Onsager relations in a two-dimensional electron gas with spin-orbit coupling

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Motivations and Results

Let me start with two questions:

- What is the correct definition of the spin current?
- Are Onsager relations satisfied for SHE and ISHE?

Why to ask?

- Proposal to replace the conventional definition (which is not generally conserved because of e.g. spin-orbit interaction) with a conserved one (Shi et al. PRL **96**, 076604 (2006))
- Observation that a spin current (which may induce a ISHE) may be driven by different external spin-dependent ($SU(2)$) potentials (Wang et al. PRB **85**, 165201 (2012)), as a charge current may be driven by a scalar or vector potential

We will see that the two issues are intimately related (Gorini, Raimondi, Schwab, arXiv:1207.1289)

- Spin-orbit interaction as a $SU(2)$ gauge field with scalar and vector components (Mathur and Stone PRL **68**, 2964 (1992), Tokatly PRL **101**, 106601 (2008), Gorini et al. PRB **82**, 195316 (2010) not an exaustive list!)
- All definitions are legitimate (provided being careful)
- Connection between spin current definition and external driving field

Spin current definition

- Conventional (and most natural) definition:

$$J_i^a = \frac{1}{2}\{v_i, s^a\}$$

See for instance: Burkov et al. PRB **70**,155308 (2004); Mishchenko et al. PRL **93**, 226602 (2004); Raimondi et al. PRB **74**, 035340 (2006)

- Spin is not conserved (torque):

$$\partial_t s^a + \partial_i J_i^a = T^a$$

Conserved current definition (Shi et al. PRL **96**, 076604 (2006))

$$\mathscr{J}_i^a = J_i^a + P_i^a, \quad \partial_i P_i^a = -T^a$$

Question

How the spin Hall conductivity changes depending on the definition adopted? See: Sugimoto et al. PRB **73**, 113305 (2006) where both definitions are used in a number of models (linear Rashba, cubic Rashba, short- and long-range impurity scattering, cf. Table 1)

Onsager relations PR **37**, 405 (1931); **38**, 2265 (1931))

Microscopic reversibiity (in Onsager words)

A value of a fluctuating variable α_1 at time t followed by a value of another fluctuating variable α_2 at time $t+\tau$ occurs as often as the value of the second variable α_2 at time t followed by the value of the first variable α_1 at time $t+\tau$

Recent discussions

- emphasizing AHE: Nagaosa et al. RMP **82**, 1539 (2010)
- emphasizing symmetries and thermal transport: Jacquod et al. arxiv:1207:1629

To be specific

- Coupling of currents to external fields

$$\delta H = J_i A_i$$

- Response functions

$$\delta J_i = K_{ij} A_j, \quad K_{ij} = \langle J_i J_j \rangle$$

- Symmetry relations

$$K_{ij}(B) = \varepsilon_i \varepsilon_j K_{ji}(-B), \qquad \varepsilon_i = \pm 1$$

Onsager relations for spin currents and spin potentials

- Coupling with charge and (conventional) spin current (due to spin-orbit interaction)

$$\delta H = J_i A_i, \quad \delta H = J_i^a \mathscr{A}_i^a$$

- Example: the spin Hall effect

$$J_y^z = \sigma^{sH} E_x, \quad E_x = -\partial_t A_x$$

- and the inverse spin Hall effect

$$J_y = -\sigma^{sH} \mathscr{E}_x^z, \quad \mathscr{E}_x^z = -\partial_t \mathscr{A}_x^z$$

Note: J_i and J_i^a behave differently under time reversal

Question

The electric field can described also by a scalar field (gauge invariance) What about the spin-electric field?

Connection between fields and spin currents

- Consider both the *spin-vector gauge*

$$H_V = H_0 + J_i^z \mathscr{A}_i^z , \quad \mathscr{A}_i^z = -t \mathscr{E}_i^z$$

- and *spin-scalar gauge*

$$H_S = H_0 + \frac{\tau^z}{2} \Psi^z , \quad \Psi^z = -r_i \mathscr{E}_i^z ,$$

- Perform a gauge transformation from the scalar to the vector one

$$U = \exp\left(\mathrm{i} \frac{1}{2} \tau^z \chi \right)$$

In general:

$$H_S' = H_0 + \mathscr{J}_i^z \mathscr{A}_i^a , \mathscr{J}_i^z = -\frac{\mathrm{i}}{\hbar} \left[\frac{\tau^z}{2} r_i , H_S \right]$$

Note: only if H_S commutes with τ^z the current reduces to the conventional one

Models and methods

- Rashba model for 2DEG (by allowing for extrinsic spin-orbit interaction as well)

$$H = \frac{p^2}{2m} + \alpha(p_y \sigma^x - p_x \sigma^y) + V(\mathbf{r}) - \frac{\lambda_0^2}{4} \sigma \times \nabla V(\mathbf{r}) \cdot \mathbf{p}$$

- Rashba model for 2DHG

$$H = \frac{p^2}{2m} + \tilde{\alpha}(p_y(3p_x^2 - p_y^2)\sigma^x + p_x(3p_y^2 - p_x^2)\sigma^y) + V(\mathbf{r})$$

- Short-range Impurity scattering

$$\langle V)\mathbf{r})V(\mathbf{r}')\rangle = u^2 \delta(\mathbf{r} - \mathbf{r}')$$

- Long-range impurity scattering

$$FT\left(\langle V)\mathbf{r})V(\mathbf{r}')\rangle\right) = W(\mathbf{p} - \mathbf{p}')$$

Diffusive regime:

Fermi energy $\gg$ scattering rate $\gg$ spin-orbit splitting

Rashba 2DEG

Using the conventional current

$$\sigma^{sH}(\omega) = -\frac{\gamma\sigma}{e}\frac{-i\omega+\tau_s^{-1}}{-i\omega+\tau_{DP}^{-1}+\tau_s^{-1}}$$

Using the conserved current

$$\tilde{\sigma}^{sH}(\omega) = -\frac{\gamma\sigma}{e}\frac{-i\omega}{-i\omega+2\tau_{DP}^{-1}}\frac{-i\omega-\tau_{DP}^{-1}+\tau_s^{-1}}{-i\omega+\tau_{DP}^{-1}+\tau_s^{-1}}.$$

- σ Drude conductivity
- $\gamma = \gamma_{intr}+\gamma_{ss}+\gamma_{sj}$ charge-spin coupling parameter

$$\gamma_{intr} = -m\alpha^2\tau, \quad \gamma_{sj} = \frac{\lambda_0^2 m}{4\tau}$$

$$\gamma_{ss} = -\frac{(\lambda_0 p_F)^2}{16}(2\pi N_0 v)$$

- τ_{DP}^{-1} Dyakonov-Perel relaxation
- τ_s^{-1} Elliott-Yafet relaxation

Rashba 2DHG

$$\sigma^{sH} = -\frac{\gamma\sigma}{e}, \quad \tilde{\sigma}(\omega) = -\frac{\gamma\sigma}{e}\frac{-i\omega}{-i\omega+\tau_{DP}^{-1}}, \quad \gamma = 6\tilde{\alpha}^3 k_F^4 m\tau$$

(Hughes et al. PRB **74**, 193316 (2006))

Notice: the response associated to the conserved current always vanishes in the zero frequency limit

- Static response of the density ρ to an external field ϕ with κ the *compressibility*
- Drift current due an external field with σ the *conductivity*
- Diffusion current due to a density gradient with D the diffusion coefficient
- Einstein relation $\sigma = \kappa D$

$$\rho(q,\omega) = -\frac{\sigma q^2}{Dq^2 - \mathrm{i}\omega}\phi(q,\omega)$$

Static limit: density response

$$\rho = \kappa\phi$$

Dynamic limit: current response

$$j = \sigma E$$

Comparison with the literature: Sugimoto et al. PRB **73**, 113305 (2006)

Rashba model

Impurity potential	Born approximation	Conventional	Conserved
$\delta(\mathbf{x})$	1st / higher	0	0
$V(\mathbf{p}-\mathbf{p}')$	1st	0	0
	higher	0	Finite

Cubic Rashba model

Impurity potential	Born approximation	Conventional	Conserved
$\delta(\mathbf{x})$	1st / higher	Finite	0
$V(\mathbf{p}-\mathbf{p}')$	1st	Finite	0
	higher	Finite	Finite

Rashba model with extrinsic spin-orbit coupling

Impurity potential	Born approximation	Conventional	Conserved
$\delta(\mathbf{x})$	1st	*Finite*	0

Red-Typed cases can be directly compared and there is agreement

Conclusions

- Current definition associated to the gauge choice
- Onsager relations hold irrespective of the choice
- Big question: how to experimentally excite a given spin current, or, said otherwise, how to create spin-vector and/or a spin-scalar potential
  How classify existing and future experiments?
  - H-like device for non-local measurement: SHE and ISHE HgTe (Brüne et al. Nat. Phys. **6**, 448 (2010). Would it be possible frequency dependent?
  - Optical excitation of polarized carriers: spin current driven by spin-dependent vector potential ? (Wunderlich et al. NAt. Phys. **5**, 675 (2009))