Glassy behaviours in a-thermal systems : the case of granular media

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Many thanks to J.P. Bouchaud, E. Vincent, G. Biroli, J. Kurchan, J.M. Lück, C. Godrèche, A. Barrat, M. Nicodemi and A. Coniglio for inspiring discussions

Specificity of dense granular materials

- Insensitive to the thermal environment
- Dissipative dynamics at the grain level
  - ⇒the stationary dynamics of an isolated system is the arrest
  - ⇒mechanical driving is necessary to generate any sort of dynamics (stationary or aging)
- But strong similarities with glasses …

	Thermal systems	**a-thermal systems**
Stationary dynamics	Gibbs equilibrium	a-thermal stationary states
Aging dynamics	thermal glasses	a-thermal glasses

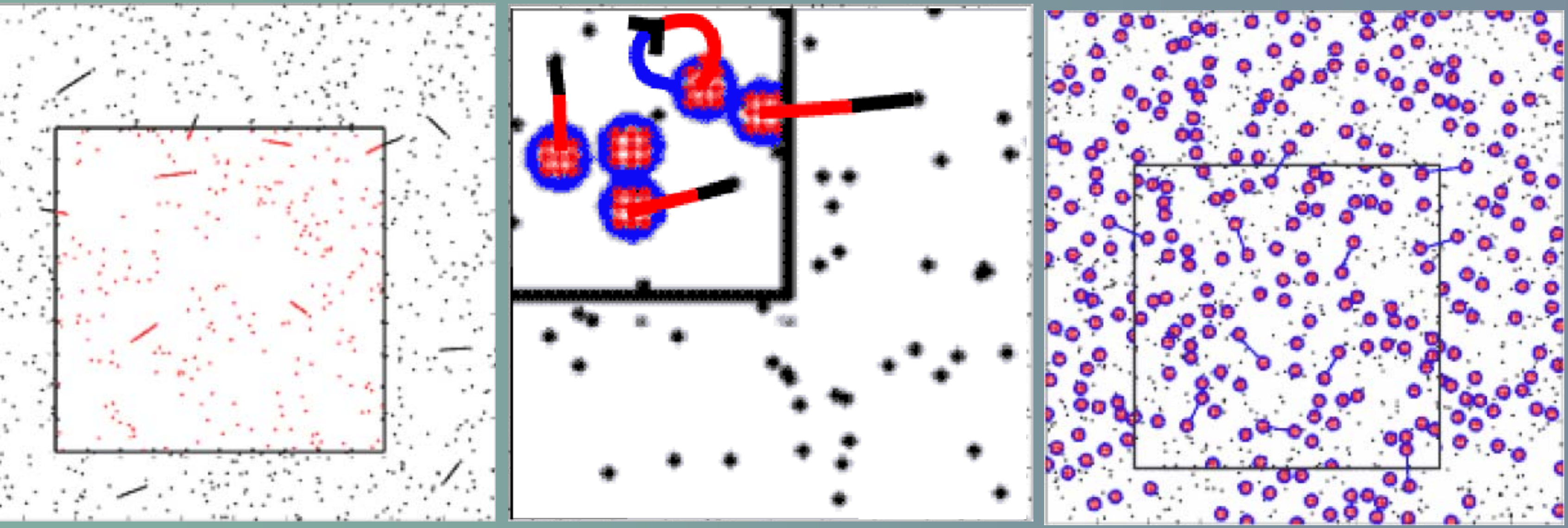
Overview

- **1 Thermal vs. a-thermal systems**
  - 1.1 Definitions and general considerations
  - 1.2 Illustration in the context of stochastic dynamics
- **2 Glassy behaviour of granular media**
  - 2.1 Experimental evidence of the analogy at the macroscopic scale
  - 2.2 Recent experimental results at the grain scale
  - 2.3 Partial Conclusion
- **3 Looking for a statistical description**
  - 3.1 Edwards' proposal
  - 3.2 Experimental test of Edwards' proposal?

Part 1

Thermal versus a-thermal systems

Thermal vs. a-thermal systems : Definitions and general considerations



- What is meant by thermal is a question of scale
- Dissipation : flux of energy towards the internal degrees of freedom which are not included in the description

hermal vs. a-thermal systems : llustration in the context of stochastic dynamics

	Gibbs Equilibrium	Stationary state for a dissipative system
Micro-states α	$\alpha = \{q_i, p_i\}$, i=1…N	$\alpha = \{x_i\}$, i=1…N
Dynamics	Hamiltonian structure Liouville's theorem	Dissipative
Conserved quantity	Conservation of energy	Conservation of some other extensive quantity **U**
Time reversal symmetry	Micro-reversibility $w_{\alpha\beta} = w_{\beta\alpha}$	No micro-reversibility
Probability $P(\alpha)$	uniform over config. of energy E	a priori not uniform f_α

Thermal vs. a-thermal systems : micro-canonical distributions

- For thermal systems :

$$P(\alpha)=\frac{1}{\Omega(E_0)}\delta(E_\alpha-E_0)$$ with

$$\Omega(E_0)=\sum_\alpha \delta(E_\alpha-E_0)$$ the total number of configuration of energy E_0

- In the present case :

$$P_\mu(\alpha)=\frac{1}{Z_\mu(U_0)}f_\alpha\delta(U_\alpha-U_0)$$ with

$$Z_\mu(\alpha)=\sum_\alpha f_\alpha\delta(U_\alpha-U_0)$$ a micro-canonical partition function

Thermal vs. a-thermal systems : construction of a "temperature"

- For thermal systems :

E_1 | E_2 $\quad E_{TOT}=E_1+E_2$

$$P(\{E_1,E_2\}) = \Omega(E_1)\Omega(E_2) = \Omega(E_1)\Omega(E_{TOT}-E_1)$$

maximization vs. E_1 => $\frac{1}{T_1} \equiv \left.\frac{\partial \ln \Omega(E_1)}{\partial E_1}\right|_{E_1^*} = \left.\frac{\partial \ln \Omega(E_2)}{\partial E_2}\right|_{E_2^*} \equiv \frac{1}{T_2}$

- In the present case :

U_1 | U_2 $\quad U_{TOT}=U_1+U_2$

$$P(U_1|U_{TOT}) = \sum_\alpha P_\mu(U_{TOT})\delta(U_{\alpha_1}-U_1) = \frac{1}{Z_\mu(U_{TOT})}\sum_\alpha f_\alpha \delta(U_\alpha - U_{TOT})\delta(U_{\alpha_1}-U_1)$$

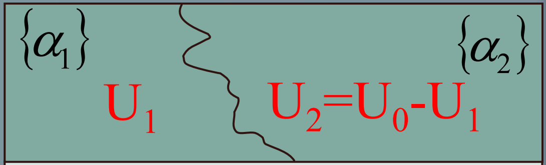
assuming the factorisation $\boxed{f_\alpha(U_1+U_2) = f_{\alpha_1}(U_1) f_{\alpha_2}(U_2)}$

$P(U_1|U_{TOT}) = \frac{Z_{\mu 1}(U_1) Z_{\mu 2}(U_2)}{Z_\mu(U_{TOT})}$ maximization $\boxed{\frac{1}{Y_1} \equiv \left.\frac{\partial \ln Z_{\mu 1}(U_1)}{\partial U_1}\right|_{U_1^*}}$

Thermal vs. a-thermal systems : canonical distributions

- For thermal systems : $P_c(\alpha) = \dfrac{1}{Z_c(T)} e^{-E_\alpha / kT}$

- In the present case :



$$P_\mu(\{\alpha_1,\alpha_2\}) = \frac{1}{Z_\mu(U_0)} f(\{\alpha_1,\alpha_2\})\,\delta(U_{\{\alpha_1,\alpha_2\}} - U_0)$$

$$P_c(\alpha_1) = \sum_{\alpha_2} P_\mu(\{\alpha_1,\alpha_2\}) = \frac{1}{Z_\mu(U_0)} \sum_{\alpha_2} f_{\alpha_1,\alpha_2}\,\delta(U_0 - U_1 - U_2)$$

$+$ factorisation $\quad f_{\alpha_1,\alpha_2} = f_{\alpha_1} f_{\alpha_2}$

$$P_c(\alpha_1) = \frac{f_{\alpha_1}}{Z_\mu(U_0)} \sum_{\alpha_2} f_{\alpha_2}\,\delta(U_0 - U_1 - U_2) = \frac{f_{\alpha_1}}{Z_\mu(U_0)} Z_\mu^R(U_0 - U_1)$$

$+$ expansion $\quad \ln(Z_\mu^R(U_0 - U_1)) = \ln(Z_\mu^R(U_0)) - \dfrac{1}{Y} U_1$

$$P_c(\alpha_1) = \frac{1}{Z_c(Y)} f_{\alpha_1} \exp(-U_1/Y) \text{ with } Z_c(Y) = \frac{Z_\mu^R(U_0)}{Z_\mu(U_0)} \text{ and } Y = \left.\frac{\partial \ln Z_\mu^R(U_2)}{\partial U_2}\right|_{U_2^*}$$

Thermal vs. a-thermal systems : thermo-dynamical relations

- One can define a dynamical entropy :

$$S(U,t) \equiv \sum_{\alpha} P_{\mu}(\alpha,t) \ln\left(\frac{P_{\mu}(\alpha,t)\delta(U_{\alpha}-U)}{f_{\alpha}} \right) \quad \nearrow \quad S(U) = \ln\left(Z_{\mu}(U)\right)$$

- It is straightforward that :

$$\langle U \rangle = -\frac{\partial \ln Z_c(U)}{\partial \gamma} \text{ and } \langle U^n \rangle - \langle U \rangle^n = (-1)^n \frac{\partial^n \ln Z_c(U)}{\partial \gamma^n} \text{ with } \gamma = \frac{1}{Y}$$

- One naturally introduces :

$$F(Y) = -Y \ln\left(Z_c(Y)\right) = < U > -YS(< U >)$$

Comments!

Thermal vs. a-thermal systems : comments !

- For a stochastic dynamics, which does not conserve energy and which does either not satisfy micro-reversibility but conserves another extensive quantity :
  - one looses the property of uniformity for the probability distribution in the micro-canonical ensemble;
  - if the micro-canonical distribution factorizes, one can still define an intensive parameter associated with the conserved quantity;
  - this intensive parameter equilibrates between subsystems;
  - one can compute a canonical distribution, which is different from but similar to the Gibbs distribution
- Drawbacks : very similar indeed !

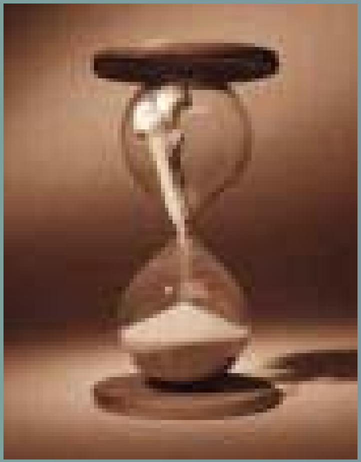
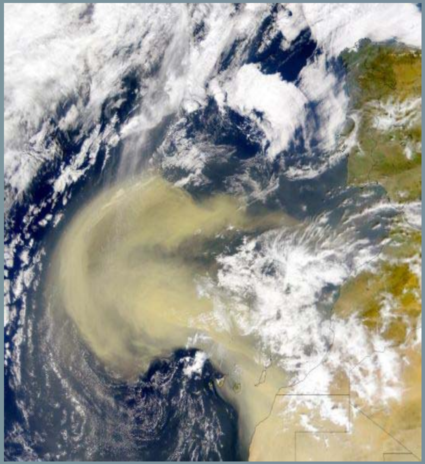
Part 2
Glassy behaviour of granular media

- The jamming transition

- Macroscopic behaviours
  - Relaxation towards a stationary state
  - Fluctuations and critical slowing down
  - Aging and Memory effects

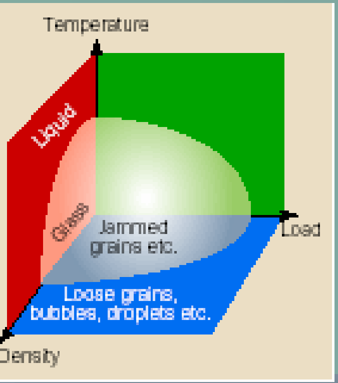
- At the scale of the grain
  - Internal structure & Diffusion properties
  - Dynamical heterogeneities

Glassy behaviour of granular media : jamming

- In this review : from liquid to solid, the jamming transition and its analogy with the glassy transition
- "Jamming is not just cool anymore"

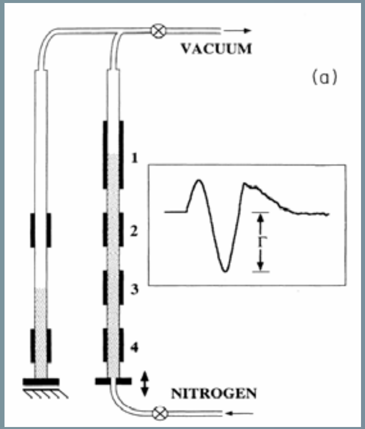
Liu and Nagel, Nature, vol. 396, (1998)



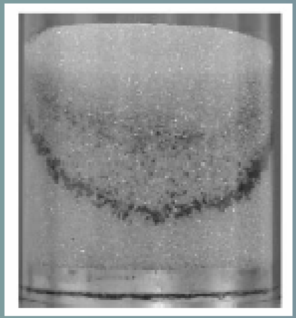
- Are the dynamics of different systems approaching the jammed state similar?
- If temperature and applied stress play similar roles […] is it possible that driven athermal systems might be described by an effective temperature?
- Is statistical mechanics useful at all in describing these systems?

Relaxation towards a stationary state

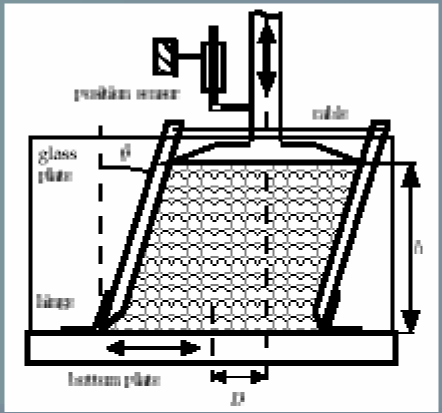
- Compaction under vibration
  - Knight et al.
    PRE 51 (5), 1995.



  - Philippe and Bideau
    Europhys. Lett 60 (5), 2002

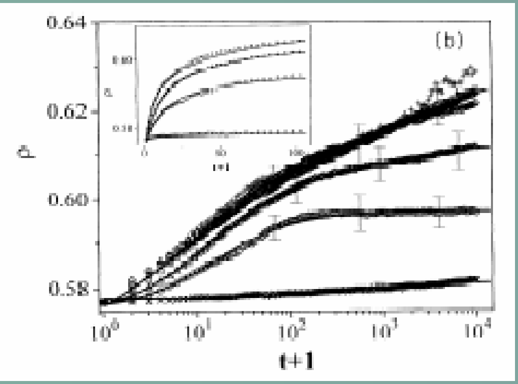


- Compaction under cyclic shear
  - Nicolas et al.
    Eur. Phys. J. E 3, 2000

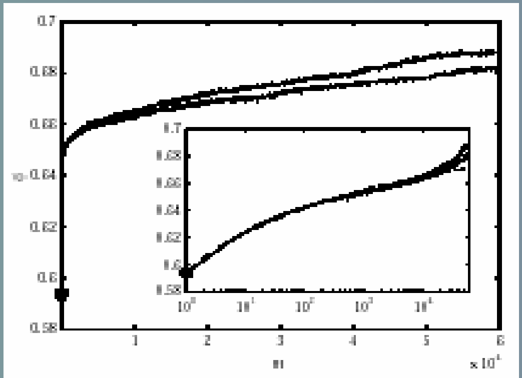


Very Slow compaction

Knight et al.



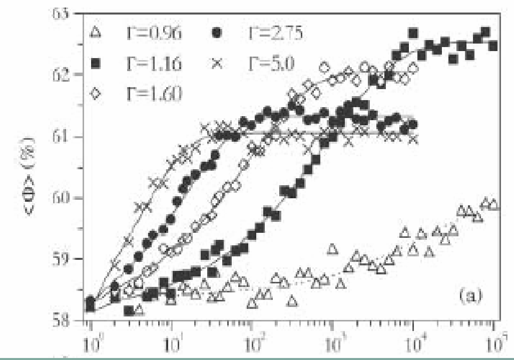
Nicolas et al.

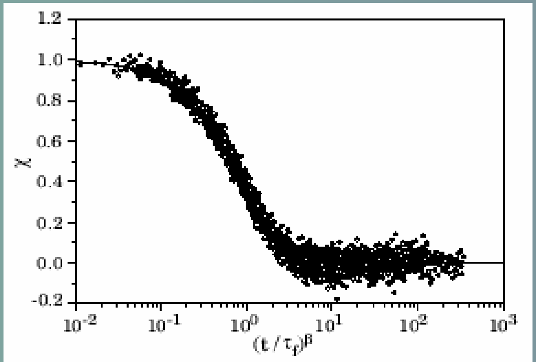


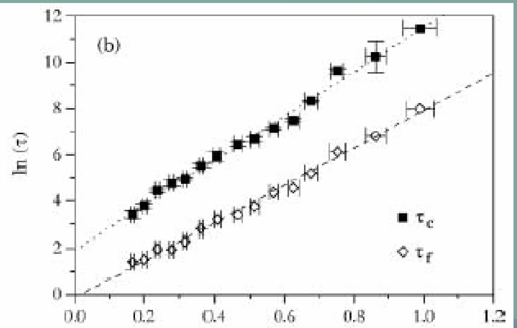
Heuristic law

$$\rho(t)=\rho_f-\frac{\Delta\rho_\infty}{1+B\ln\left[1+\frac{t}{\tau}\right]}\,,$$

Philippe et al.







Stretched exponential

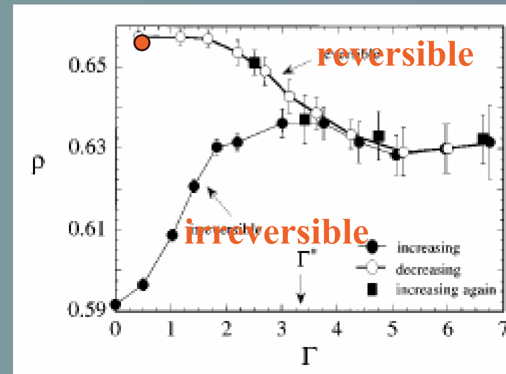
$$X(t)=X_\infty-(X_\infty-X_0)\exp\left[-(t/\tau_{\rm f})^\beta\right]$$

with Arrhénius timescale

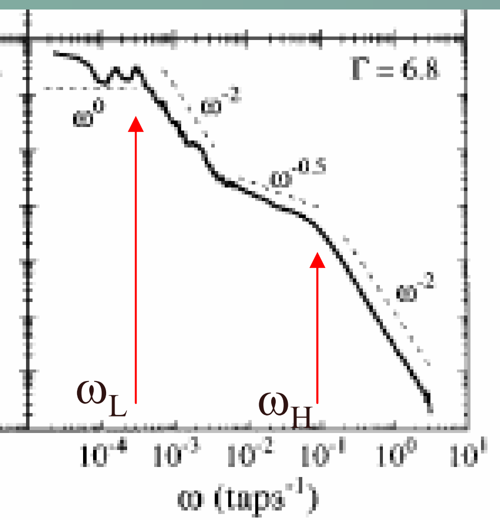
$$\tau_{\rm c,f}(\Gamma)=\tau_0\exp\left[\frac{\Gamma_0}{\Gamma}\right].$$

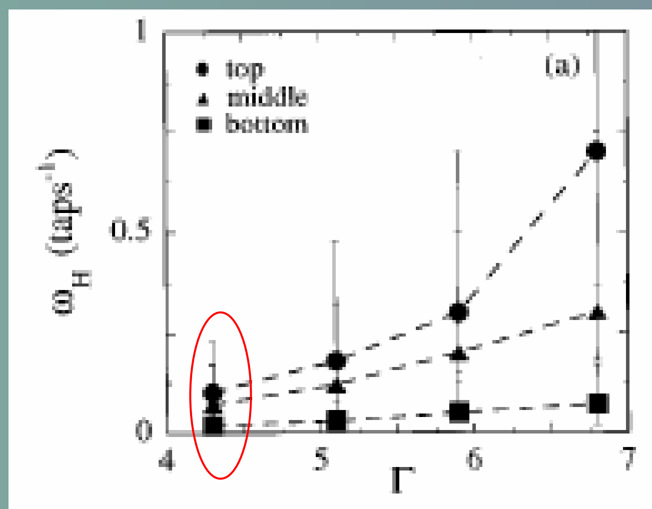
Fluctuations around the steady state

- Density fluctuations
  Nowak et al. PRE 57 (2) 1998
  - Reaching the steady state



  - Power spectra of the fluctuations



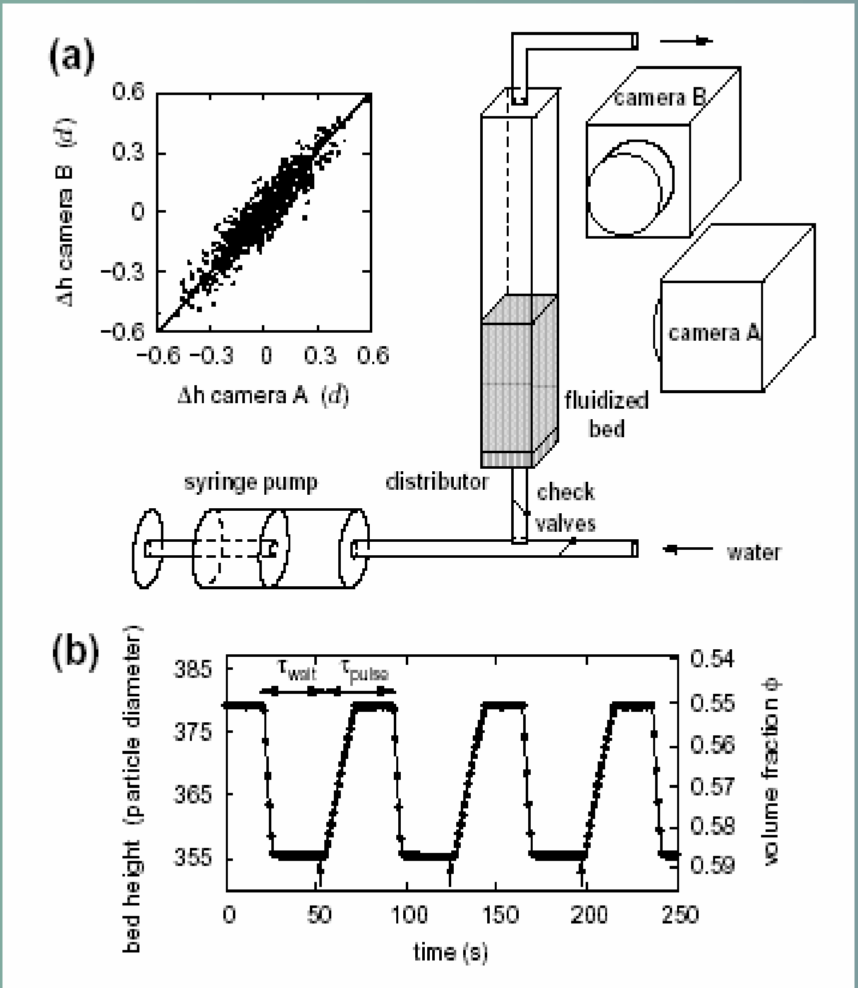


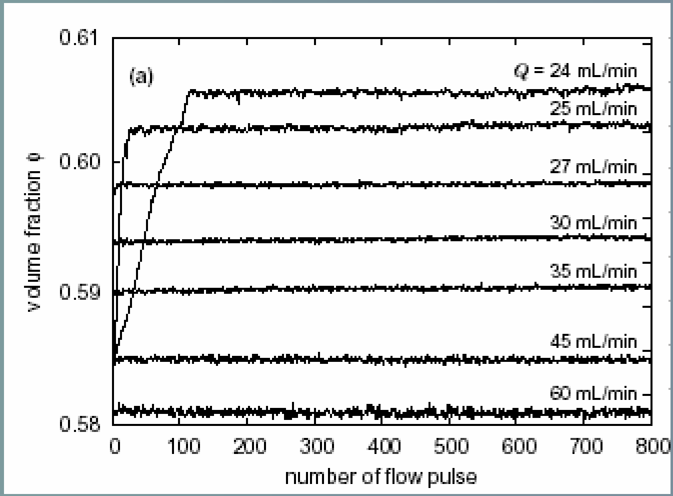
$$\omega_H = \omega_0 \exp(-\Gamma_0/\Gamma) = \omega_0 \exp[-m\Gamma_0/(\rho_{max}-\rho)]$$

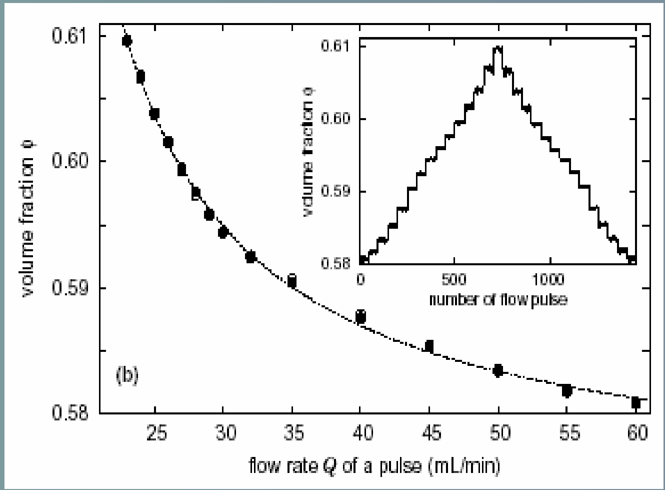
Vogel-Fulcher-Tamman dependence of the characteristic relaxation times

Fluctuations around the steady state

- Volume fraction fluctuations
  - Schröter et al. condmat0501264

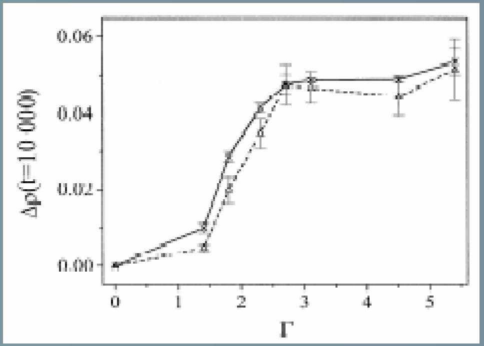


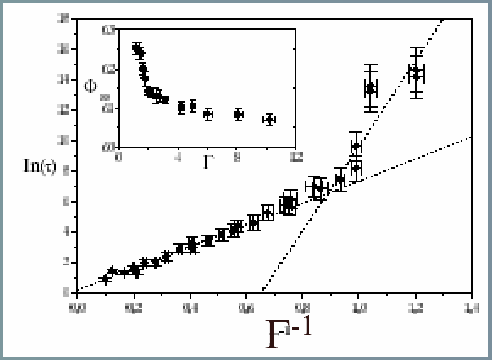


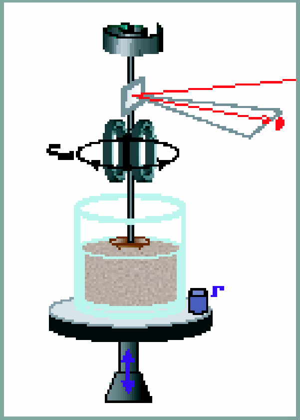


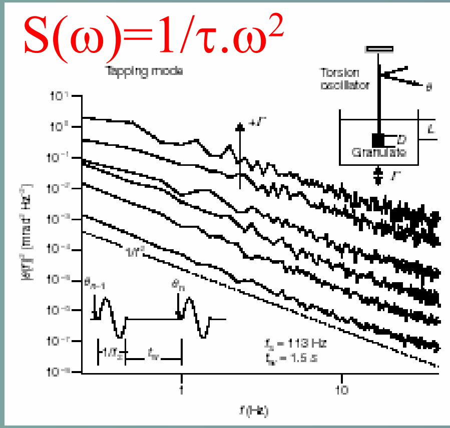
Towards the jammed state

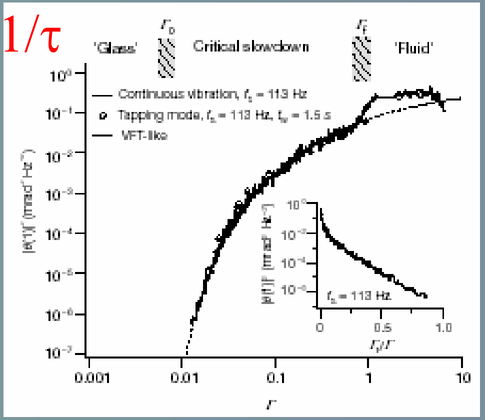
- Transition around $\Gamma=1$
  - Knight et al.
    PRE 51 (5), 1995
  - Philippe and Bideau
    Phys. Rev. Lett 91 (10), 2003
  - D'Anna and Gremaud
    Nature 413, 2001







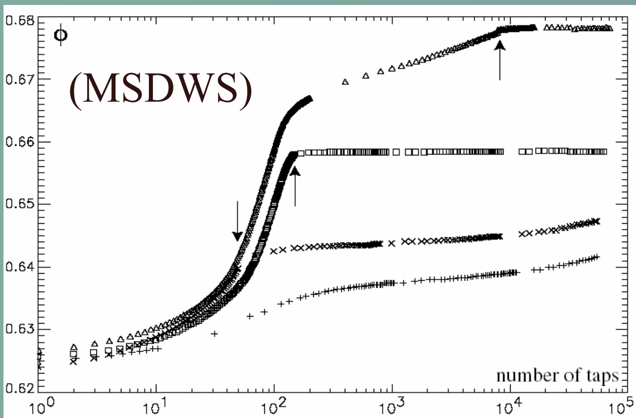




Critical slow down at the liftoff acceleration threshold

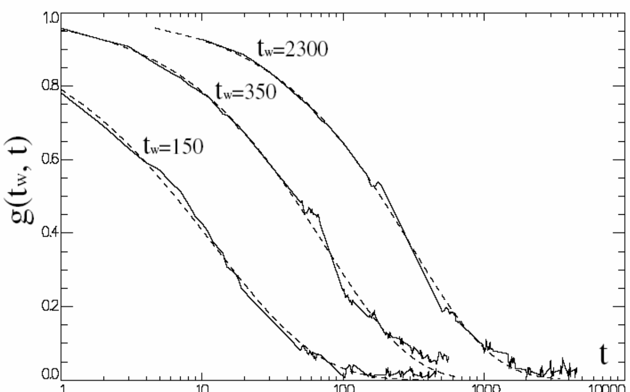
Aging and Memory effect

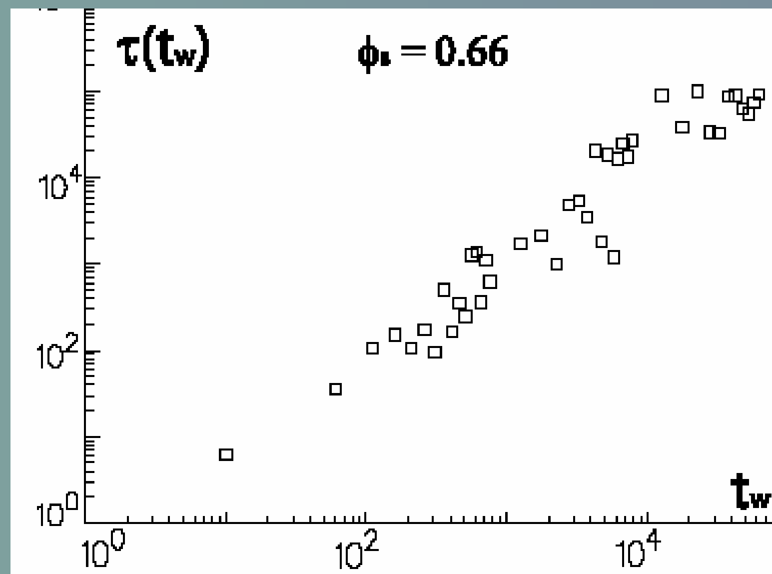
- Aging at very low vibration rate
  Kabla and Debregeas PRL 92 (3) 2004



$$g(t_w, t) = \exp(-(t/\tau(t_w))^{\alpha(t_w)})$$

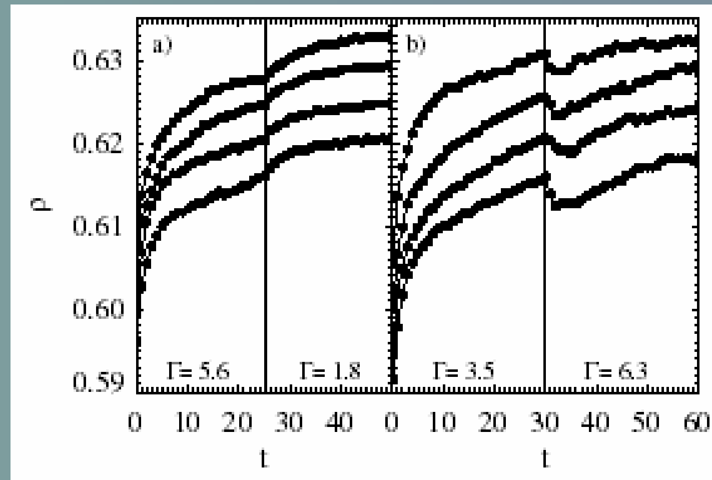
$$g(t_w, t) = \frac{\langle I(t_w + t) I(t_w) \rangle_{\text{spkl}} - \langle I(t_w) \rangle^2_{\text{spkl}}}{\langle I(t_w)^2 \rangle_{\text{spkl}} - \langle I(t_w) \rangle^2_{\text{spkl}}}$$



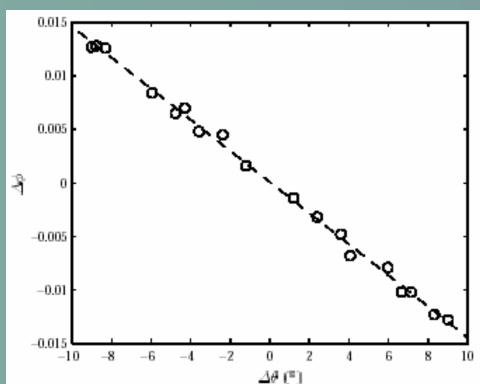


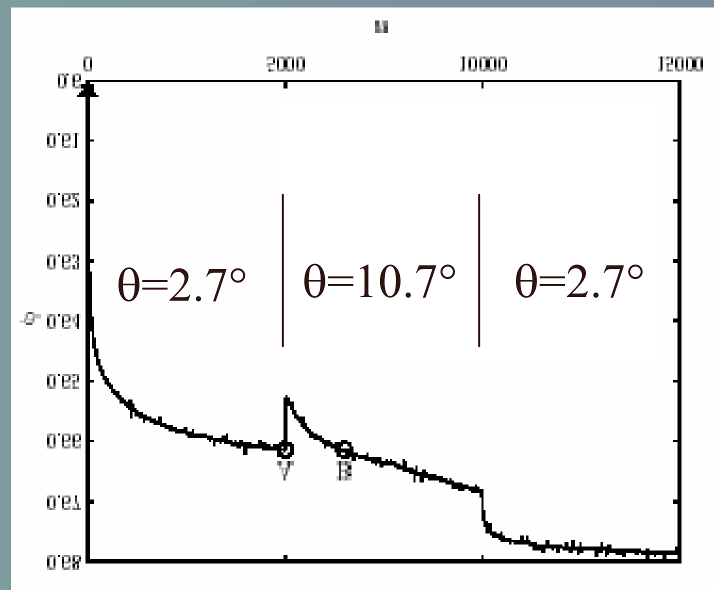
Aging and Memory effect

- Both under vibration
  Josserand et al.
  PRL 85 (17) 2000.



- and cyclic shear
  Nicolas et al.
  Eur. Phys. J. E 3, 2000

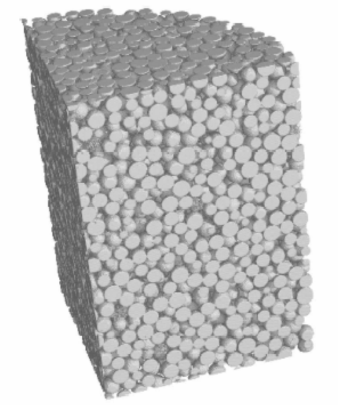




"The next important step would be to experimentally relate the compaction process with the evolution of the internal structure of the packing"

Internal structure

- X-ray microtomography
  Richard et al. PRE 68, 020301 (2003)



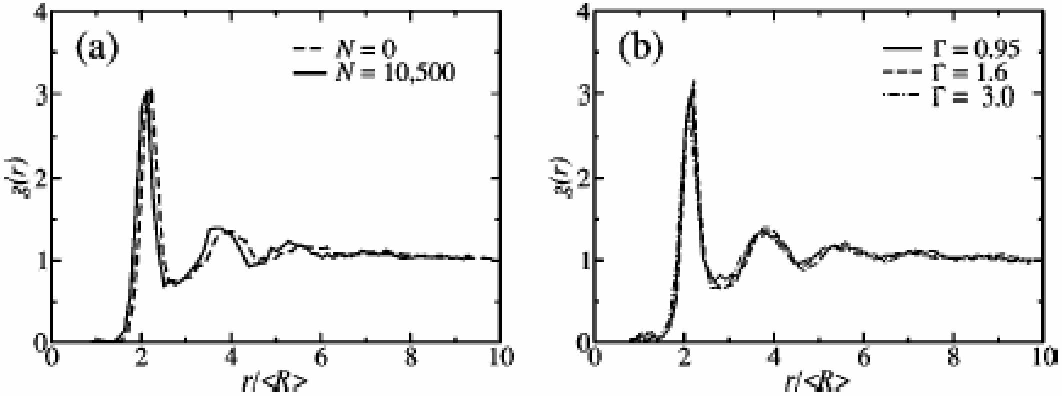


FIG. 2. (a) Evolution of the pair correlation function for $\Gamma = 3.0$. (b) Steady-state pair correlation functions obtained for $\Gamma = 0.95$, 1.6, and 3.0.

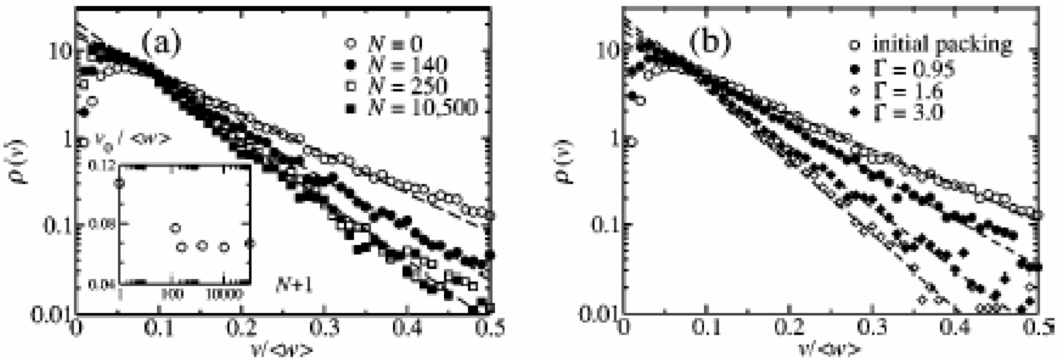
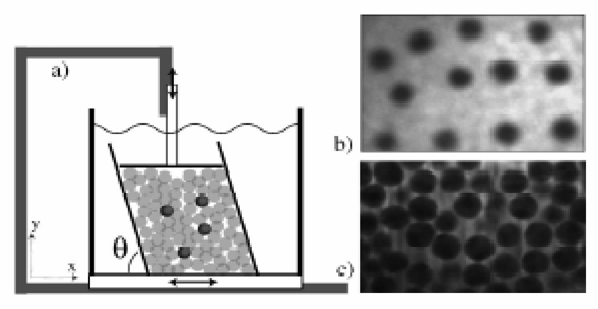


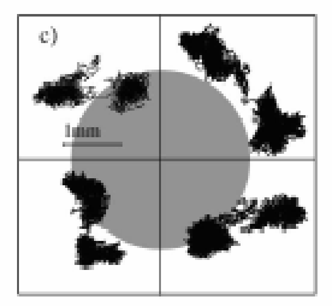
FIG. 3. (a) Evolution of the volume distribution of the pores during compaction with $\Gamma = 3.0$. Inset of (a): decrease of $v_0/\langle w\rangle$ with the number of oscillations N and for $\Gamma = 3.0$. (b) Volume distributions of the pores for the initial packing and for three different steady-state packings obtained for $\Gamma = 0.95$, 1.6, and 3.0 (b).

Diffusion properties : Cage effect

- Both during compaction under cyclic shear

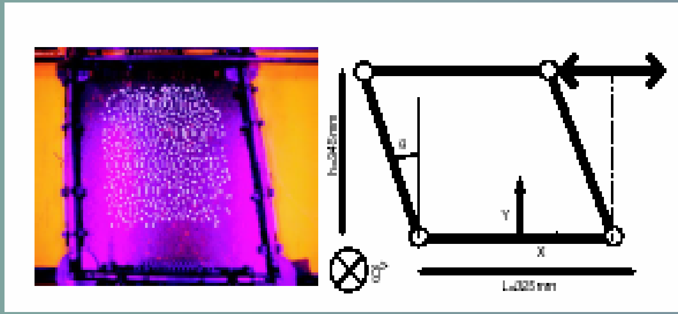
Pouliquen et al.
PRL 91, 014301
(2003)

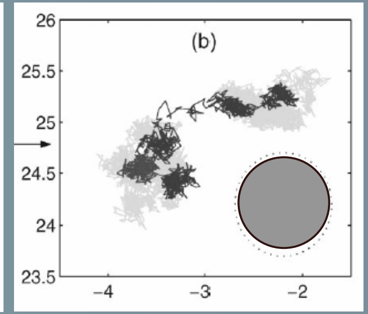




- And during cyclic shear at constant volume

Marty and Dauchot
PRL 94, 015701
(2005)



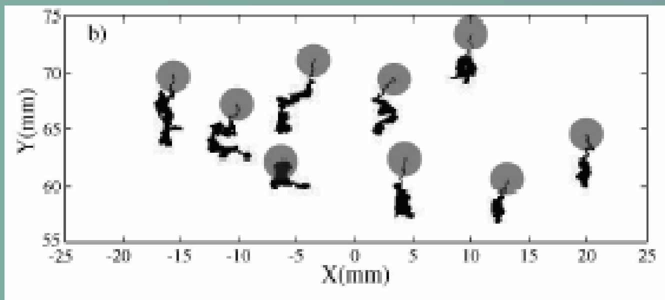


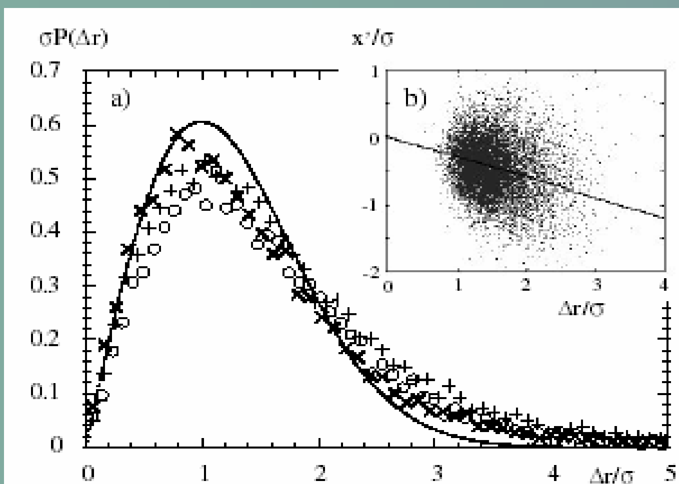
- Very similar to the colloidal glass transition

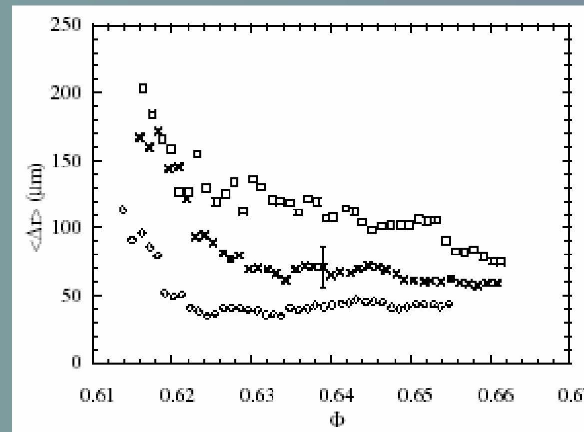
Weeks and Weitz
PRL 89 (9)
(2002)

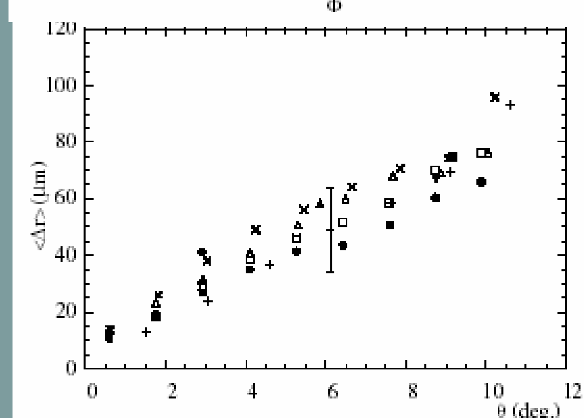
More details on diffusion (Pouliquen et al.)

Cyclic shear under compaction (I)





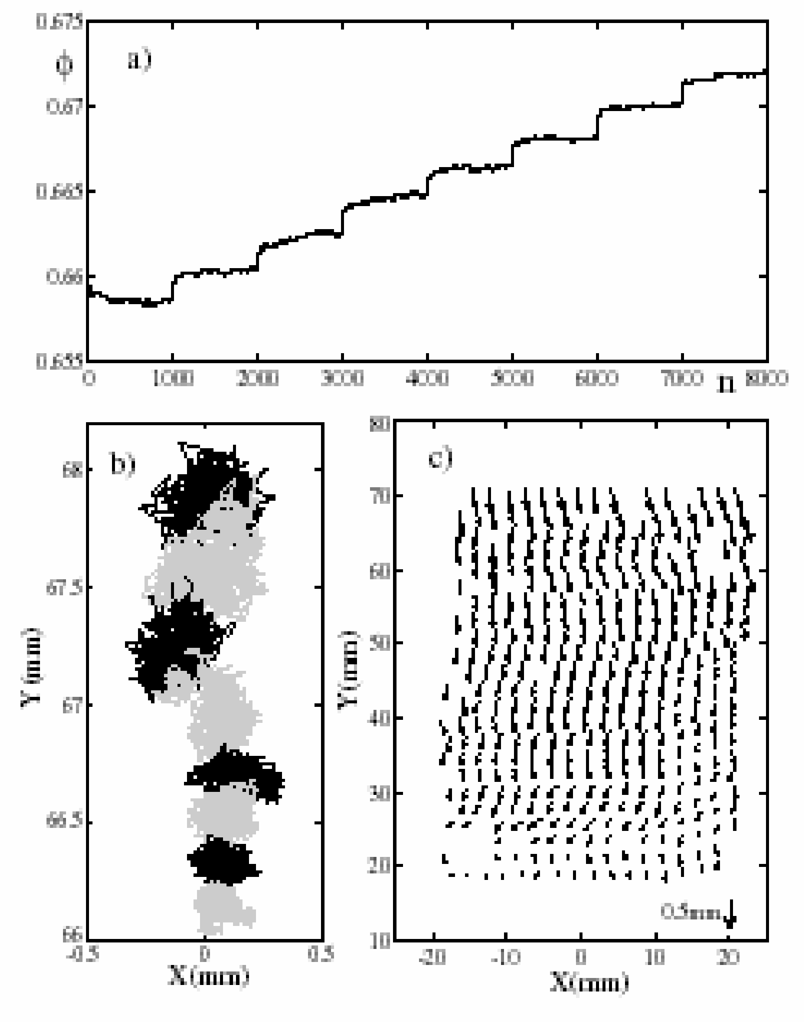




- Distribution of the particle displacement are larger than Gaussian.
- The process in not stationary <Δr> decreases during compaction.
- In the steady state, <Δr> is proportional to the shear amplitude

More details on diffusion

Cyclic shear under compaction (II)



<u>A 2 scales motion scenario</u>

- **Within the cage** : reversible random motion whose extend is directly proportional to the shear amplitude, responsible for the rapid change of compaction when varying the shear amplitude

- **From cage to cage** : irreversible structural re-arrangements, responsible of the slow compaction dynamics

=> The observed memory effect

Summary of the first lecture

1) Granular media = A-thermal + Dissipative

No forcing from the thermal environment

Energy lost towards the thermal environment

⇓

2) Need for a statistical description of dissipative systems with mechanical excitation but no thermal noise

Stochatic dynamics without detailed balance (time irreversibility) BUT

1. A conserved quantity
2. Factorization of the microcanonical distribution

⇒ Thermo generalization

3) Is there any chance to succeed in the case of dense granular media ?

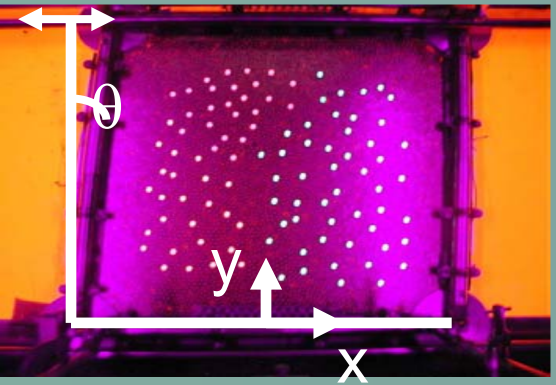
They are indeed very similar to supercooled liquids close to the glass transition, both at the macroscopic and at the microscopic scales

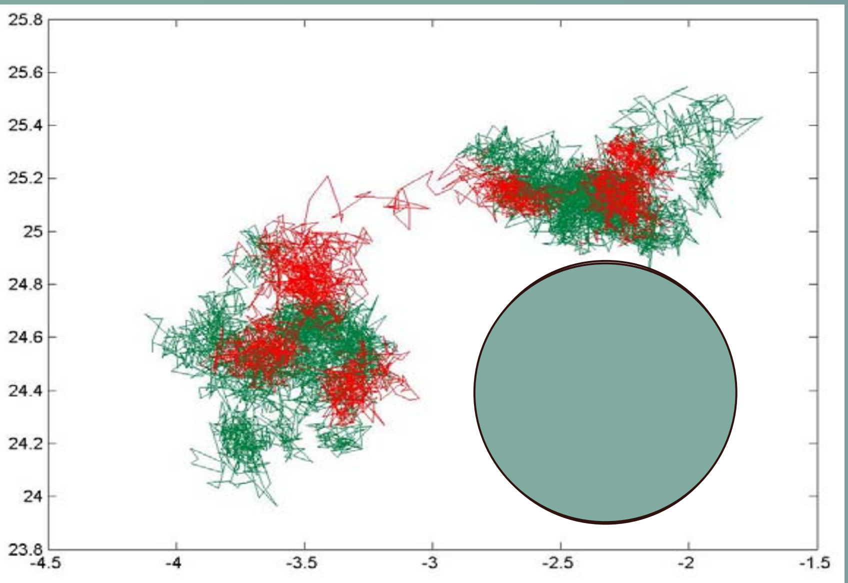
4) Let's try !

- Edwards' proposal
- Experimental investigation

More details on diffusion (Marty & Dauchot)

Cyclic shear at constant volume (I)





The system

8000 particles
Bi-disperse (Ø=4 and 5mm)
Quasi-static shear
Constant Volume (Φ=0.86)

The protocol

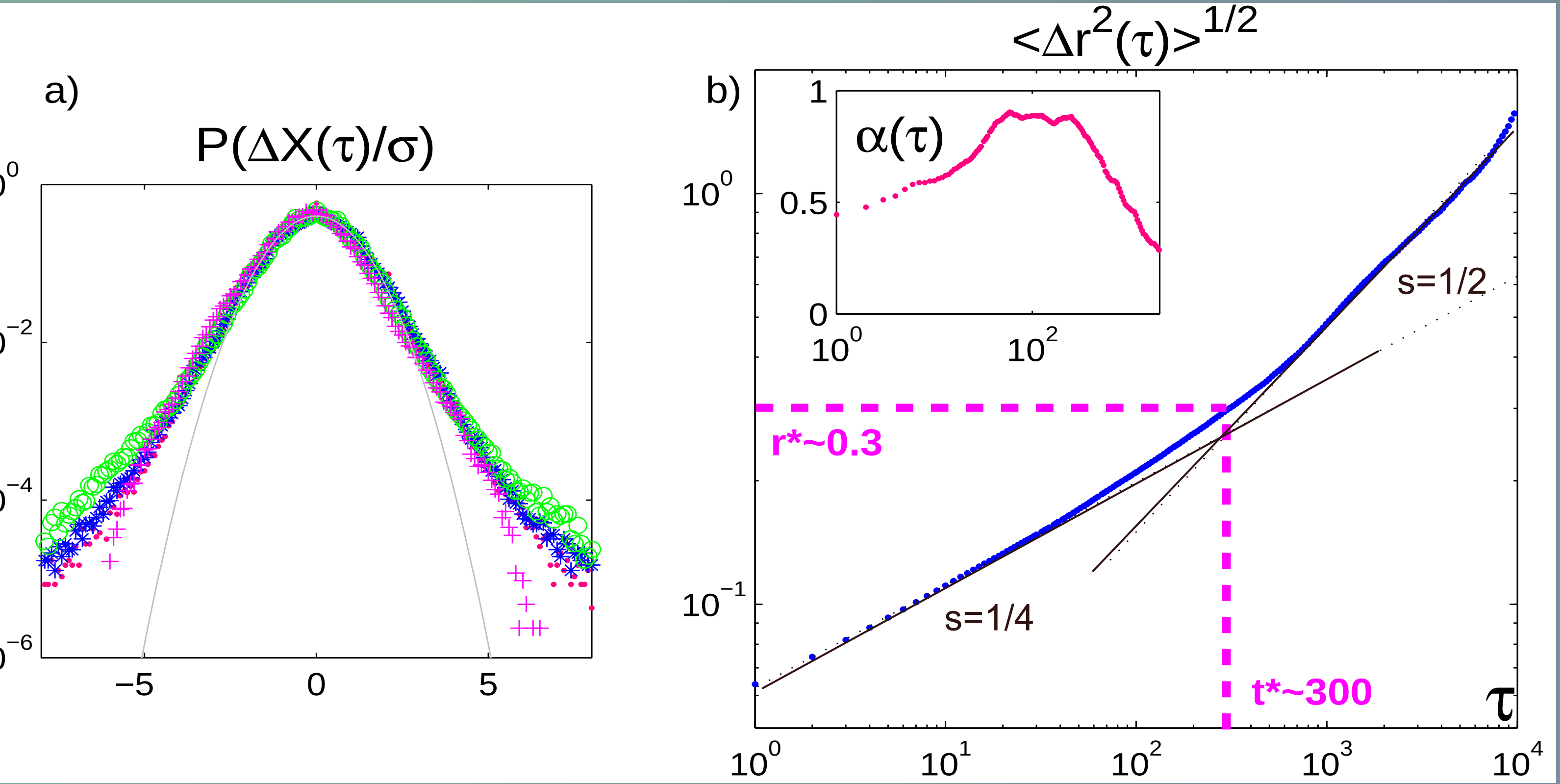
10 000 cycles
θmax=10°; max strain=0.3
500 tracers are followed
A snapshot is taken at each cycle

Particle trajectories

Small cages compared to the grain size

More details on diffusion

- Cyclic shear at constant volume : sub-diffusion

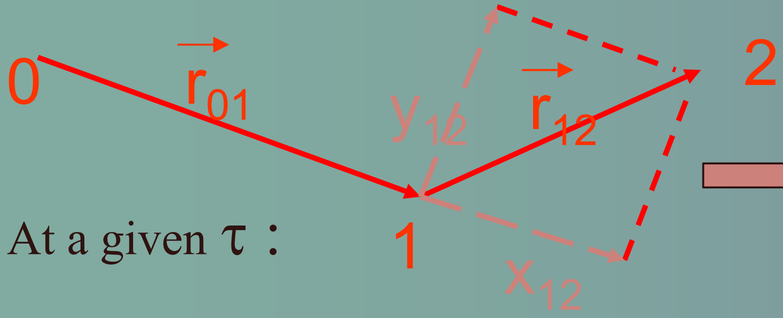


- Fat tails, Intermittency
- Non gaussianity factor depends on the timescale
- Crossover between sub-diffusive and diffusive motion at **r*=0.3 and t*=300**

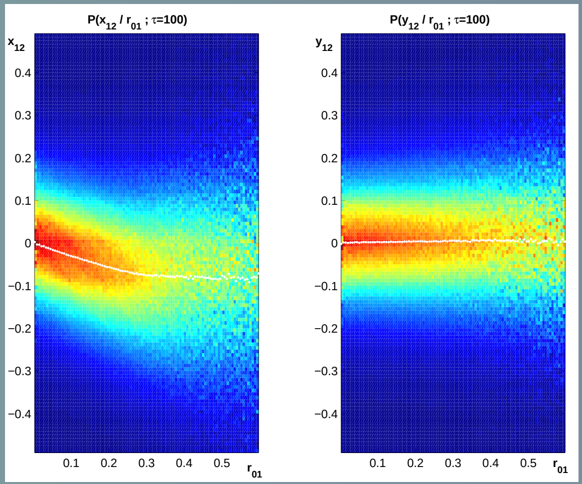
More details on diffusion

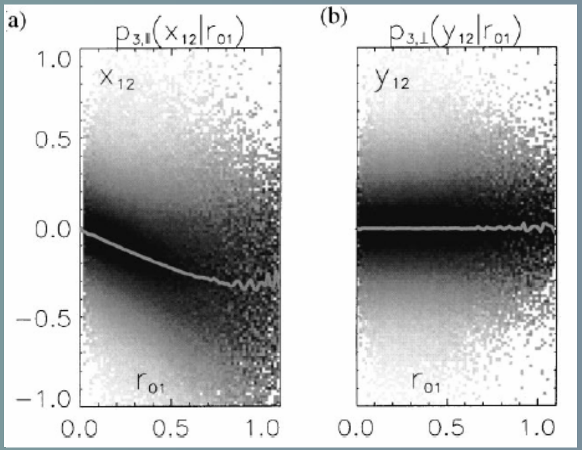
- Cyclic shear at constant volume : anti-correlations

At a given τ :



$P(x_{12}|r_{01};\tau)$ $P(y_{12}|r_{01};\tau)$





- $\langle y_{12}\rangle = 0$
- $\langle x_{12}\rangle < 0$
- for $r_{01}<r^*$

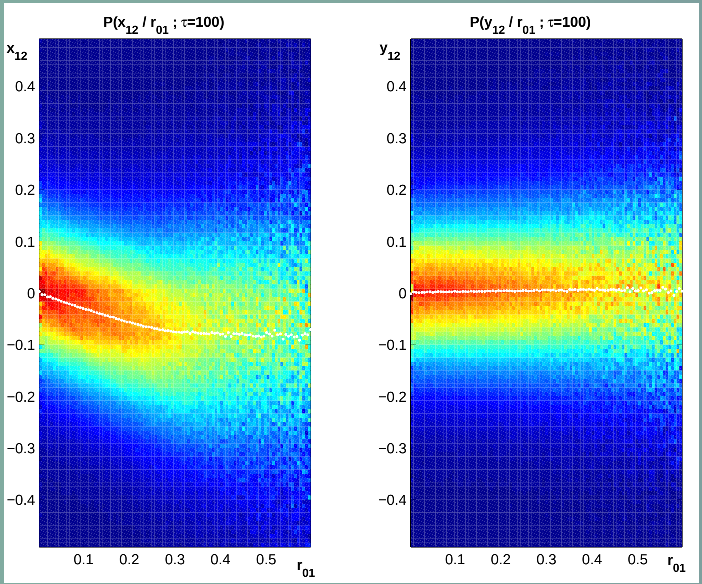
  $\langle x_{12}\rangle = c(\tau)\, r_{01}$
- for $r_{01}>r^*$

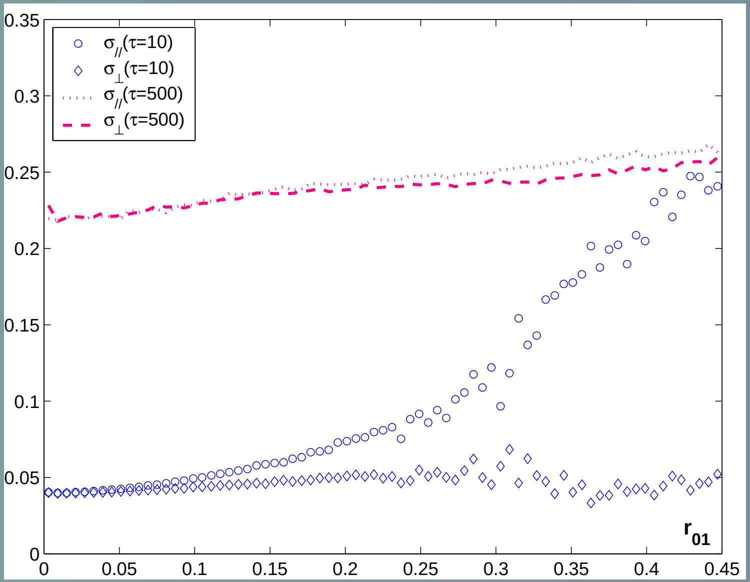
  $\langle x_{12}\rangle = cte$
- $\forall\tau<t^*$, the saturation occurs for $r_{01}=r^*$
- For $\tau>t^*$, the anti-correlations vanishes

(Doliwa and Heuer : thermal hard spheres simulation)

More details on diffusion

- Cyclic shear at constant volume : heterogeneities

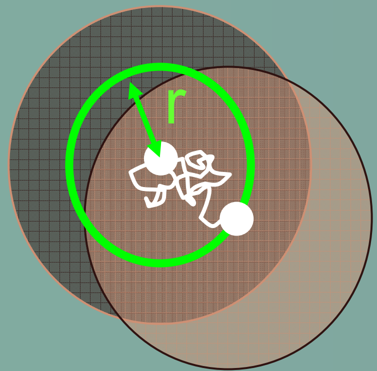




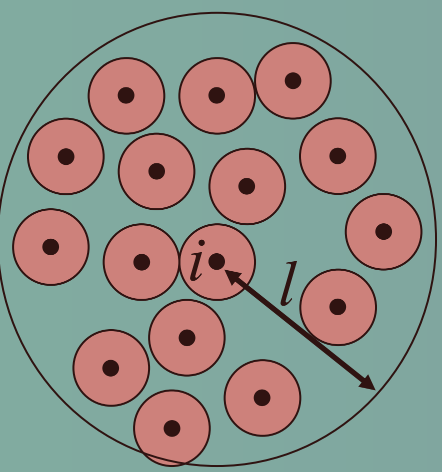
- For $\tau<t^*$, rms (x_{12}) increases with r_{01} : propensity to move is larger for previously rapidly moving particles
- While rms(y_{12}) remains constant : preferentially parallel to the previous move
- => suggest the string-like cooperation observed by Donati et al.

More details on diffusion

- Cyclic shear at constant volume : correlations

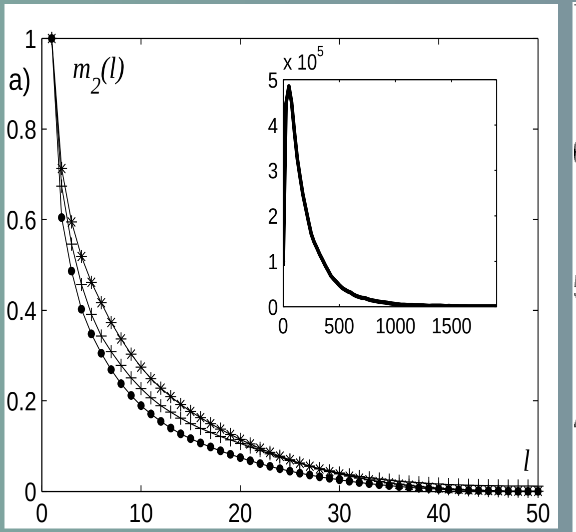


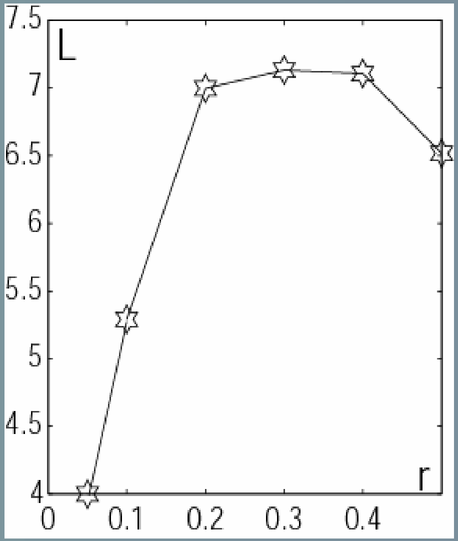
$T_i(r)$: time for grain i to reach the circle of radius r



$T_{i,l}(r) = \langle T_i \rangle$ inside C(i,l)

$$m_2(l) = \langle (T_{i,l} - T_{av})^2 \rangle / \langle (T_{i,1} - T_{av})^2 \rangle$$





The correlation length is maximum at the scale of the cage re-arrangements => up to 7 particles diameters

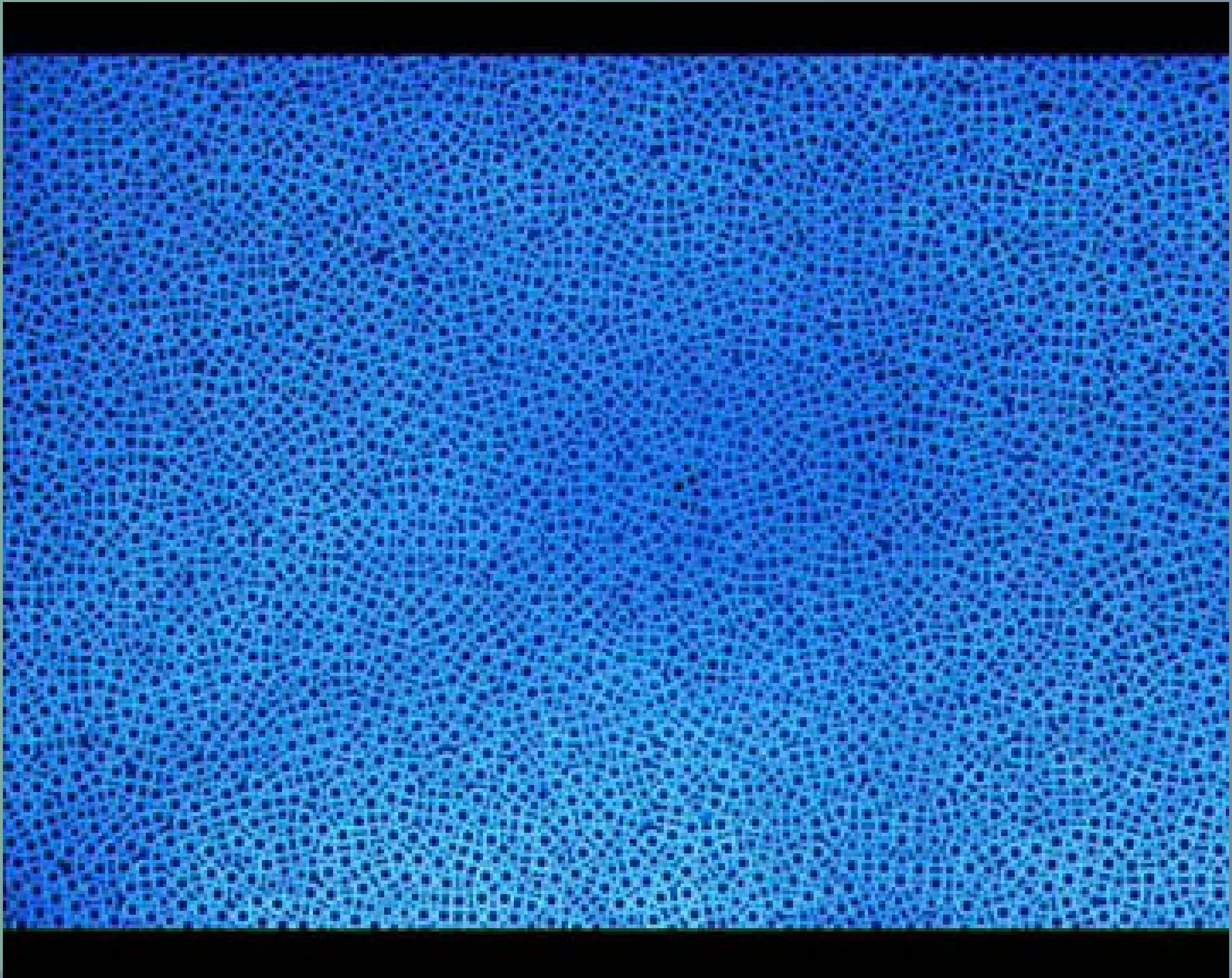
Hurley and Harrowell

Dynamical heterogeneities (Dauchot Marty and Biroli)

- Open questions :
  - What are the details of the structure?
  - How does it evolves in time?
  - Could one identify dynamical heterogeneities?
  - How do they relate to the cage dynamics?

- Need to follow all particles

Dynamical heterogeneities



Dynamical heterogeneities

- Instantaneous density field:

$$\rho(r,t)=\sum_i \delta\left(r-r_i(t)\right)\ ;\ \bar{\rho}=\langle\rho(r,t)\rangle = cste \text{ and } \int dr\rho(r,t)=N \Rightarrow \bar{\rho}=N/V$$

- Generalized density correlation function

$$\overline{W_a(t)}=\langle W_a(t)\rangle=\frac{1}{N}\int drdr'\langle\delta\rho(r,t)w_a(r-r')\delta\rho(r',0)\rangle \text{ with } \begin{array}{l}\delta\rho=\rho-\bar{\rho}\\ w_a(r-r')=\text{kernel}\end{array}$$

$$=\frac{1}{N}\left\langle\sum_{i,j} w_a\left(r_j(t)-r_i(0)\right)\right\rangle-\bar{\rho}\int dr\, w_a(r)$$

NB: self-part

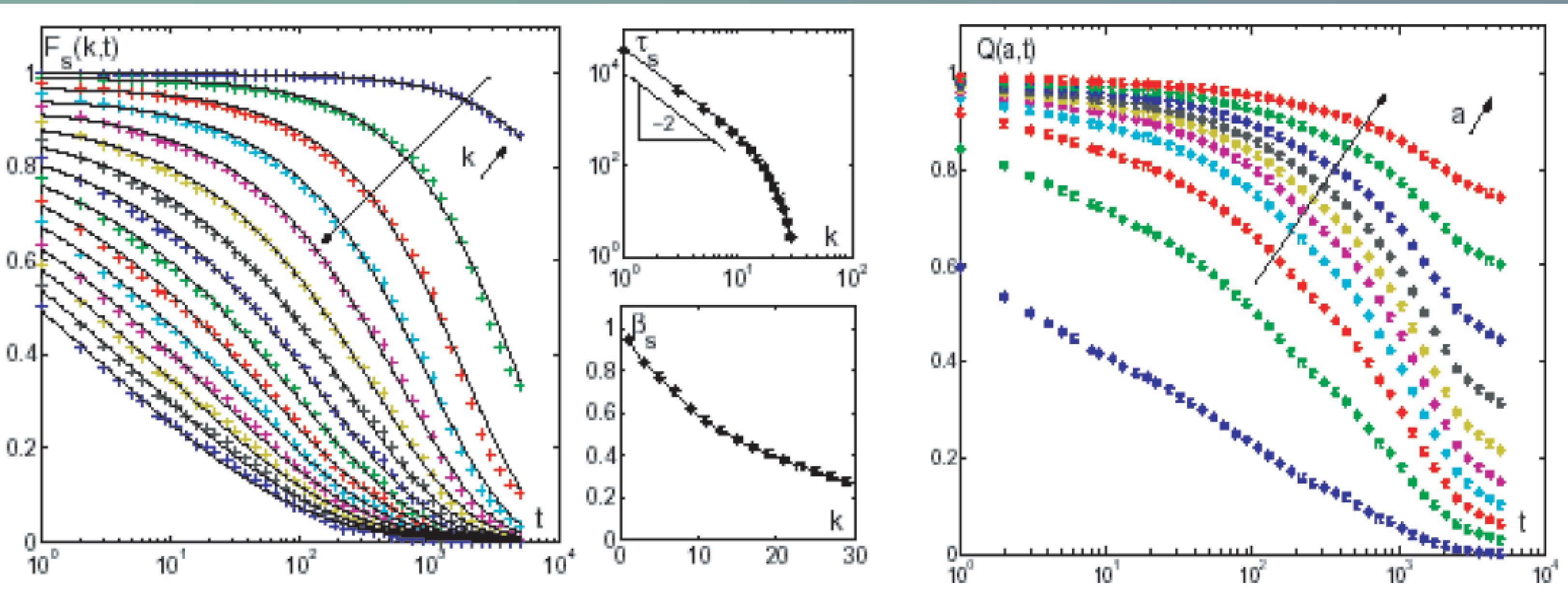
- For instance :
  - F(k,t): Intermediate scattering function

$$w_a(r)=\exp(ikr);\, k=2\pi/\text{a}$$

  - Q(a,t): Density overlap correlation function

$$w_a(r)=\exp\left(-\frac{r^2}{2a^2}\right)$$

Dynamical heterogeneities



- Stretched exponentials $\exp[-(t/\tau(k))^{\beta(k)}]$.
- At small k : $\tau(k) \sim k^{-2}$ and $\beta(k) \sim 1$. Brownian motion on large length and time scales and therefore $F_s(k, t) \exp(-Dk^2 t)$
- At large k : $\tau(k)$ steepens, $\beta(k)$ decreases : subdiffusion
- Very similar behaviour for F(k,t), Q(a,t) and their self part.

Dynamical heterogeneities

- Fluctuations of the temporal relaxation

$$\mathrm{X}_4^W(t) = N\left\langle \left(W_a(t) - \left\langle W_a(t)\right\rangle\right)^2 \right\rangle$$

- Relation to spatial heterogeneities

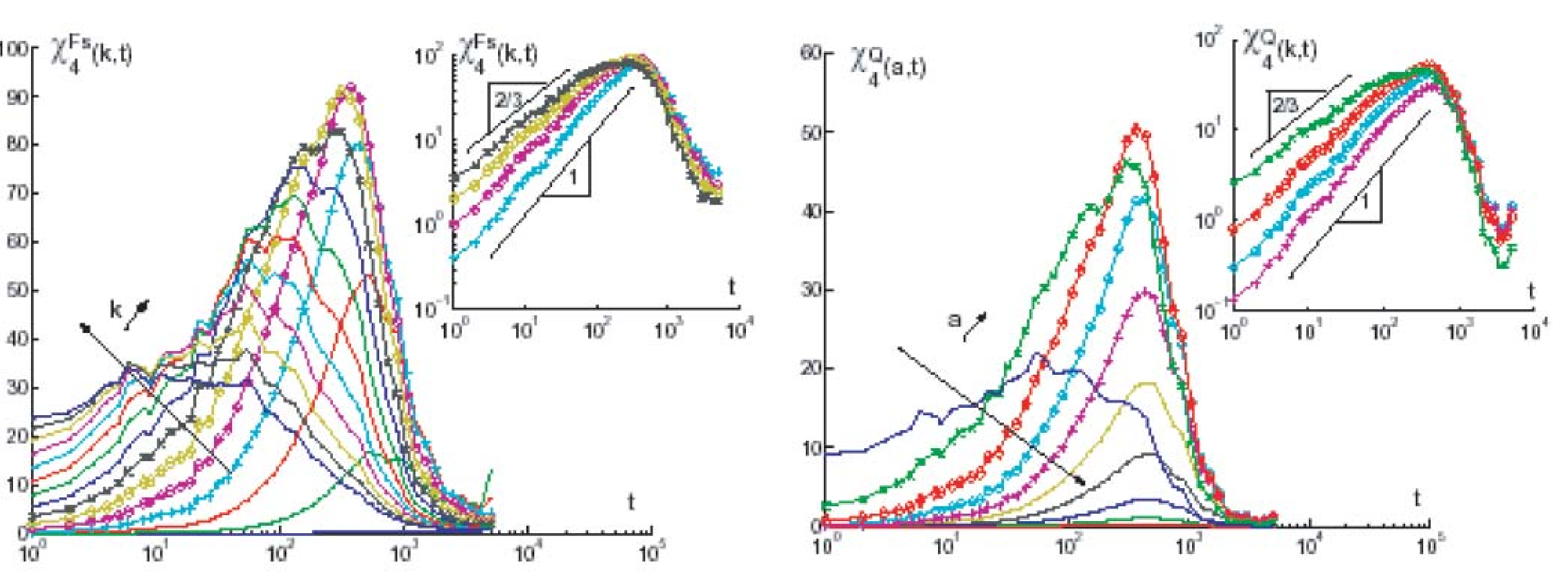
$$W_a(t) = \frac{1}{N}\int dr dr'\, \delta\rho(r,t) w_a(r-r')\delta\rho(r',0) = \frac{1}{V}\int dr\, w_a^{loc}(r,t)$$

with $w_a^{loc}(r,t) = \frac{1}{\rho}\int dr'\, \delta\rho(r,t) w_a(r-r')\delta\rho(r',0)$

=> $\mathrm{X}_4^W(t) = \rho\int dr\, G_4^W(r,t)$

with $G_4^W(r,t) = \left\langle \left(W_a^{loc}(r,t) - \left\langle W_a^{loc}(r,t)\right\rangle\right)\left(W_a^{loc}(0,t) - \left\langle W_a^{loc}(0,t)\right\rangle\right)\right\rangle$

Dynamical heterogeneities

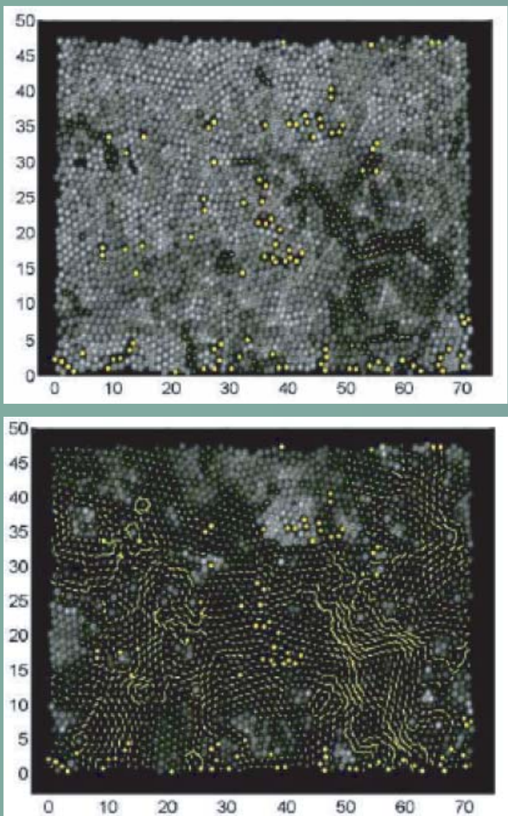


- X_4 is of the order of one at small and large times and displays a peak at a time somewhat larger than the cage lifetime.
- The largest X_4 is obtained for $k = 9$ corresponding to a length of the order of the cage size.
- The peak of the order of 100 => ξ_{het}~*6 grains*

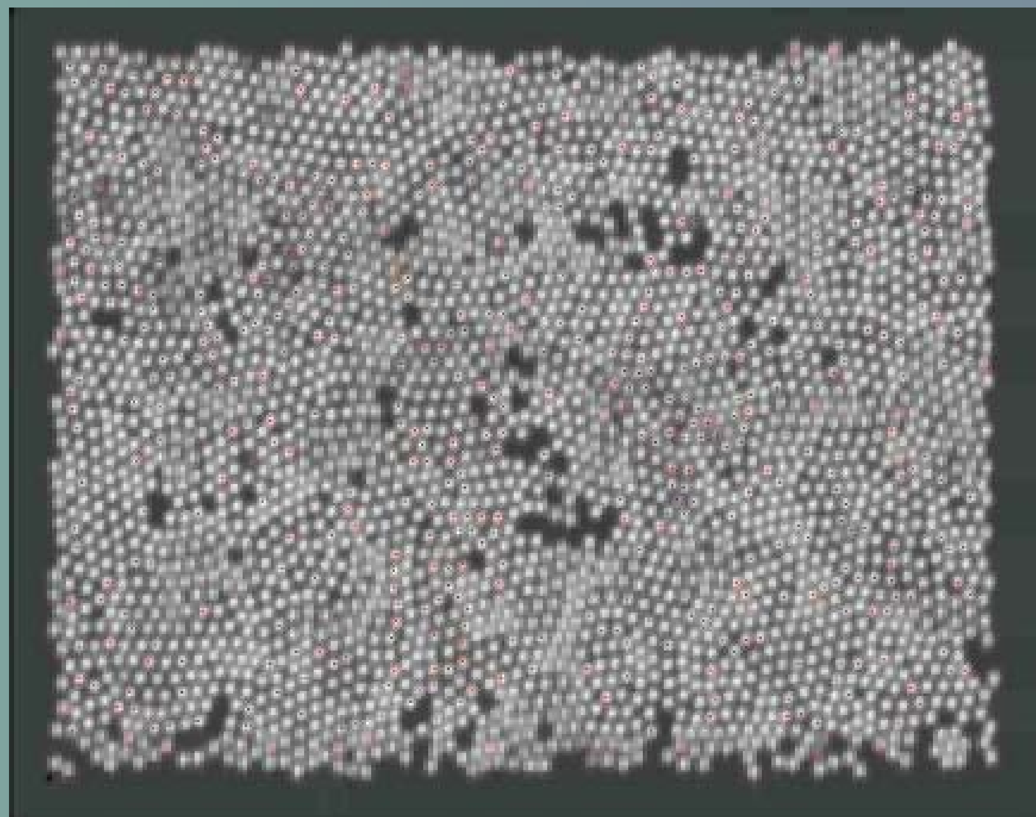
Dynamical heterogeneities

- Spatio-temporal evolution of $q_a^{loc}(r,t)$

t = 154 and t = 2526

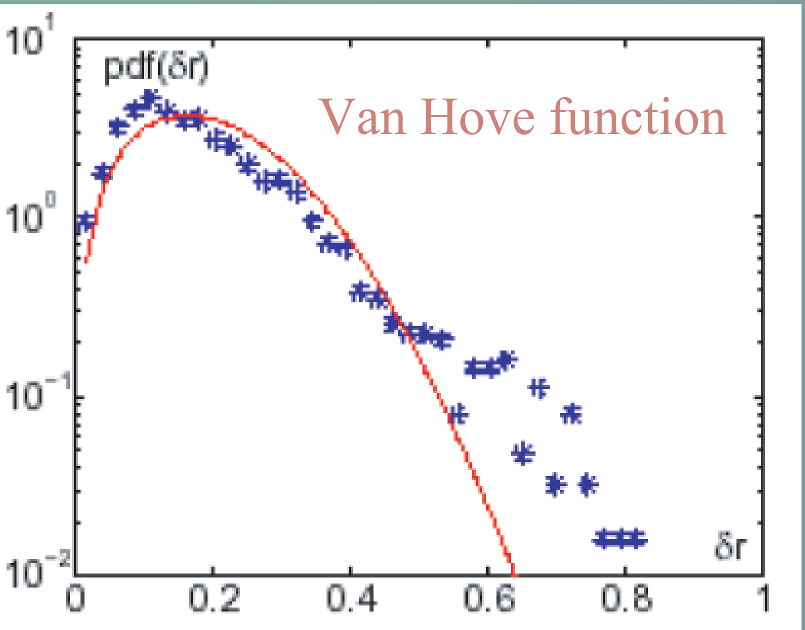


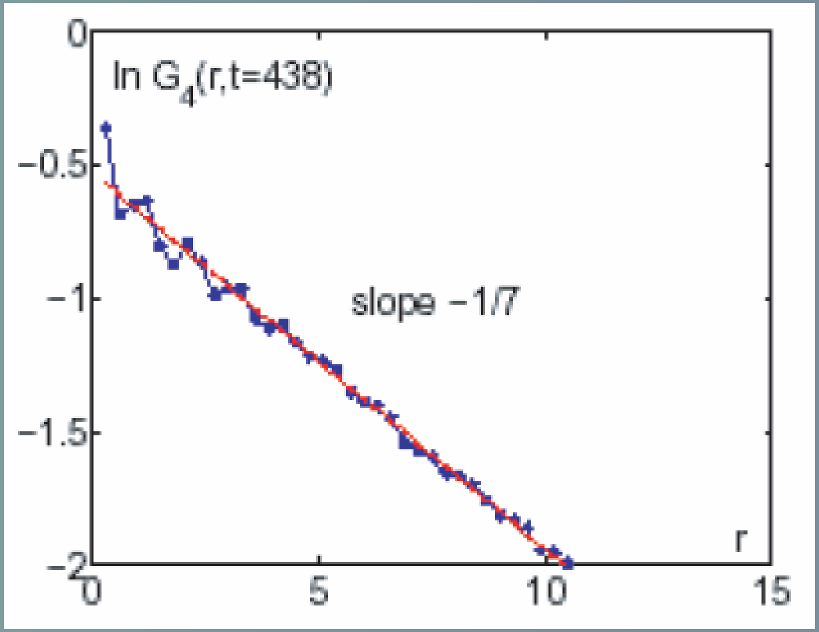
t runs from 1 to 5000



Dynamical heterogeneities

- Further characterization





- Excess of fast particles compare to a Gaussian distribution
- $G_4(r,438)$ decays exponentially over a characteristic dynamical length $\xi = 7$, in agreement with the value obtained from the peak of X_4

Partial conclusion and perspective

- The strong similarities between granular media close to the jamming transition observed at the macroscopic scale, are also present at the scale of the grain irrespectively of the kind (thermal vs. athermal forcing)
- Dense granular media is a good playground to test theoretical ideas from the field of glasses
- These similarities call for a common description, hence a statistical ground for the thermodynamics of granular media and more generally athermal systems

Part 3
Looking for a statistical description

- Edwards' proposal
- Experimental test ?
  - Volume fluctuations
  - Free volume distributions

Towards a statistical description I

General description

	Classical Gas	Granular Media
Micro-states $\mathcal{C}$	$\{x_i,p_i\}$, i=1…N	$\{r_i,f_\alpha\}$, i=1…N α=1…Nc
Accessible configurations	Conservation of energy	Conservation of volume Blocked states
Probability $P(\mathcal{C})$	Gibbs measure : uniform over all accessible configurations	Edwards' hypothesis : uniform over all blocked configurations

Micro-canonical distribution

$$P_\mu(C) = \frac{1}{Z_\mu(V_0)}\delta(V_C - V_0)\,1\,Q(C)$$

Towards a statistical description II

- Consequence on the volume distribution for a subsystem of N grains.

$$P_\mu(C) = \frac{1}{Z_\mu(V_0)} f(C) Q(C) \delta(V_C - V_0)$$

$$\Rightarrow$$

$$P_c(C) = \frac{1}{Z_c(X)} f(C) Q(C) e^{-(V(C)/X)}$$

with $\frac{1}{X} = \left.\frac{\partial \ln(Z_\mu)}{\partial V}\right|_{V^*}$

$$P_c(V_N) = \int dC P_c(C) \delta(V_C - V_N) = \frac{Z_\mu(V_N)}{Z_c(X)} e^{-V_N/X}$$

with

$$Z_\mu(V_N) = \int dC f(C) Q(C) \delta(V_C - V_N)$$

$$= \int \prod_{i=1}^{N} dw_i \rho(w_1, \ldots w_N) f(\{w_i\}) Q(\{w_i\}) \delta\left(\sum_{i=1}^{N} w_i - V_N\right)$$

Towards a statistical description III

- A "simple" example:
  - Consider a dynamics (such as tapping) for which by construction the explored configurations are blocked
  - Assume $f(\{w_i\}) = \prod_{i=1}^{N} w_i^{\eta-1}$ (see Bertin et al PRL 93 230601)

    NB: η=1 is the uniform measure
  - Considering that $\rho(\lambda\omega_1,\ldots\lambda\omega_N) = \lambda^{\gamma N}\rho(\omega_1,\ldots\omega_N)$
  - Introducing $w_i = \frac{V_N}{N}\omega_i = v_N\omega_i$ one obtains :

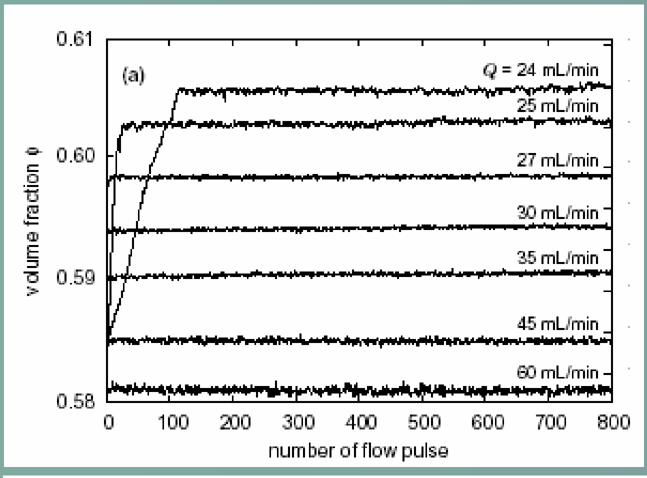
$$Z_\mu(V_N) = \left(\frac{V_N}{N}\right)^{N-1}\left(\frac{V_N}{N}\right)^{\gamma N}\left(\frac{V_N}{N}\right)^{N(\eta-1)} \int\prod_{i=1}^{N} d\omega_i \rho(\omega_1,\ldots\omega_N)\delta\left(\sum_{i=1}^{N}\omega_i - N\right)Q(\{\omega_i\})$$

$$P(v_N) = \frac{1}{Z_c(X)} A(N)(v_N)^{(\gamma+\eta)N-1} e^{-\frac{v_N}{X/N}}$$

Volume fluctuations around the steady state

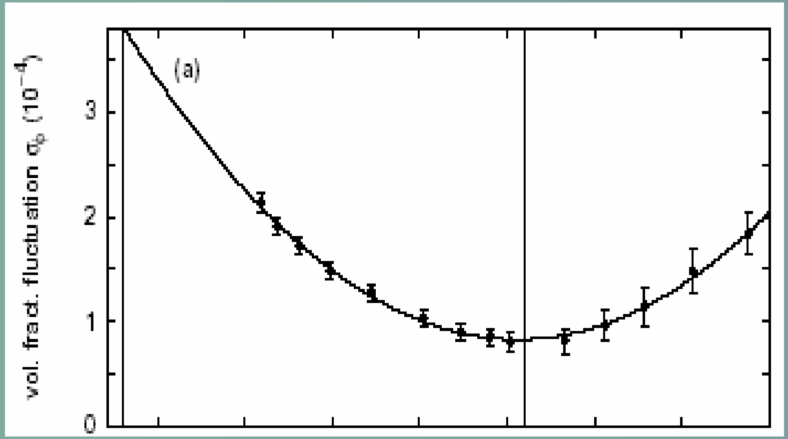
- Volume fraction fluctuations are Gaussian

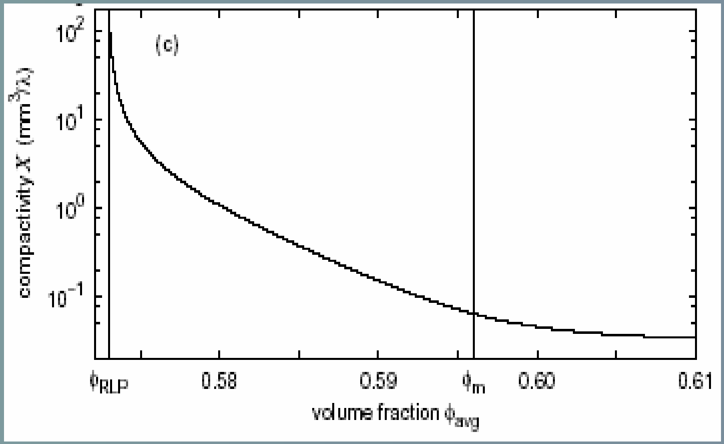
Schröter et al. condmat0501264



$$\langle \Delta V^2 \rangle = f(\langle V \rangle)$$

$$X^2 \frac{\partial \langle V \rangle}{\partial X} = \langle \Delta V^2 \rangle$$

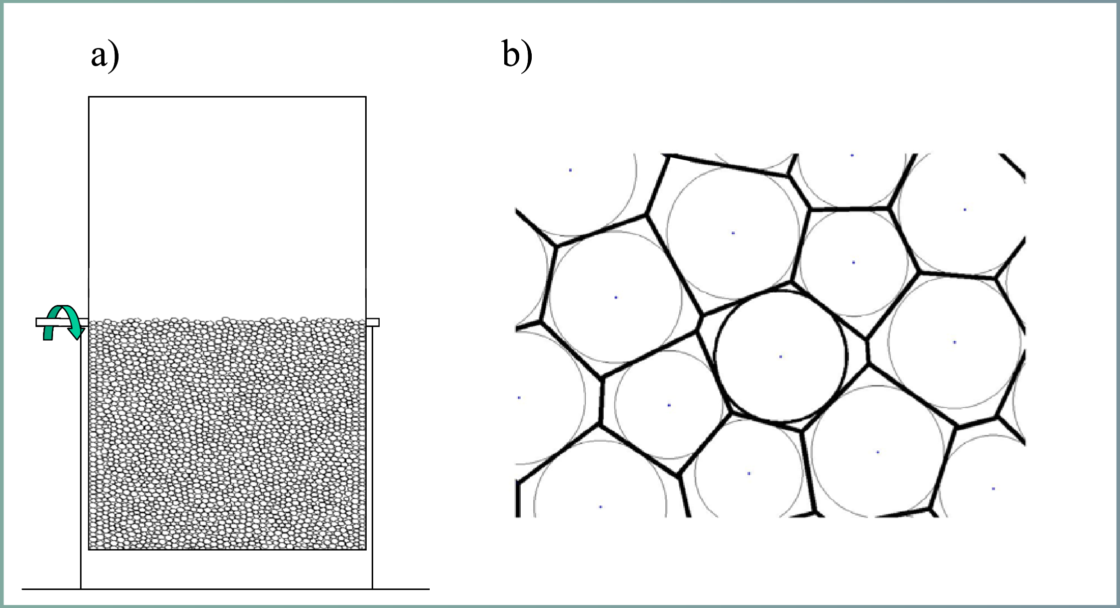




- Unfortunately not a test!

Volume distributions I

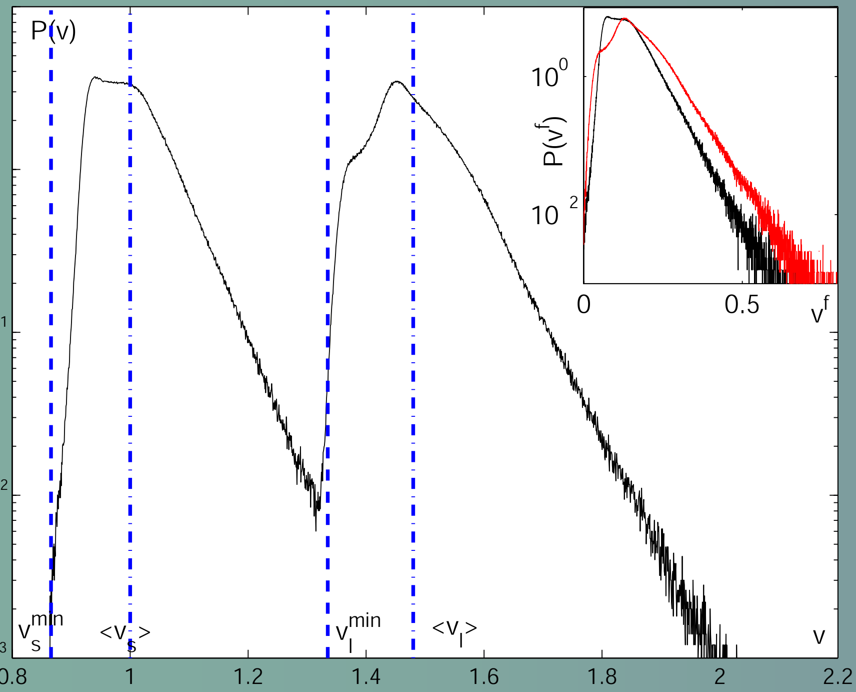
- Experimental set up

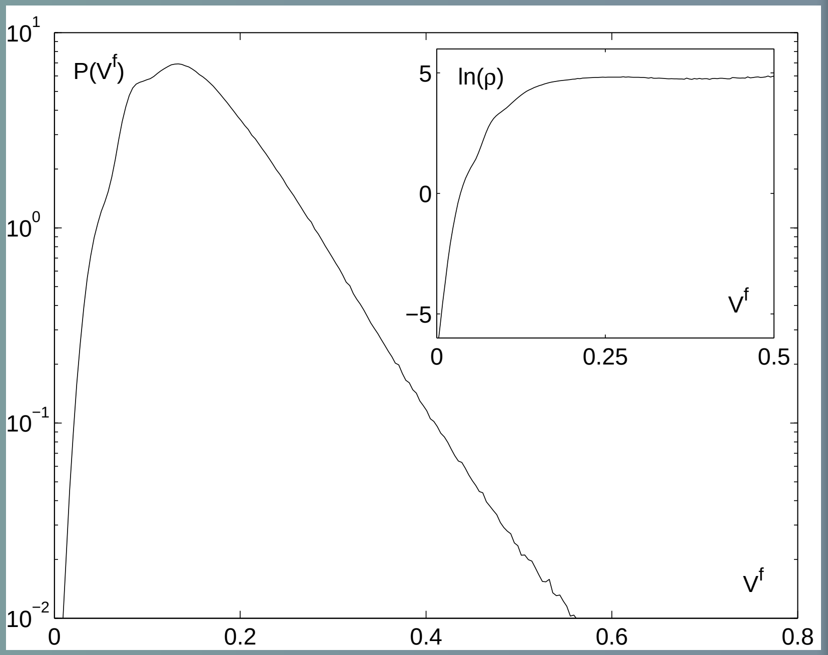


- 5000 cylindrical spacers of two sizes; d_s=4mm and d_l=5mm (3mm thick) in a 2D cell
- The cell, half filled, is rotated to reinitialize the packing.
- No dynamics => blocked configurations only

Volume distributions II

- One grain volume and free volume distributions

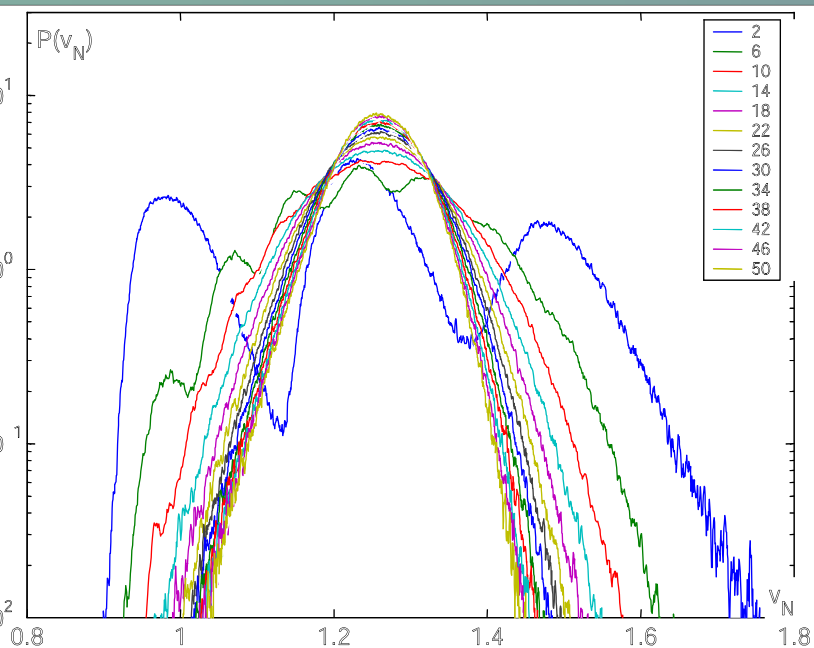


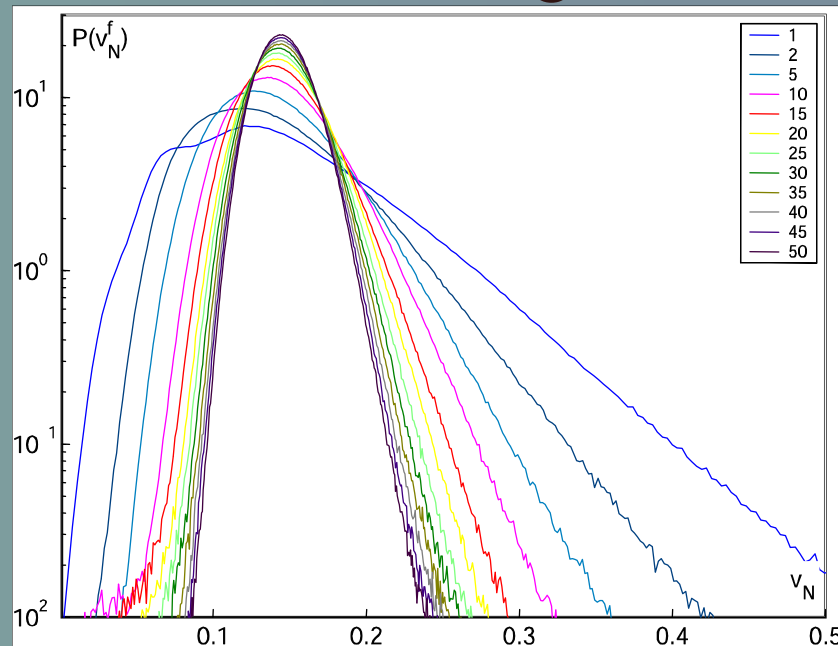


- There are indeed exponential tails => for one grain distribution, in the case of a uniform measure, one would estimate $\rho(v^f)$, the density of state for one grain defined as $P(v_1^f) = \rho(v_1^f) e^{-v_1^f / X}$

Volume distributions III

- Free volume distributions for clusters of N grains

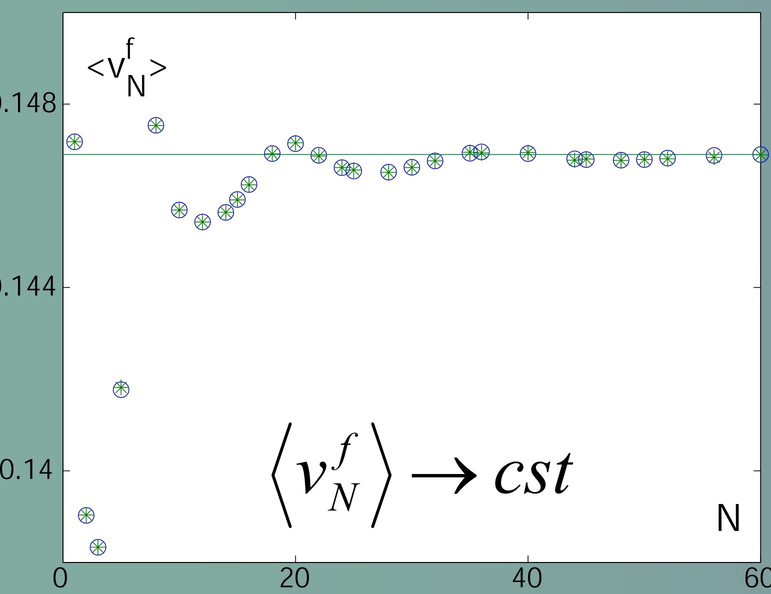


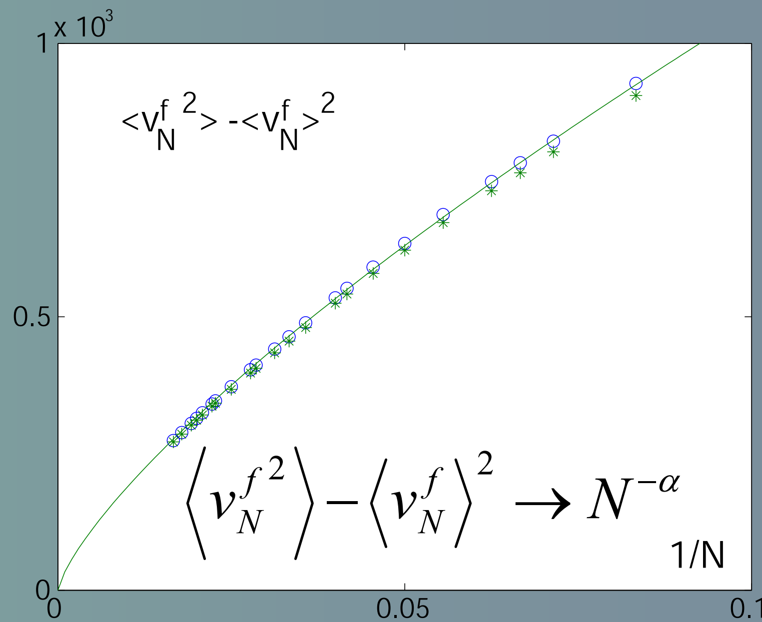


- Volume distributions suffer from combinatory effects induced by the bidispersity of the grains
- Free volume distributions evolve towards Gamma laws:

$$P(v_N^f) = \frac{1}{X_N^{\eta_N}\Gamma(\eta_N)}\left(v_N^f\right)^{\eta_N - 1} e^{-v_N^f / X_N}$$

Volume distributions IV





- In the case of Gamma law $\left\langle v_N^f \right\rangle = \eta_N X_N$

$$\left\langle v_N^{f\,2} \right\rangle - \left\langle v_N^f \right\rangle^2 = \eta_N X_N^2$$

$$\Rightarrow \eta_N = \eta_{eff} N^{\alpha} \quad X_N = X_{eff} N^{-\alpha} \Rightarrow$$

$$P(v_N^f) = \frac{1}{X_N^{\eta_N} \Gamma(\eta_N)} \left(v_N^f\right)^{\eta_{eff} N^{\alpha} - 1} e^{-\frac{v_N^f}{X_{eff} / N^{\alpha}}}$$

Discussion

- In the limit of large N, $\log P(v_N^f) = N^{\alpha} g(v_N^f, \eta_{eff}, X_{eff})$

with $g(v,\eta,X) = \eta\left(\ln(\frac{v}{\eta X}) - \frac{v}{\eta X} + 1\right)$ η_{eff}=7/2, X_{eff}=0.04 α=0.8

- Interpretation
  - α=0.8 can be understood as a non extensivity factor => the evidence of long range correlations ($1/r^{2\alpha}$)
  - Consequence for the use of thermo-dynamical relations
- Comparison with the simple example

$$P(v_N) = \frac{A(N)}{Z_c(X)} (v_N)^{(\gamma+\eta)N-1} e^{-\frac{v_N}{X/N}}$$

$$P(v_N^f) = \frac{1}{X_N^{\eta_N}\Gamma(\eta_N)} (v_N^f)^{\eta_{eff} N^{\alpha}-1} e^{-\frac{v_N^f}{X_{eff}/N^{\alpha}}}$$

Further interpretation

- The apparent validity of a statistical description (irrespective of the micro-canonical measure) suggest to write :

$$P(v_N^f) = \frac{1}{Z(X,N)} e^{-N^{\alpha}\left(\frac{v_N^f - Xs(v_N^f)}{X}\right)}$$

with $s(v_N^f) = \eta \ln(v_N^f)$

and thereby $\frac{1}{X} = \left.\frac{\partial s}{\partial v^f}\right|_{<v^f>} = \frac{\eta}{\left\langle v^f \right\rangle}$

- The micro-canonical partition function for one grain is not simply proportional to the free volume per grain
- A typical activated process within such a system has an activation rate *à la* Vogel Fücher.

Partial Conclusion and perspectives

- The distribution of the free volumes per grain inside clusters of N grains follows a Gamma law, whose parameter suggest :
  - the existence of long range correlation responsible for some non-extensivity; (nota bene impact on thermo relation use).
  - to effectively resume the macroscopic properties of the sample within a “free energy function per grain”

- The analysis does not require to make any hypothesis on the micro-canonical measure, so that it actually does not address Edwards hypothesis

The full story

Granular media = A-thermal + Dissipative

1) No forcing from the thermal environment | Energy lost towards the thermal environment

Need for a statistical description of dissipative systems with mechanical excitation but no thermal noise

2) Stochastic dynamics without detailed balance (time irreversibility) BUT

1. A conserved quantity
2. Fac...
...ribution

Thermo generalization

3) Is there any chance to succeed in the case of dense granular media ?

They are indeed very similar to supercooled liquids close to the glass transition, both at the macroscopic and at the microscopic scales

4) Let's try !

•Edwards' proposal — V is conserved (OK), P_μ is uniform (irrelevant)

•Experimental investigation

→ Intensive parameters ☺ but non extensivity ☹

General Conclusion : main messages

- The definition of a "temperature" is not related to the scale of the particles, but to the existence of an extensive conserved quantity.
- The idea of a unified description for the glass and the jamming transition has indeed strong evidences at the scale of the individual particles.
- From an experimental point of view, testing the uniformity of the measure over the blocked configurations is a chimera, in the absence of a full microscopic description of the system.
- However, looking for relevant extensive and intensive thermo-dynamical parameters is a key step. Take care with potential long range correlations and associated non-extensivity!

Some Perspectives

- How are related the “temperature” for stationary non Hamiltonian dynamics with a conserved quantity and the effective temperature obtained in the glassy regime via the fluctuation-dissipation theorem?
- It would be of great benefit to further investigate the mechanisms underlying the development of the dynamical heterogeneities and how do they relate to the structure
- Given the possibility of extracting intensive parameters from the free volume distributions inside a granular packing, it is now a priority to test whether some of these parameters equilibrate between subsystems.