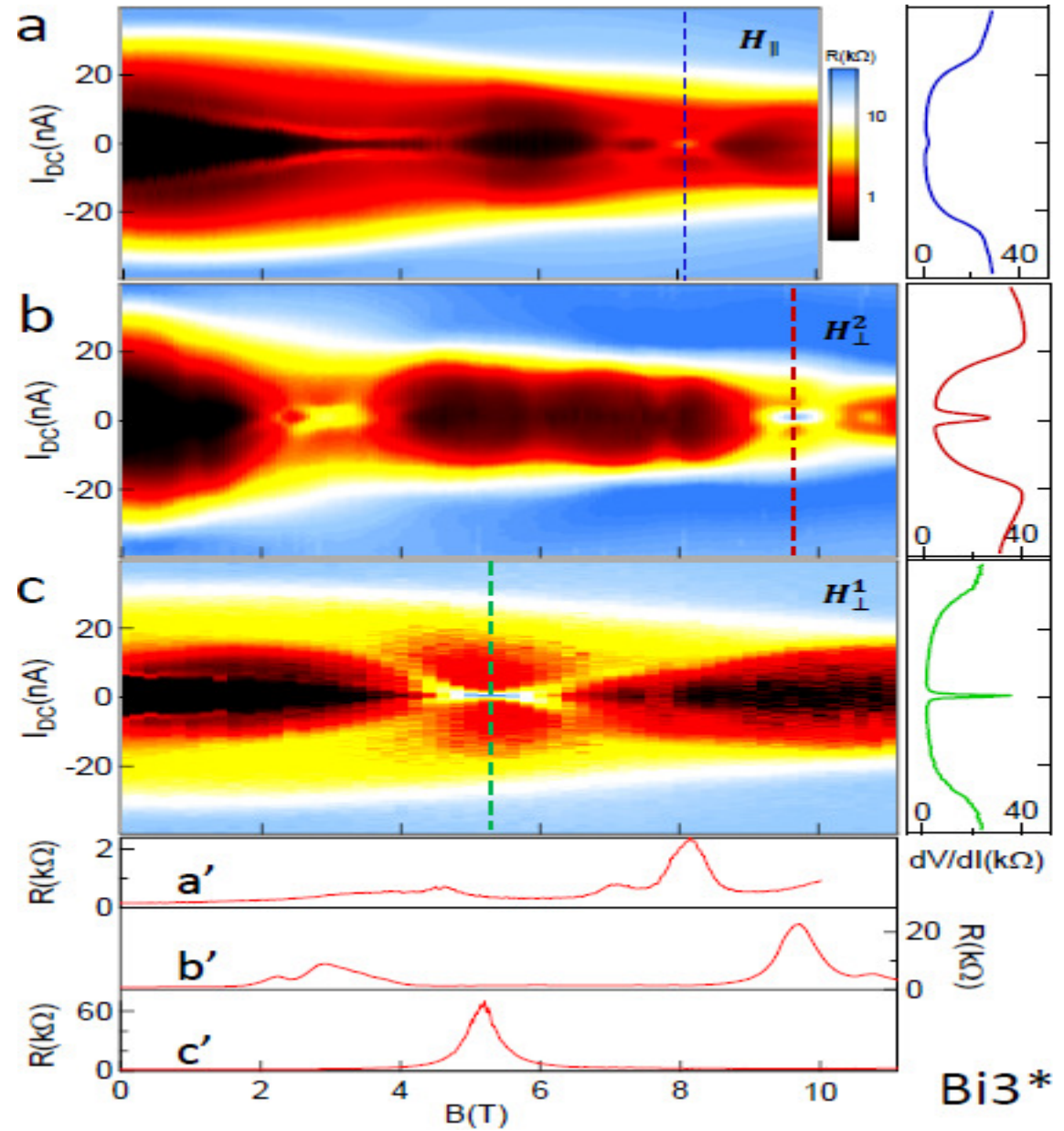
sensitive to Andreev level crossings at 0 and  $\pi$

Large Field modulations of  $I_c$ , depend on the direction of magnetic field

after 1 month at room T...

regions where  $I_c=0$  And  $R > R_N$

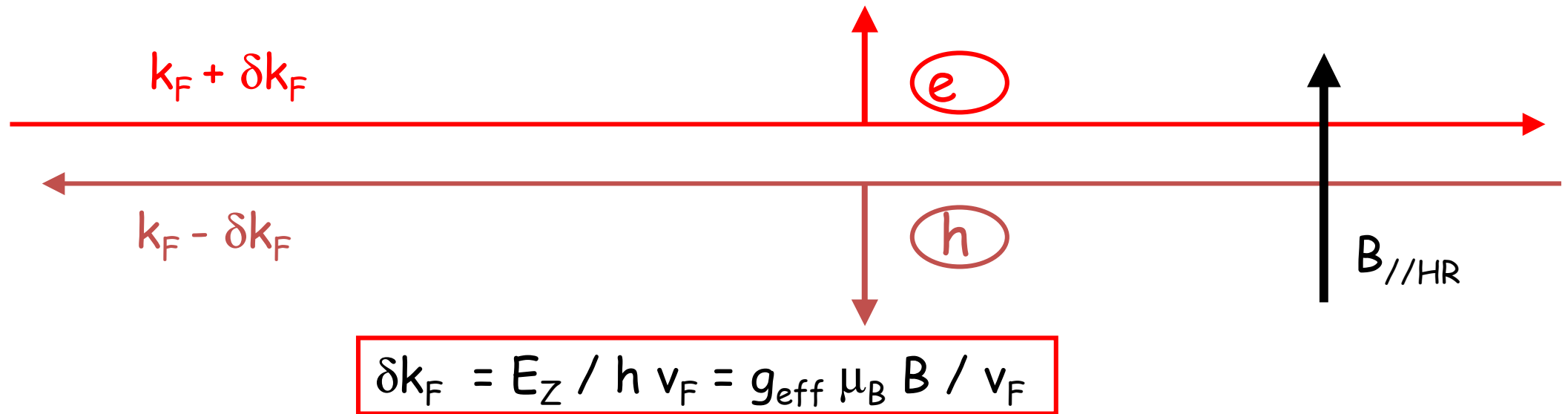
$$R_N = 30\text{k}\Omega$$





# Large field modulations of the Supercurrent

eman  $e/h$  phase shift along a ballistic channel of length  $L$



$$\phi = 2\pi L \delta k_F = 2 g_{\text{eff}} \mu_B B L / v_F \sim \pi \text{ for } 1\text{T}$$

$$v_F = 10^5 \text{ m/s} \quad g_{\text{eff}} = 10 \quad L = 2 \mu\text{m}$$

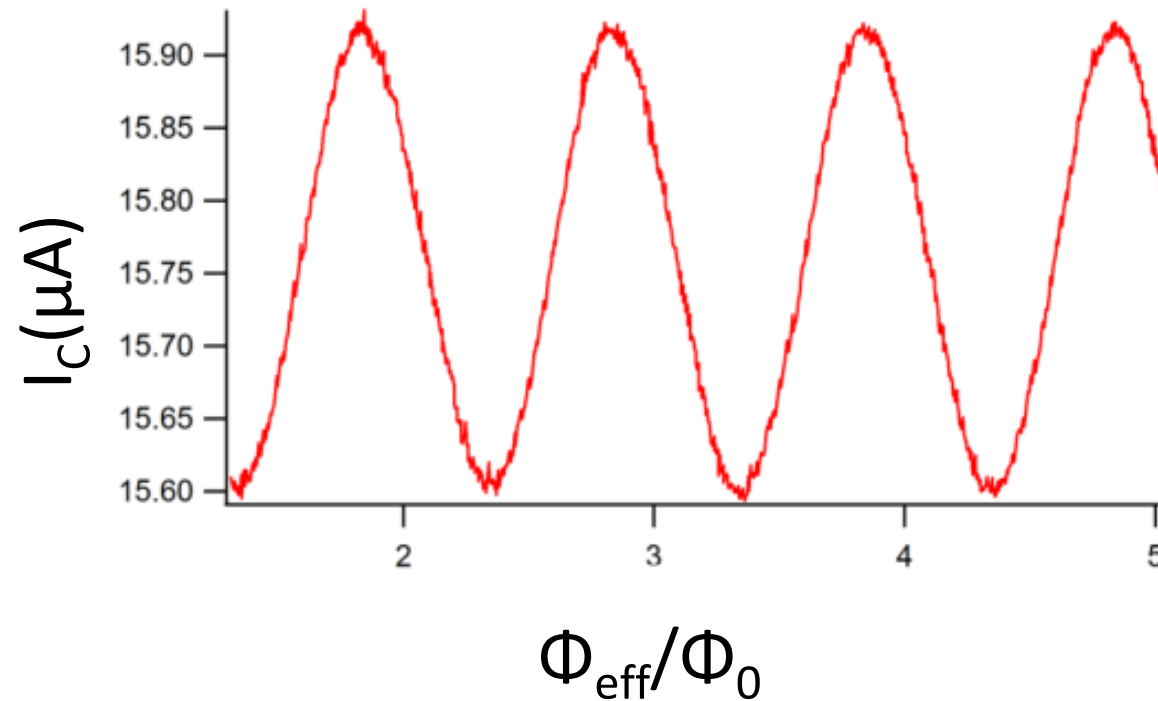
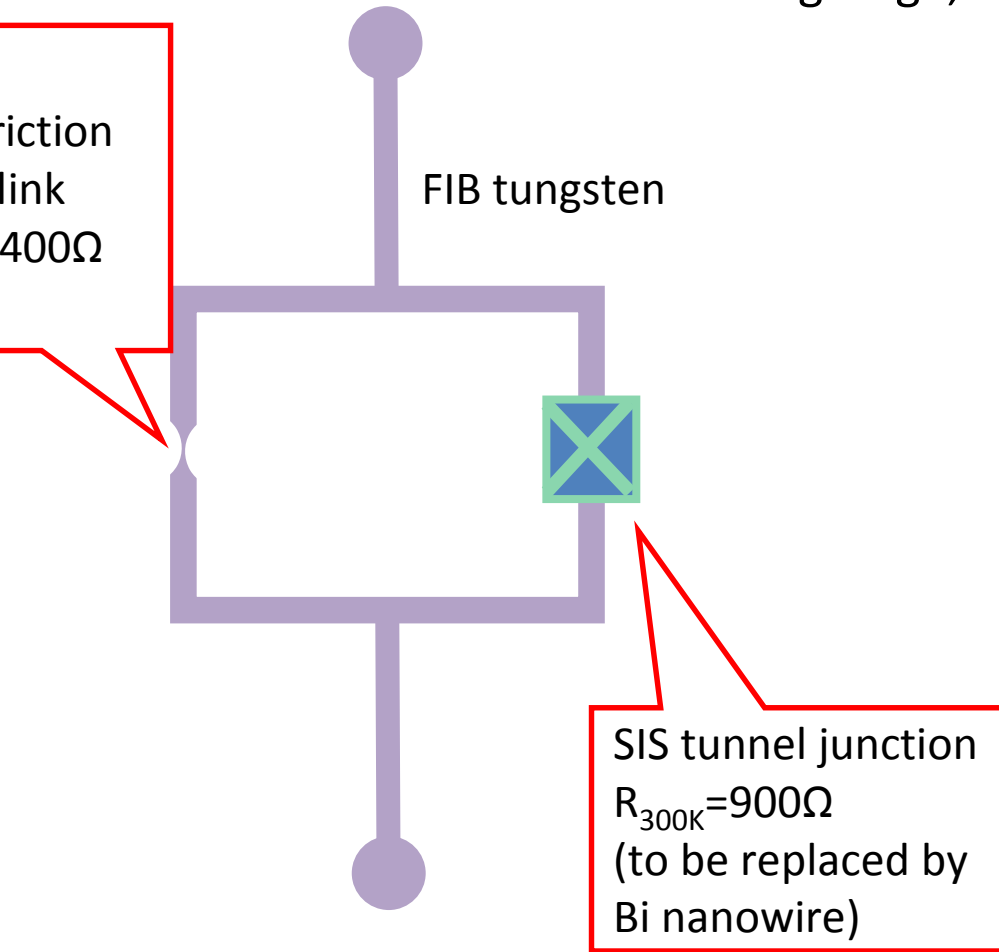
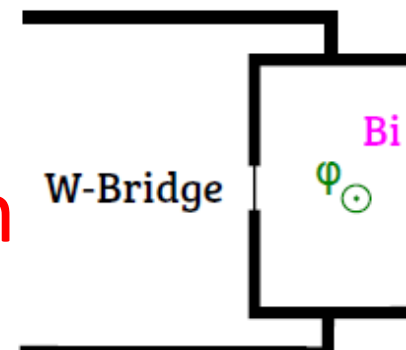
Large field modulations of  $I_c$  can be explained!  
Full modulation in the single channel limit

# Current-phase relation ?

## to be measured in asymmetric squid configuration

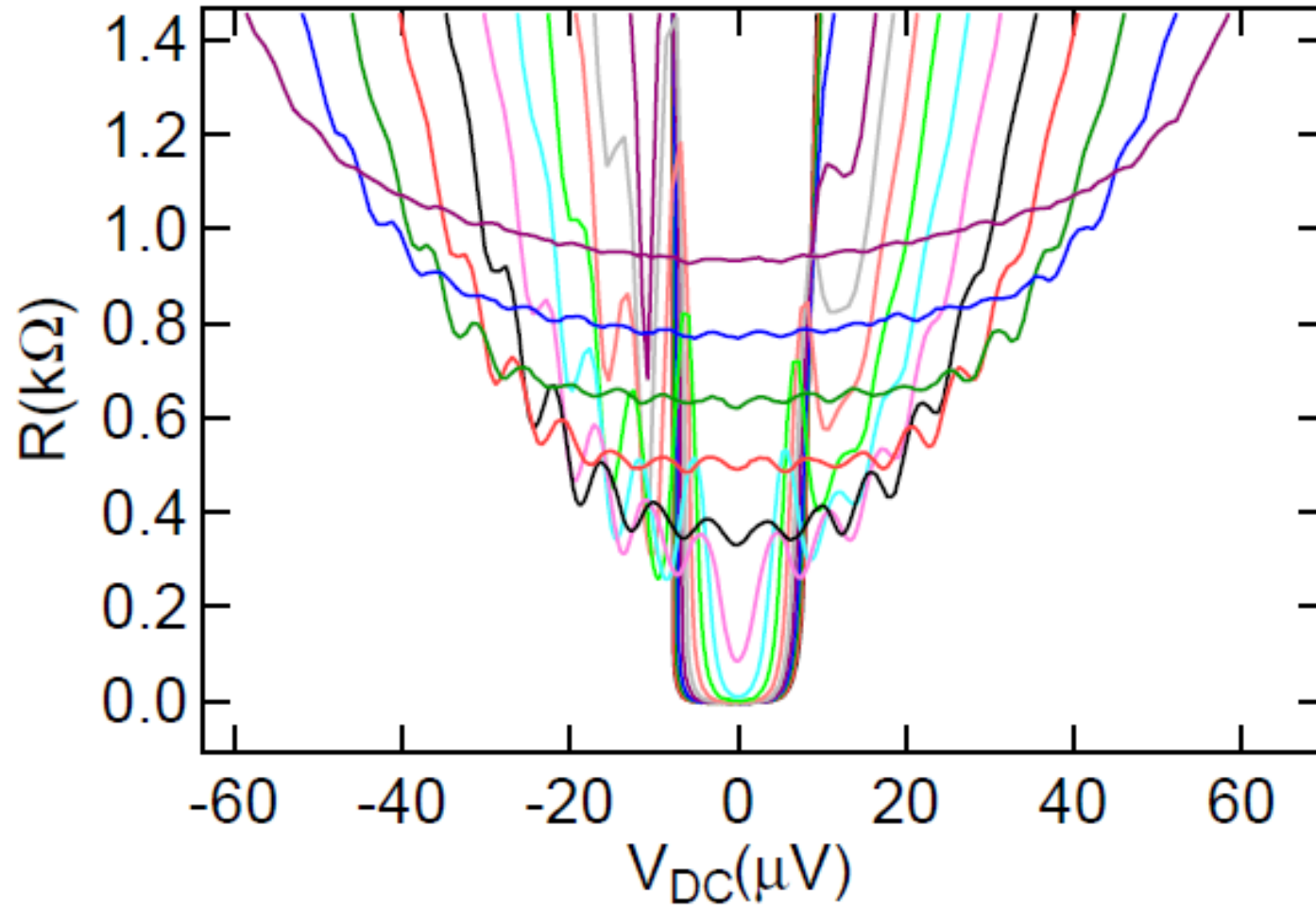
### Preliminary experiment

R. Delagrance, A. Murani, R. Deblock, A. Kasumov 2014



Bi3

Shapiro steps  $\nu=2.2 \text{ GHz}$

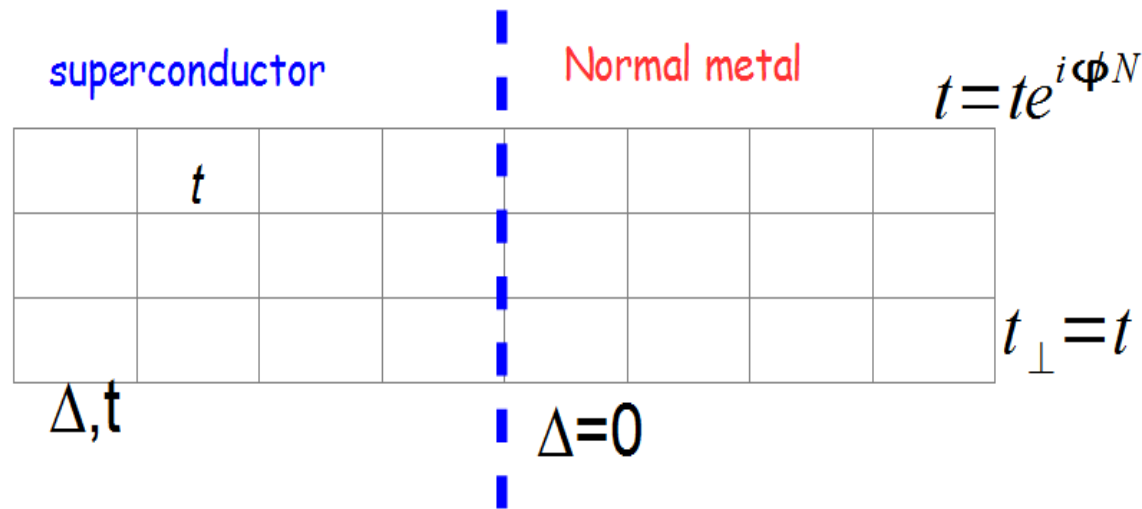


# Understanding contribution of spin orbit, superconductivity, disorder, number of channels

tight binding model

square or hexagonal lattice (H. Bouchiat, A. Chepelianskii, A. Murani, M. Ferrero)  
 just disorder, spin-orbit strength, junction length, ...

$$H = \Delta c_i^\dagger c_i^\dagger - t \sum c_{i,j}^\dagger c_{i+1,j} + c_{i,j}^\dagger c_{i,j+1}$$



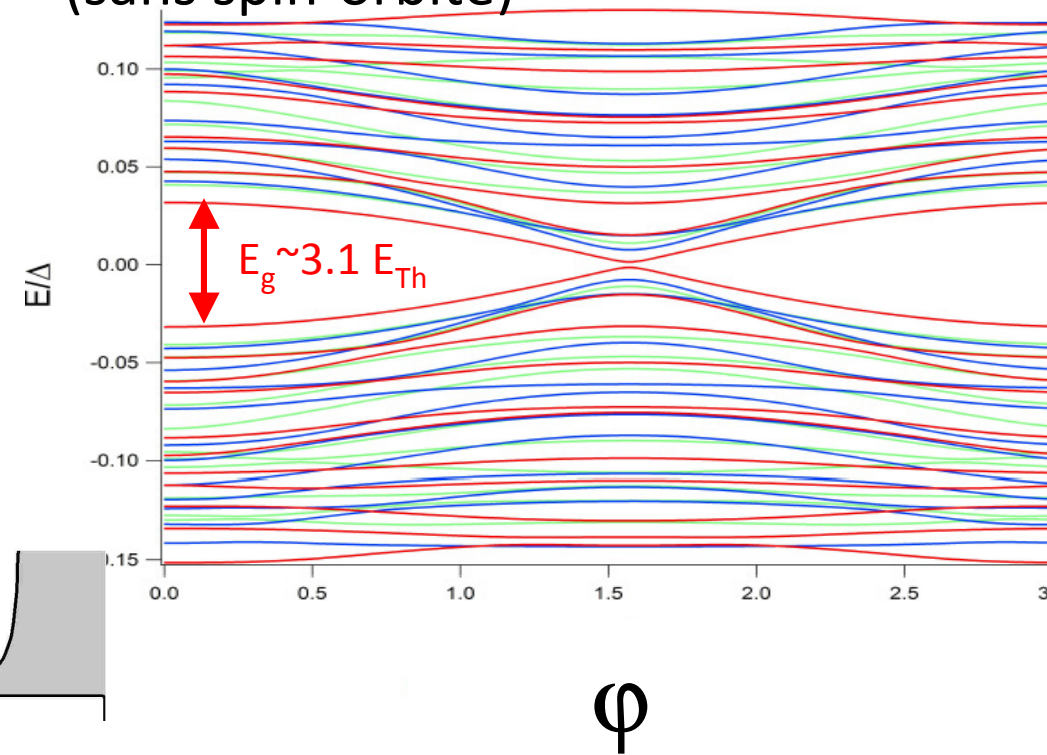
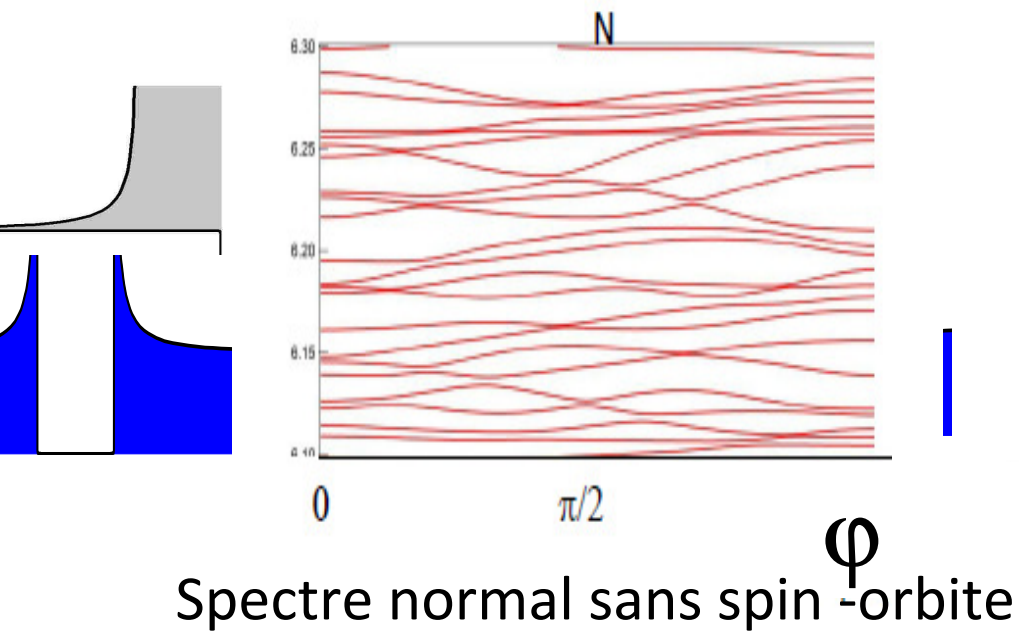
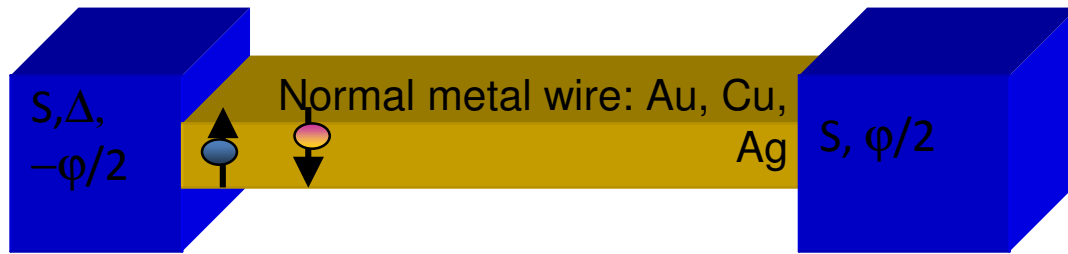
+ periodic boundary condition = SNS ring

Possible to add disorder

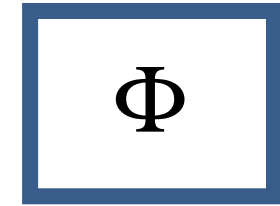
# Spectre sans spin-orbite

H. Bouchiat, M. Ferrier

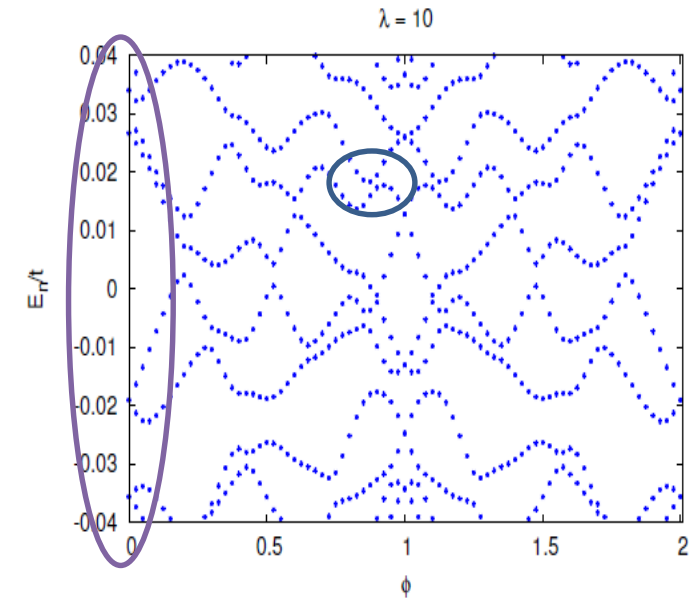
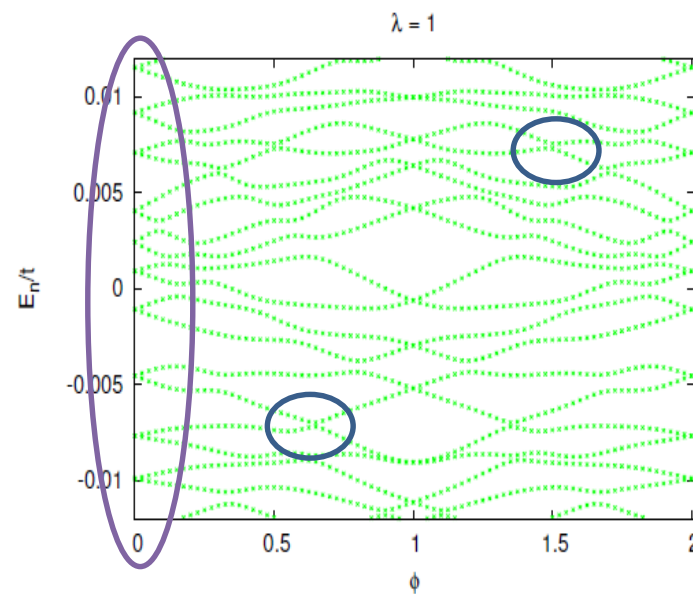
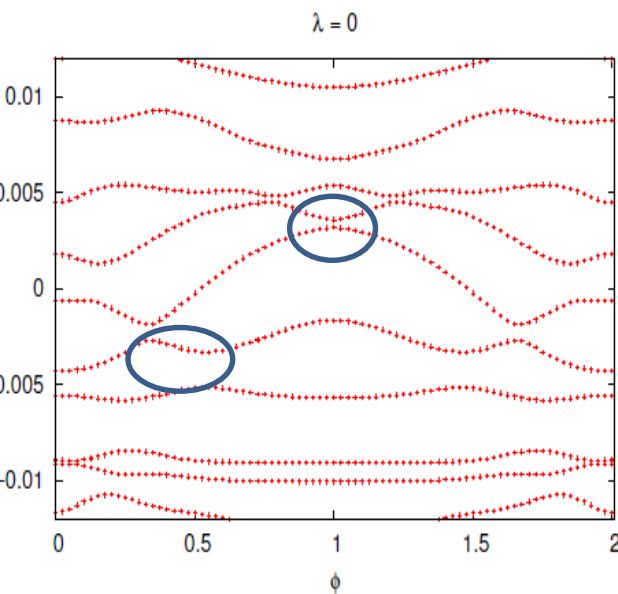
Spectre avec contacts supra  
(sans spin-orbite)



# Spectre normal (anneau): effet du spin-orbite



the strength of the spin-orbit interaction, hopping  $t = 4$ , superconducting gap  $\Delta = 0$ , disorder  $W = 4$ ,  $N_x = 80$ ,  $N_{supra} = 0$ .



de spin-orbite

ats dégénérés en spin

oisements évités (couplage par désordre)

Faible spin-orbite

- Dégénérescence de Kramers, à phase nulle.

-« Splitting » de spin

-Certains croisement autorisés : « disparition du désordre »

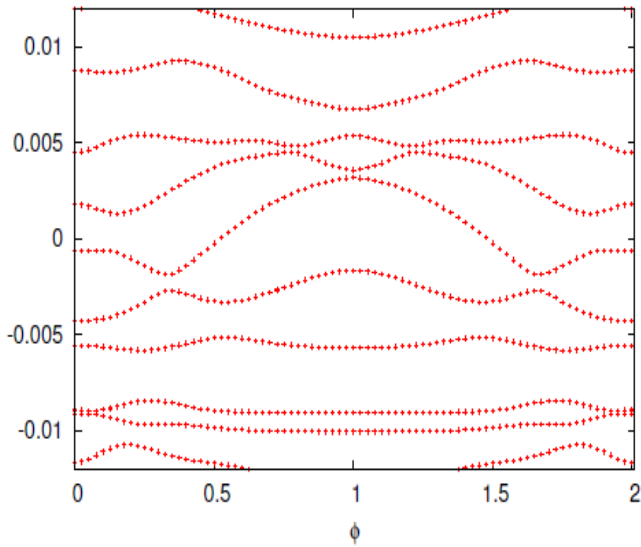
Fort spin-orbite

# N ring

$\Phi$

$$\varphi = 2\pi\Phi/\Phi_0$$

$\lambda = 0$



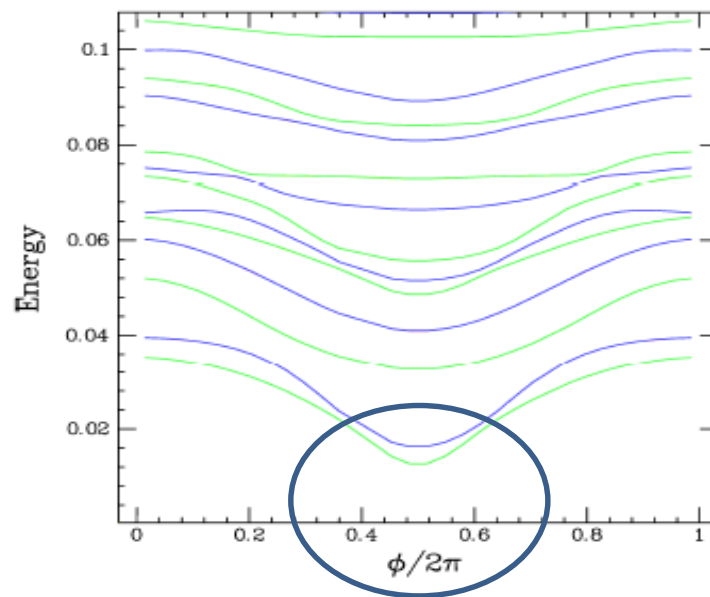
N seul, Pas de spin-orbite  
 Etats dégénérés en spin  
 Croisements évités (couplage par désordre)

# SNS Junction

S,  
 $-\varphi/2$

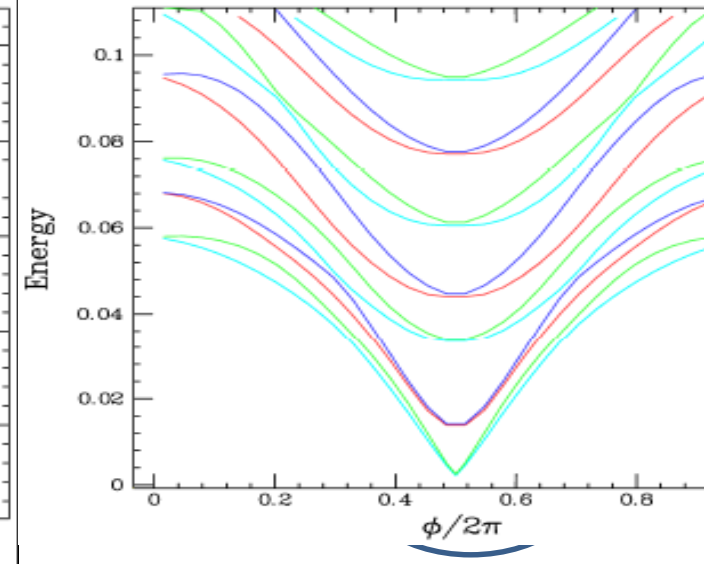
Spin-orbite  
 désordre

S,  
 $\varphi/2$



SNS, Pas de spin-orbite  
 - Etats dégénérés en spin  
 - Gap induit, ne se ferme pas à  $\pi$

$\lambda = 3$

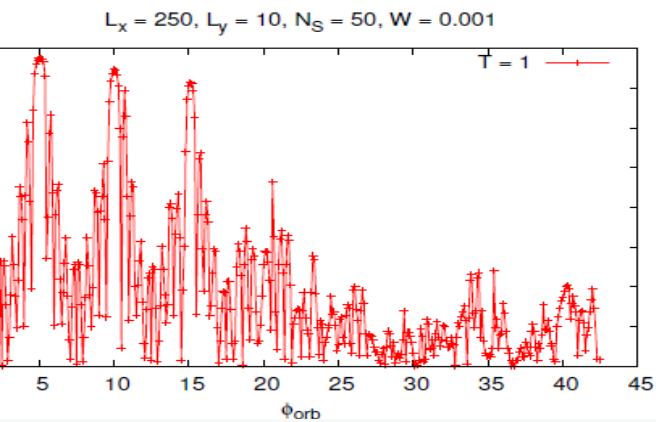


SNS, avec spin-orbite  
 -Kramers ,  
 -levée de la dégénérescence de  
 -Gap induit se ferme à  $\pi$ :  
 disparition du désordre! (certain)

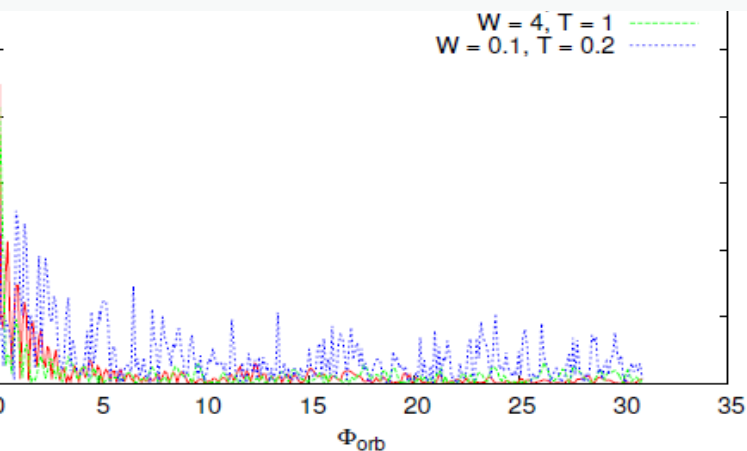
supercurrent  $I_s \propto \frac{\partial \epsilon_n}{\partial \phi}$

# Alexei Chepelianskii: test non extinction du courant critique à fort champ

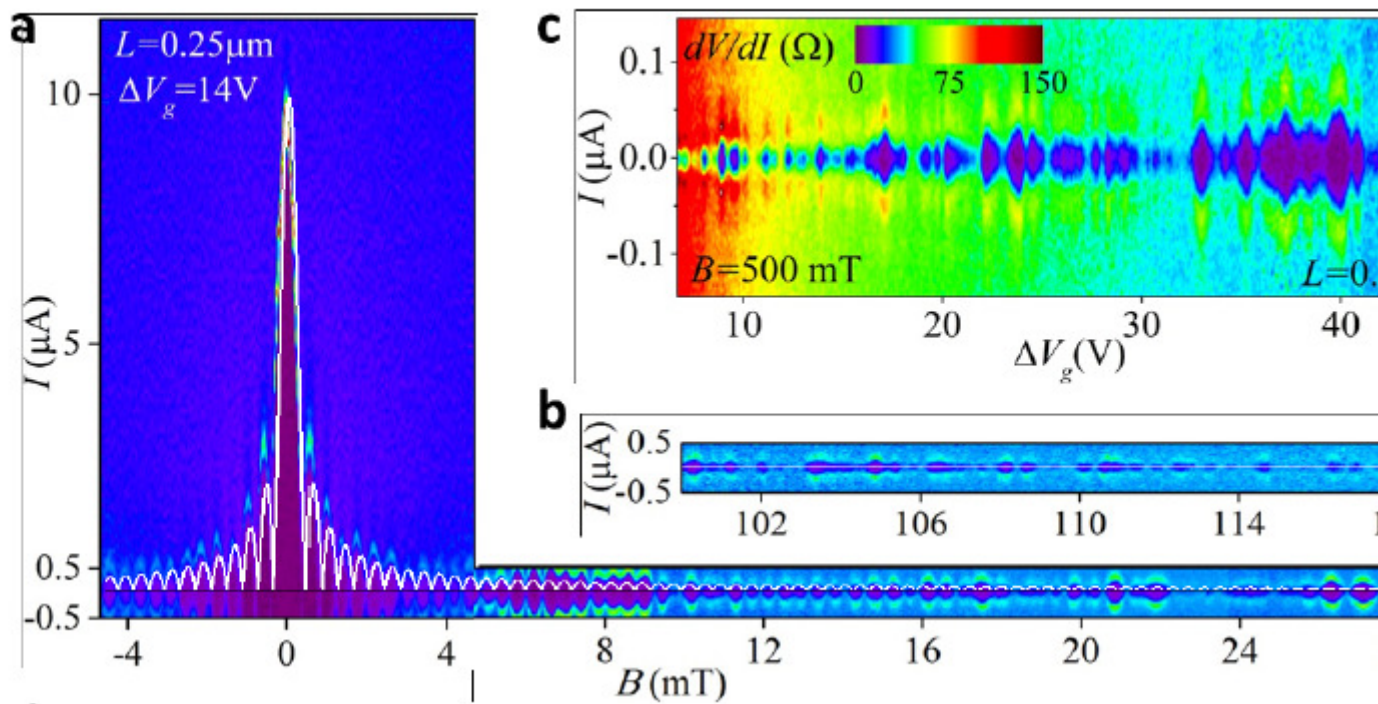
peu de canaux de conduction, courant critique oscille, ne décroît pas avant 50 quanta de



peu de canaux de conduction,  
 courant critique décroît à qq quanta, r  
 t courant critique subsiste jusqu'à  
 beaucoup plus fort champ



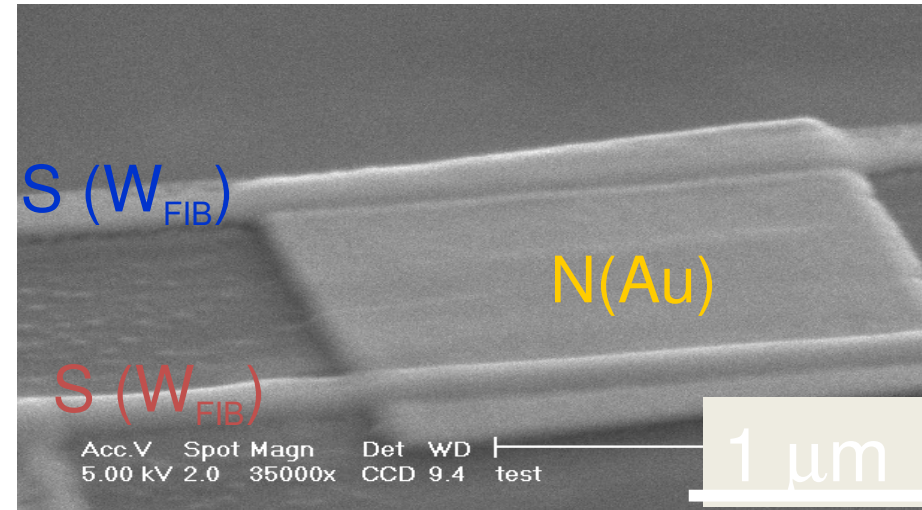
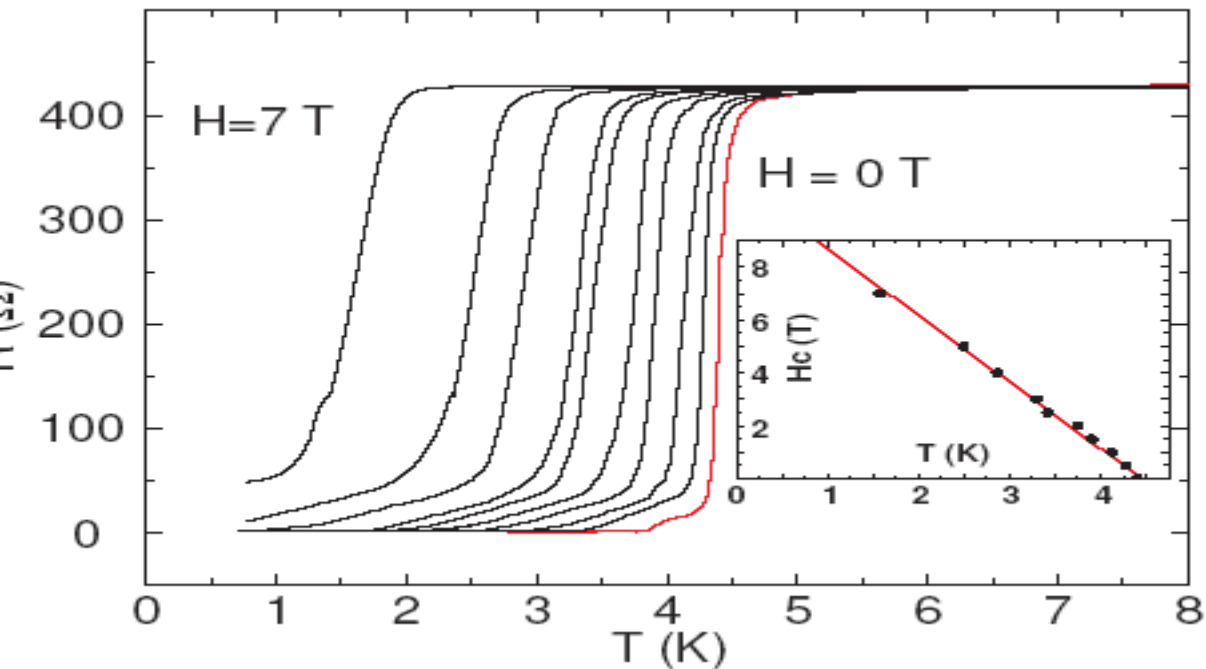
Ballistique, 10 canaux



Exp S/ballistic Graphene/S, Geim, Avril 2015



# Properties of FIB deposited « tungsten » nanowires



Energy analysis :

W  
% carbon  
% Ga  
oxygen

$T_c = 4-5$  K,  $H_c = 12$  T (pure W has  $T_c$  of less than 100 mK !)  
Length = several microns, typical width = 100 nm to 500nm

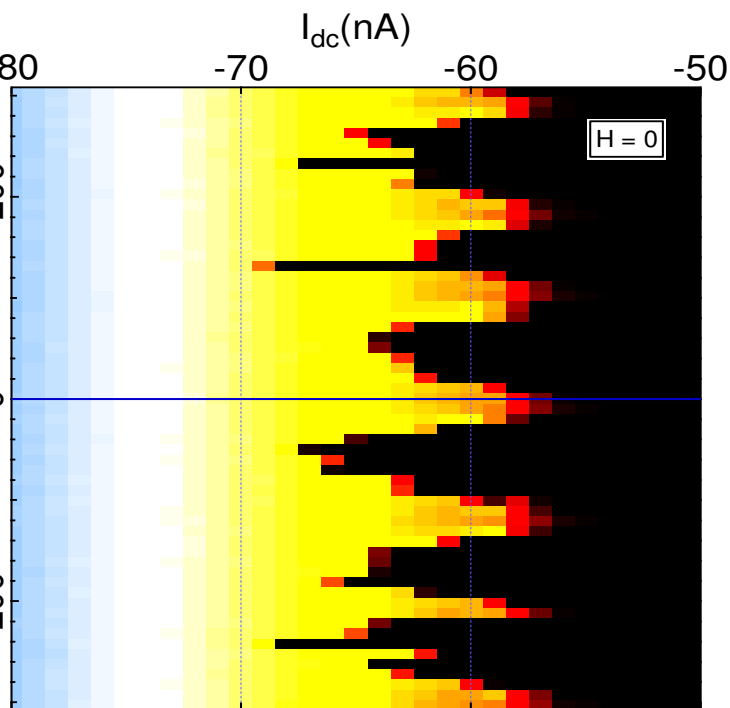
Very reproducible superconducting parameters

Nice electrodes for investigation of proximity effect...

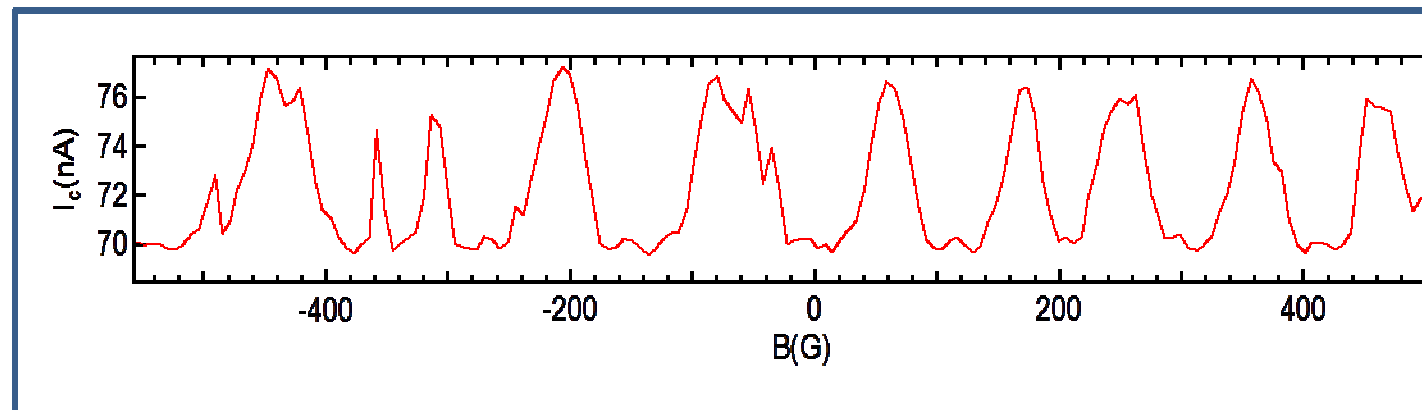
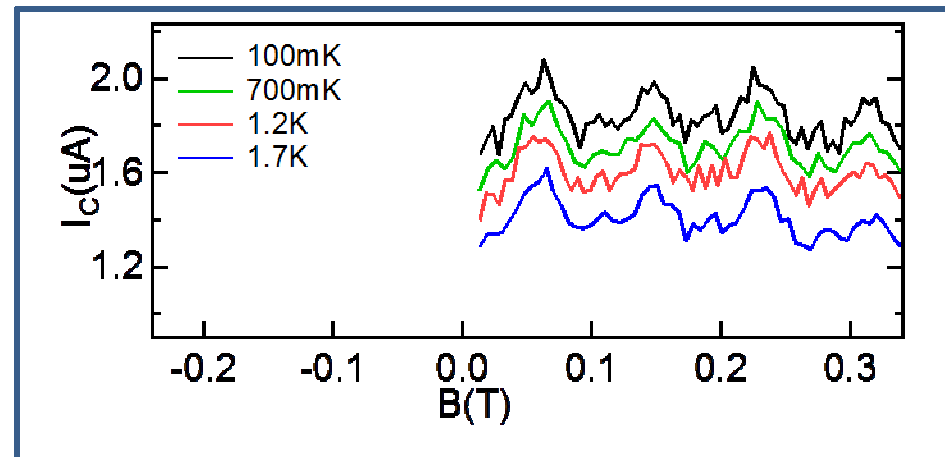
Kasumov et al: 2005

open questions...

## Junction SQUID?

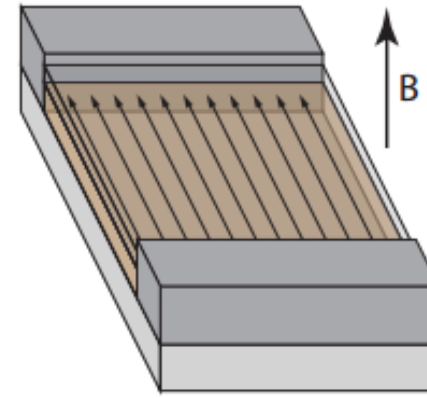
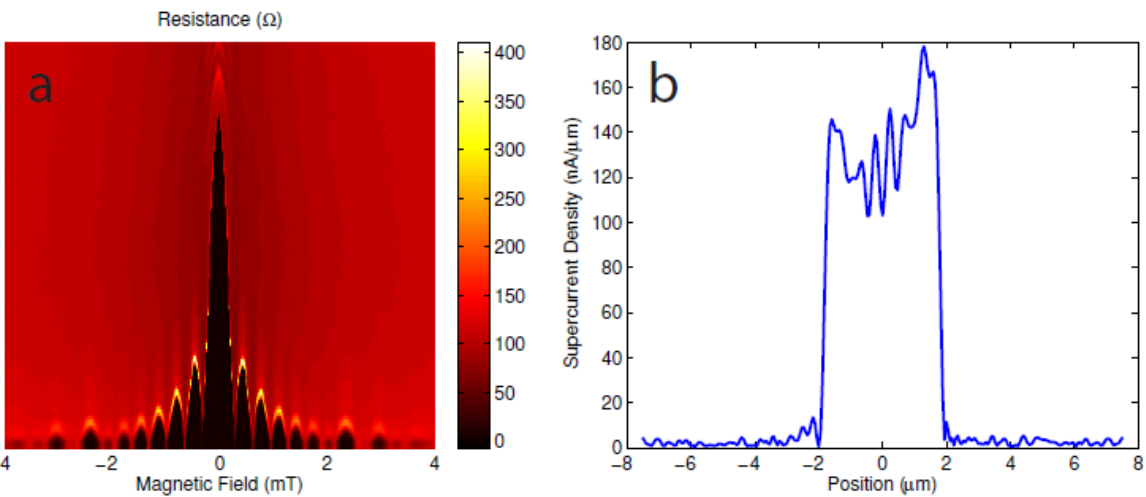


different samples which have  $I_c$  oscillations show always minimum at zero field.

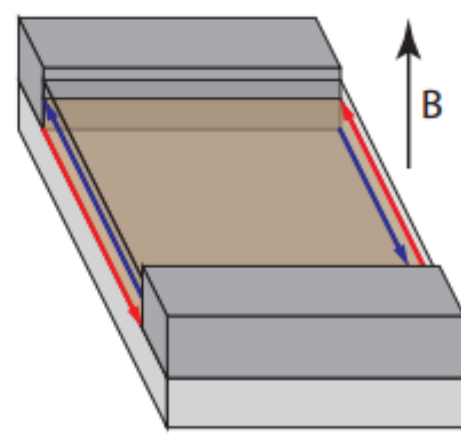
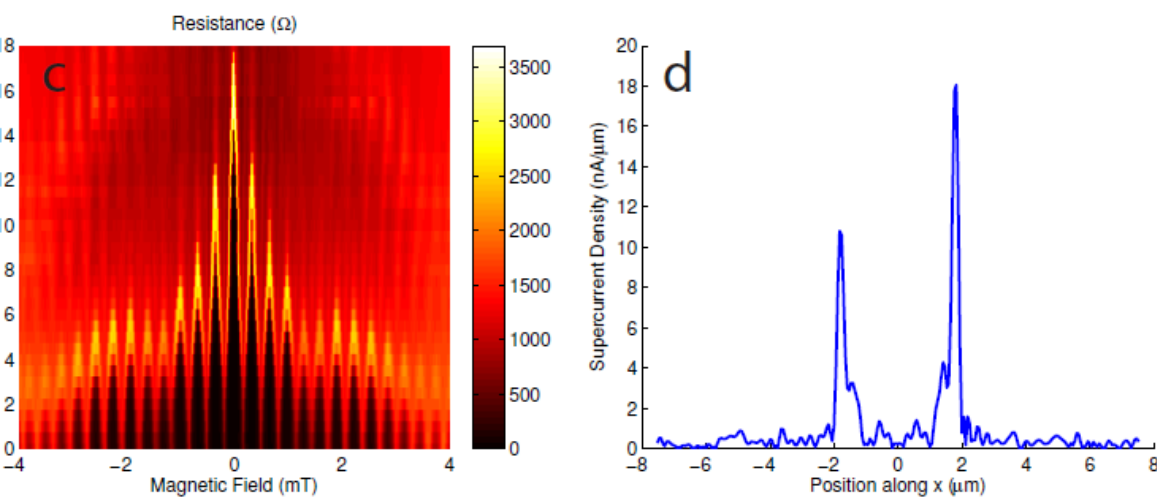


# Similar to proximity effect in Quantum Spin Hall Effect

HgTe/HgCdTe quantum wells, Yacoby Molenkamp, 2014

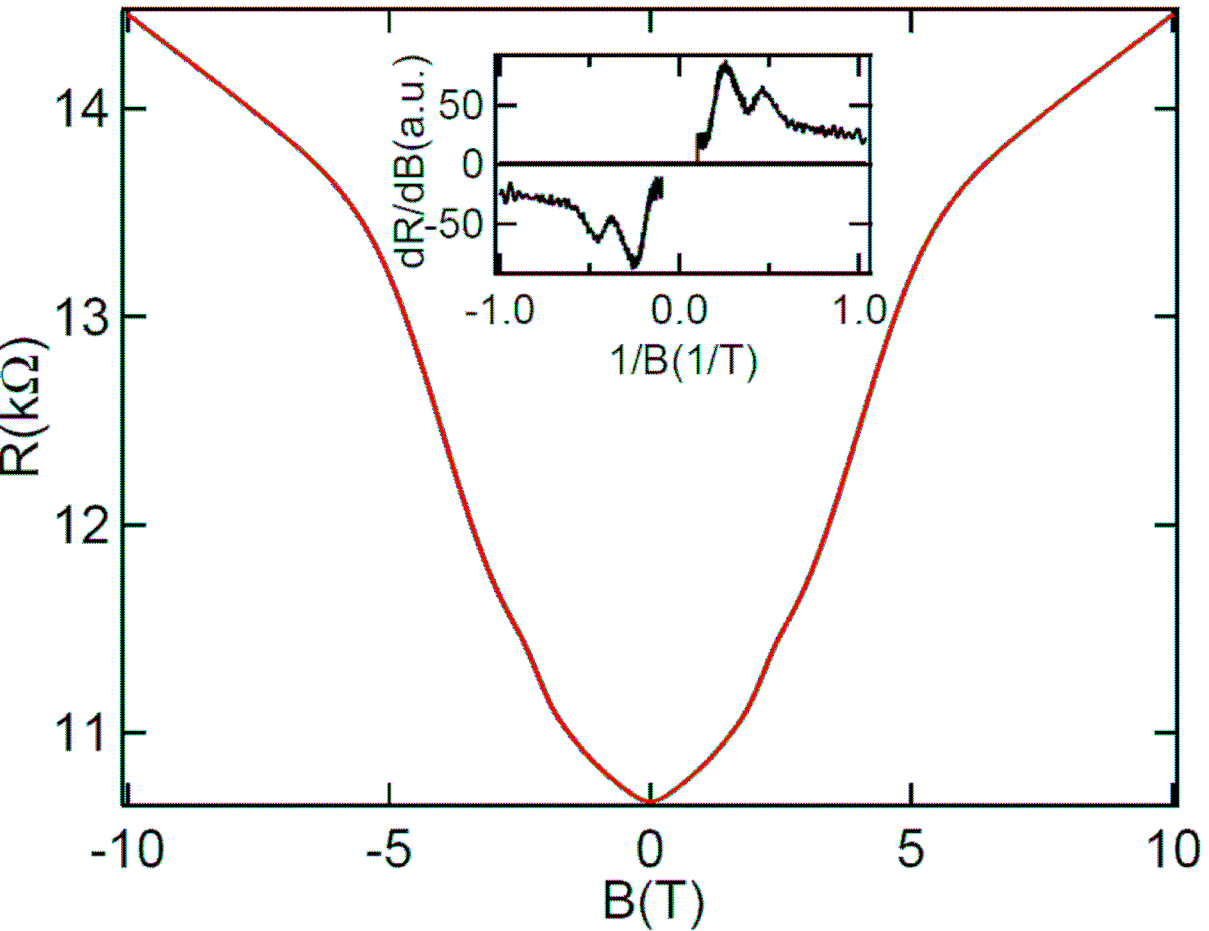


$V_g = 1.05\text{V}$   
trivial transport,  
uniform  
supercurrent  
density



$V_g = -0.425\text{V}$ ,  
supercurrent  
flows via **helical  
edge states!**

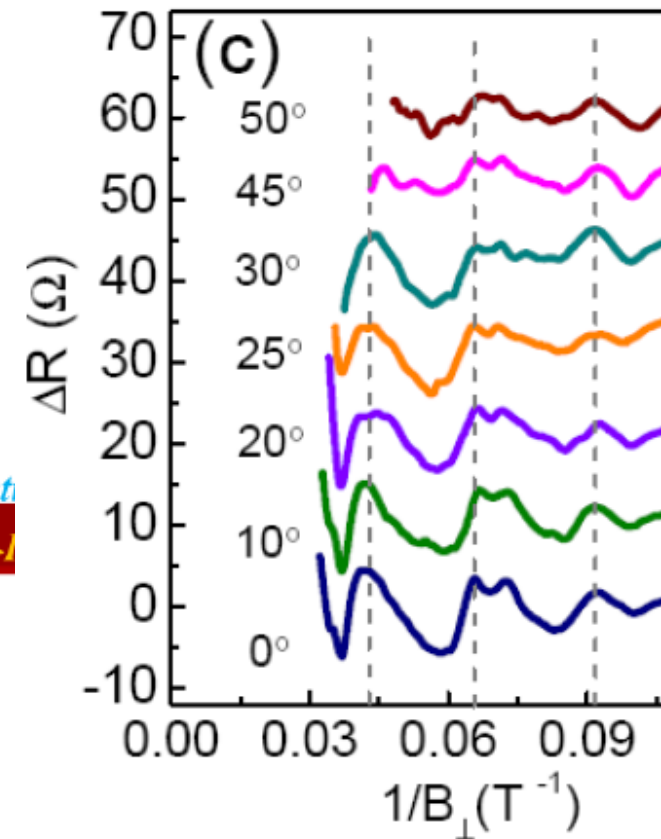
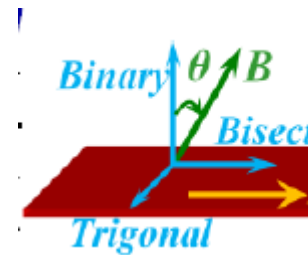
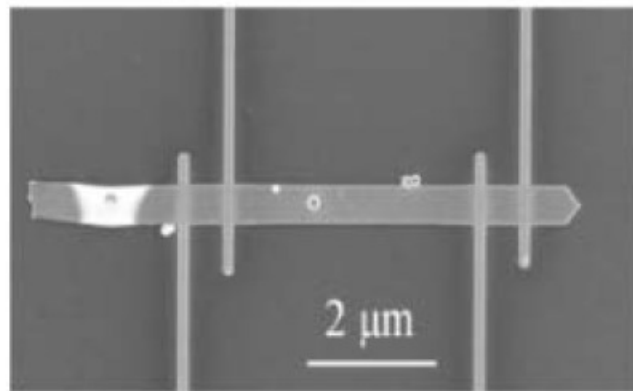
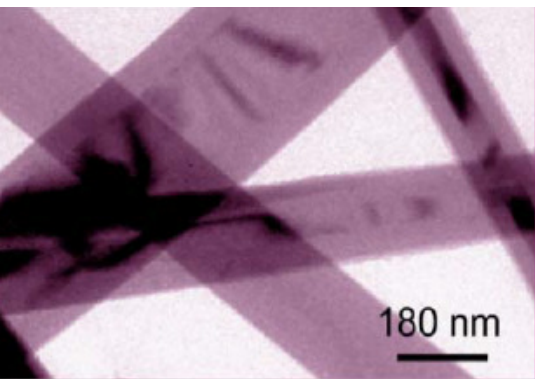
# Shubnikov de Haas oscillations seen at $T > T_c$



$T=6K$

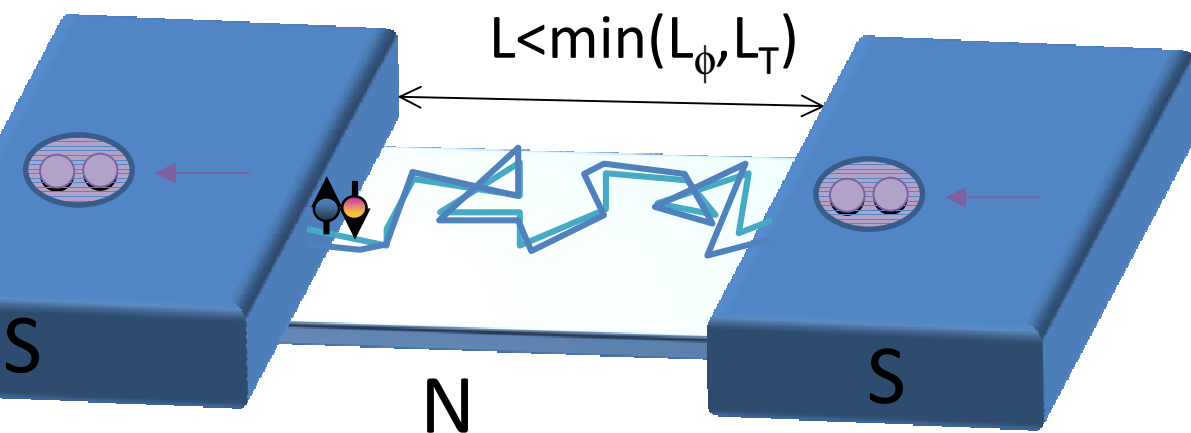
# Dirac-type surface states on Bi nanoribbons

Yang et al. (2014) arXiv:1404.5702:  
(10), 50 nm thick



- SdH oscillations with 1/2-shifted behavior (i.e.  $\gamma=1/2$ ): Dirac electrons on the surface

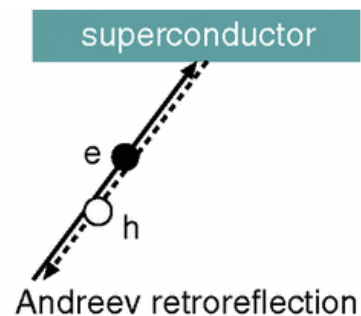
# Proximity effect



$$\Delta\varphi = k\Delta l + \frac{e}{\hbar} \oint \vec{A} \cdot d\vec{l} = k\Delta l + 2\pi \frac{\phi}{\phi_0}$$

Proximity effect is a very sensitive probe of coherence, spin properties in different systems

## Andreev reflection



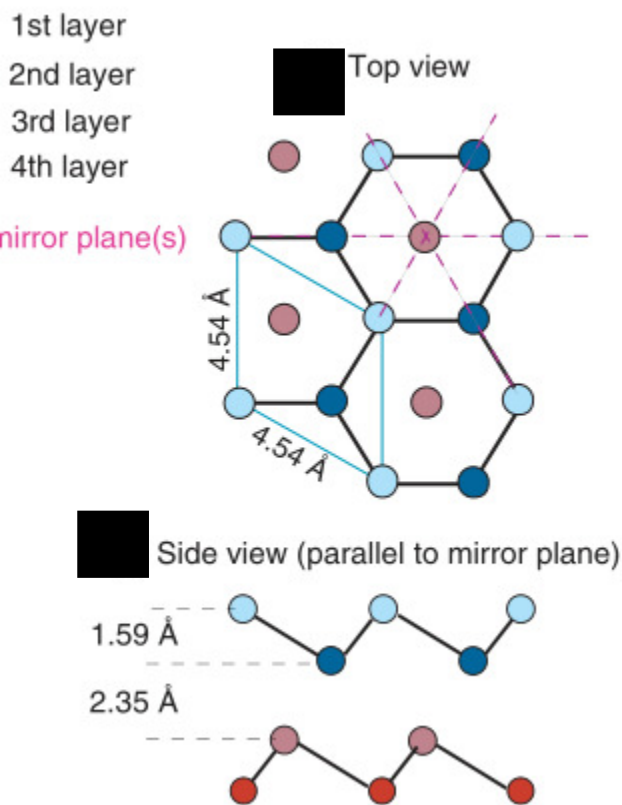
## Time reversal symmetry

$$\Delta\varphi = \int \overbrace{\vec{k}}^e d\vec{l} - \int \overbrace{(-\vec{k})}^h d\vec{l} = 0$$

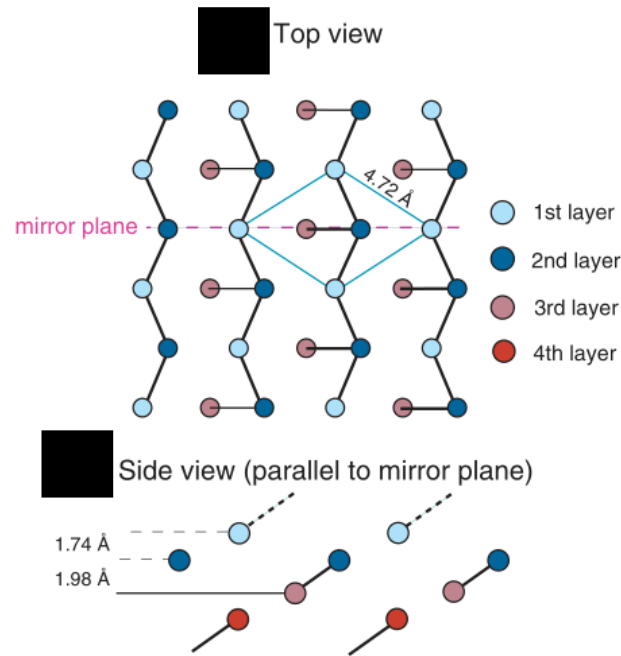
➔ Constructive interferences

# Bi thin film and surface structure

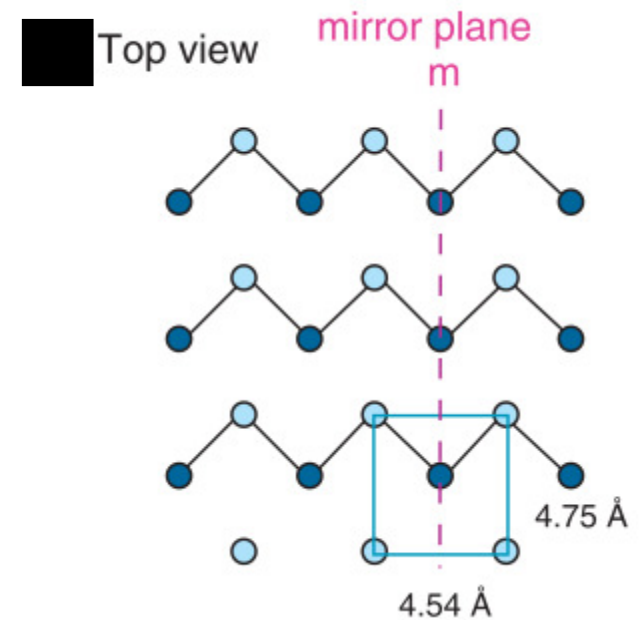
Truncated-bulk for (111)



Truncated-bulk for (110)



Truncated-bulk for (100)

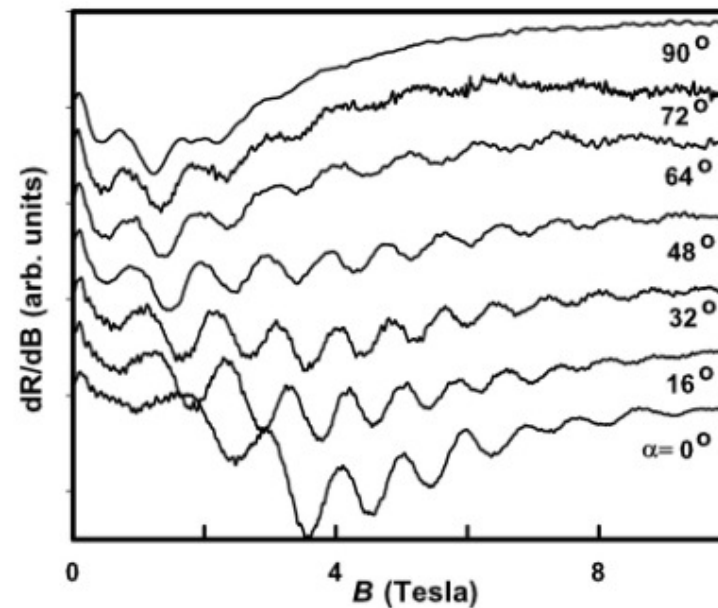
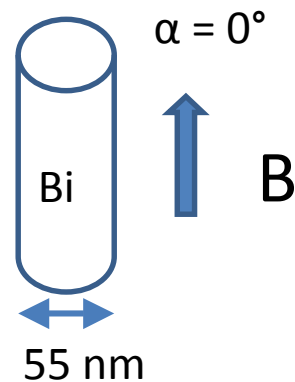


Long de Broglie wavelength  $\sim 120\text{\AA}$   
**Semimetal – semiconductor transition** at a  
 critical thickness:  $320\text{\AA}$

# What do you get with Bi nanowires?

Diameter  $\lesssim (\lambda_F^{\text{bulk}} = 50 \text{ nm})$ , wire is only surface!

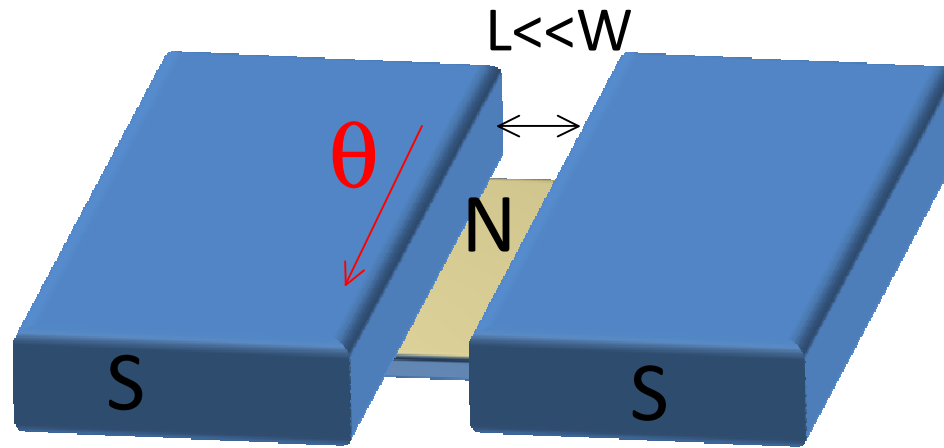
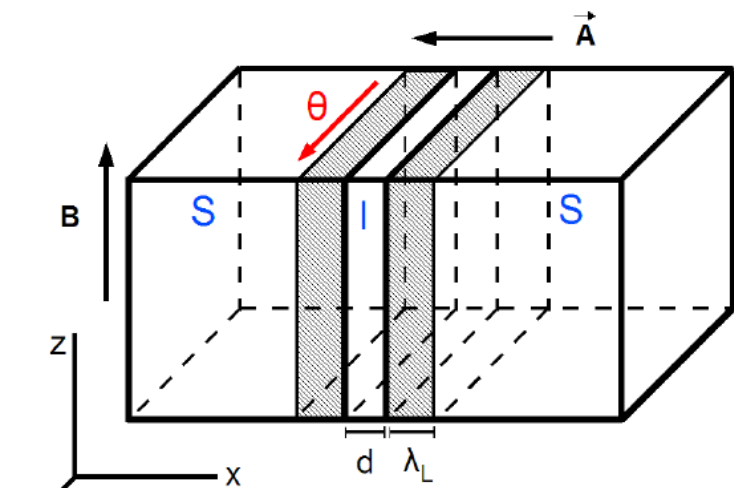
of: Aharonov-Bohm oscillations in parallel field (Nikolaeva 2008)



Clear period: as if cylinders are hollow: only surfaces?



# Oscillatory decay for a planar SIS or wide SNS junction

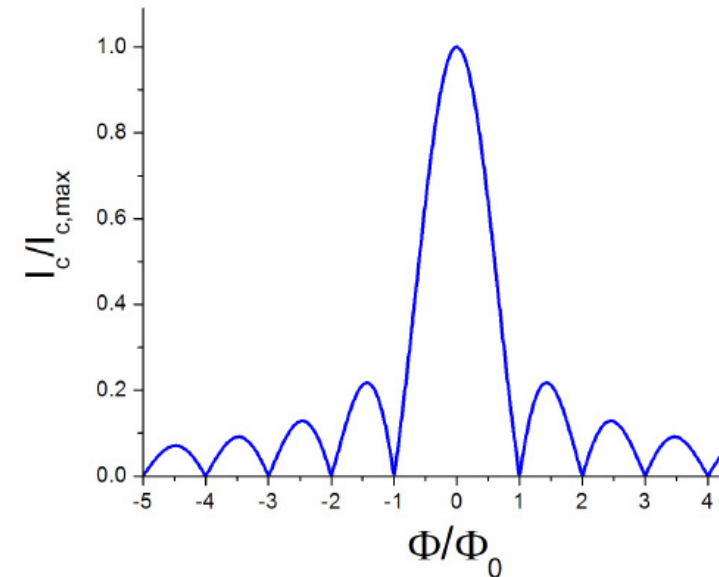


$x$  dependent phase variation at boundary  $\theta(y)$

$$\Delta\theta = 2\pi \frac{2e}{h} \int_{-\lambda_{L,d}}^{d+\lambda_{L,d}} A_x dx = 2\pi \frac{2e}{h} B l y = 2\pi \frac{\Phi_J(y)}{\Phi_0}$$

$$j = j_c \sin \left( \delta + 2\pi \frac{\Phi_J(y)}{\Phi_0} \right)$$

$$I_c = I_c(0) \frac{\Phi_0}{\pi \Phi_J} \left| \sin \left( \frac{\pi \Phi_J}{\Phi_0} \right) \right|$$



Fraunhofer pattern for a wide junction