Josephson Photonics:

From Coulomb Blockade to Non-linear Quantum Dynamics

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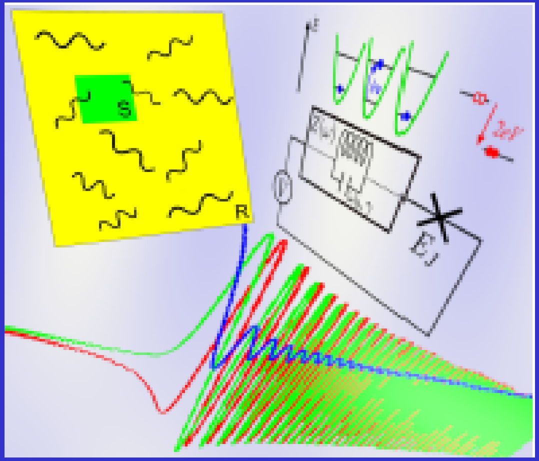


Quantronics30, 24-06-2015

Annalen der Physik 17, 132 (1905)

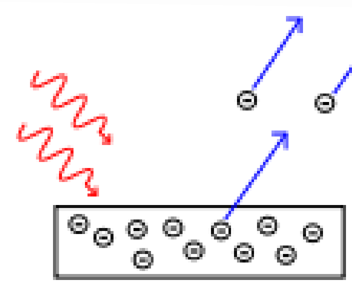
On a Heuristic Viewpoint Concerning the Production and Transformation of Light

6. *Über einen die Erzeugung und Verwandlung des Lichtes betreffenden heuristischen Gesichtspunkt;*

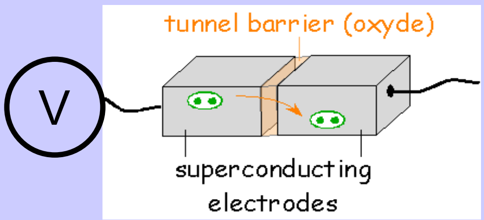
von A. Einstein. * Ulm, March 14, 1879

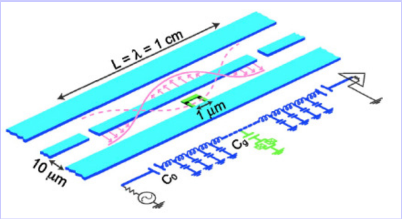
Zwischen den theoretischen Vorstellungen, welche sich die Physiker über die Gase und andere ponderable Körper gebildet haben, und der Maxwellschen Theorie der elektromagnetischen Prozesse im sogenannten leeren Raume besteht ein tiefgreifender formaler Unterschied. Während wir uns nämlich den Zustand eines Körpers durch die Lagen und Geschwindigkeiten einer zwar sehr großen, jedoch endlichen Anzahl von Atomen und Elektronen für vollkommen bestimmt

Light – matter interaction

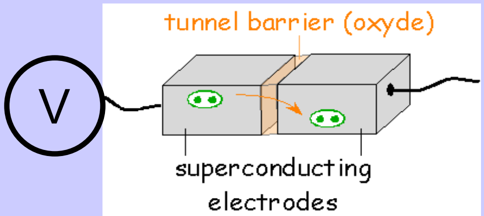


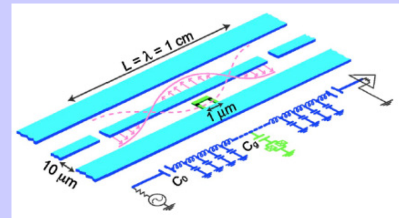
Photon – charge interaction



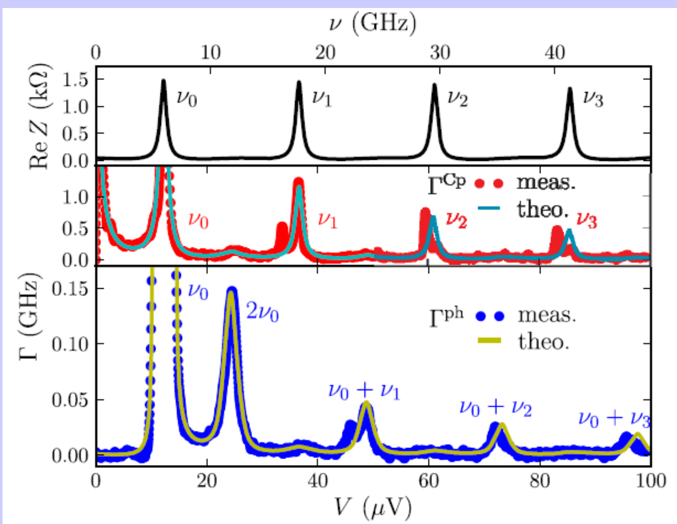


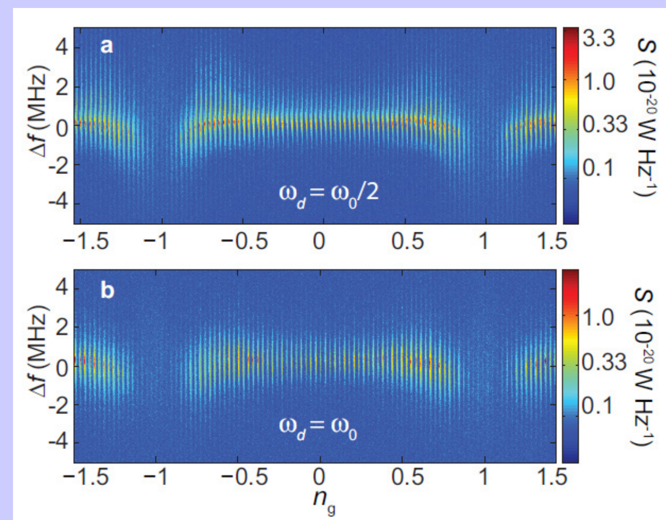
Photon - charge interaction





weak → strong





Hofheinz et al, PRL 106, 217005 (`11)
Altimiras, Esteve, Portier et al, PRL 112, 236803 (`14)
Rimberg, Armour et al PRB 90, 020506(R) (`14)

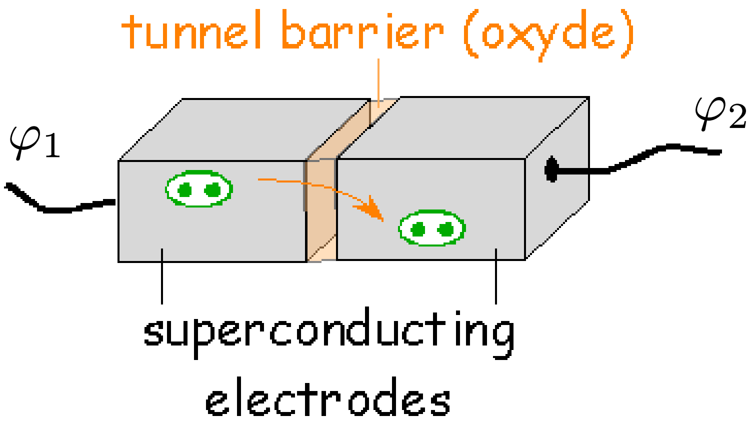
Leppäkangas, Johannson et al, PRL 110, 267004 (`13)
Armour, Blencowe et al, PRL 111, 247001 (`13)
Gramich, Kubala, JA et al, PRL 111, 247002 (`13)

$$\Psi \sim |\Psi_1| \, e^{i\varphi_1} + |\Psi_2| \, e^{i\varphi_2}$$

$$|\Psi\rangle \sim |\Psi_1 \Psi_2\rangle + |\Psi_2 \Psi_1\rangle$$

Equilibrium → Steady state

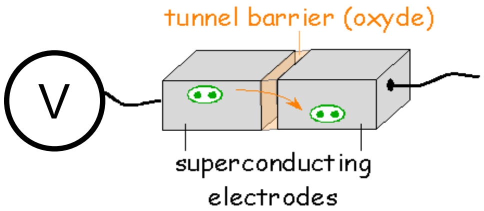
Josephson physics

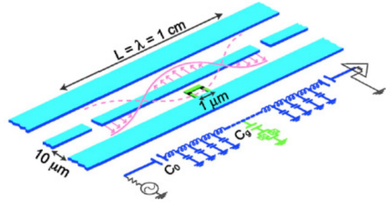


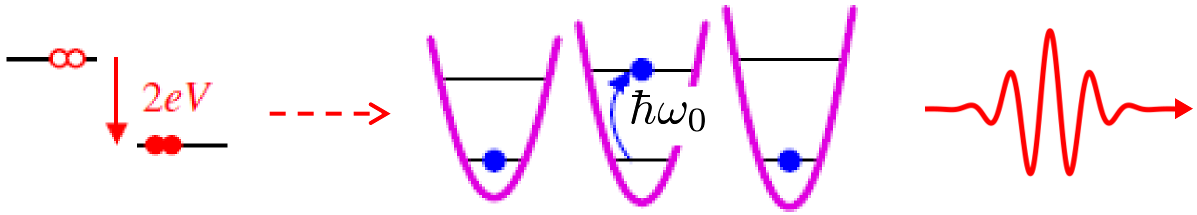
Interaction with environment –

dissipation, fluctuations, …

$$\hat{H}_J = E_C\,\hat{N}^2 - \frac{E_J(\Phi_{\rm ext})}{2}\left(e^{i\hat{\varphi}} + e^{-i\hat{\varphi}}\right) \qquad [\hat{N}, \hat{\varphi}] = -i\hat{\varphi}$$





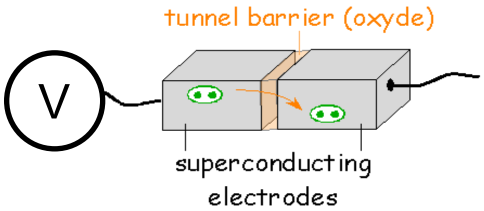


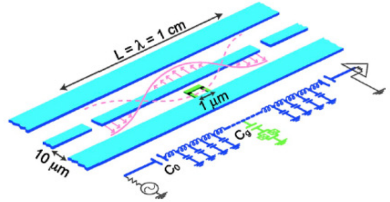
Sequential charge transfer

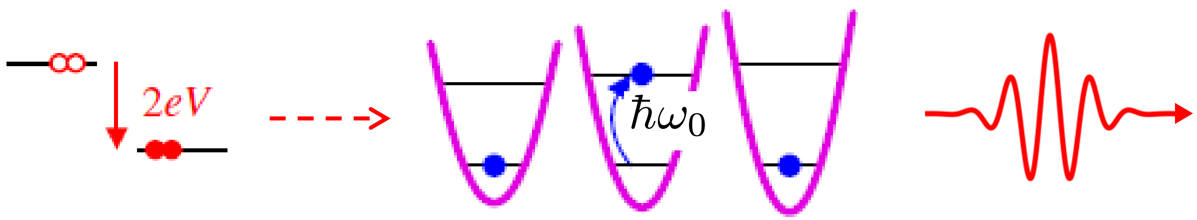
N is a good quantum number

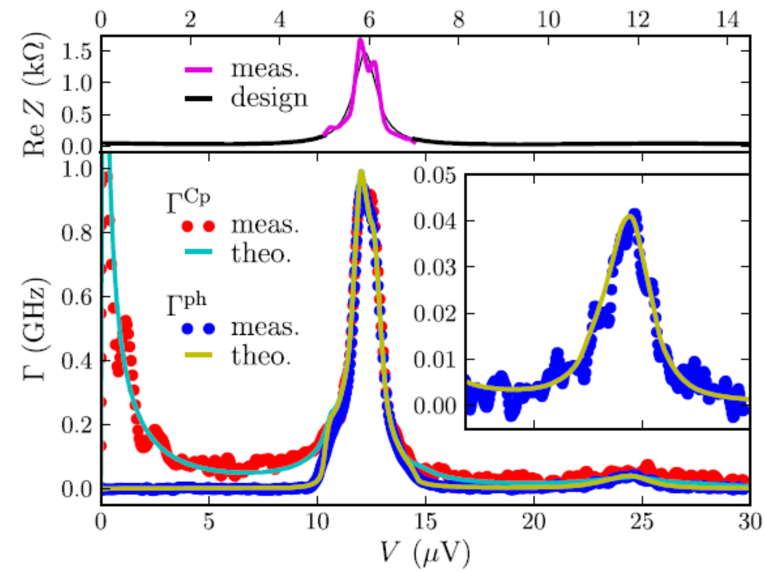
Coulomb blockade $\kappa \langle n \rangle \ll 1$

$$\kappa = \frac{E_C}{\hbar\omega_0} = \pi \frac{Z_i}{R_Q}$$









Sequential charge transfer

N is a good quantum number

Coulomb blockade $\kappa \langle n \rangle \ll 1$

$$\kappa = \frac{E_C}{\hbar\omega_0} = \pi \frac{Z_i}{R_Q}$$

$$I_J \sim (eE_J^2/\hbar)\, P(2eV) \;, \quad T = 0$$

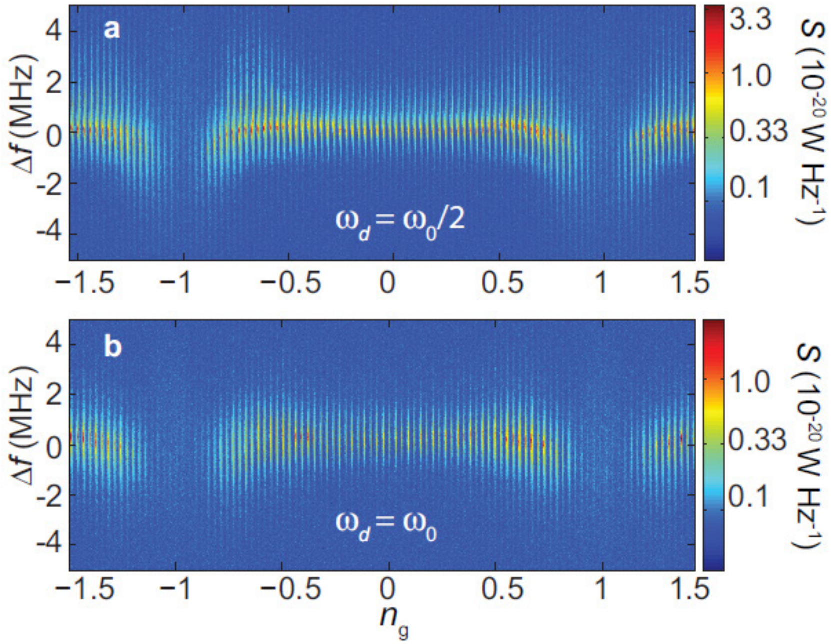
Strong coupling

Coherent charge transfer

φ is a good quantum number

Josephson regime $\kappa \langle n \rangle \gg 1$

$$m = \phi_0^2 C,\ \omega_0^2 = 1/LC$$



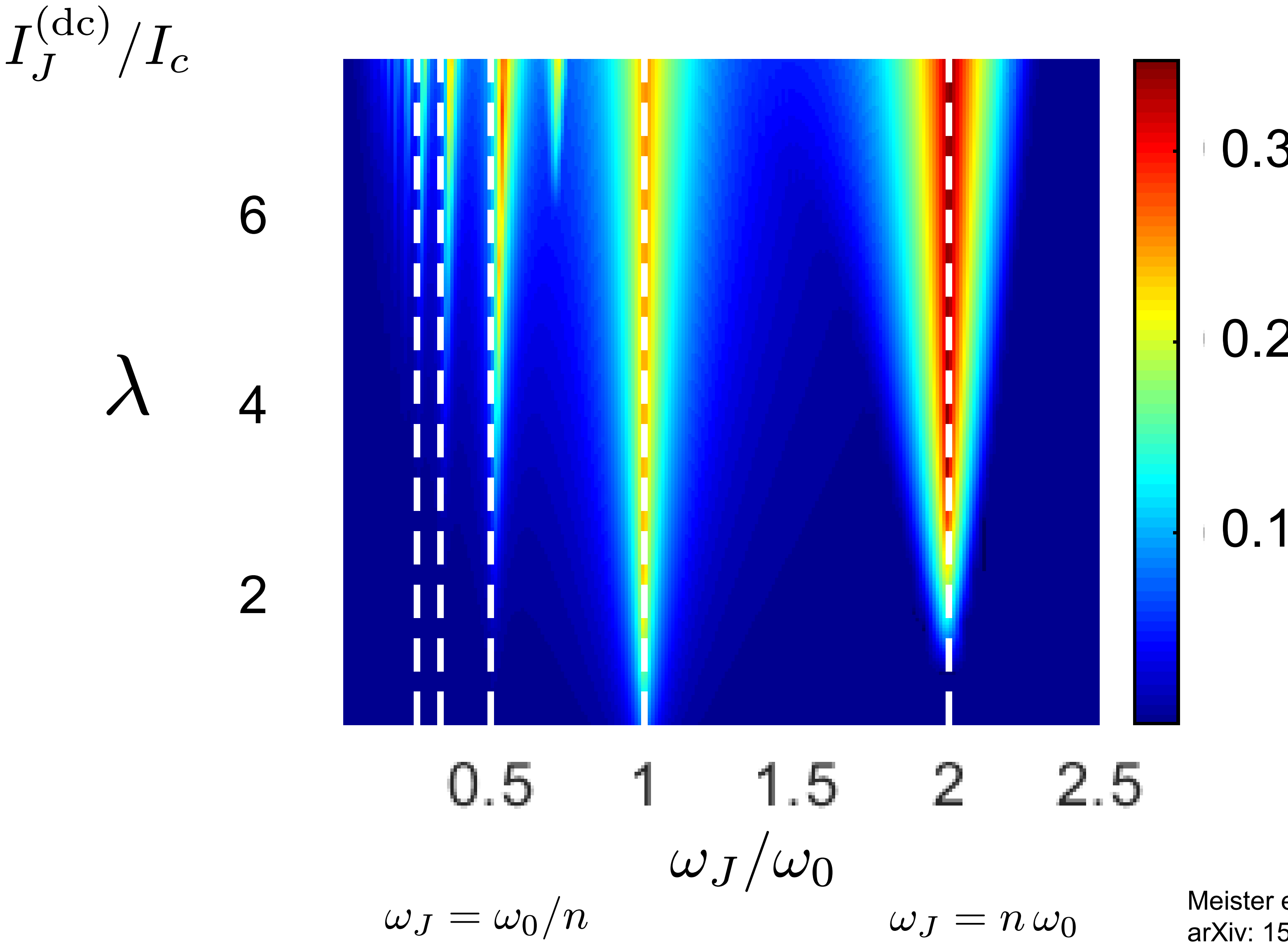
Rimberg et al, PRB 2014

$$H = \frac{p^2}{2m} + \frac{m\omega_0^2}{2}\phi^2 - E_J \cos(\phi + \omega_J\, t)$$

$$\omega_J \equiv \frac{2eV}{\hbar} = \frac{\hbar}{2e}(\dot{\varphi} - \dot{\phi})$$

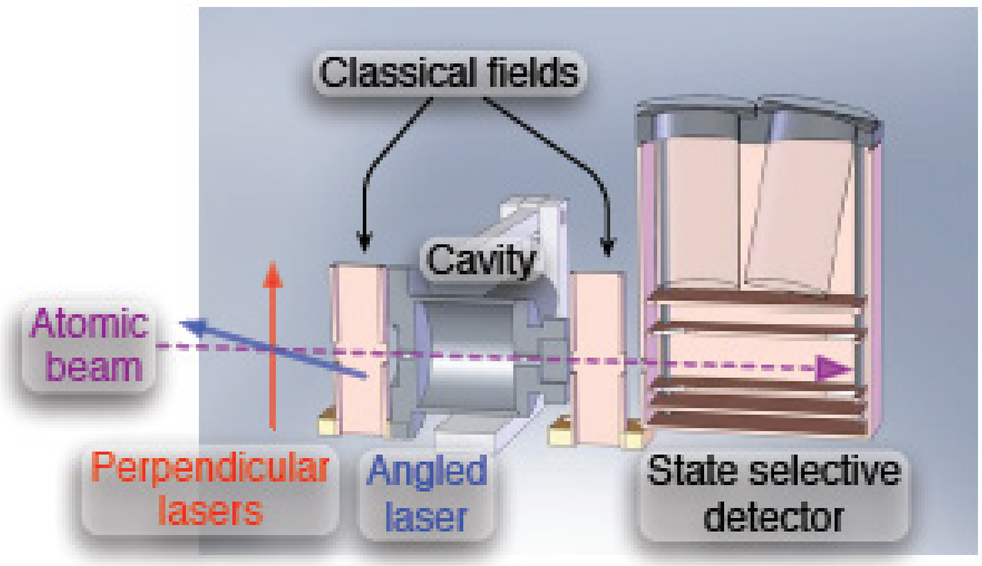
$$\lambda = \frac{E_J}{m\omega_0\gamma} = \frac{E_J\, Q}{E_L}$$

Classical nonlinear dynamics



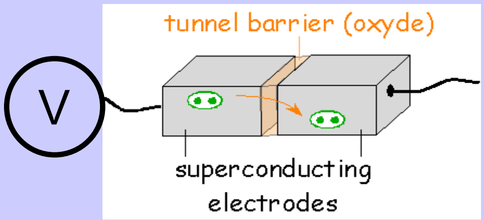
Meister et al.
arXiv: 1506.06908

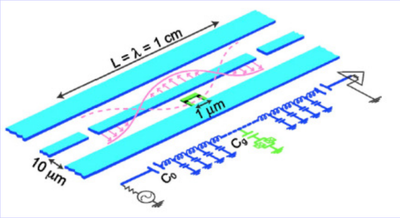
One atom maser - micromaser



- Excited atoms ⟷ Voltage driven Cooper pairs
- Cavity ⟷ LC resonator

Quantum description





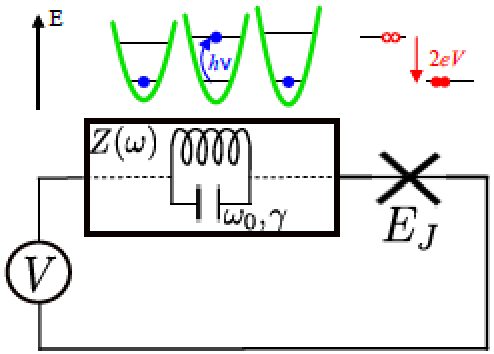
From Coulomb blockade to quantum microwaves

Charge noise and nonlinear quantum dynamics

Statistics of photon radiation

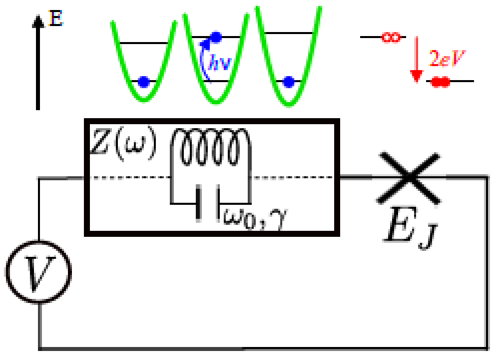
Two cavity set-up and entanglement

Coulomb blockade to quantum microwaves



Fundamental resonance (1-photon resonance)

Coulomb blockade to quantum microwaves



Fundamental resonance (1-photon resonance)

$$H_1 = \hbar\Delta a^\dagger a + i\frac{E_J^*}{2} : \left(a^\dagger e^{i\eta} - a\, e^{-i\eta}\right) \frac{J_1(2\sqrt{\kappa\, n})}{\sqrt{n}} :$$

$$\Delta = \omega_0 - \omega_J$$

$$E_J^* = E_J\, e^{-\kappa/2}$$

+ cavity damping and local voltage noise

$$\dot{\rho} = \frac{1}{i\hbar}[H_1, \rho] + \mathcal{L}_Q[\rho] + \mathcal{L}_V[\rho]$$

Gramich, Kubala, JA et al, PRL 111, 247002 (`13)

Coulomb blockade to quantum microwaves

p-photon resonance $\omega_J = p\,\omega_0$

$$H_p = \hbar\Delta a^\dagger a + i^p \frac{E_J^*}{2} : \left[(a^\dagger)^p\, \mathrm{e}^{i\eta} + (-1)^p a^p\, \mathrm{e}^{-i\eta}\right] \frac{J_p(2\sqrt{\kappa\, n})}{n^{p/2}} :$$

Coulomb blockade to quantum microwaves

$$H_1 = \hbar\Delta a^\dagger a + i\frac{E_J^*}{2} : \left(a^\dagger\, \mathrm{e}^{i\eta} - a\, \mathrm{e}^{-i\eta}\right) \frac{J_1(2\sqrt{\kappa\, n})}{\sqrt{n}} :$$

$\Delta = \omega_0 - \omega_J$

$(\Delta = 0)$

Weak driving $\Leftrightarrow \kappa\langle n\rangle \ll 1$

Strong driving $\Leftrightarrow \langle n\rangle \gg 1$

$2\sqrt{\kappa}a \rightarrow Z\mathrm{e}^{i\theta}$

$$H_{CB} = i\frac{E_J^*\sqrt{\kappa}}{2}\left(a^\dagger\, \mathrm{e}^{i\eta} - a\, \mathrm{e}^{-i\eta}\right)$$

$$H_{\mathrm{cl}} = E_J J_1(Z)\, \sin(\theta)$$

P(E)- theory of CB

$\phi(t) = Z\cos(\omega_J t + \theta)$

Coulomb blockade to quantum microwaves

$$H_1 = \hbar\Delta a^\dagger a + i\frac{E_J^*}{2} : \left(a^\dagger \mathrm{e}^{i\eta} - a\,\mathrm{e}^{-i\eta}\right) \frac{J_1(2\sqrt{\kappa\, n})}{\sqrt{n}} :$$

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P(E)- theory of CB

$\phi(t) = Z\cos(\omega_J t + \theta)$

„Quantum amplitude“

$$Z_q = \sqrt{2\kappa\langle n\rangle}$$

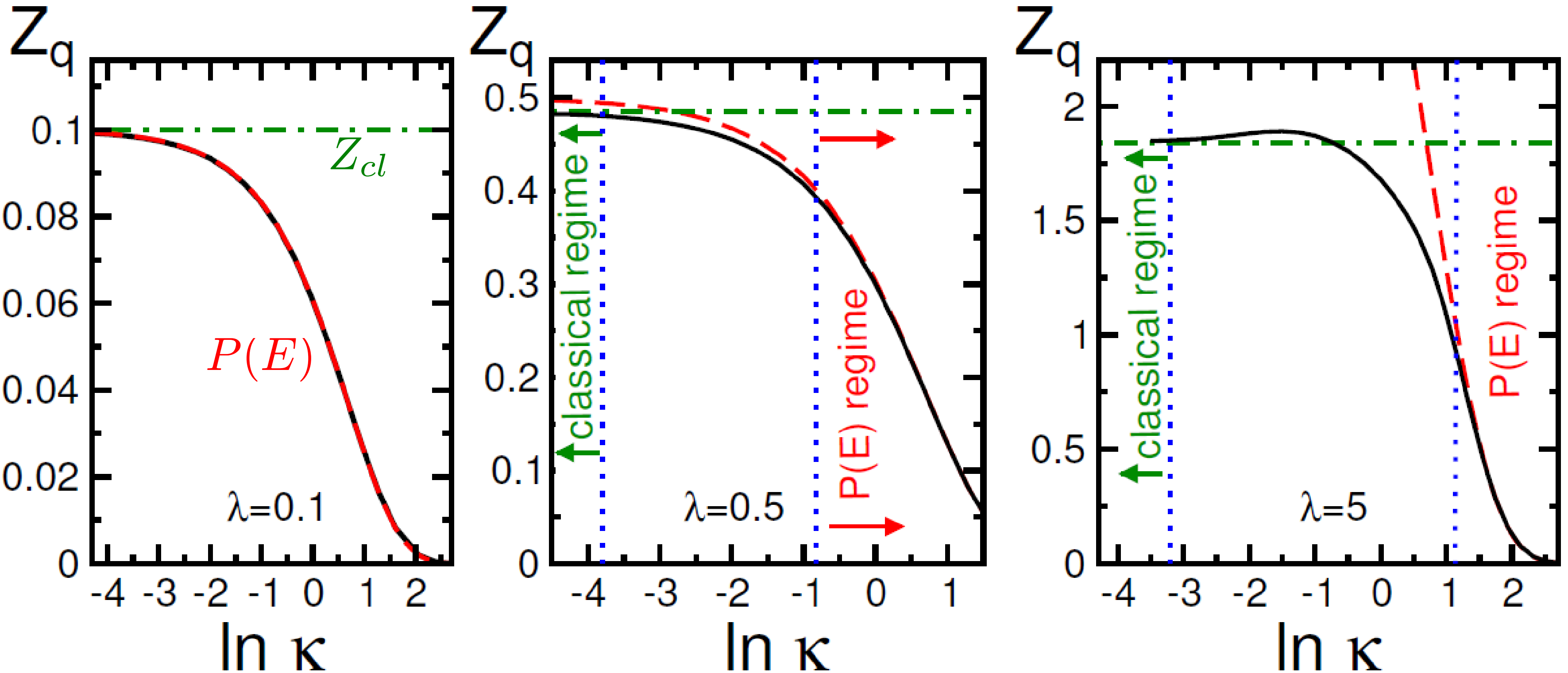
quantum – classical transition

„quantumness“ $\kappa = E_C/\hbar\omega_0$

„driving amplitude“ $\lambda = E_J/m\omega_0\gamma$

„Quantum amplitude“

$$Z_q = \sqrt{2\kappa\langle n\rangle}$$



quantum – classical transition

„quantumness“ $\kappa = E_C/\hbar\omega_0$

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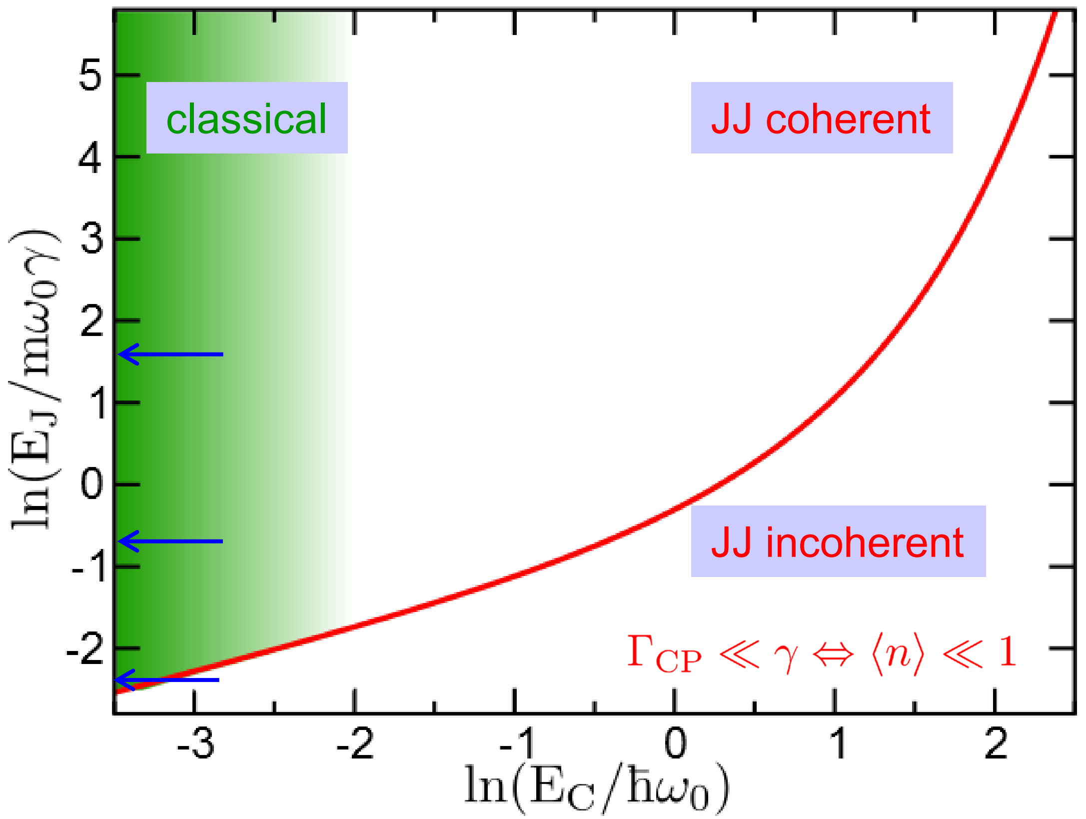
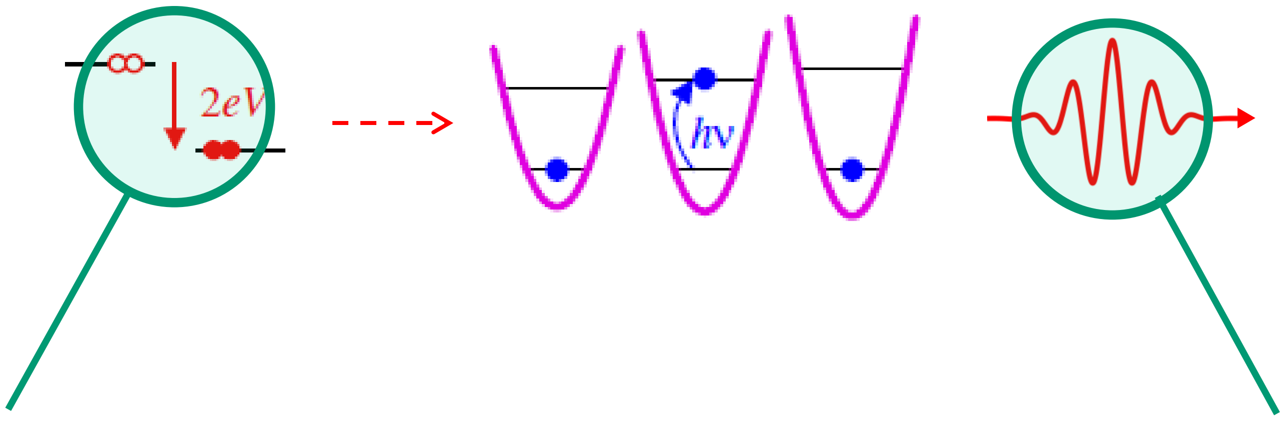


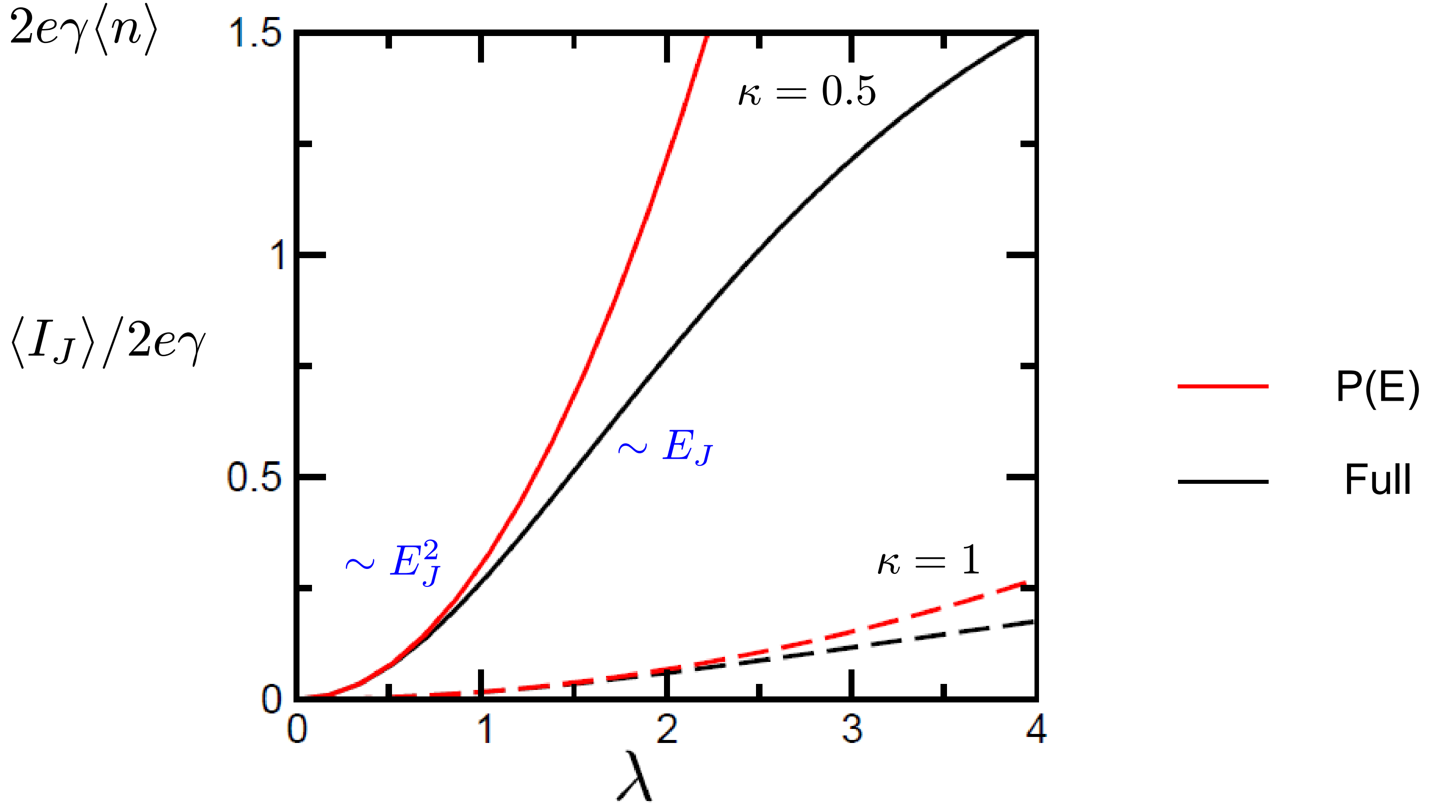
Detection: Charge flow – photon radiation



Coulomb blockade to quantum microwaves

$$\langle I_J \rangle_{\rm st} = \frac{2eE_J^*}{\hbar} \langle : (a^\dagger + a) \frac{J_1(2\sqrt{\kappa n})}{\sqrt{n}} : \rangle_{\rm st}$$

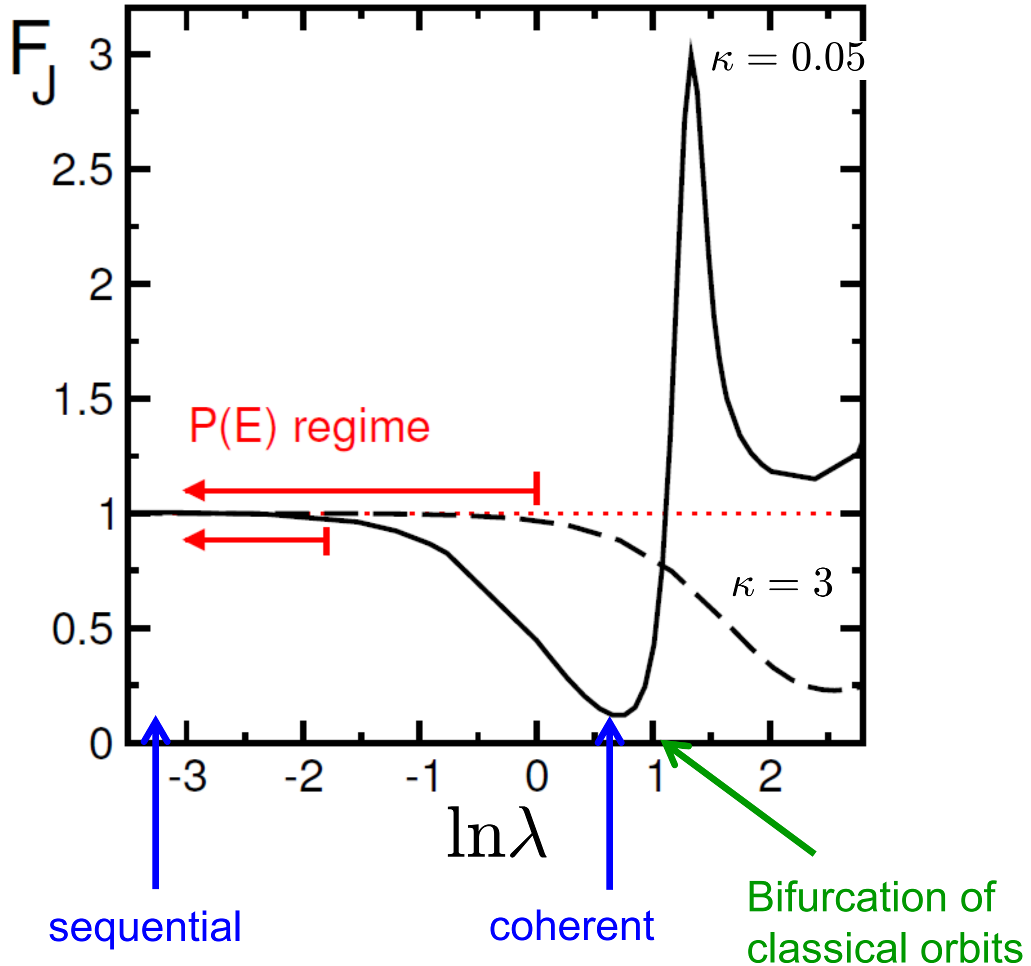
$$\langle I_J \rangle = 2e\gamma \langle n \rangle$$



Copper pair current noise

Fano factor

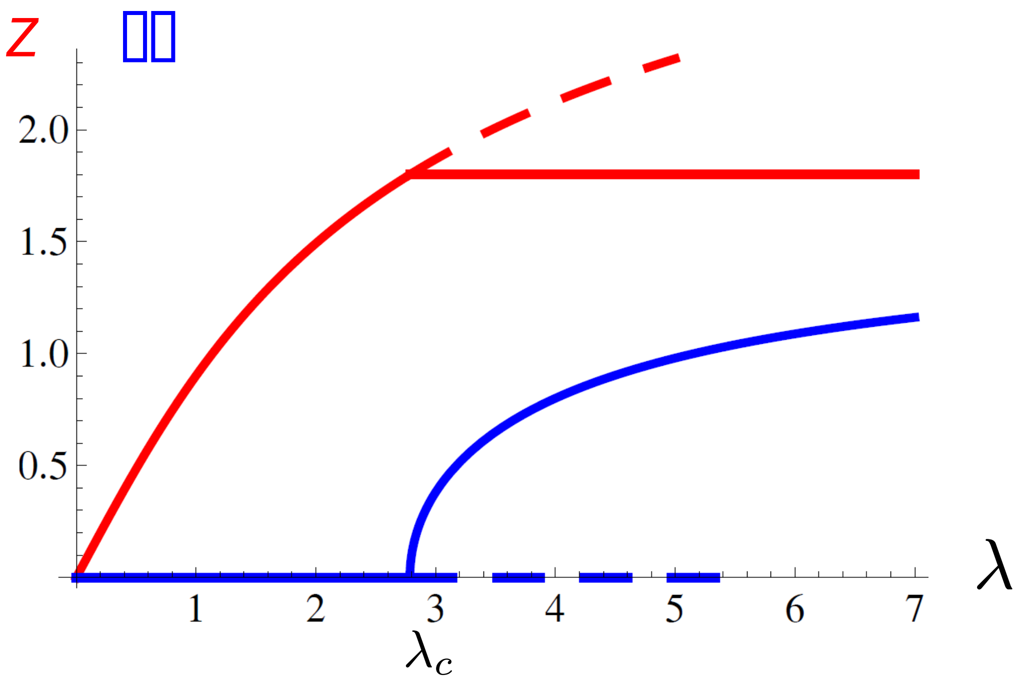
$$F_J = \frac{1}{4eI_J}\int d\tau \langle \delta I_J(t+\tau)\,\delta I_J(t)\rangle_{\rm st}$$



Gramich, Kubala, JA et al, PRL 111, 247002 (`13)

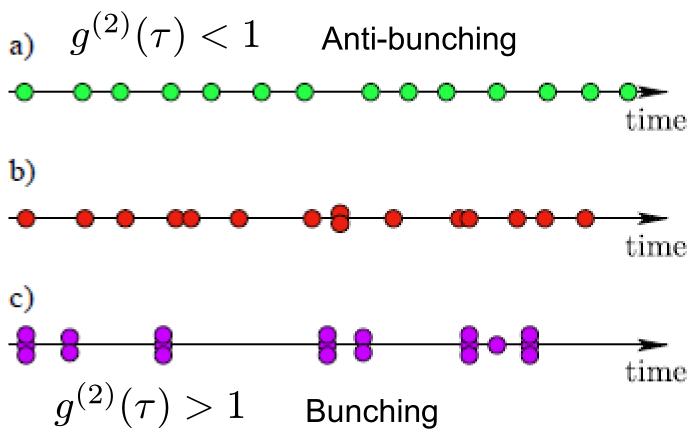
Classical nonlinear dynamics: bifurcation

Steady state $$\phi(t) = Z\cos(\omega_J t + \theta)$$

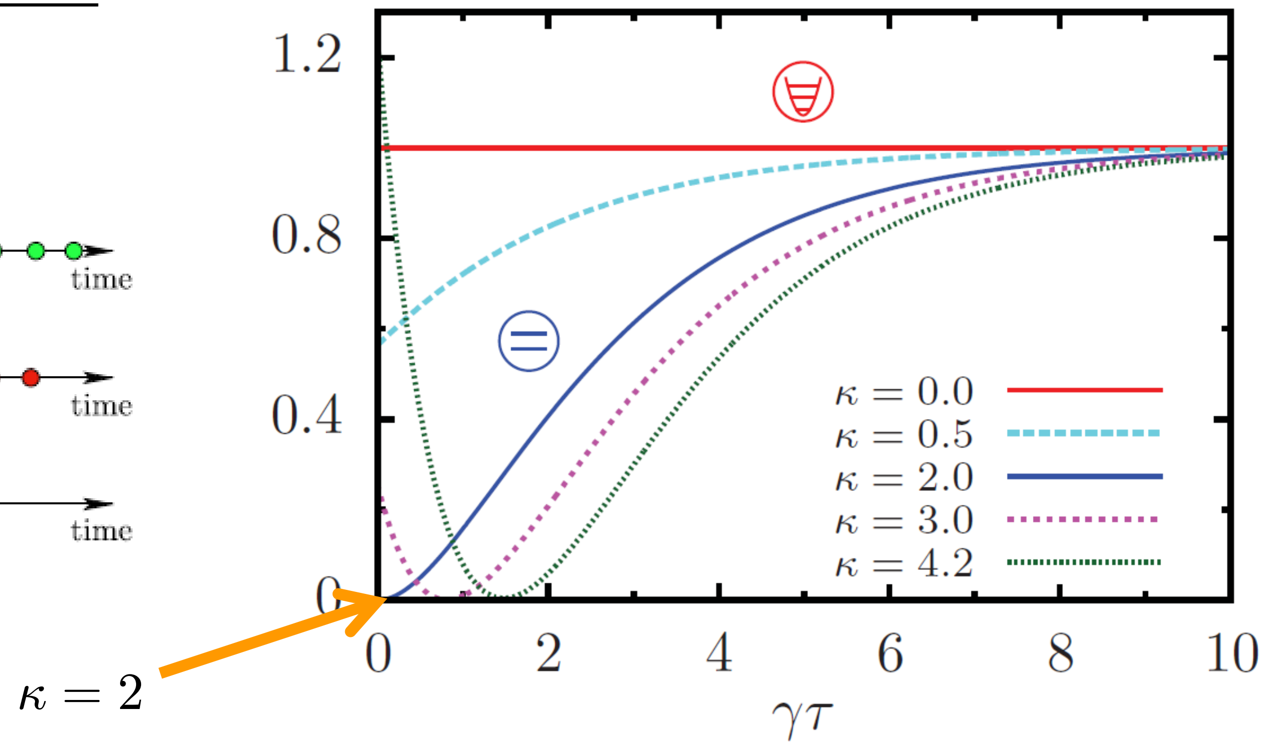


Photonic perspective

$$g^{(2)}(\tau) = \frac{\langle :n(t)n(t+\tau): \rangle}{\langle n \rangle_{\rm st}^2}$$



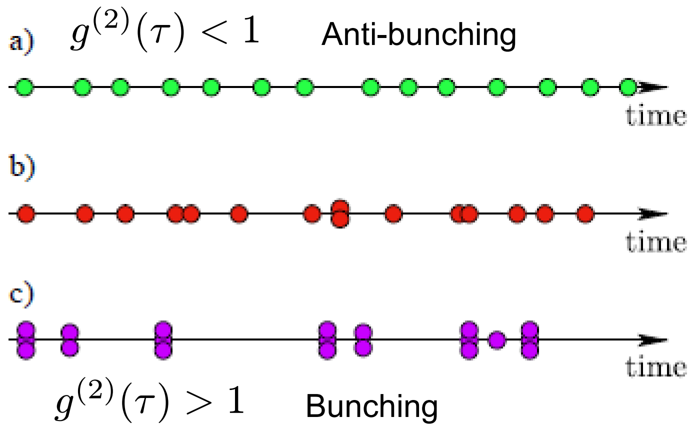
$\lambda \ll 1$

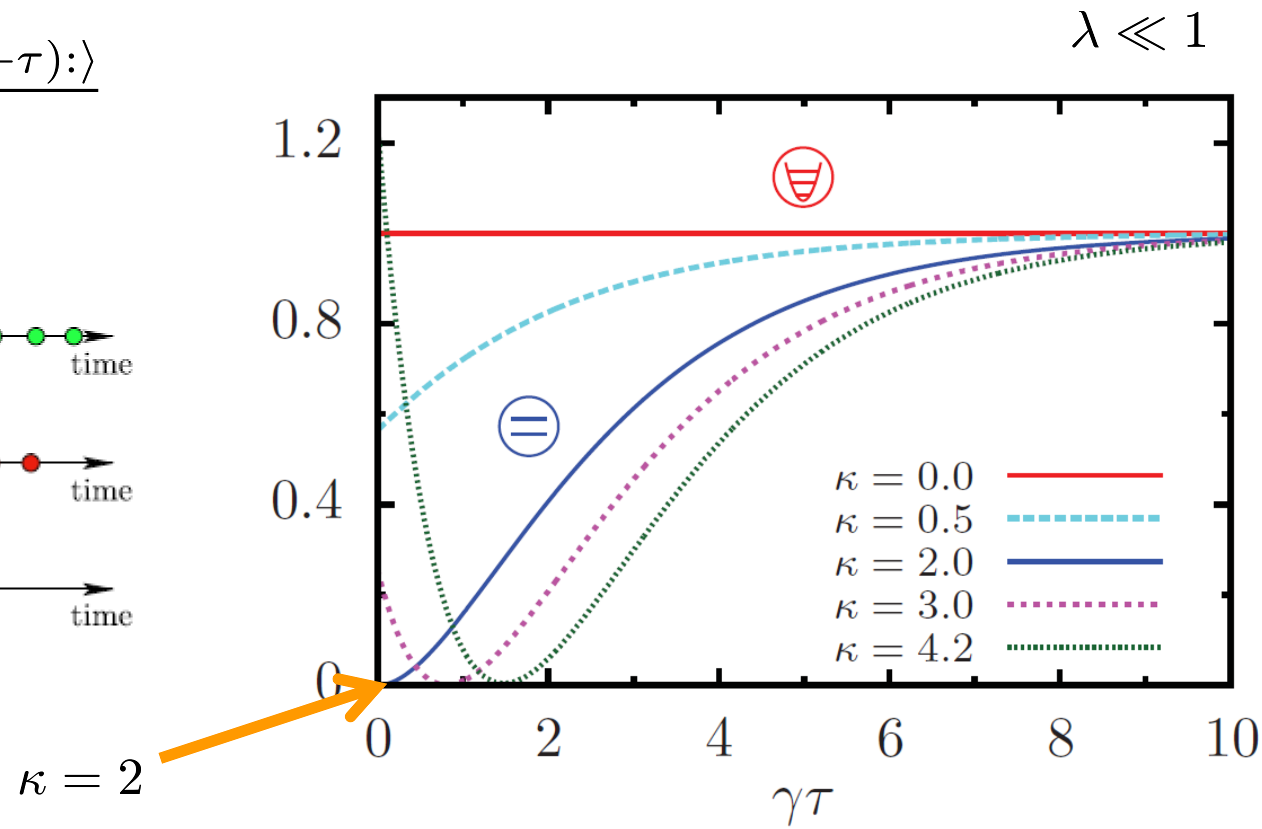


Dambach, Kubala, JA , arXiv: 1506.05626

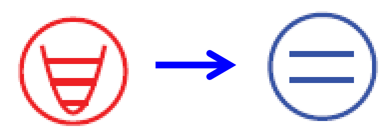
Photonic perspective

$$g^{(2)}(\tau) = \frac{\langle :n(t)n(t+\tau): \rangle}{\langle n \rangle_{\mathrm{st}}^2}$$





$$T_{1\to 2} = \langle 2|H_1|1\rangle = 0$$

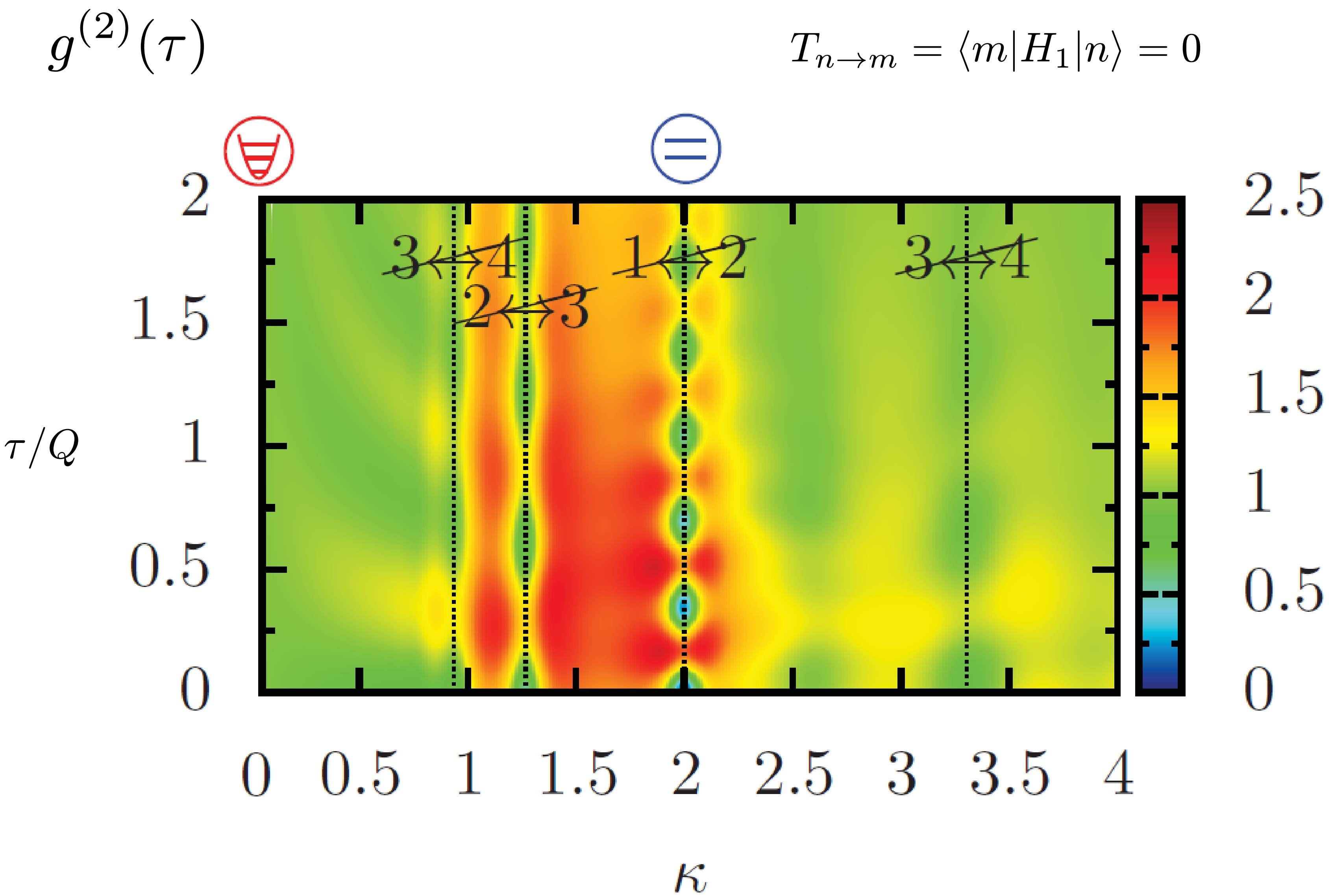


Ideal single photon source

Dambach, Kubala, JA , arXiv: 1506.05626

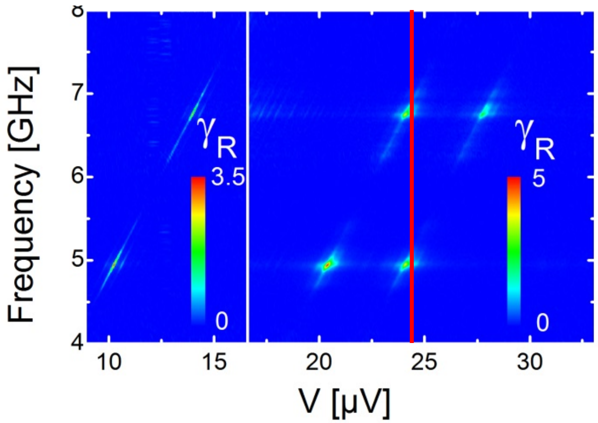
Photon noise: stronger driving

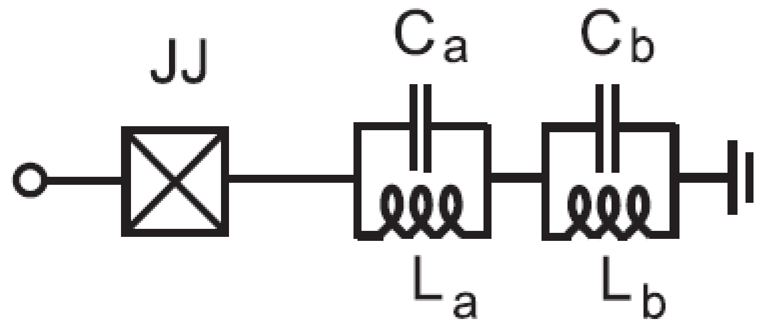
$$T_{n\to m} = \langle m|H_1|n\rangle = 0$$



Two cavities: correlated photons

$$2eV = \hbar(\omega_a + \omega_b)$$





$$g_{ab}^{(2)}(0) = \frac{1}{2\langle n \rangle} + \frac{1}{2}\left[g_{aa}^{(2)}(0) + g_{bb}^{(2)}(0)\right]$$

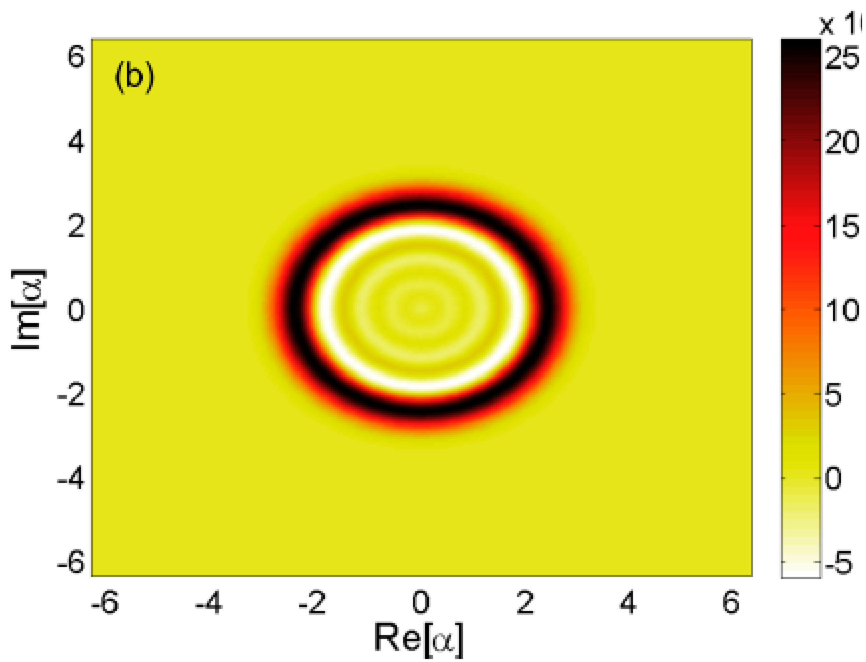
Non-classical light: violation of Cauchy-Schwartz $\quad g_{ab}^{(2)}(0) \geq \sqrt{g_{aa}^{(2)}(0)\, g_{bb}^{(2)}(0)}$

Armour, Kubala, JA, PRB 91, 184508 (2015)
Single cavity: Leppäkangas et al, NJP 2013
Paduriariu et al, PRB (2012)

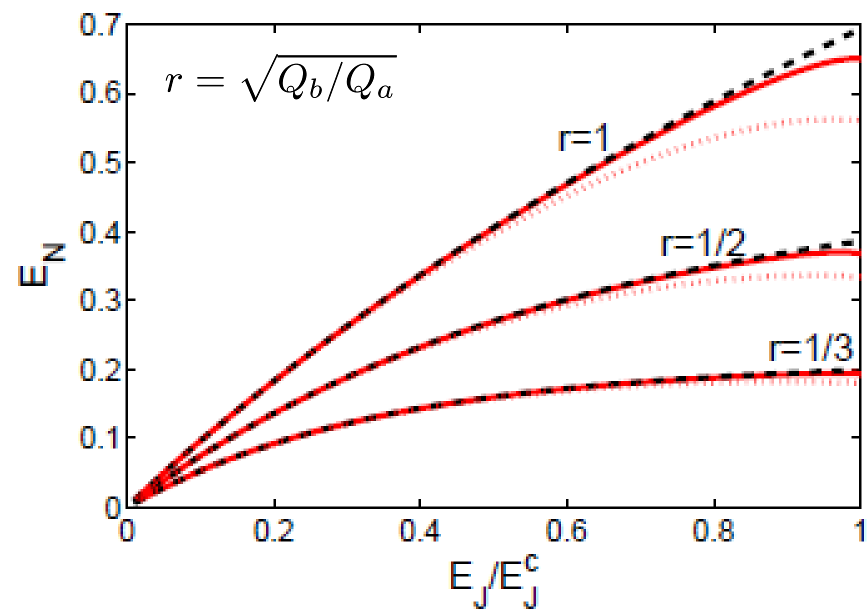
Two cavities: correlated photons

Phase-entangled: $$|\theta_1 + \theta_2\rangle \sim |\theta_1\rangle_a|\theta_2\rangle_b + |\theta_2\rangle_a|\theta_1\rangle_b$$

Wigner function for cavity a

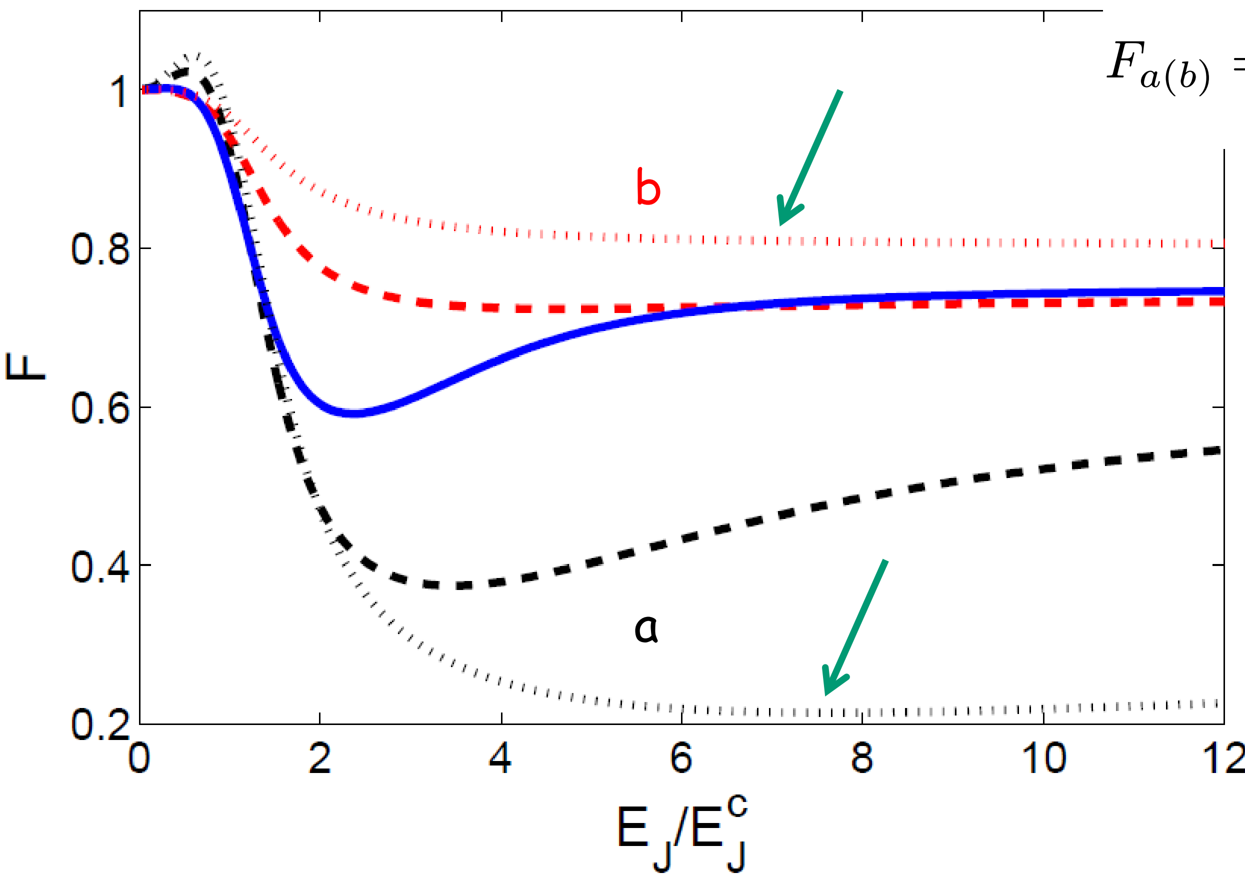


Log Negativity



Armour, Kubala, JA, PRB 91, 184508 (2015)
Armour, Kubala, JA, in preparation

Higher photon occupancy



$$F_{a(b)} = \frac{\langle n_{a(b)}^2 \rangle - \langle n_{a(b)} \rangle^2}{\langle n_{a(b)} \rangle}$$

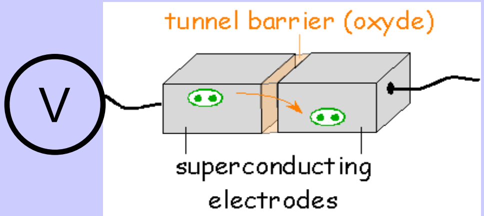
$r = 1$

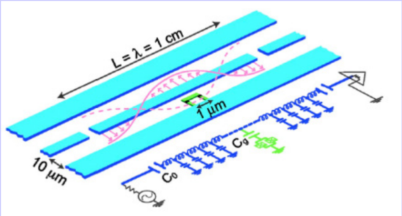
$r = 1/2$

$r = 1/3$

$r = \sqrt{Q_b/Q_a}$

Summary & Future experiments





Josephson photonics

Replace JJ by atomic point contact

Relation between current noise and photon statistics

on-demand quantum microwave sources

Nonlinear quantum dynamics

…

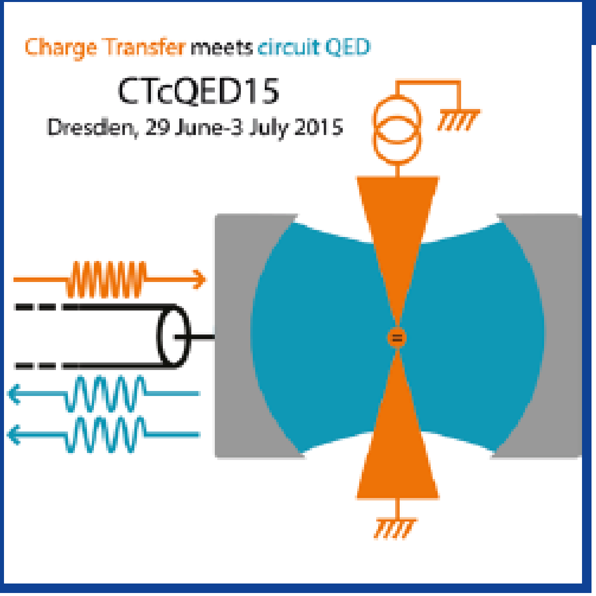
MAX-PLANCK-GESELLSCHAFT

Charge Transfer meets Circuit Quantum Electrodynamics

Complex Systems

International Workshop
29 June - 3 July 2015

Recent experiments have considered various mesoscopic transport devices embedded within microwave cavities to investigate correlated photon-charge transfer processes. This workshop will discuss experimental and theoretical aspects relevant to these and related set-ups.



Ulm

V. Gramich
B. Kubala
S. Dambach
S. Meister
M. Mecklenburg

Beyond

F. Potier, D. Esteve (Saclay)
A. Armour (Nottingham)
M. Blencowe, A. Rimberg (Dartmouth)
M. Hofheinz (Grenoble)
J. Leppäkangas, G. Johansson (Göteborg)


HAPPY
BIRTHDAY

Life is not measured by the number of breath we take,

but by the number of moments that take our breath away

Thank you - Quantronics