Clusters in granular flows : elements of a non-local rheology

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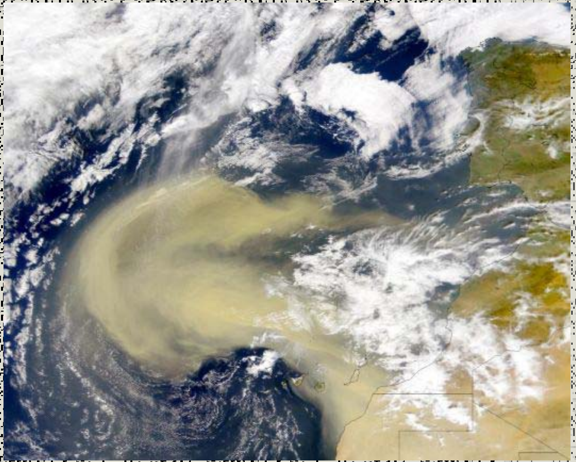
Geneve, November 2003

CEA-Saclay/SPEC: Group Instabilities and Turbulenc

Introduction: Granular materials

- Three typical behaviours, depending on the stress/strain imposed to the material



- <u>Specific properties</u>
  - Complex interactions : solid friction
  - Inelastic collision => dissipative dynamics
  - Rather large number of particles => statistical description
  - But : Weak scale separation and $kT \ll mgd$

Overview

- Dense flows :
  The need for a non-local rheology

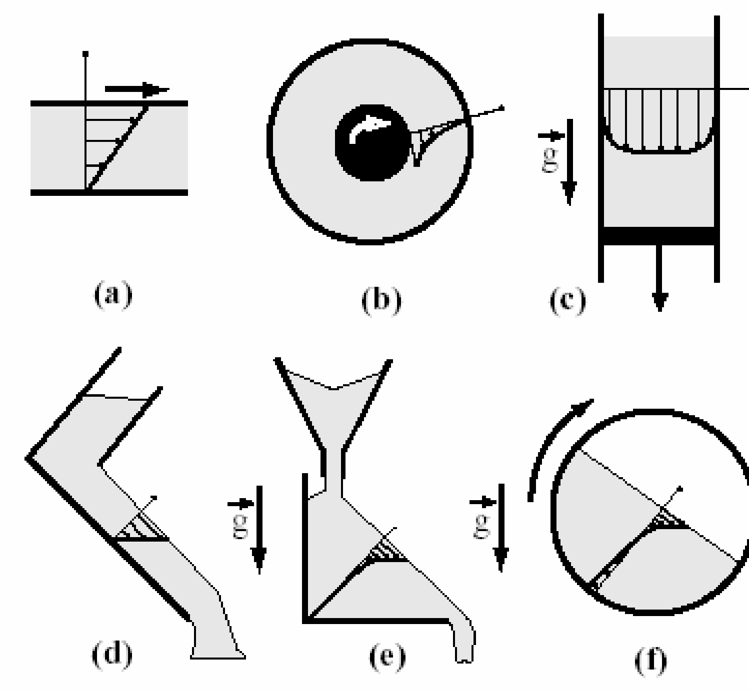
- Experimental evidence of clusters : a first step towards a non-local rheology

  - Dynamical Clusters in stationnary flow (rapidly)

  - Intermittent clusters during relaxation towards mechanical equilibrium (in more details)

Studying dense flows rheology

- Several configurations
  - given state of stress /strain
  - measure the kinematic properties
  - extract the rheology



- Questions are
  - Relevant timescales?
  - Dependance on microscopic properties?
  - Dimensionless parameters?
  - Influence of the geometry?

Relevant parameters (I)

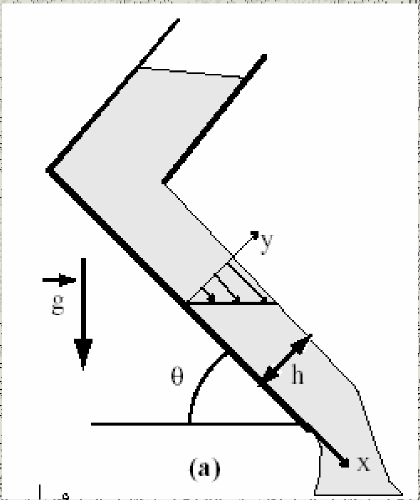
- Three scales of different natures:
  - The microscopic scale : the contact scale
  - The grain scale
  - The flow scale
- At the microscopic scale:
  - The roughness and the interparticle friction: fully encoded into some effective friction
  - During a collision : a collision time (elastic properties) and a dissipation timescale (internal elastic vibrations damping rate)
  - => For slow enough flows, the grains are equivalent to rigid inelastic spheres

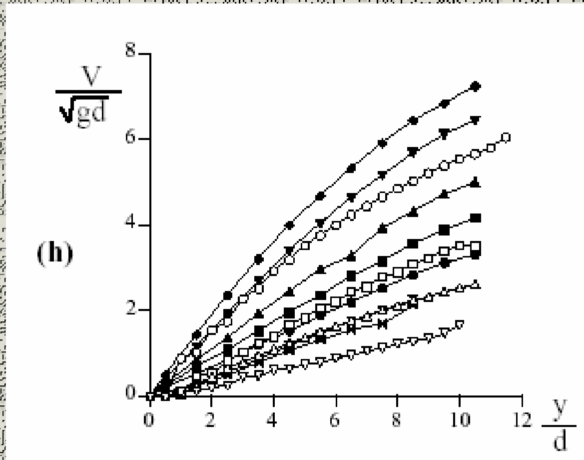
Relevant parameters (II)

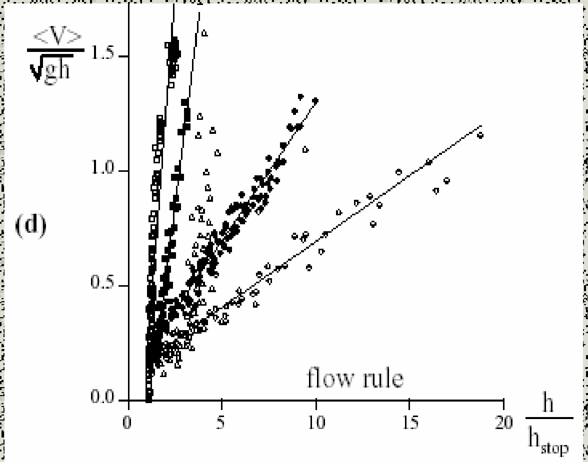
- At the grain level :
  - Natural length scale : grain diameter d
  - In homogeneous shear flows :
    - Strain tensor reduces to shear rate $\dot{\gamma}$
    - Stress tensor reduces to normal stress P and shear stress τ
  - Two independant dimensionless numbers
    - the rescaled shear rate $I = \dot{\gamma} d \Big/ \sqrt{\dfrac{P}{\rho}}$
    - the effective friction coefficient $\mu_{eff} = \tau / P$
- => The rheological relation is given by the relation between τ / P and I

Flow down an inclined plane

(O. Pouliquen)

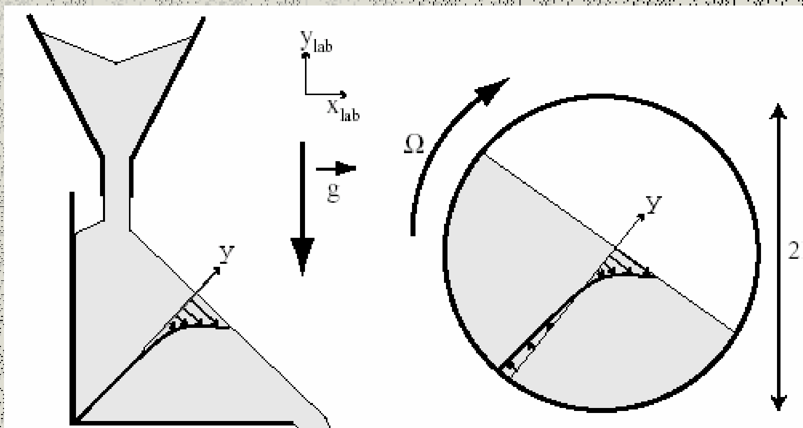


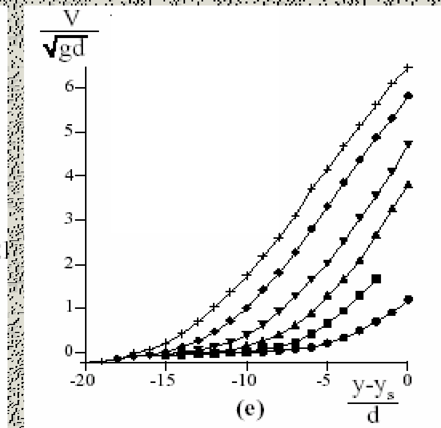


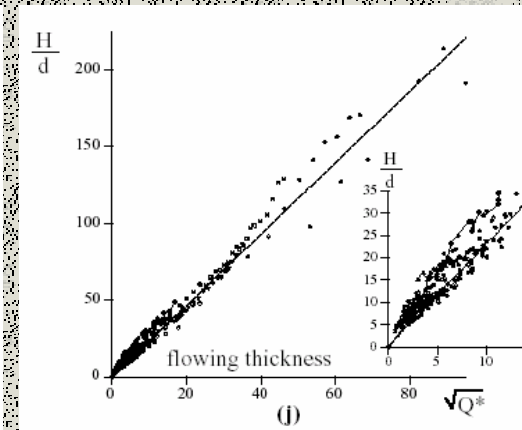


- Flow thresholds :the flow starts at $\theta_{start}(h)$ and is sustained above $\theta_{stop}(h) < \theta_{start}(h)$ => hysteretic
- The velocity profile is Bagnold like : $\frac{V(y)}{\sqrt{gd}} = A(\theta)\frac{(h^{3/2} - (h-y)^{3/2}}{d^{3/2}}$
- A scaling law is obtained : $\frac{<V>}{\sqrt{gh}} \approx \beta \frac{h}{h_{stop}(\theta)}$

Flow down a heap (D. Bonamy)







- Flow thresholds :the flow starts at θ_{start} and is sustained above θ_{stop} => hysteretic
- The velocity profile is linear : $\frac{V(y)}{\sqrt{gd}} = \dot{\gamma}\frac{(h-y)}{d}$
- A scaling law is obtained : $\frac{h}{d} \propto \sqrt{Q^*}$ ⇔ $\frac{\langle V \rangle}{\sqrt{gh}} \propto \sqrt{\frac{h}{d}}$

in agreement with the direct observation on the profiles of a constant shear rate

Rheology

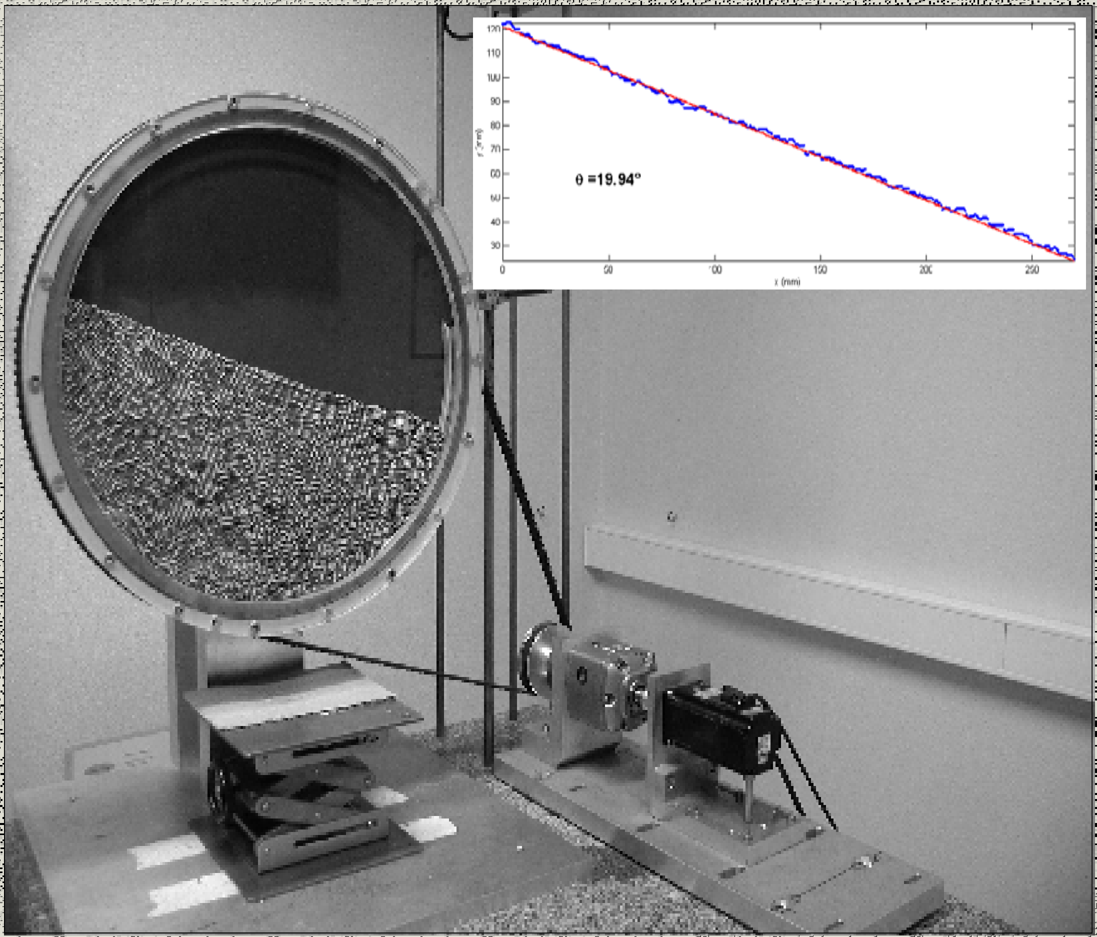
- In *both* flows $$P(y) = \rho g(h-y)\cos\theta; \qquad \frac{\tau}{P} = \tan\theta$$
- Assuming a *local* rheology : τ/P=μ(I), integrating $I = \frac{\dot{\gamma}(y)d}{\sqrt{P(y)/\rho}} = \mu^{-1}(\tan\theta)$

  leads to the Bagnold profile $$\frac{V(y)}{\sqrt{gd}} = A(\theta)\frac{(h^{3/2}-(h-y)^{3/2})}{d^{3/2}}$$
  with $A(\theta) = \frac{2}{3}I(\theta)\sqrt{\cos\theta}$
- Integrating the profile on the flow thickness leads to the scaling $$\frac{\langle V \rangle}{\sqrt{gh}} = \frac{3}{5}\frac{h}{d}A(\theta)$$

First Conclusion:

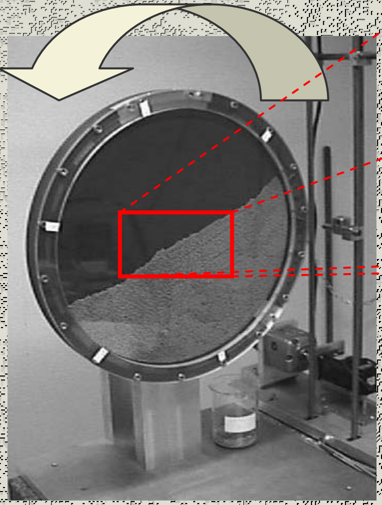
- At the first order, a local rheology is compatible with the kinematic properties of a dense granular flow down an inclined plane
- On the contrary, it is incompatible with the observations made in the case of the flow down a heap!
- As a matter of fact, a closer inspection of the inclined plane case reveals that the boundary layers escape the Bagnold type profile and therefore also escape the local rheology assumption.

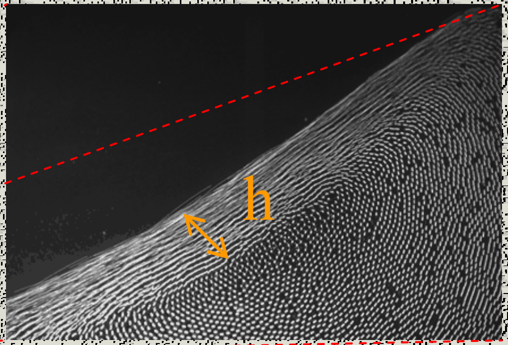
Experimental evidence of clusters : a first step towards a non-local rheology



- Rotating drum
  - D = 450 mm
  - δ = 3 to 22 mm
- Steel beads
  - d = 3 mm
  - m = 0.11 g
- Angular velocity
  - Ω

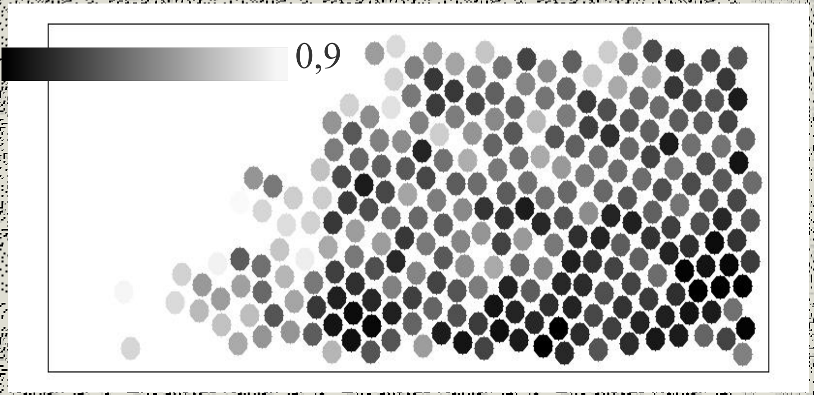
Dynamical clusters in steady flow



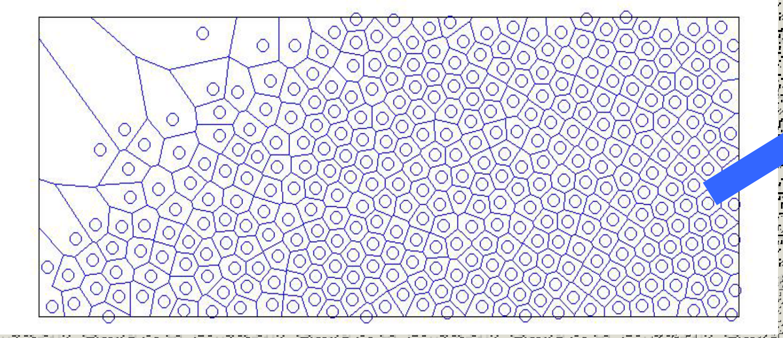


(2D flow)

Local density field at the grain scal



Voronoï cells around each grain

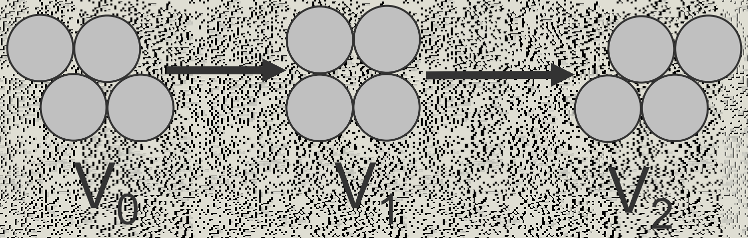


A priori, rather weak variatio

Rigid clusters induced by the Reynolds dilatancy property

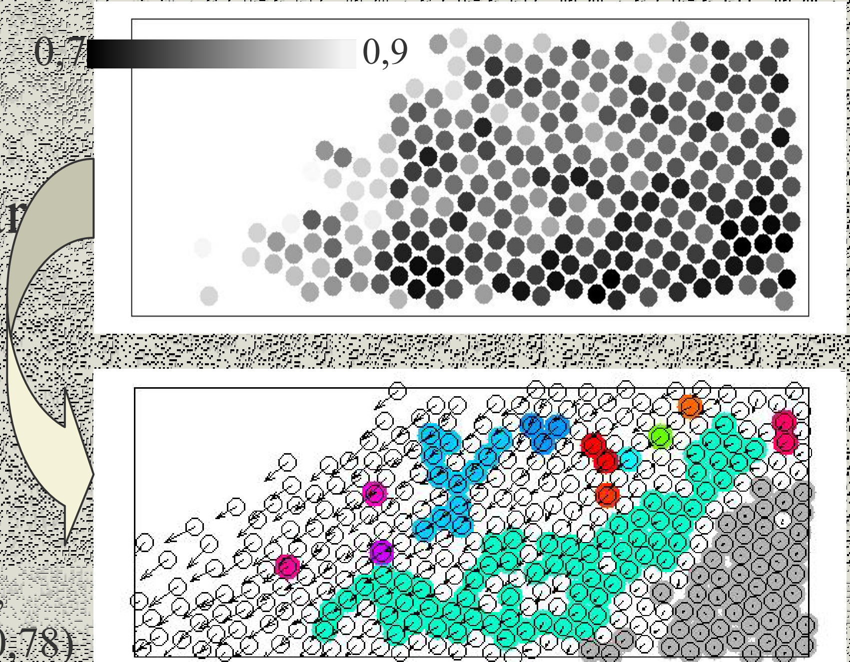
- Reynolds dilatancy

  $V_0 < V_1 > V_2$

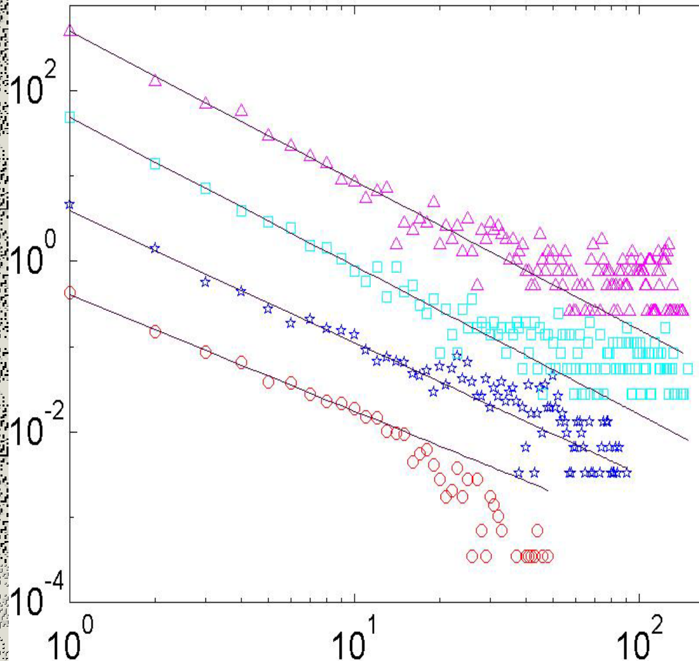


- Very intermittent rigid clusters of all sizes

$\rho > \rho_c$
no shear



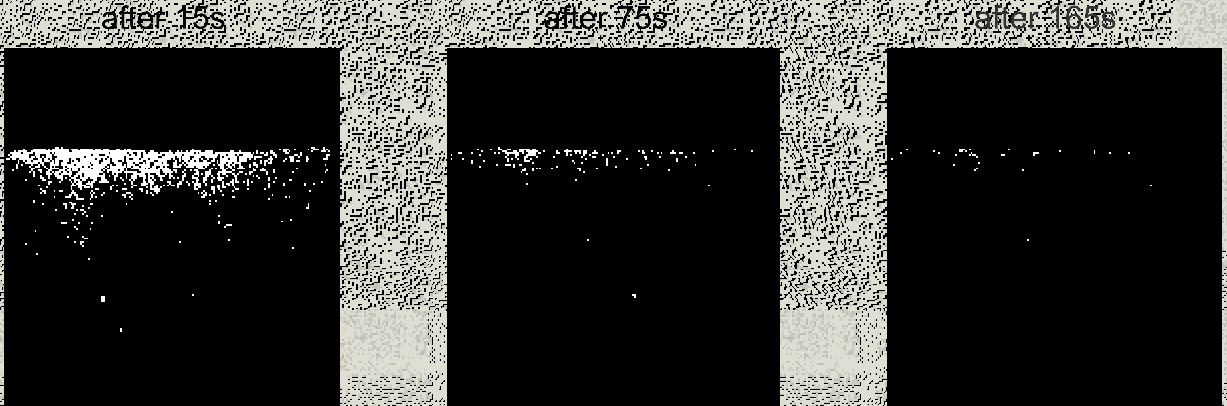
(in 2D, $\rho_c = \pi/4 = 0{,}78$)



Clusters during relaxation

- The drum is rotated for a while, stopped and the pile slope is set to $\theta_i < \theta_{stop}$ at t=0, ($\theta_{stop} = 19.2°$)
- The pile relaxes towards mechanical equilibrium:
  - Average images (25 im at 5 Hz) are recorded every 15 s.
  - Image differences allows to observe where displacements occur.

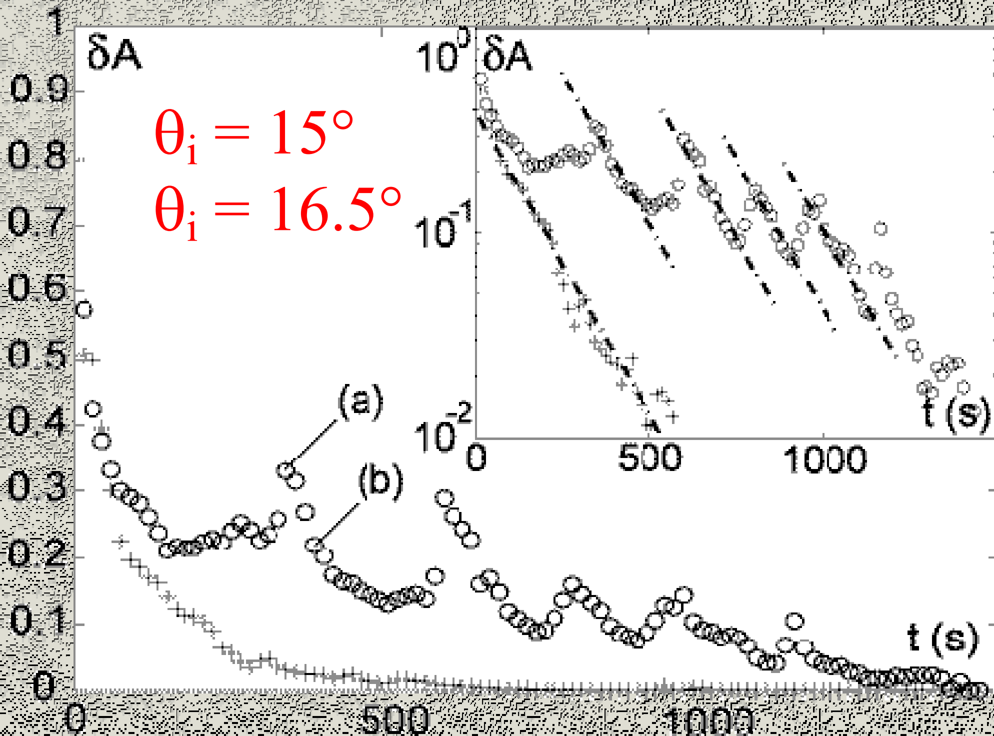
$\theta_i = 0.3°$



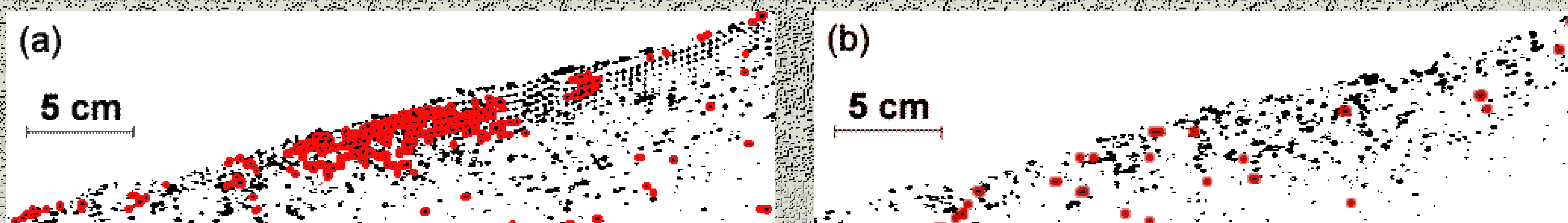
An intermittent relaxation dynamic

- First a rapid relaxation of the bulk
- An intermittent relaxation of the surface layer

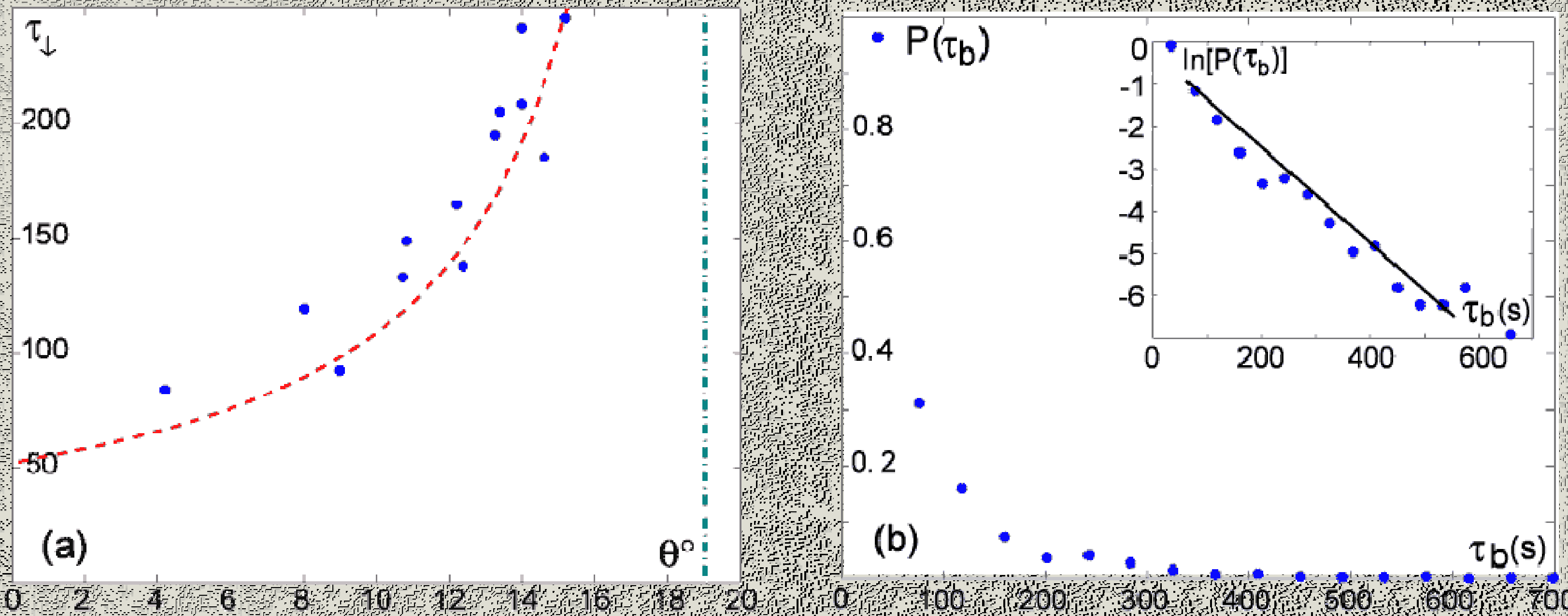
Intermittent bursts relates to correlated displacements



- A dynamics composed of
  - exponential decay
  - short intermittent burst
- Exponential decay : Individual displacements
- Intermittent bursts : Correlated displacements



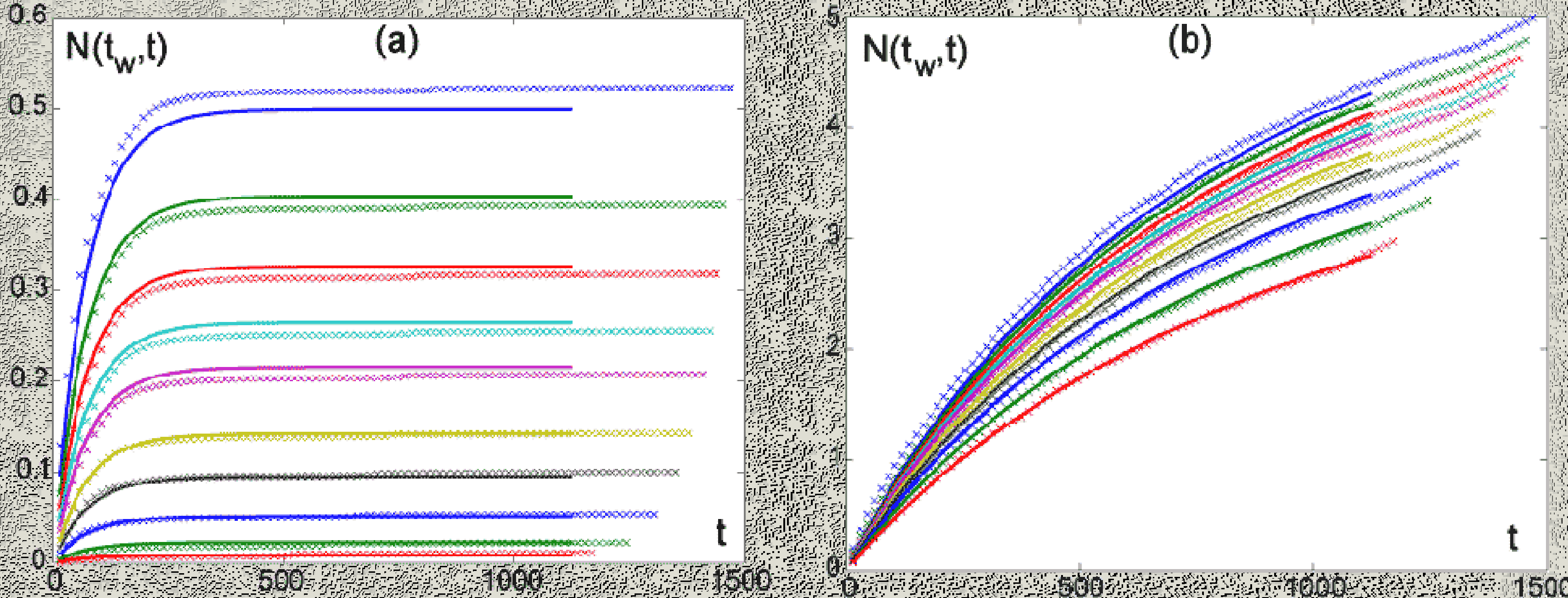
Further characterization : characteristic times



- Exponential decay rate : $\tau_{\downarrow}$ increases with θ_i.
- Bursts intervals distribution essentially invariant, with characteristic time : τ_b=100s.
- Bursts become significant when $\tau_{\downarrow} > \tau_b$.

Further characterization: two-time relaxation function

$$N(t_w,t) = \left\langle \int_{t_w}^{t_w+t} \delta A(u)\,du \right\rangle$$



- Exponential decay : rapid saturation and average number of move smaller than one
- In the presence of bursts : no saturation and average number of move larger than one.

An over-simplified model

- Beads can be in active (1->n) or inactive (0) states
  - active state : the bead transits to another active state with rate α' and to inactive state with rate α resulting in a global rate of transition from an active state $\gamma=\alpha+(n-1)\alpha'$. These transition are assimilated to actual individual move of the beads and thus contribute to $N(t_w, t)$.
  - inactive state : the bead do not evolve spontaneously.
  - the reactivation process : randomly chosen beads are instantaneously set in the active state with probability ν – independently of their state. This process is assumed not to involve displacement of the bead but rather a rearrangement of its environment and thus do not contribute to $N(t_w, t)$.

Model : results

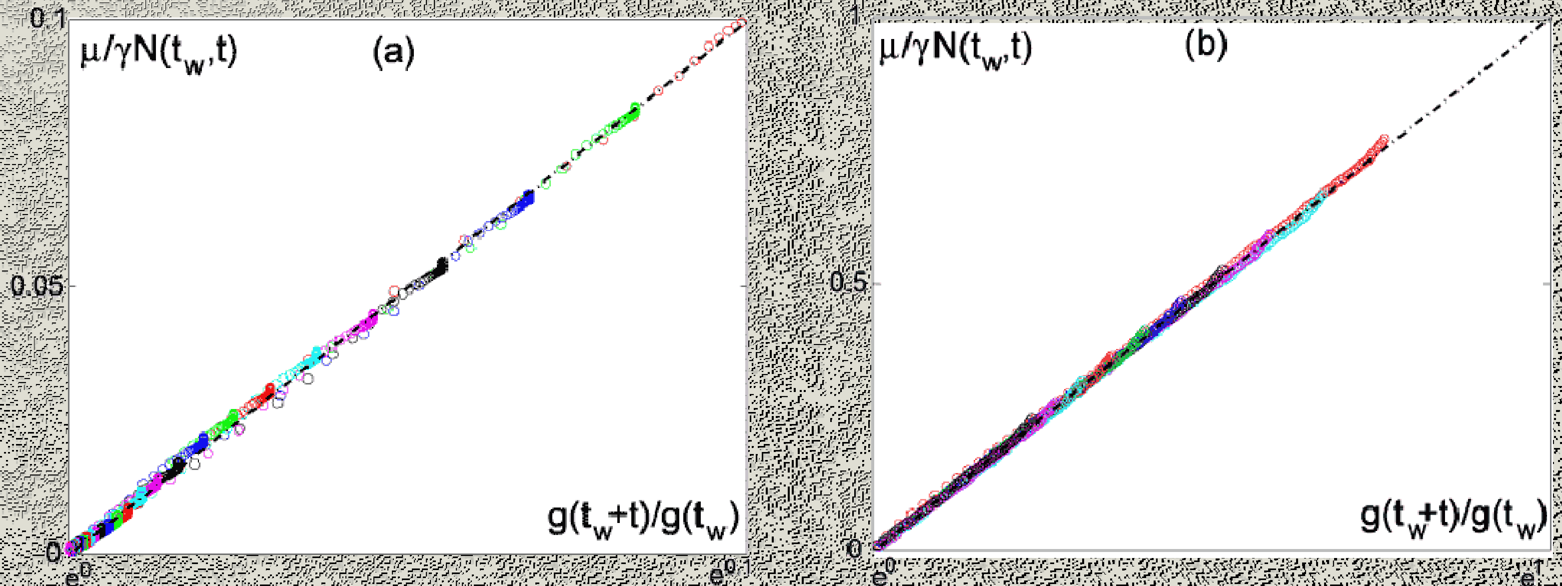
- The fraction of active beads is given by : $P_m(t) = \sum_1^n P_i(t)$ where $P_i(t)$ is the probability of being in active state i.
- Accordingly, $N(t_w, t) = \int_{t_w}^{t_w+t} \gamma P_m(t)dt$
- The dynamics is given by $\frac{dP_m}{dt} = -\alpha P_m + \nu P_0 = -\alpha P_m + \nu(1 - P_m)$ where the key ingredient of the model is now introduced: $\nu = \nu(P_m) = \mu P_m$
- After some calculations, one finds: $N(t_w, t) = \frac{\gamma}{\mu} \ln\left(\frac{g(t_w + t)}{g(t_w)}\right)$ with a reparametrization of time:

$$g(t) = (\alpha - \mu) + \mu P_m(0)\left[1 - e^{-(\alpha-\mu)t}\right]$$

Model discussion :

applying to experimental data

- A scaling function is proposed for $N(t_w,t)$: It is valid



- It provides experimental values for the parameters.

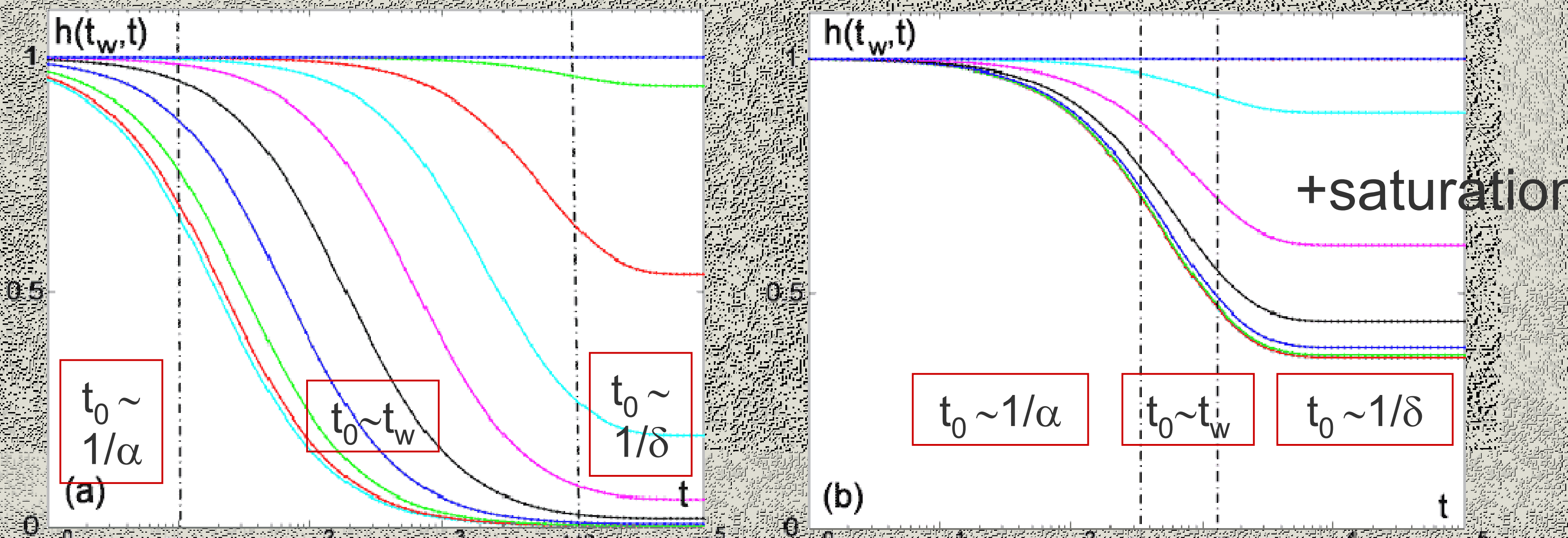
	(a)	(b)
$1/\alpha$	56 s	294 s
$1/(\alpha-\mu)$	75 s	1250 s
γ/α	1	4.7

Model discussion: interpretation

- A correlation function can be defined as the probability not to have changed state between t_w and t_w+t.
- It reads : $C(t_w,t) = P_m(t_w)e^{-\gamma t} + (1-P_m(t_w))\left[\frac{g(t_w+t)}{g(t_w)}\right]^{-1}$

rapid relaxation

relaxation on time $t_0(t_w)$



Conclusion

- The kinematic properties of dense granular flows are incompatible with a simple local reology
- Experimental evidence of clusters support the need for a non-local rheology:
  - In steady flows : rigid clusters induced by the Reynold's dilatancy property
  - In relaxation processes : intermittent bursts of correlated displacement
- Investigating clusters has driven us towards the description of long term but finite aging behaviours
- Granular rheology, in very dense or slow flow may have a lot to do with glassy dynamics.