Jamming transition of a granular pile : a first step towards athermal glass?

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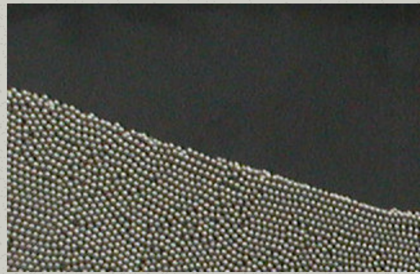
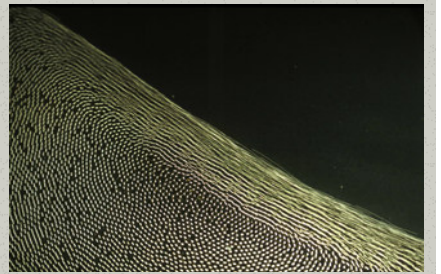
Intro: granular flow down a heap

- Static pile below the angle of avalanche θ_a.
- Surface flowing layer above the angle of repose θ_r.

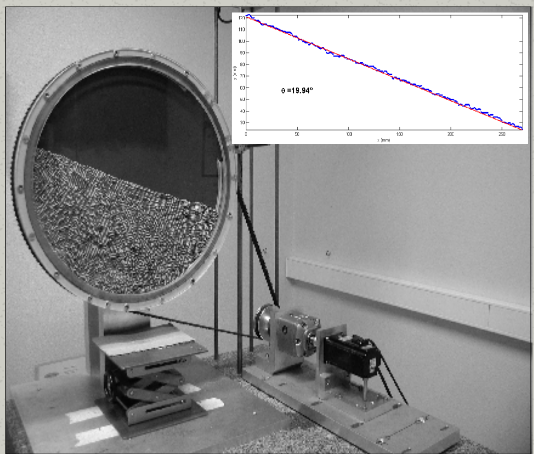




- Subcritical (1[rst] order) transition : $\theta_r < \theta_a$.
- Present study :

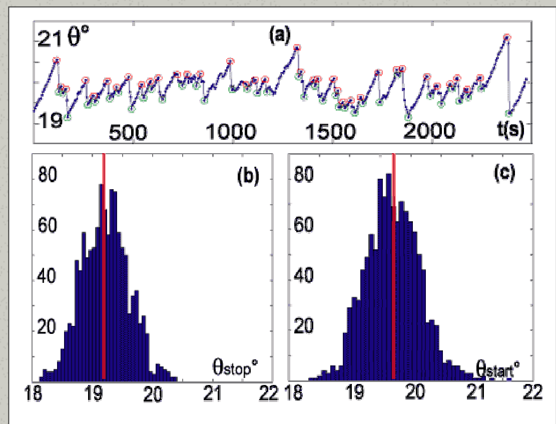
characterize the dynamics back to equilibrium

Experimental set up



- Rotating drum
  - D = 450 mm
  - δ = 22 mm
- Steel beads
  - d = 3 mm
  - m = 0.11 g
- Angular velocity
  - Ω

Measure of θ_r: intermittent regime



Ω=0.01°/s
f_acq.= 0.2 Hz
acq. time = 5.10[4] s.

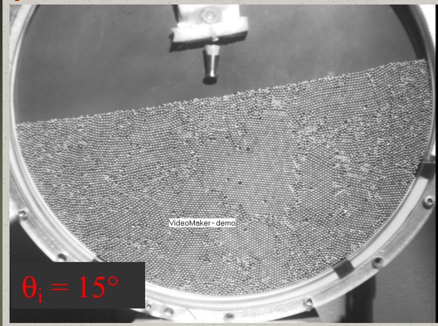
$< \theta_{stop} > = 19.2°$ $< \theta_{start} > = 19.7°$

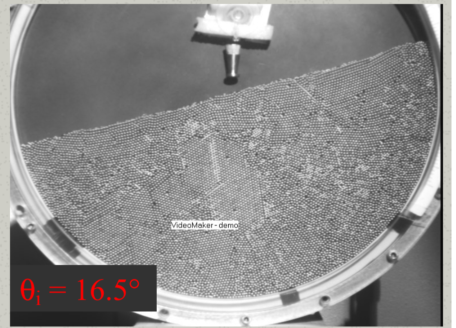
Experimental procedure

- The drum is rotated for a while, stopped and the pile slope is set to $\theta_i < \theta_r$ at t=0.
- The pile relaxes towards mechanical equilibrium:
  - Average images (25 im at 5 Hz) are recorded every 15 s.
  - Image differences allows to observe where displacements occur.

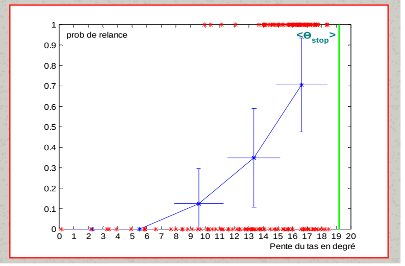
$\theta_i = 0.3°$

Very different relaxation dynamics

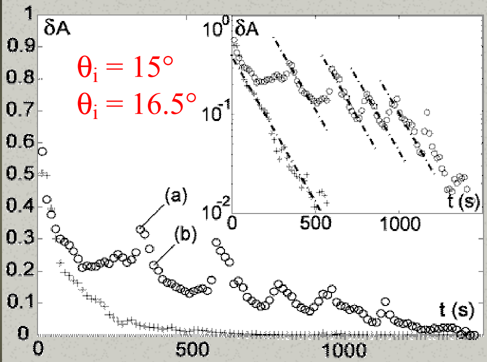




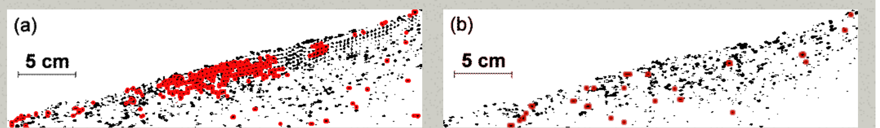
- First a rapid relaxation of the bulk (from bottom to top)
- A monotonous / intermittent relaxation of the surface layer
- Probability of intermittent dynamics increases with θ_i .



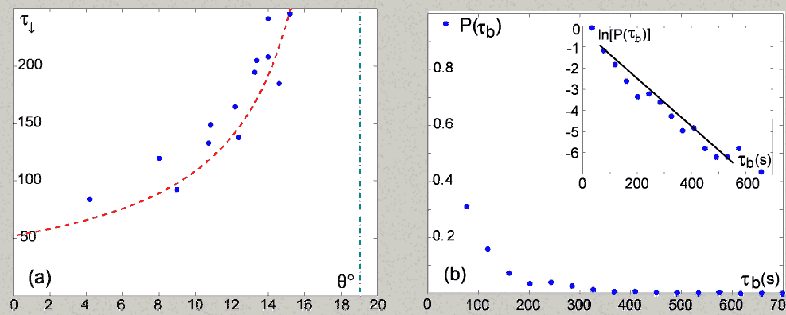
Further investigation of the surface layer relaxation ([10-20] beads diameter thick)



- A dynamics composed of:
  - exponential decay
  - short intermittent bursts
- Exponential decay : Individual displacements
- Intermittent bursts : Correlated displacements

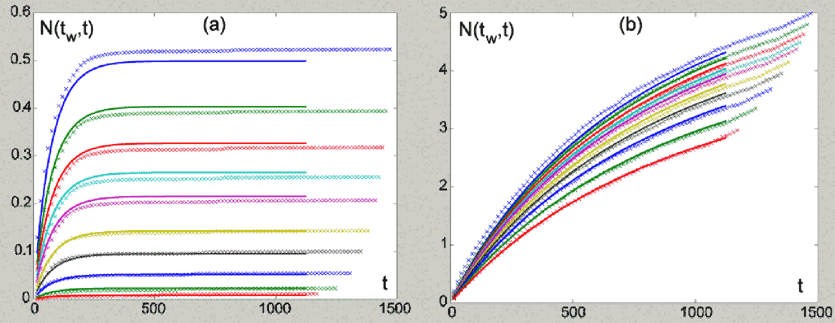


Further characterization : characteristic times



- Exponential decay rate : $\tau_\downarrow$ increases with θ_i.
- Bursts intervals distribution essentially invariant, with characteristic time : τ_b=100s.
- Bursts become significant when $\tau_\downarrow > \tau_b$.

Further characterization: two-time relaxation description $N(t_w,t)=\left\langle \int_{t_w}^{t_w+t} \delta A(u)\,du \right\rangle$



- Exponential decay : rapid saturation and average number of move smaller than one
- In the presence of bursts : no saturation and average number of move larger than one.

Summary : experimental results

- After a rapid relaxation of the bulk, the relaxation slows down in a subsurface layer of thickness [10-20] bead diameters
- The relaxation is essentially exponential with a characteristic time increasing from 50 to 250 s. Grains relax independently on very short time and very small displacements scales.
- The relaxation is interrupted by collective motion of correlated clusters, which reactivate the dynamics of the layer.
- These bursts only occur when the layer is still relaxing.

Discussion

- Why are the time scales $\tau_\downarrow$ and $\tau_b \approx 100$ s so large compare to the bead-scale time $(d/g)^{(0.5)} \approx 0.01$ s ?

  (out of reach of the present study)

- Is there a simple model which allows a direct calculation of $N(t_w, t)$ and proposes an interpretation of the relaxation dynamics of the pile?

An over-simplified model

- Beads can be in active (1->n) or inactive (0) states
  - active state : the bead transits to another active state with rate α' and to inactive state with rate α resulting in a global rate of transition from an active state $\gamma=\alpha+(n-1)\alpha'$. These transition are assimilated to actual individual move of the beads and thus contribute to $N(t_w, t)$.
  - inactive state : the bead do not evolve spontaneously.
  - the reactivation process : randomly chosen beads are instantaneously set in the active state with probability ν – independently of their state. This process is assumed not to involve displacement of the bead but rather a rearrangement of its environment and thus do not contribute to $N(t_w, t)$.

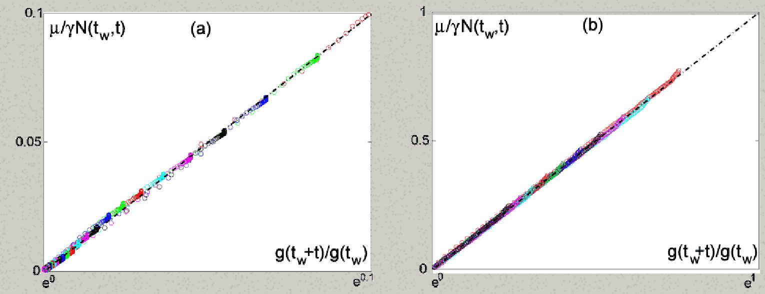
Model : results

- The fraction of active beads is given by : $P_m(t) = \sum_1^n P_i(t)$
  where $P_i(t)$ is the probability of being in active state i.
- Accordingly, $N(t_w, t) = \int_{t_w}^{t_w+t} \gamma P_m(t) dt$
- The dynamics is given by $\frac{dP_m}{dt} = -\alpha P_m + \nu P_0 = -\alpha P_m + \nu(1-P_m)$.
  where the key ingredient of the model is now introduced:
  $\nu = \nu(P_m) = \mu P_m$
- After some calculations, one finds: $N(t_w, t) = \frac{\gamma}{\mu} \ln\left(\frac{g(t_w+t)}{g(t_w)}\right)$
  with a reparametrization of time:
  $$g(t) = (\alpha - \mu) + \mu P_m(0)\left[1 - e^{-(\alpha-\mu)t}\right]$$

Model discussion :

applying to experimental data

- A scaling function is proposed for $N(t_w,t)$: It is valid



- It provides experimental values for the parameters.

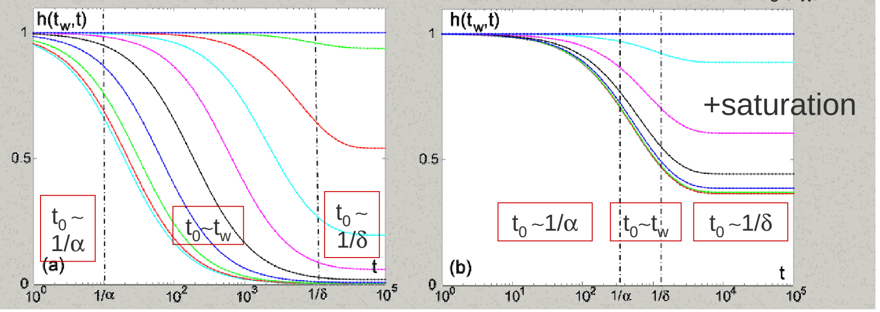
	(a)	(b)
1/α	56 s	294 s
1/(α-μ)	75 s	1250 s
γ/α	1	4.7

Model discussion: interpretation

- A correlation function can be defined as the probability not to have changed state between t_w and t_w+t.
- It reads : $C(t_w, t) = P_m(t_w) e^{-\gamma t} + (1 - P_m(t_w)) \left[\frac{g(t_w+t)}{g(t_w)}\right]^{-1}$

rapid relaxation

relaxation on time $t_0(t_w)$



Conclusion

- The relaxation dynamics of a granular pile below the angle of repose exhibits non trivial behaviors, including intermittent reactivation bursts.
- As a result, a long-time dynamics occurs.
- A simplistic model is proposed of which the key ingredient is that the reactivation process depends on the population of the active state themselves.
- This model allows us to identify in the experiment a scaling function of time and to give it a precise meaning in terms of aging via the computation of a correlation function.
- This aging behavior is transient, which can be interpreted as the signature of an athermal system.
- The micro-mechanics at the origin of the reactivation process (collective events /intermittent bursts) remain to be clarified.