Scattering approach to heat transport. Application to a *single*-electron emitter for chiral wave-guides

Michael V. Moskalets
NTU "KhPI", Kharkiv, Ukraine
michael.moskalets@gmail.com

21-27 October 2012, Cargèse, Corsica

Overview

- General scattering approach
    - Heat generation and heat pump effect
- Single-channel chiral emitter
    - Quantized emission regime : Energy of emitted particles understood from different perspectives
- Heat fluctuations and how to measure them
- Energy dissipated by the two-particle emitter and two emission regimes, adiabatic and non-adiabatic
- Conclusion

General formalism

Continuity equation for energy

$$H = -\frac{\hbar^2}{2m}\nabla^2 + U\,, \qquad i\hbar\frac{\partial}{\partial t}\Psi = H\Psi\,,$$

$$\langle E\rangle_V = \frac{1}{2}\int_V dV\left\{\Psi^* H\Psi + h.c.\right\},$$

$$\frac{\partial}{\partial t}\langle E\rangle_V + \int_\Sigma d\Sigma I^E = \int_V dV S^E\,,$$

energy current density

energy production

$$S^E = \mathrm{Re}\left\{\Psi^*\frac{\partial U}{\partial t}\Psi\right\},$$

J.S. Lim, D. Sanchez, M. M. (unpublished)

Energy current from wave function

energy current :

$$I^E = \frac{\hbar^2}{2m}\mathrm{Re}\left\{\Psi^*\frac{\partial\nabla\Psi}{\partial t} - \frac{\partial\Psi^*}{\partial t}\nabla\Psi\right\} ,$$

particle current :

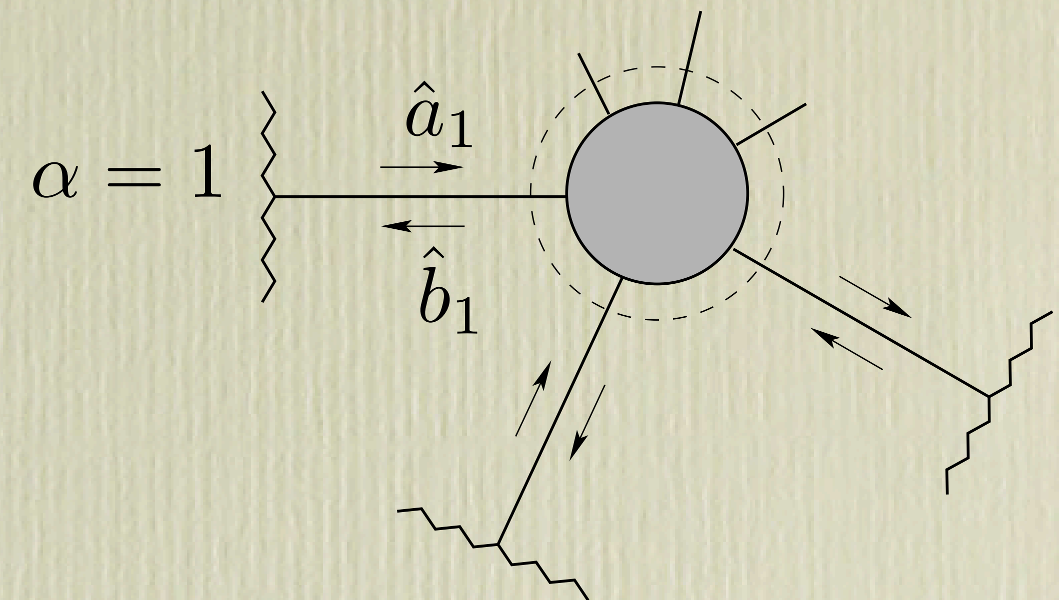
$$i\hbar\frac{\partial}{\partial t}$$

$$I = \frac{\hbar}{m}\mathrm{Im}\left\{\Psi^*\nabla\Psi\right\} ,$$

i) A. Brataas, Y. Tserkovnyak, and G. E. W. Bauer, Physical Review B 84, (2011),
ii) G. A. Levin, W. A. Jones, K. Walczak, and K. L. Yerkes, arXiv cond-mat.mes-hall, (2012),
(D. Bohm, Quantum Theory, Dover Pub. New York (1989) p. 89.)

Heat current in scattering language

$$\hat{\Psi}_\alpha = \frac{1}{\sqrt{2\pi}} \int_0^\infty dE\, e^{-i\frac{E}{\hbar}t} \left\{ \hat{a}_\alpha(E)\, \frac{\psi_\alpha^{(in)}(E)}{\sqrt{\hbar v_\alpha(E)}} + \hat{b}_\alpha(E)\, \frac{\psi_\alpha^{(out)}(E)}{\sqrt{\hbar v_\alpha(E)}} \right\},$$



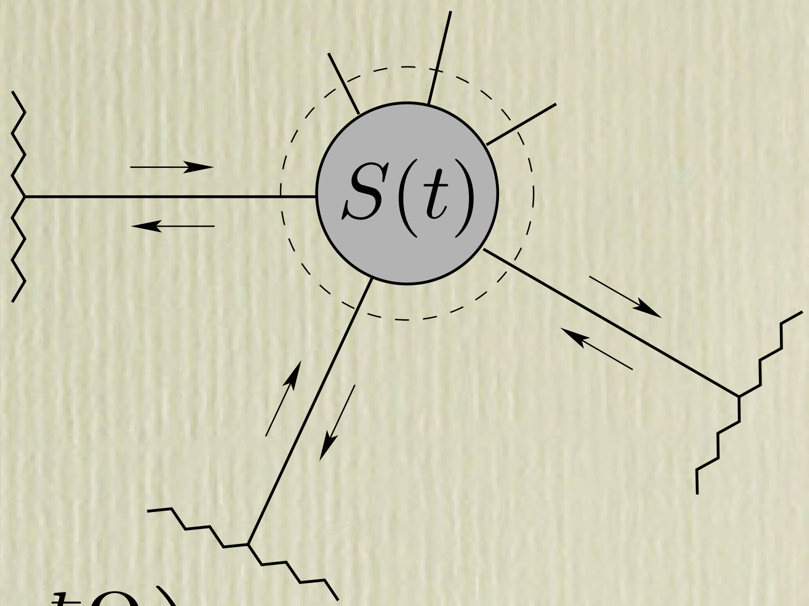
M. Büttiker, Physical Review B 45, 3807 (1992).

$$\hat{I}_\alpha^E(t) = \frac{1}{h} \iint dE\, dE'\, \frac{E+E'}{2} e^{i\frac{E-E'}{\hbar}t} \left\{ \hat{b}_\alpha^\dagger(E)\, \hat{b}_\alpha(E') - \hat{a}_\alpha^\dagger(E)\, \hat{a}_\alpha(E') \right\},$$

$$\hat{I}_\alpha(t) = \frac{e}{h} \iint dE\, dE'\, e^{i\frac{E-E'}{\hbar}t} \left\{ \hat{b}_\alpha^\dagger(E)\, \hat{b}_\alpha(E') - \hat{a}_\alpha^\dagger(E)\, \hat{a}_\alpha(E') \right\},$$

$$\hat{I}_\alpha^Q(t) = \frac{1}{h} \iint dE\, dE' \left\{ \frac{E+E'}{2} - \mu_\alpha \right\} e^{i\frac{E-E'}{\hbar}t} \left\{ \hat{b}_\alpha^\dagger(E)\, \hat{b}_\alpha(E') - \hat{a}_\alpha^\dagger(E)\, \hat{a}_\alpha(E') \right\},$$

Heat current in terms of Floquet



$$S(t) = S(t+\mathcal{T})\,,$$

$$\hat{b}_\alpha(E) = \sum_\beta \sum_n S_{F,\alpha\beta}(E, E_n)\hat{a}_\beta(E_n)\,,$$

$$E_n = E + n\hbar\Omega\,, \quad \Omega = 2\pi/\mathcal{T}\,,$$

$$I_\alpha^Q(t) = \frac{1}{h}\sum_{\ell=-\infty}^{\infty} e^{-i\ell\Omega t}\int dE\left(E - \mu_\alpha + \ell\frac{\hbar\Omega}{2}\right)$$

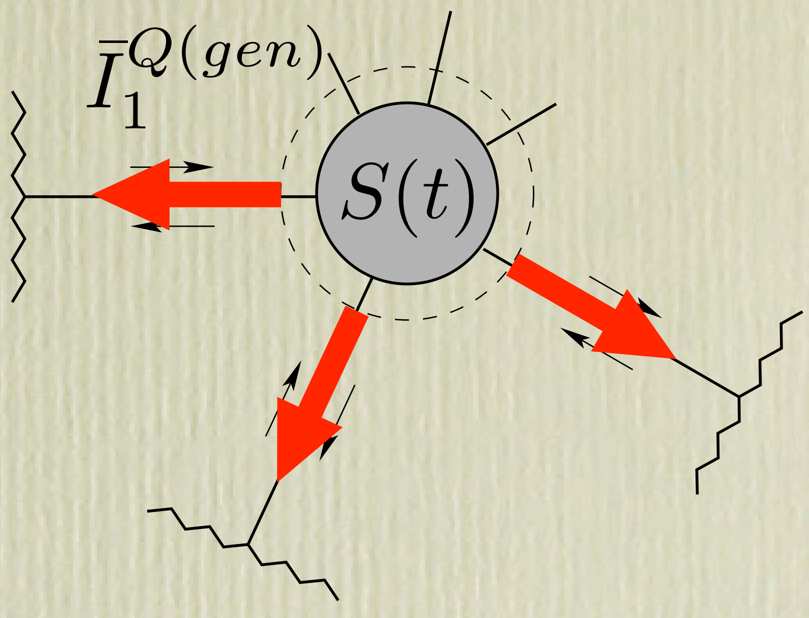
$$\times \sum_n \sum_\beta \left\{f_\beta\left(E_n\right) - f_\alpha(E)\right\} S_{F,\alpha\beta}^*\left(E,\, E_n\right)\, S_{F,\alpha\beta}\left(E_\ell,\, E_n\right)\,,$$

$$\bar{I}_\alpha^Q = \frac{1}{h}\int dE\,(E-\mu_\alpha)\sum_n\sum_\beta \left\{f_\beta\left(E_n\right) - f_\alpha(E)\right\}\left|S_{F,\alpha\beta}\left(E,\, E_n\right)\right|^2\,,$$

a DC heat is in: M. M. and M. Büttiker, Physical Review B 66, 205320 (2002).

Heat generation

pumping set-up : $f_\alpha = f_0\,, \quad \forall\alpha\,,$



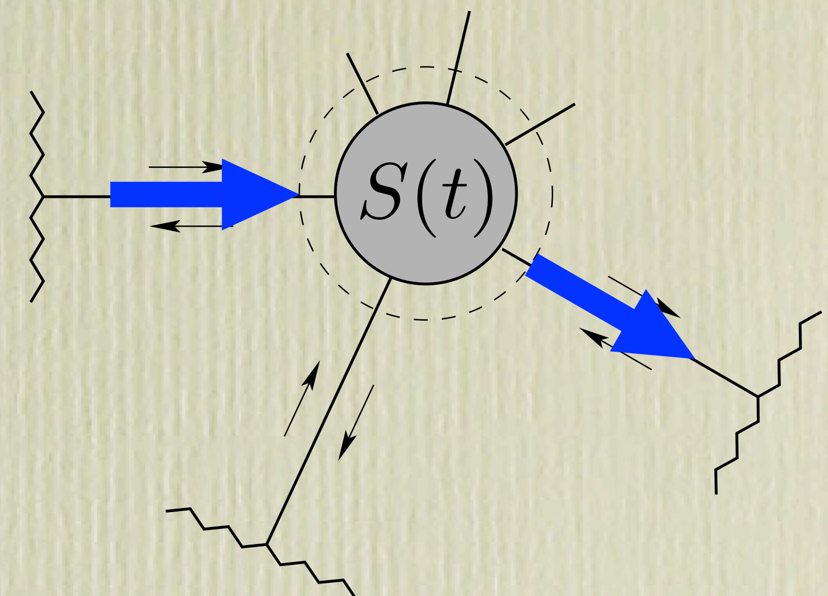
generated DC heat :

$$\bar{I}^Q_{tot} \equiv \sum_{\alpha=1}^{N_r} \bar{I}^Q_\alpha = \frac{\Omega}{2\pi}\int_0^\infty dE\, f_0(E) \sum_{n=-\infty}^{\infty} n \sum_{\alpha=1}^{N_r}\sum_{\beta=1}^{N_r} |S_{F,\alpha\beta}(E_n,E)|^2 \neq 0\,,$$

while no generated charge ! $\bar{I}_{tot} \equiv \sum_{\alpha=1}^{N_r} \bar{I}_\alpha = 0\,,$

L. Arrachea, M.M. Energy transport and heat production in quantum engines, in Handbook of Nanophysics: Nanomedicine and Nanorobotics, ed. by Klaus D. Sattler (CRC Press, Taylor & Francis Group), (2010).

Pumped heat



transferred heat :

$$I_\alpha^{Q(pump)} = I_\alpha^Q - I_\alpha^{Q(gen)} \,,$$

$$I_\alpha^{Q(pump)} = \frac{1}{h} \int\limits_0^\infty dE (E - \mu_0)\, f_0(E) \left\{ \sum_{n=-\infty}^{\infty} \sum_{\beta=1}^{N_r} |S_{F,\alpha\beta}(E_n, E)|^2 - 1 \right\} \,,$$

$$\sum_{\alpha=1}^{N_r} I_\alpha^{Q(pump)} = 0 \,,$$

L. Arrachea, M.M. Energy transport and heat production in quantum engines, in Handbook of Nanophysics: Nanomedicine and Nanorobotics, ed. by Klaus D. Sattler (CRC Press, Taylor & Francis Group), (2010).

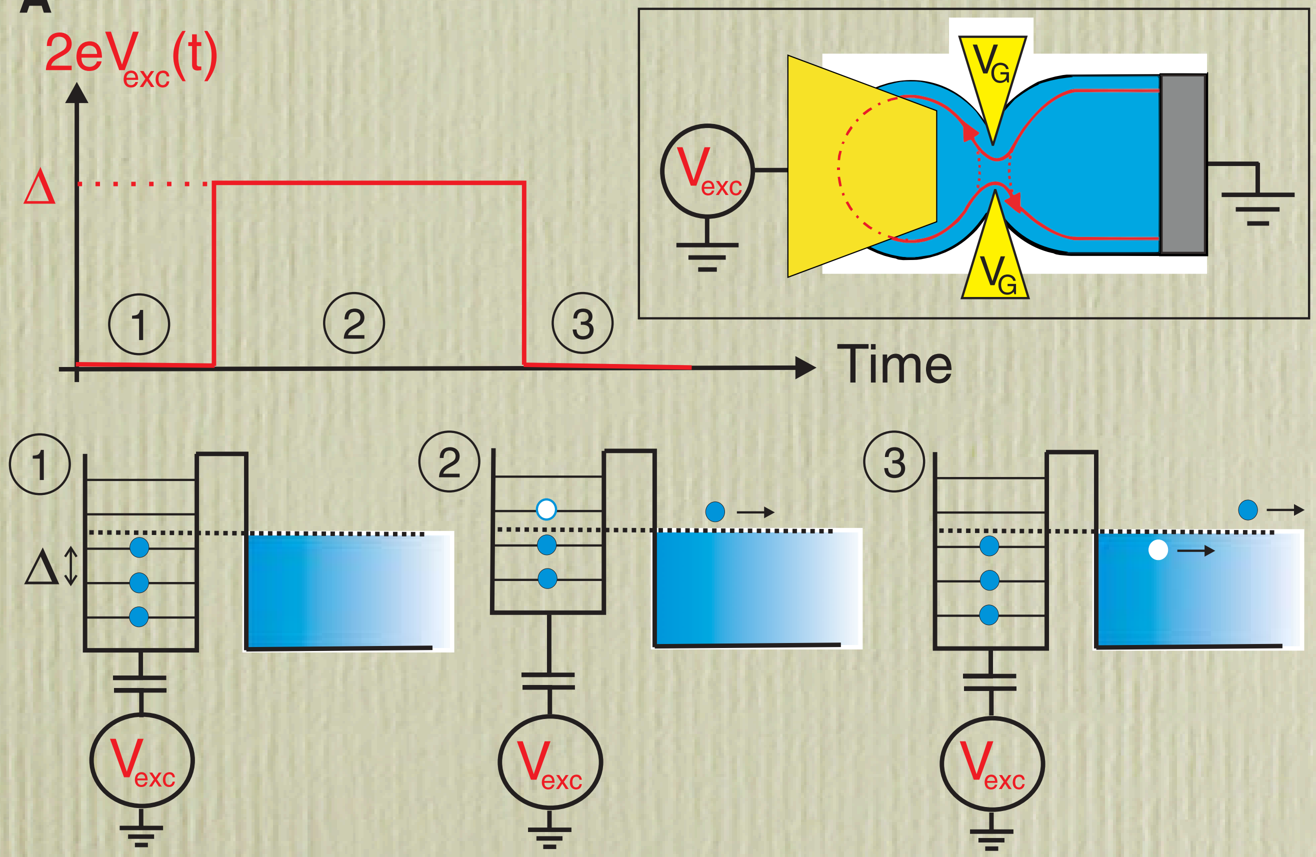
Applications :
Single-channel chiral emitter

Motivation

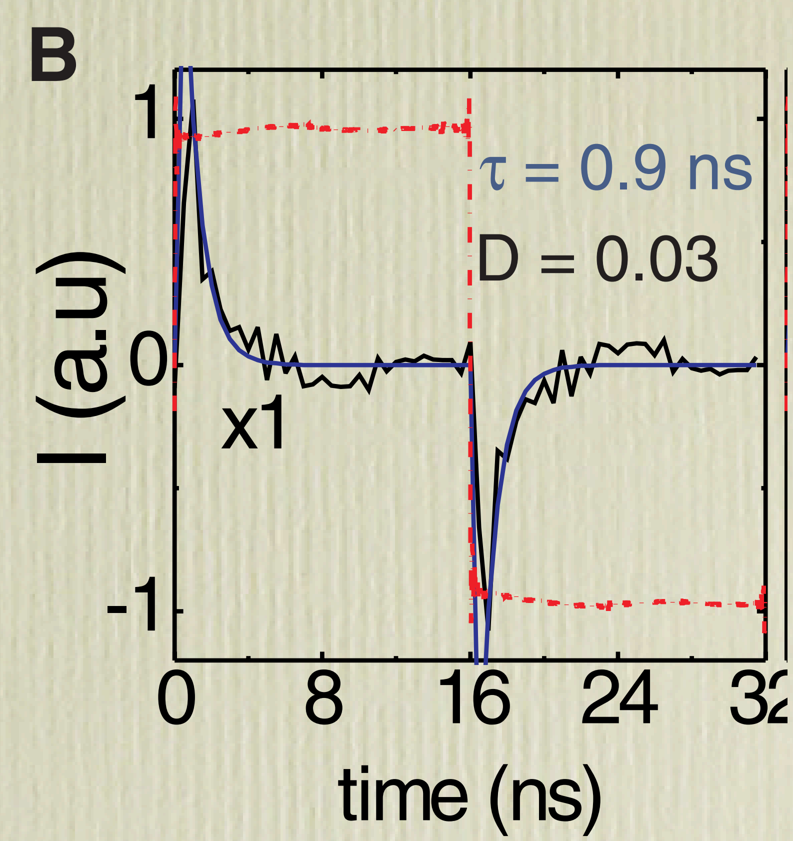
Experiment

$$\Delta + e^2/C \approx \Delta \approx 2.5K$$

A



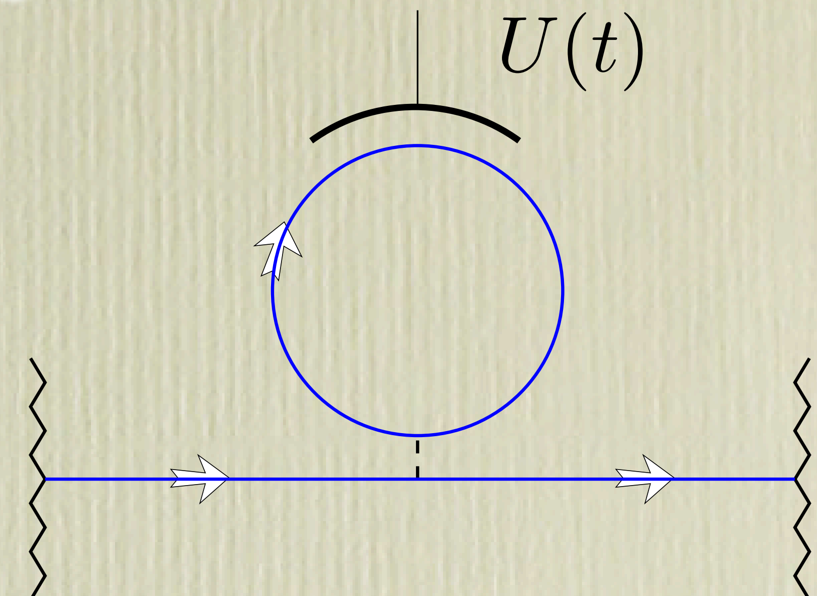
B

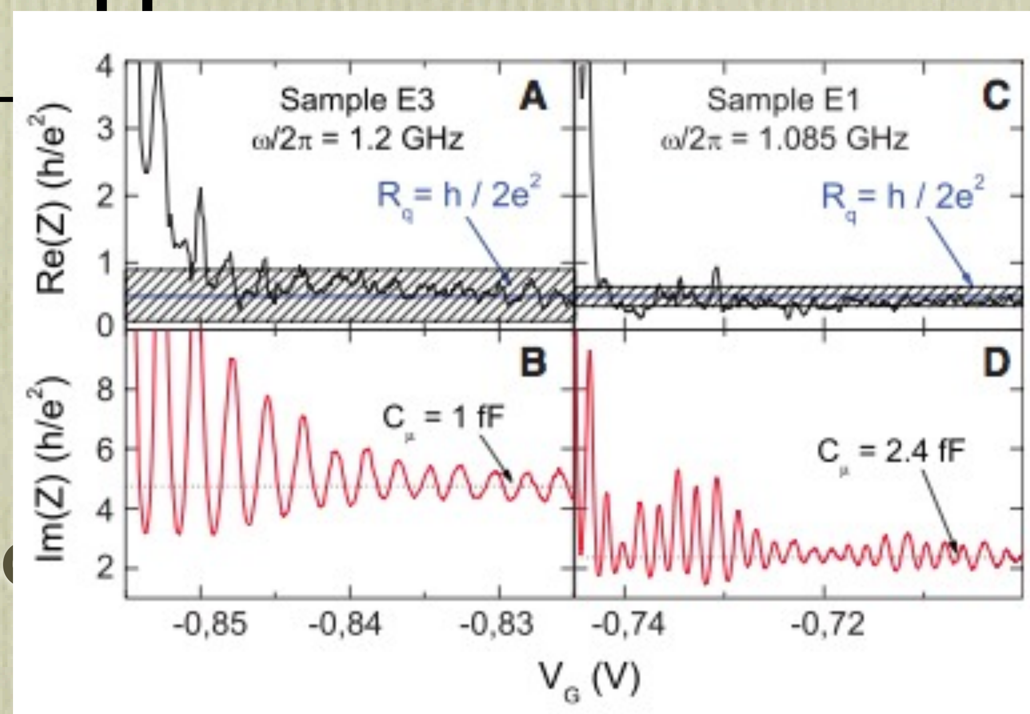


G. Fève, A. Mahé, J.-M. Berroir, T. Kontos, B. Plaçais, D. C. Glattli, A. Cavanna, B. Etienne, Y. Jin, Science 316, 1169 (2007)

Equivalent (low frequency) electric circuit

no DC charge current :





a linear respons

$$C_q = \ e^2\nu(\mu_0)\,, \quad R_q = \frac{h}{2e^2}\,,$$

charge relaxation resistance quantum

Predicted: M. Büttiker, H. Thomas, and A. Prêtre, Physics Letters A 180, 364 (1993).

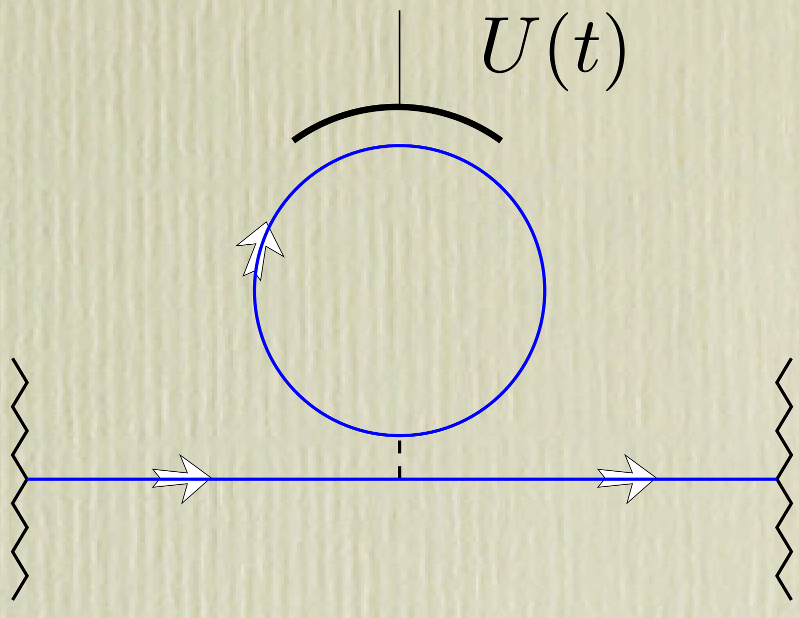
Confirmed: J. Gabelli, G. Fève, J.-M. Berroir, B. Plaçais, A. Cavanna, B. Etienne, Y. Jin, and D. Glattli, Science 313, 496 (2006).

Theory

Floquet scattering amplitude

(of a Fabry-Pérot type)

$$S_F(E_n, E) = \int_0^{\mathcal{T}} \frac{dt}{\mathcal{T}} e^{in\Omega_0 t} S_{in}(t, E)\,,$$



$$S_{in}(t, E) = \sum_{j=0}^{\infty} A_j(t),$$

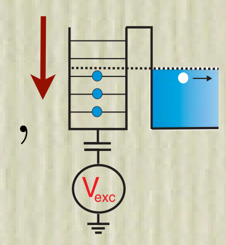
$$A_j = \frac{\bar{t}^2}{r}\left(re^{ikL}\right)^j e^{-\frac{ie}{\hbar}\int\limits_{t-j\tau}^{t} dt' U(t')}$$

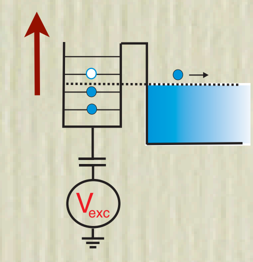
M.M., P. Samuelsson, and M. Büttiker, PRL **100**, 0866012 (2008)

Quantized adiabatic regime :

$$U(t) = U\cos(\Omega t + \varphi)$$

$$S(t,\mu) = e^{i\theta_r}\begin{cases} \dfrac{t - t^{(+)} - i\Gamma_\tau}{t - t^{(+)} + i\Gamma_\tau}\,, & \left|t - t^{(+)}\right| \lesssim \Gamma_\tau\,, \\[2ex] \dfrac{t - t^{(-)} + i\Gamma_\tau}{t - t^{(-)} - i\Gamma_\tau}\,, & \left|t - t^{(-)}\right| \lesssim \Gamma_\tau\,, \\[2ex] 1\,, & \left|t - t^{(\mp)}\right| \gg \Gamma_\tau\,, \end{cases}$$



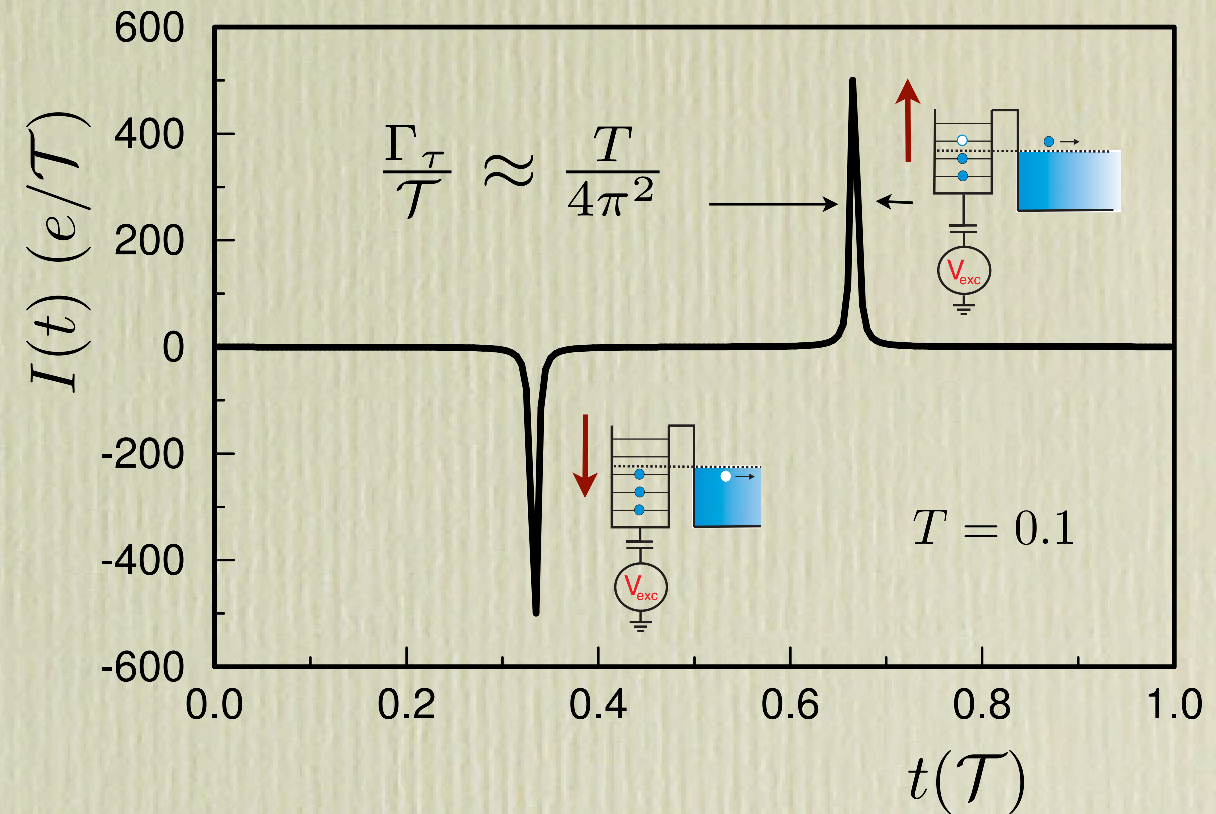


$$\Gamma_\tau = \frac{\delta}{dU/dt} = \frac{T\Delta}{4\pi}\,\frac{1}{dU/dt}\,,$$

S. Ol'khovskaya, J. Splettstoesser, M.M., and M. Büttiker, PRL **101**, 166802 (2008)

Zero temperature charge current

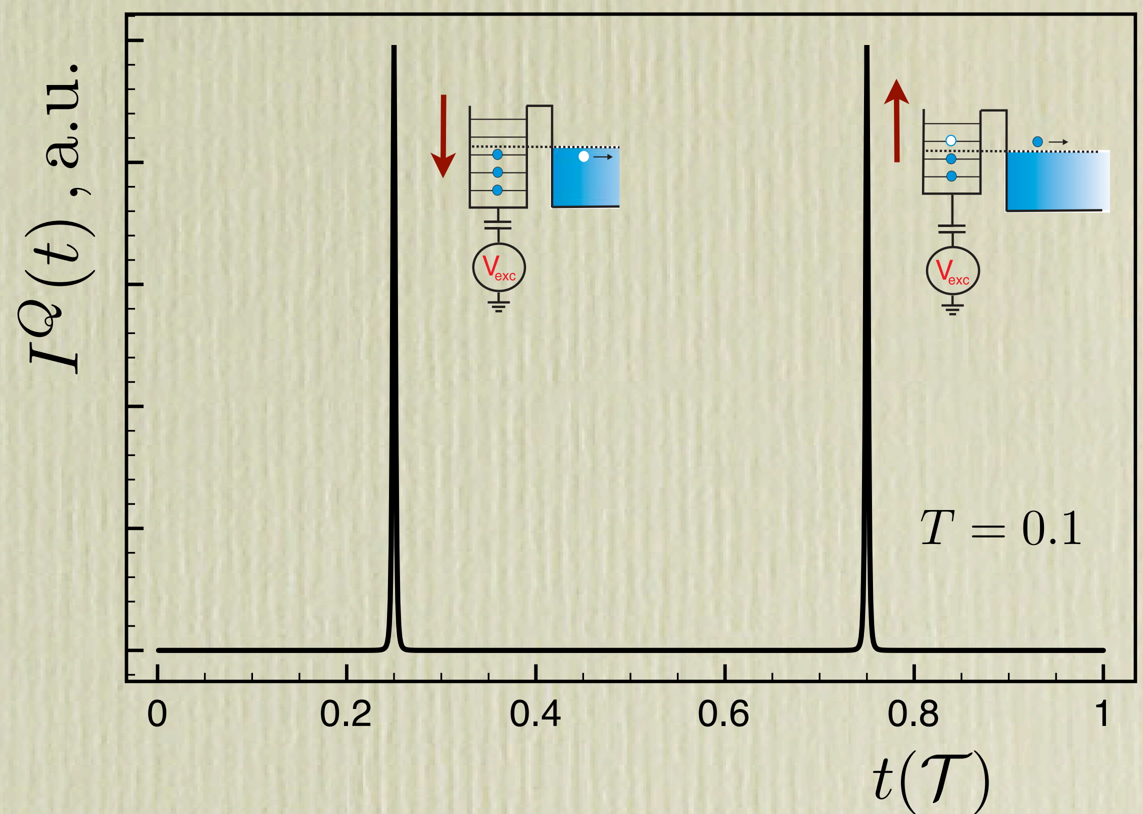
$$I(t) = \frac{e}{\pi}\left\{\frac{\Gamma_\tau}{\left(t - t_0^{(-)}\right)^2 + \Gamma_\tau^2} - \frac{\Gamma_\tau}{\left(t - t_0^{(+)}\right)^2 + \Gamma_\tau^2}\right\}$$



S. Ol'khovskaya, J. Splettstoesser, M.M., and M. Büttiker, PRL **101**, 166802 (2008)

Zero temperature heat current

$$I^Q(t) = \frac{\hbar}{2\Gamma_\tau} \sum_{\alpha=+,-} \frac{2\Gamma_\tau^3/\pi}{\left(\left(t - t^{(\alpha)}\right)^2 + \Gamma_\tau^2\right)^2},$$



$$I^Q(t) = \frac{I(t)}{e}\,\epsilon(t)\,,$$

$$\epsilon(t) = \frac{\hbar\Gamma}{(t - t^{(\alpha)})^2 + \Gamma^2} \quad \Rightarrow \quad \epsilon(t^{(\alpha)}) = 2\bar{\epsilon}\,,$$

a work done by the external force depending on emission time
(an energy of a particle emitted at a given instant of time)

J.S. Lim, D. Sanchez, M.M. (unpublished)

DC heat and particle's energy

the energy of a single-particle

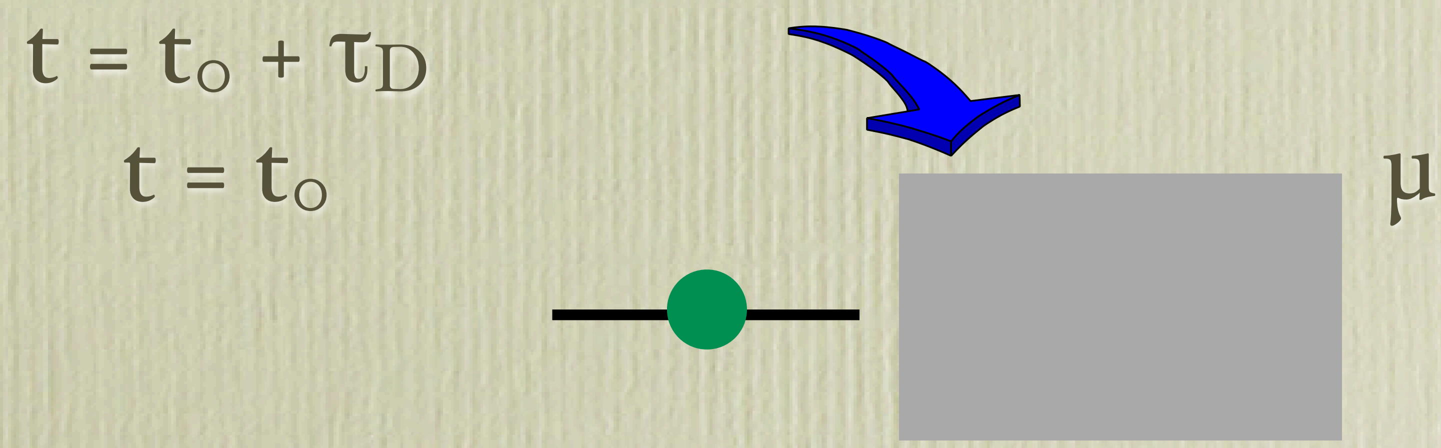
$$\mathcal{T}\bar{I}^{Q} = 2\frac{\hbar}{2\Gamma_{\tau}}\,,$$

the number of particles emitted per period

M. M. and M. Büttiker, Physical Review B **80**, 081302(R) (2009).

Particle's energy from the work done

$$\frac{\hbar}{2\Gamma_\tau} = \tau_D e \frac{dU}{dt} \, ,$$



Particle's energy from the distribution function

$$f(\mu+\epsilon) = \langle b^\dagger(\mu+\epsilon) b(\mu+\epsilon)\rangle = 2\Omega\Gamma_\tau e^{-\frac{2\Gamma_\tau}{\hbar}\epsilon}\,,$$ (Poisson distribution)

mean energy : $$\langle\epsilon\rangle = \frac{\int_0^\infty d\epsilon\,\epsilon\, f(\epsilon)}{\int_0^\infty d\epsilon\, f(\epsilon)} = \frac{\hbar}{2\Gamma_\tau}\,,$$

fluctuations :

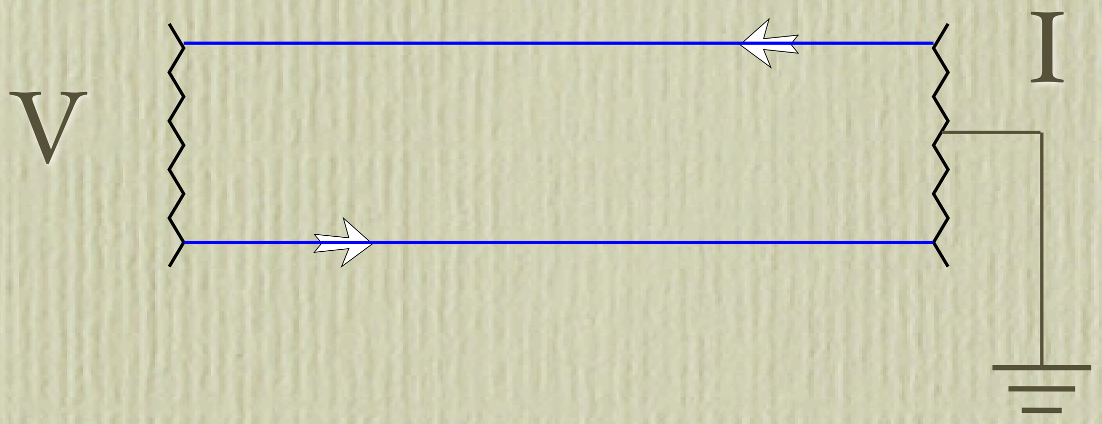
Fluctuations in energy of emitted particles result in fluctuations of a heat flux

$$\left\langle\delta^2\epsilon\right\rangle = \left\langle\epsilon^2\right\rangle - \left\langle\epsilon\right\rangle^2 = \left\langle\epsilon\right\rangle^2\,,$$

F. Battista and P. Samuelsson, M. Albert, M.M. (unpublished)

Heat fluctuations and how to measure them

Paradigmatic example : a biased Hall bar



Charge noise
Charge current
(zero frequency)

Heat noise
Heat flow
(zero frequency)

A probabilistic absorption of energy from a time-dependent field results in fluctuations of an energy flow even if the particle flow is regular

$R_q I^2$,

V(t) :

$\frac{e^2}{h} V(t)$,

$R_q I^2$, $\frac{h}{T} I_Q$,

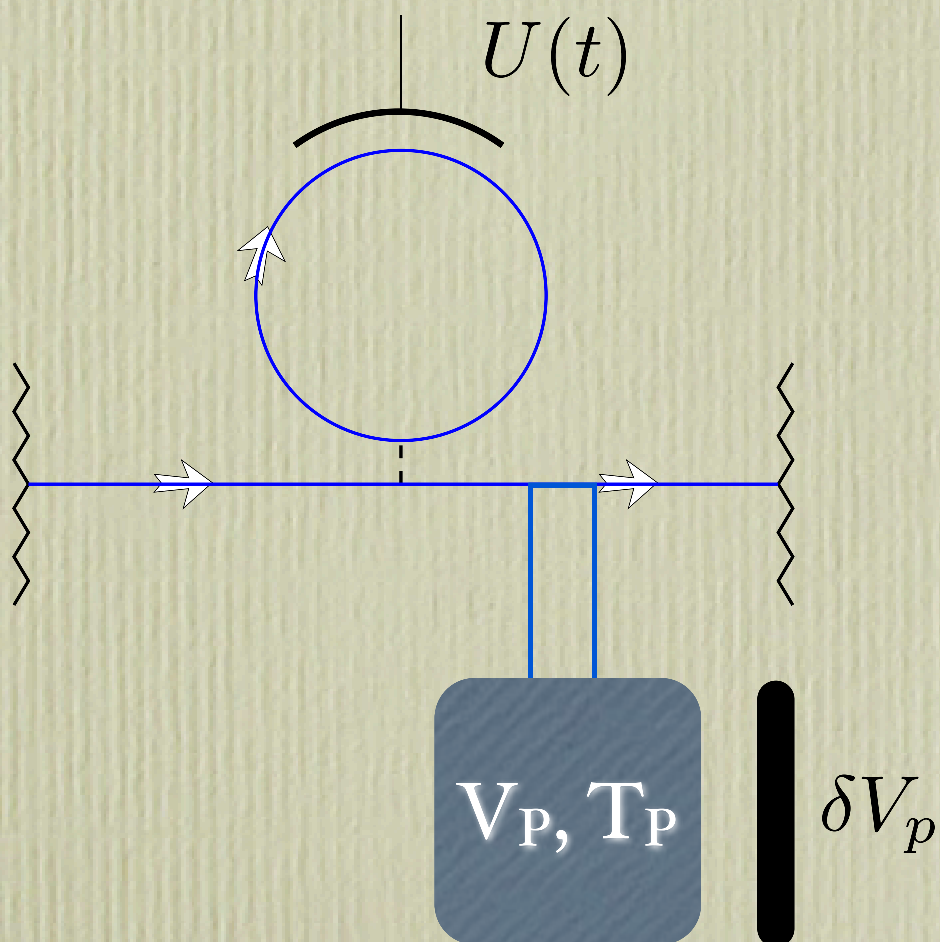
(a shot-like heat noise)

F. Battista and P. Samuelsson, M. Albert, M.M. (unpublished)

How to measure the energy fluctuations

SES generates a noiseless particle flow but noisy heat flow

(like an AC voltage does)



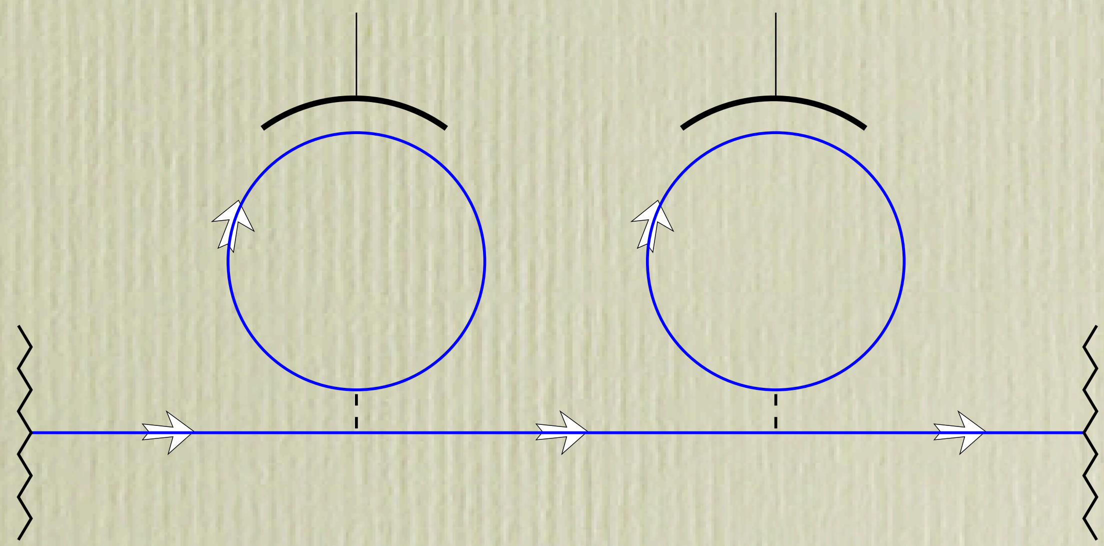
voltmeter & thermometer

$$\left\langle (\delta T_P)^2 \right\rangle \sim \left\langle (\delta \epsilon)^2 \right\rangle ,$$

$$\left\langle (\delta V_P)^4 \right\rangle \sim \left\langle (\delta T_P)^2 \right\rangle ,$$

F. Battista and P. Samuelsson, M. Albert, M.M. (unpublished)

Two-particle emitter



J. Splettstoesser, S. Ol'khovskaya, M.M, and M. Büttiker, Physical Review B **78**, 205110 (2008).

Heat is not additive :

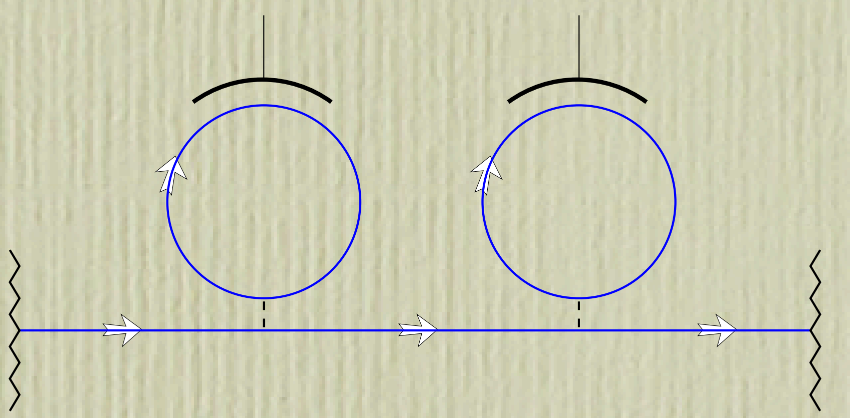
First, why it should be additive (U is constant, N is additive):

$$I^Q = dA/dt = UI = UedN/dt \,,$$

Second, heat can be non-additive if the particles overlap and, as a result of the Pauli principle, their energies become interrelated: (effective eU for different particles becomes different).

M. M. and M. Büttiker, Physical Review B **80**, 081302(R) (2009).

Utilizing the Joule-Lenz law: $I^Q \sim I^2$



separate emission of particles : heat is additive

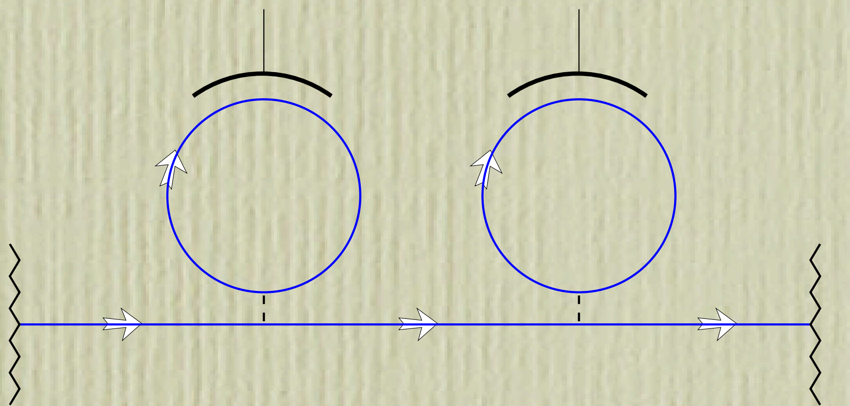
$$I_2(t) = I(t - t_{0L}) + I(t - t_{0R})\,,$$

$$I_2^2(t) = I^2(t - t_{0L}) + I^2(t - t_{0R}) \quad \rightarrow \quad I_2^Q(t) = I_L^Q + I_R^Q\,,$$

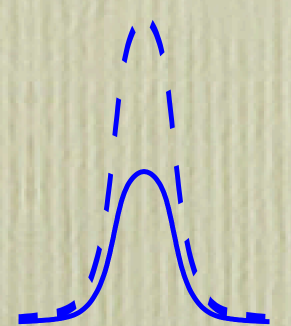
$$E_2 = E_L + E_R = 2\frac{\hbar}{2\Gamma_\tau}\,,$$

M. M. and M. Büttiker, Physical Review B **80**, 081302(R) (2009).

Utilizing the Joule-Lenz law: $I^Q \sim I^2$



e⁻e pair emission : heat is enhanced



$$I_2(t) = 2I_1(t) \quad \rightarrow \quad I_2^Q = 4I_1^Q\,,$$

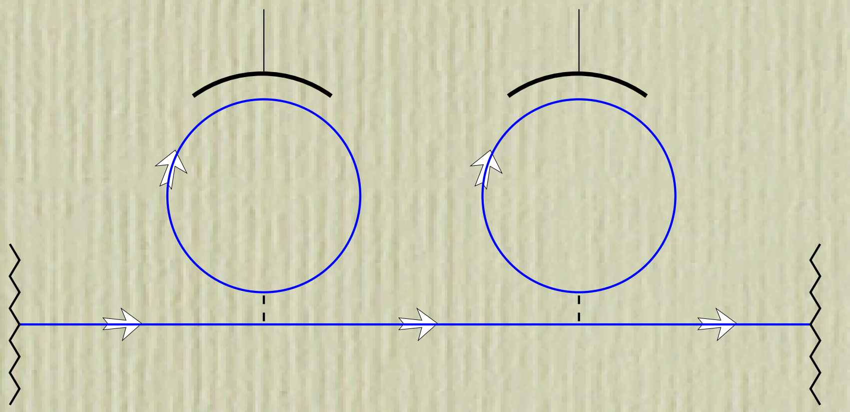
$$E_2 = 4\frac{\hbar}{2\Gamma_\tau} > E_L + E_R\,,$$

$$\frac{\hbar}{2\Gamma_\tau}\,,\, 3\frac{\hbar}{2\Gamma_\tau}\,,$$

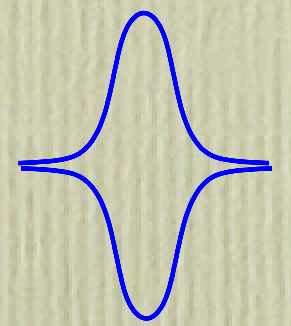
The second particle has enhanced energy
(the Pauli principle requires this)

M. M. and M. Büttiker, Physical Review B **80**, 081302(R) (2009).

Paradoxically an energy carried by an e-h pair is sensitive to the regime of emission



adiabatic regime : heat is suppressed



$$I_2(t) = 0 \quad \to \quad I_2^Q = 0\,,$$

$$E_2 = 0\,,$$

What emitted by the first source is reabsorbed by the second source !
(two positive energies "add up" to zero!)

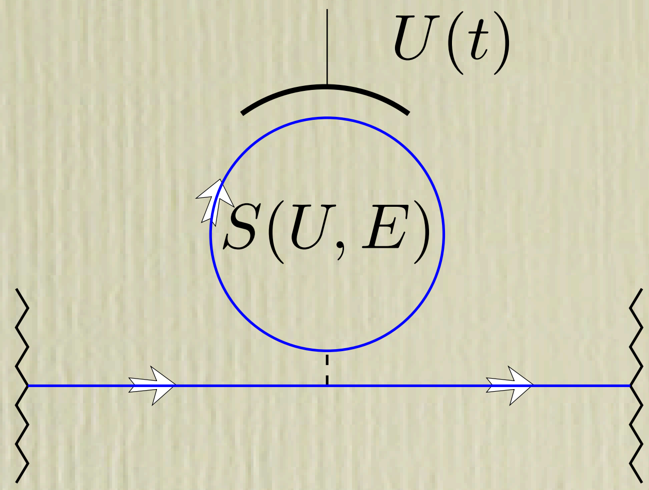
non-adiabatic regime : heat is additive

$$E_2 = E_L + E_R = 2\frac{\hbar}{2\Gamma_\tau}\,,$$

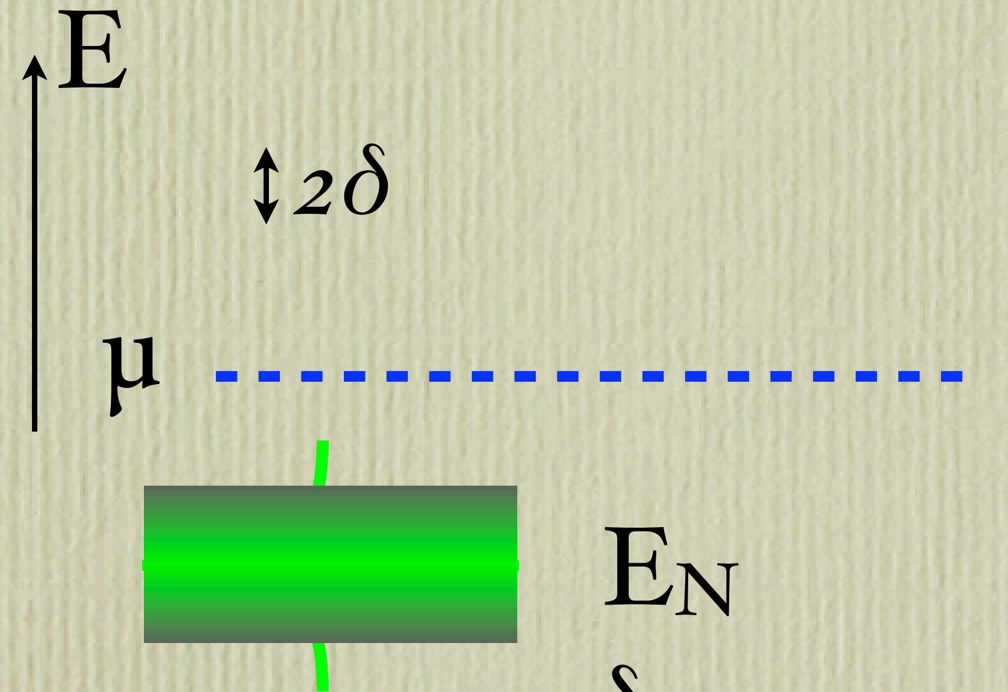
M. M. and M. Büttiker, Physical Review B **80**, 081302(R) (2009).

Adiabatic versus non-adiabatic

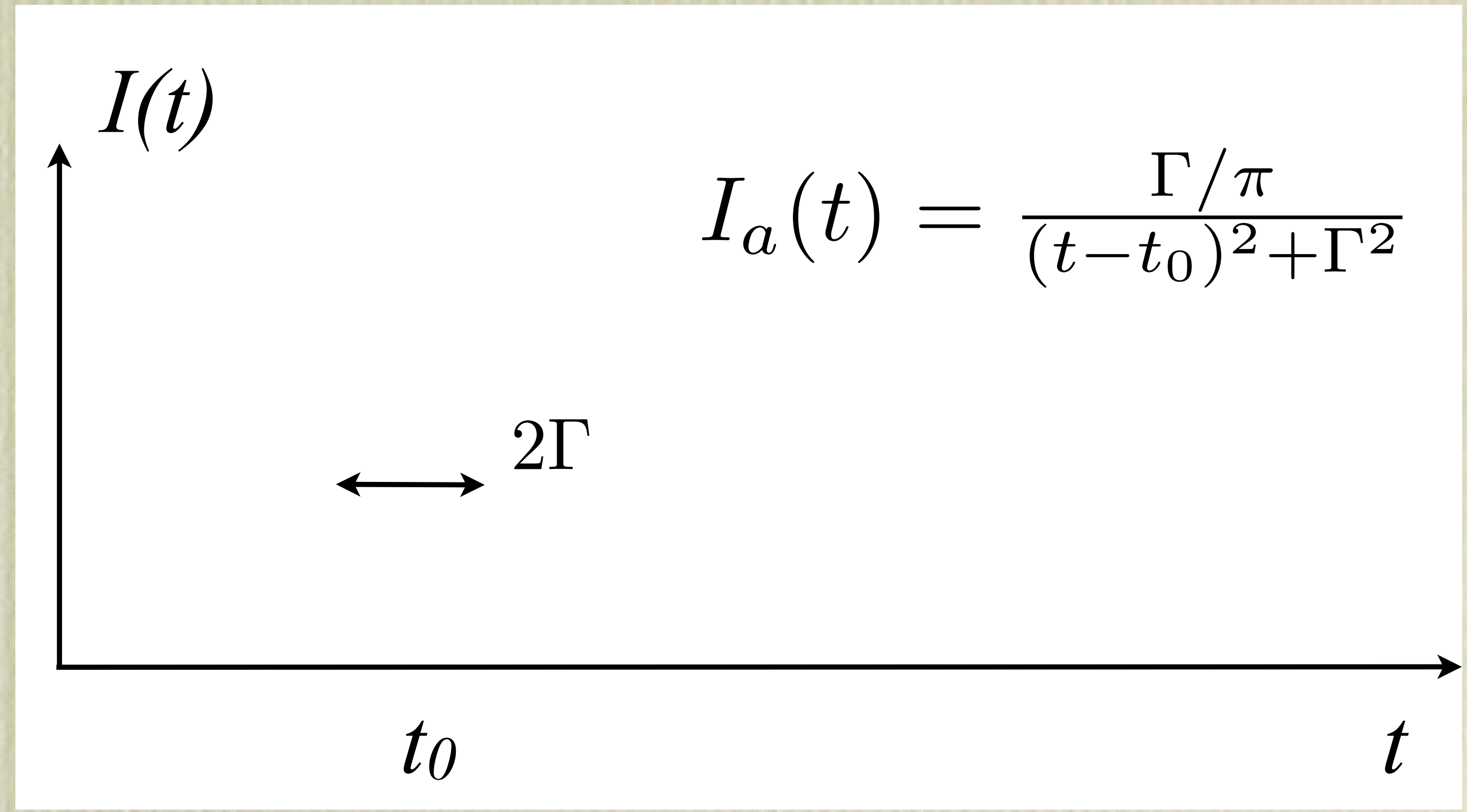
Emission : adiabatic



$$U(t) = U_o \cos(\Omega t)$$

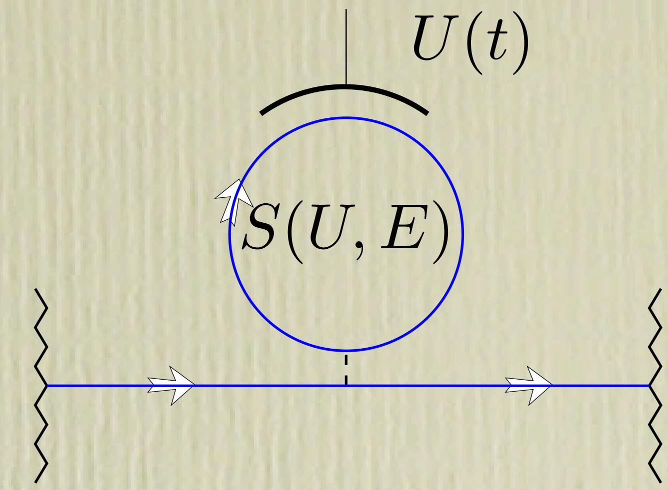


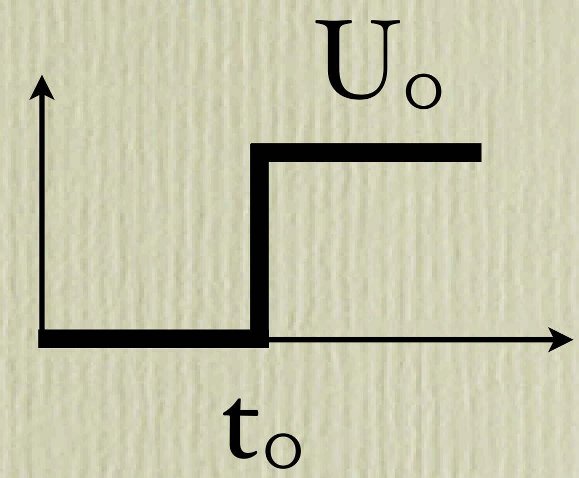
$$\Gamma = \frac{\delta}{dU/dt},$$

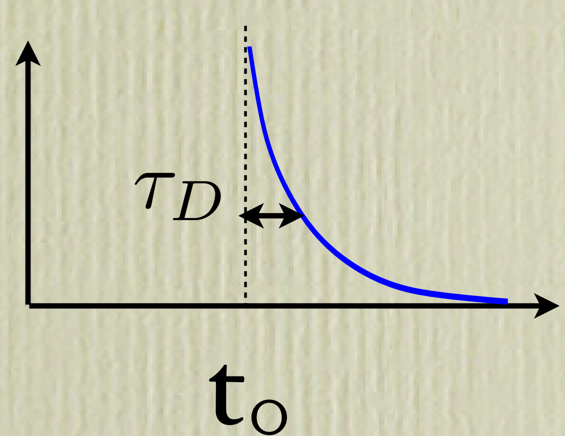


Adiabatic emission condition : $\Gamma \ll \tau_D$

Emission : non-adiabatic

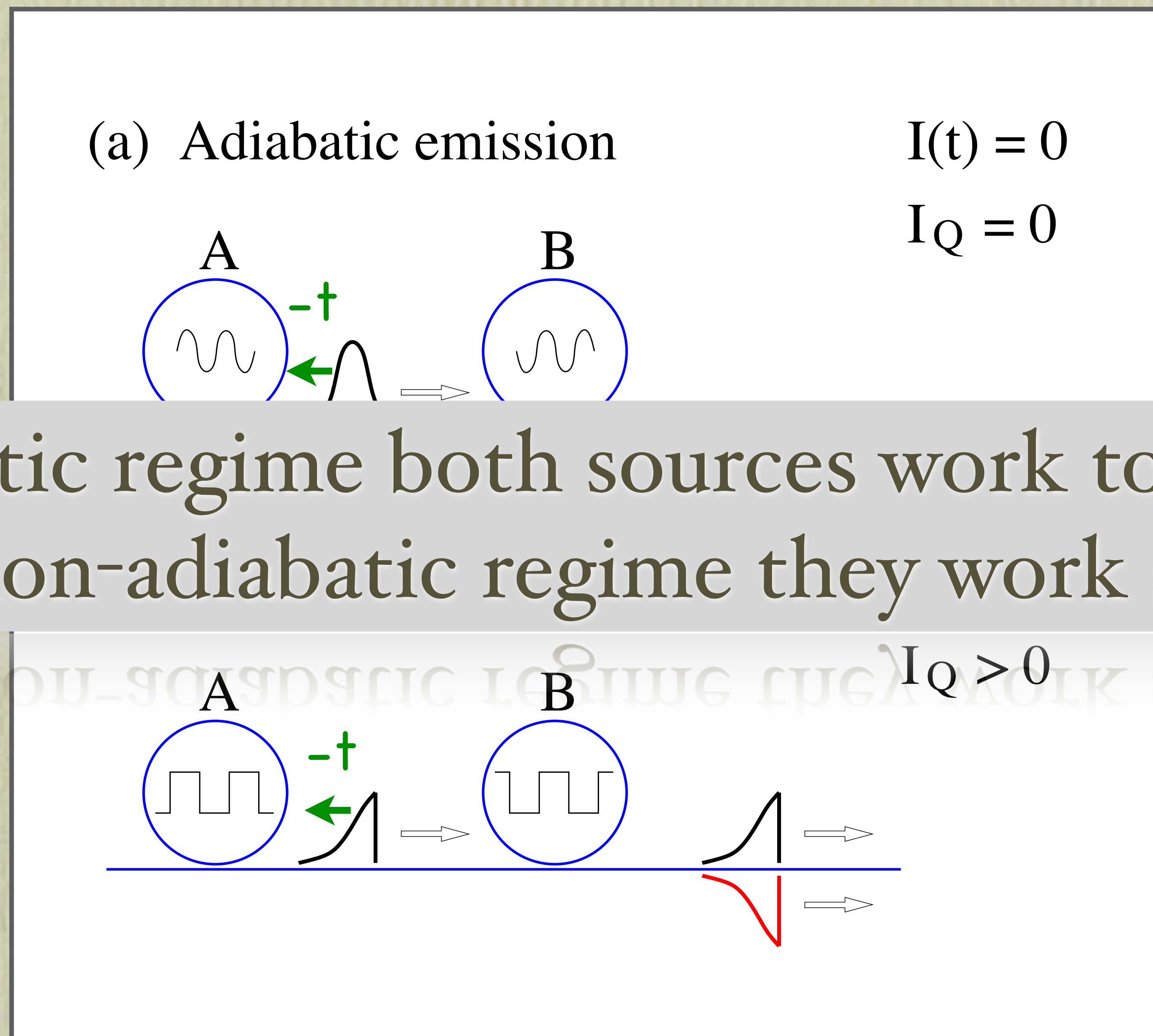






Non-adiabatic emission condition : $\frac{dU}{dt} \gg \frac{U_0}{\tau_D}$,

Reabsorption as due to the TRS



In an adiabatic regime both sources work together.
While in a non-adiabatic regime they work separately.

G. Haack, M. M. and M. Büttiker, (unpublished)

Conclusion

- We presented the scattering matrix approach to a time-dependent heat transport in mesoscopic dynamic structures.
  - There are two generic effects : Heat generation and heat pump effect.
- Probabilistic absorption of energy from a dynamical source results in heat fluctuations even if the particle flow is regular.
- Heat current is not additive (unlike a charge current). We analyzed heat generation in a two-particle source and found
  - in adiabatic regime : heat suppression at e-h emission,
  - in both regimes : heat increase at e-e (h-h) emission,
  - otherwise heat is additive.

People with whom I collaborated

Liliana Arrachea,
Universidad de Buenos Aires

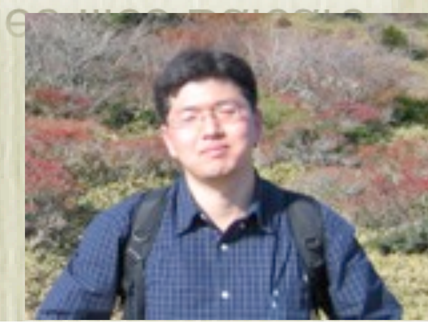
Géraldine Haack,
Freie Universität Berlin

David Sanchez,
IFISC, Universitat de les Illes Balears

Janine Splettstoesser,
ITSP, RWTH Aachen

Markus Büttiker,
Université de Genève

Peter Samuelsson,
Lunds universitet



Jong Soo Lim,
IFISC, Universitat de les Illes Balears

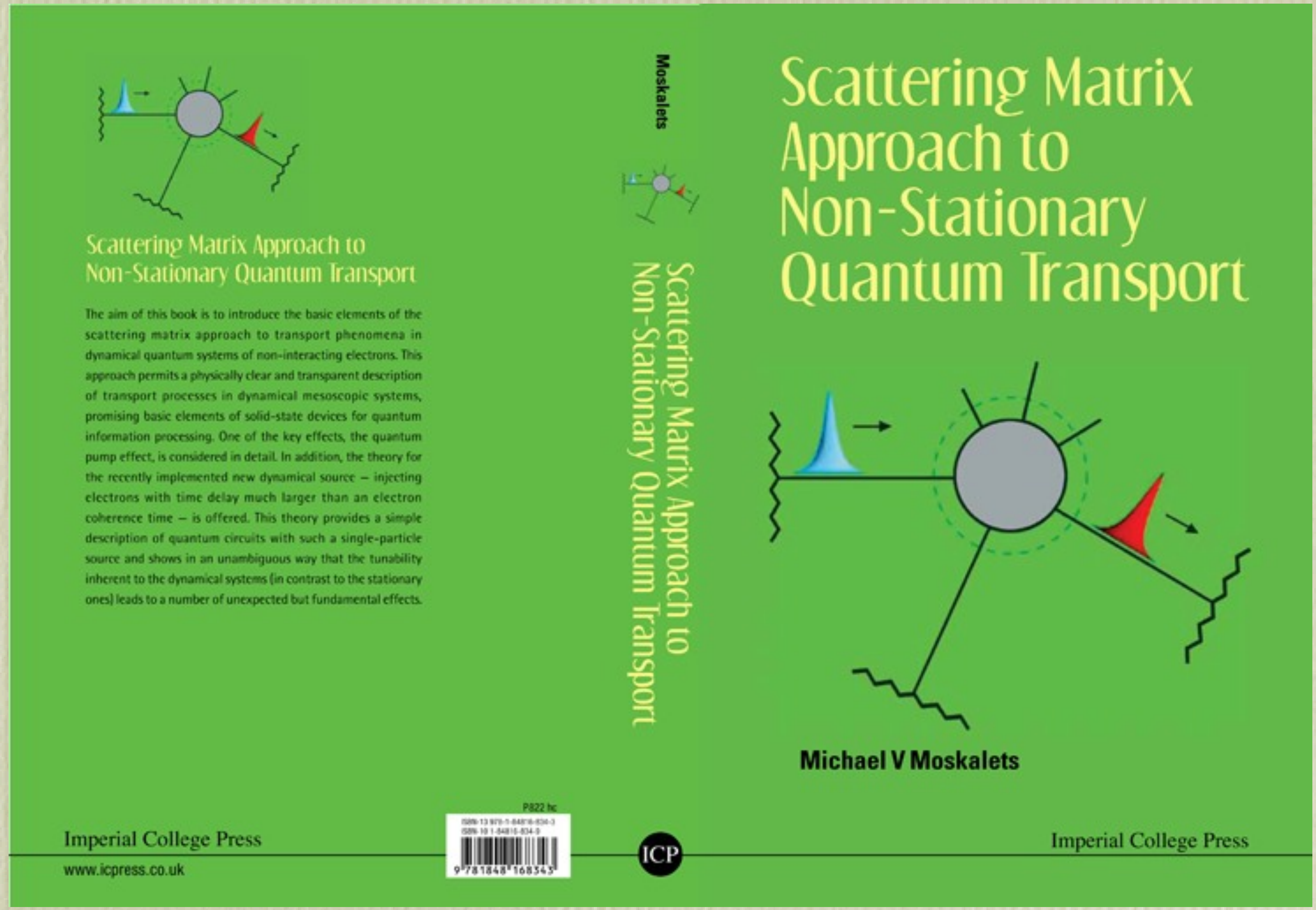
Mathias Albert,
LPS, Université Paris-Sud 11

No photo :(

Francesca Battista,
Lunds universitet

Sveta Ol'khovskaya,
NTU "KhPI", Kharkiv, Ukraine

Thank You !



http://www.worldscientific.com/worldscibooks/10.1142/p822