

Nanoscale pattern formation at surfaces under ion-beam sputtering: a perspective from continuum models

Rodolfo Cuerno

Departamento de Matemáticas &
Grupo Interdisciplinar de Sistemas Complejos (GISC)
Universidad Carlos III de Madrid

cuerdo@math.uc3m.es <http://gisc.uc3m.es/~cuerno>



Sketch of the talk

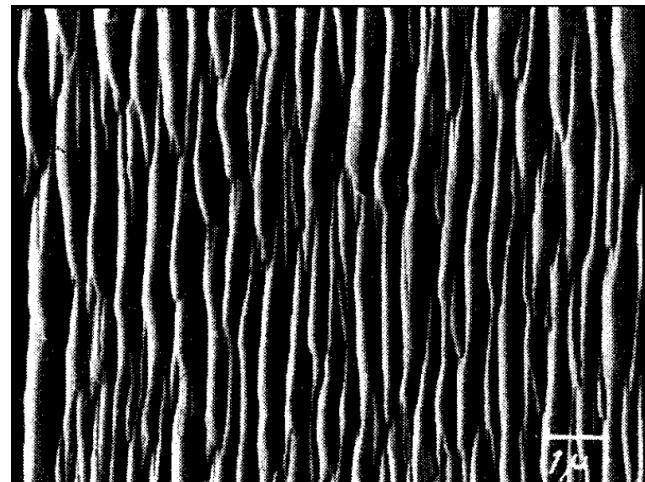
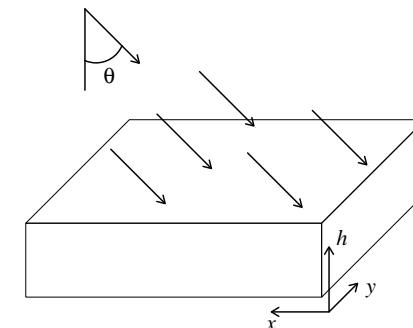
- Introduction
 - Surface nanostructuring by ion-beam sputtering (IBS)
- Pattern Formation at the Nanoscale
 - Continuum descriptions
 - “Toy” model of IBS
 - Related systems
- Recent developments
- Conclusions/Outlook

Surface nanopatterns by IBS

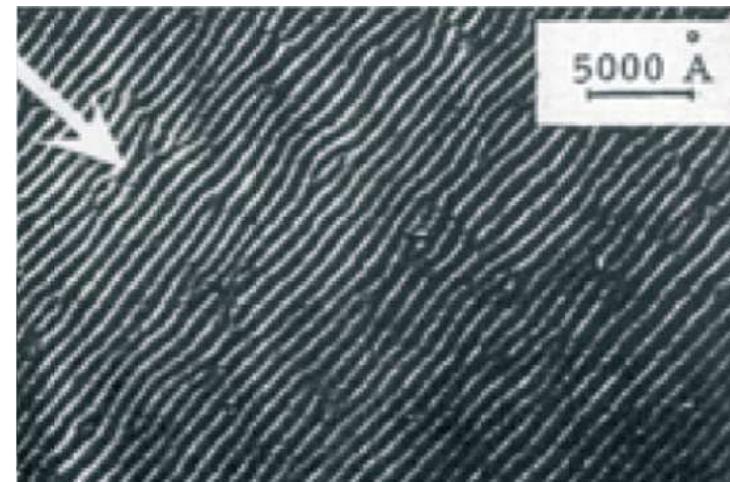
Consider amorphous/amorphizable targets and
low to intermediate energies $E = 200 \text{ eV} - 10 \text{ keV}$

Arbitrary angle of incidence

Singly charged ions



R. L. Cunningham et al. JAP '60:
8 keV Ar⁺ / polycrys. Au



M. Navez et al. CRAS '62:
4 keV air onto glass



IBS associated with “Nanotechnology” since day one

On the Basic Concept of 'Nano-Technology'

Norio TANIGUCHI
Tokyo Science University
Noda-shi, Chiba-ken, 278 Japan

Abstract

'Nano-technology' is the production technology to get the extra high accuracy and ultra fine dimensions, i.e. the preciseness and fineness of the order of 1 nm (nanometer), 10^{-9} m in length. The name of 'Nano-technology' originates from this nanometer. In the processing

In the present paper, the basic concept of 'Nano-technology' in materials processing is discussed on the basis of microscopic behaviour of materials and as a result, the ion sputter-machining is introduced as the most promising process for the technology.



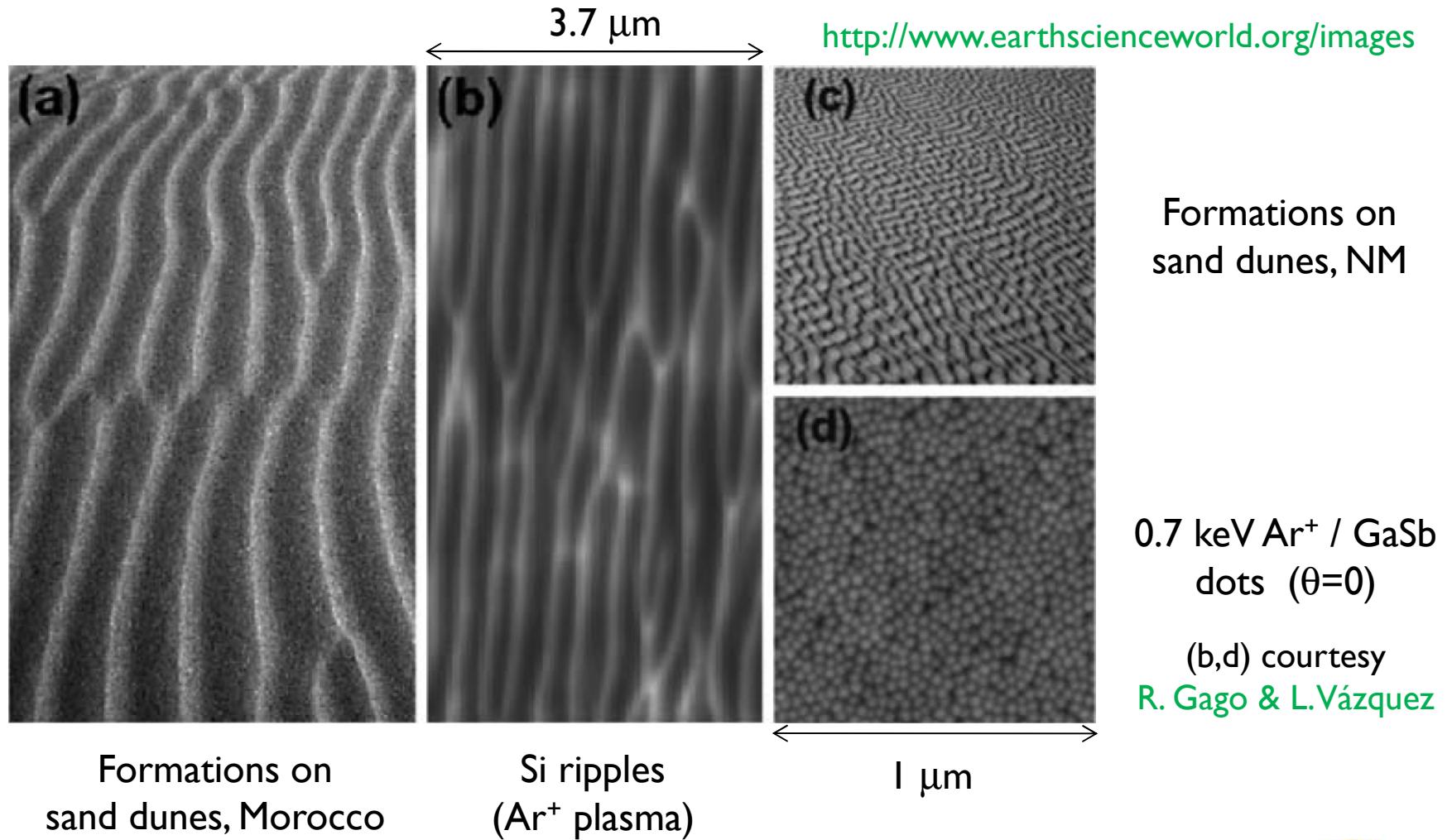
IBS associated with “Nanotechnology” since day one

Table 1. Kinds of materials processing by one atom or one molecule

mechanism	kinds of processing
(removing) separation processes	chemical decomposition chemical etching (photo-etching), chemical polishing
	electro-chemical decomposition electrolytic polishing, electro-chemical machining
	evaporation , dissolution (thermal) electron beam machining, laser ray machining, electro discharge mach., dissolution mach.,
	sputtering (dynamical) ion sputter machining
(accreting, joining) consolidation processes	physical and thermal accretion vapour deposition, sputter deposition, ionic deposition
	chemical accretion chemical plating or deposition
	electro-lytic accretion electro-plating, electrocasting
	chemical and electro-chemical composition thin film (anodic oxidation, oxidation , nitration; gaseous, liquid)
	implantation (dynamical) ion implantation
	diffusion (thermal) surface treatment, sintering
	crystal growth (thermal) epitaxial, molecular beam
(flow) deform. proc.	fusion (thermal) dip plating, thermal fusion
	surface flow (thermal) flow finishing(gas flame, high frequency, heat ray, electron beam , laser ray)
	viscous flow and abrasion (dynamical) flow finishing(vibration sliding,liquid, gas)

Surface nanopatterns by IBS: statics

Naked eye analogy with macroscopic patterns

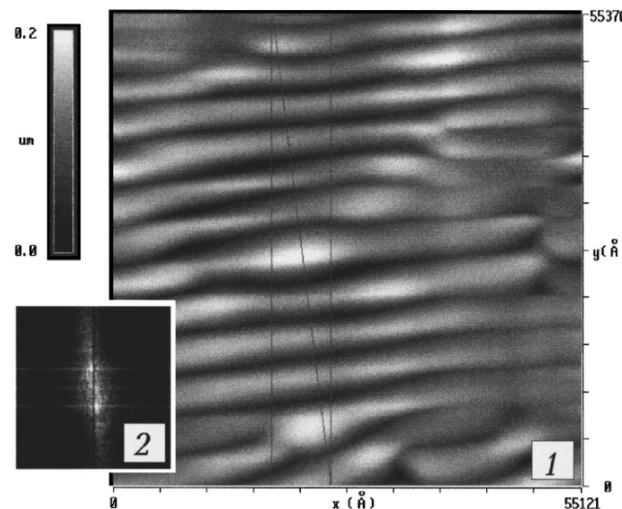


Surface nanopatterns by IBS: dynamics (i)

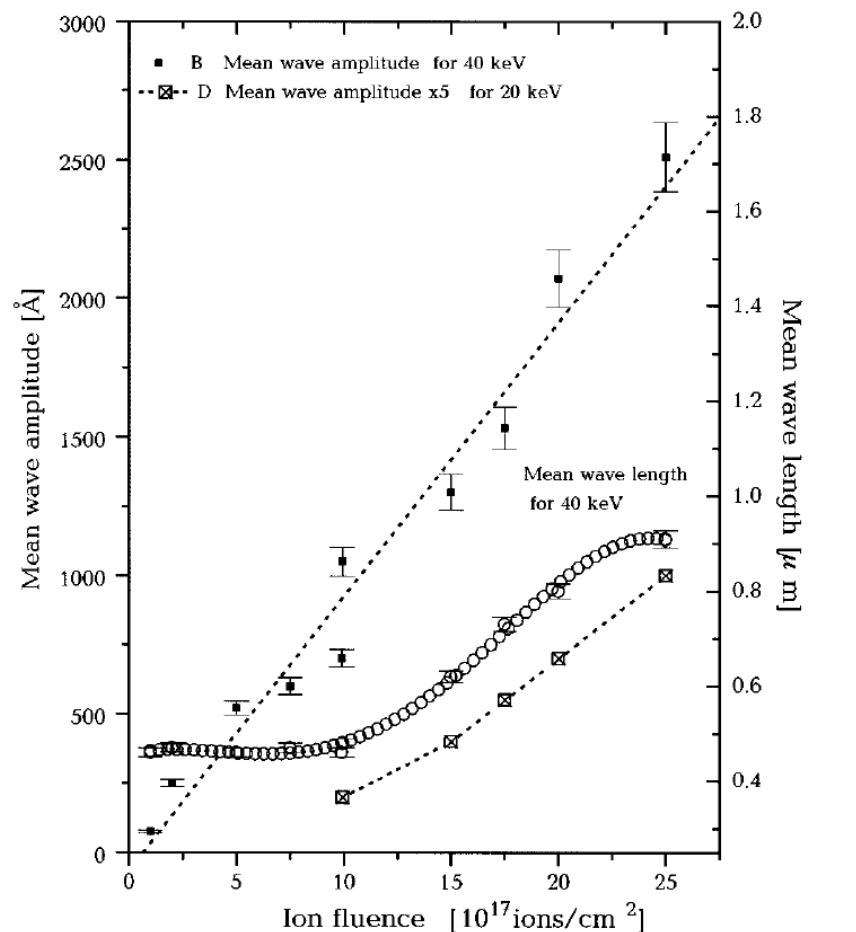
Nontrivial morphology changes
in *macroscopic* times

Typical fluxes

$$J = 60 \text{ ion s}^{-1} \text{ nm}^{-2}$$



(Linear)
Instability Nonlinear effects:
coarsening, ... Stationary
state

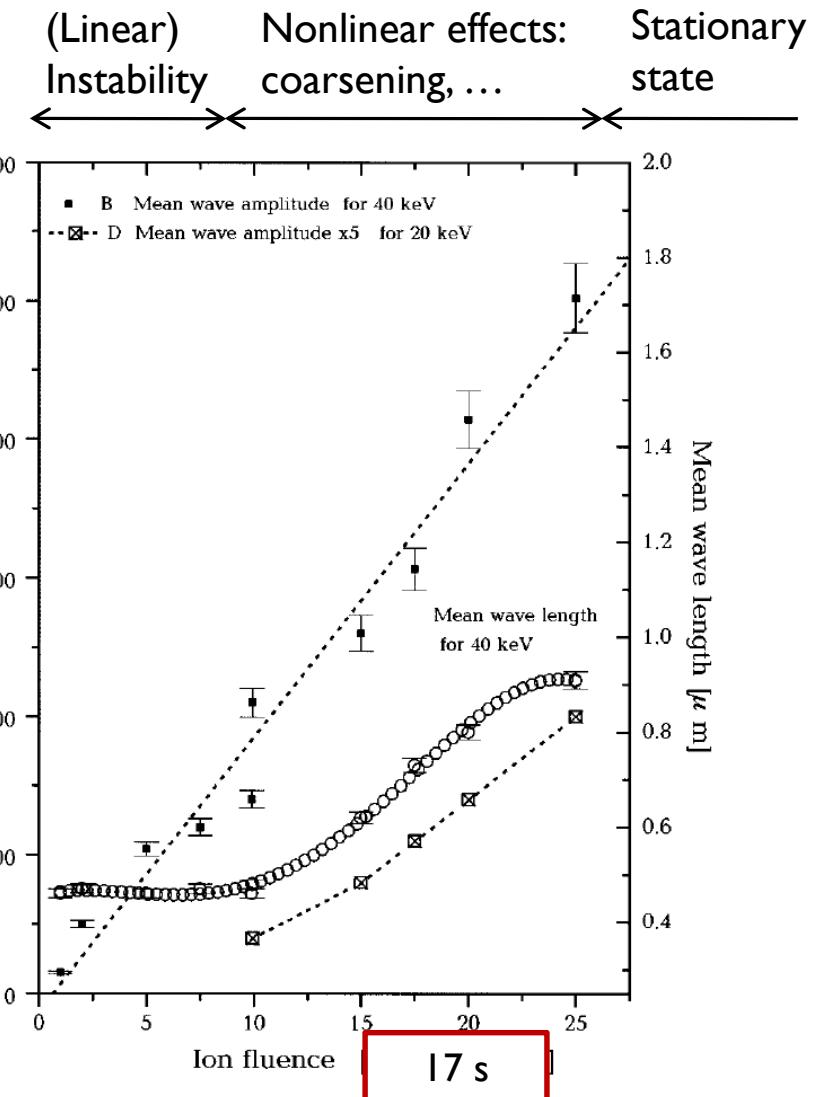
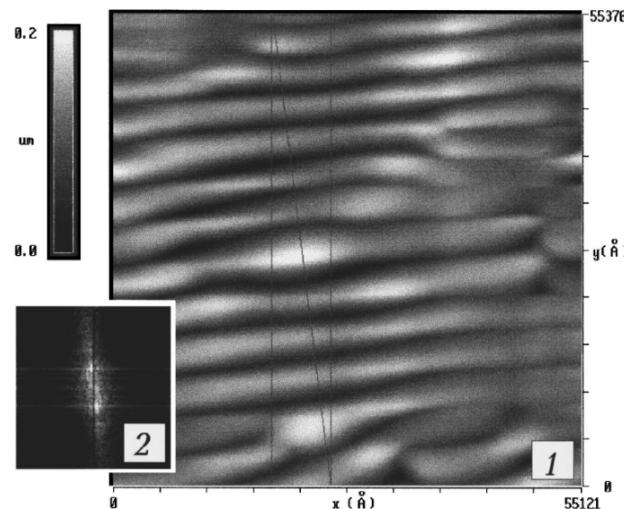


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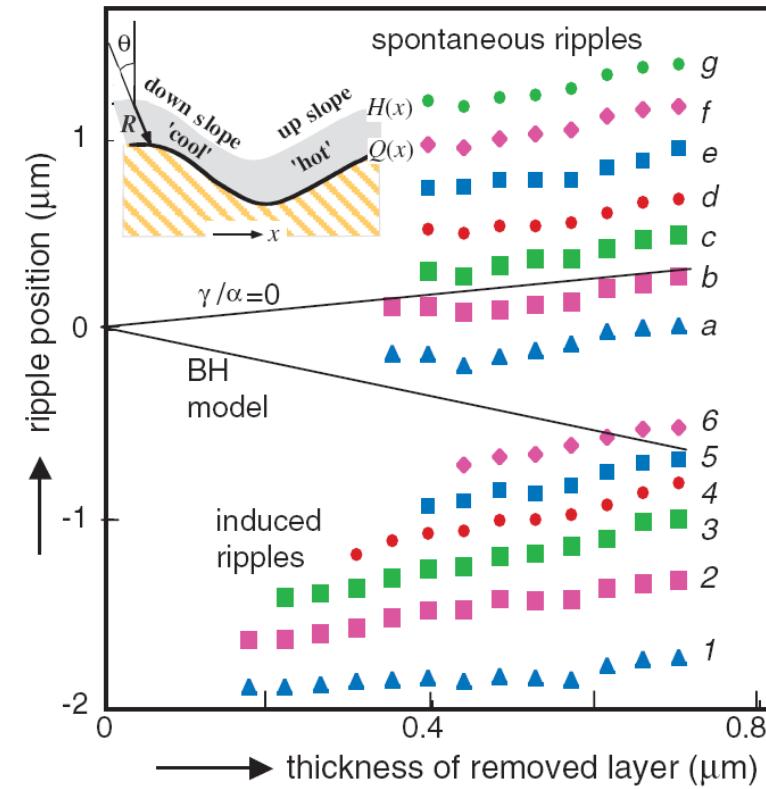
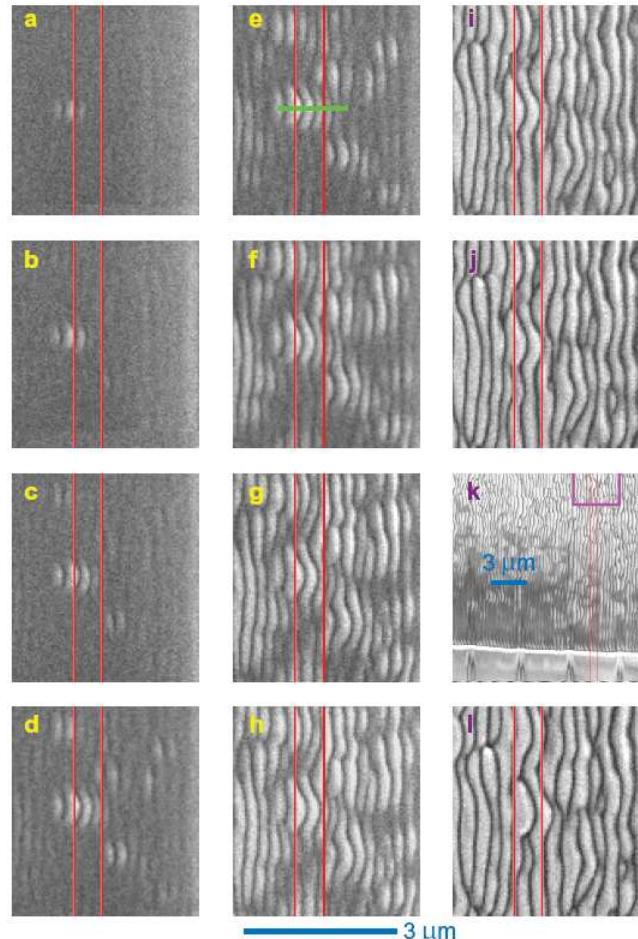
Nontrivial morphology changes
in *macroscopic* times

SMALL!!! Typical fluxes

$$J = 60 \text{ ion s}^{-1} \text{ nm}^{-2}$$

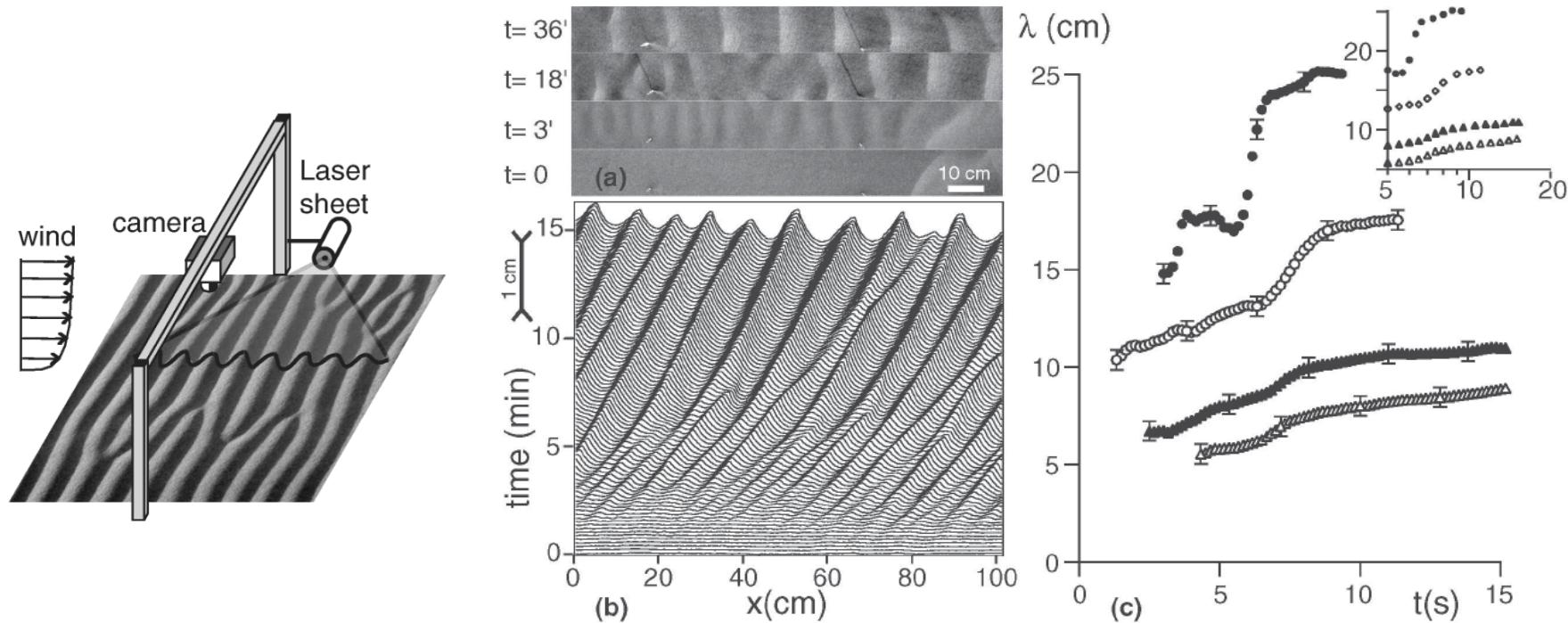


Surface nanopatterns by IBS: dynamics (ii)



Ripple formation, transport $V = 0.3 \text{ nm/s}$
& coarsening $\lambda = 200 \text{ nm}$

Naked eye analogy with macroscopic patterns (dynamics)



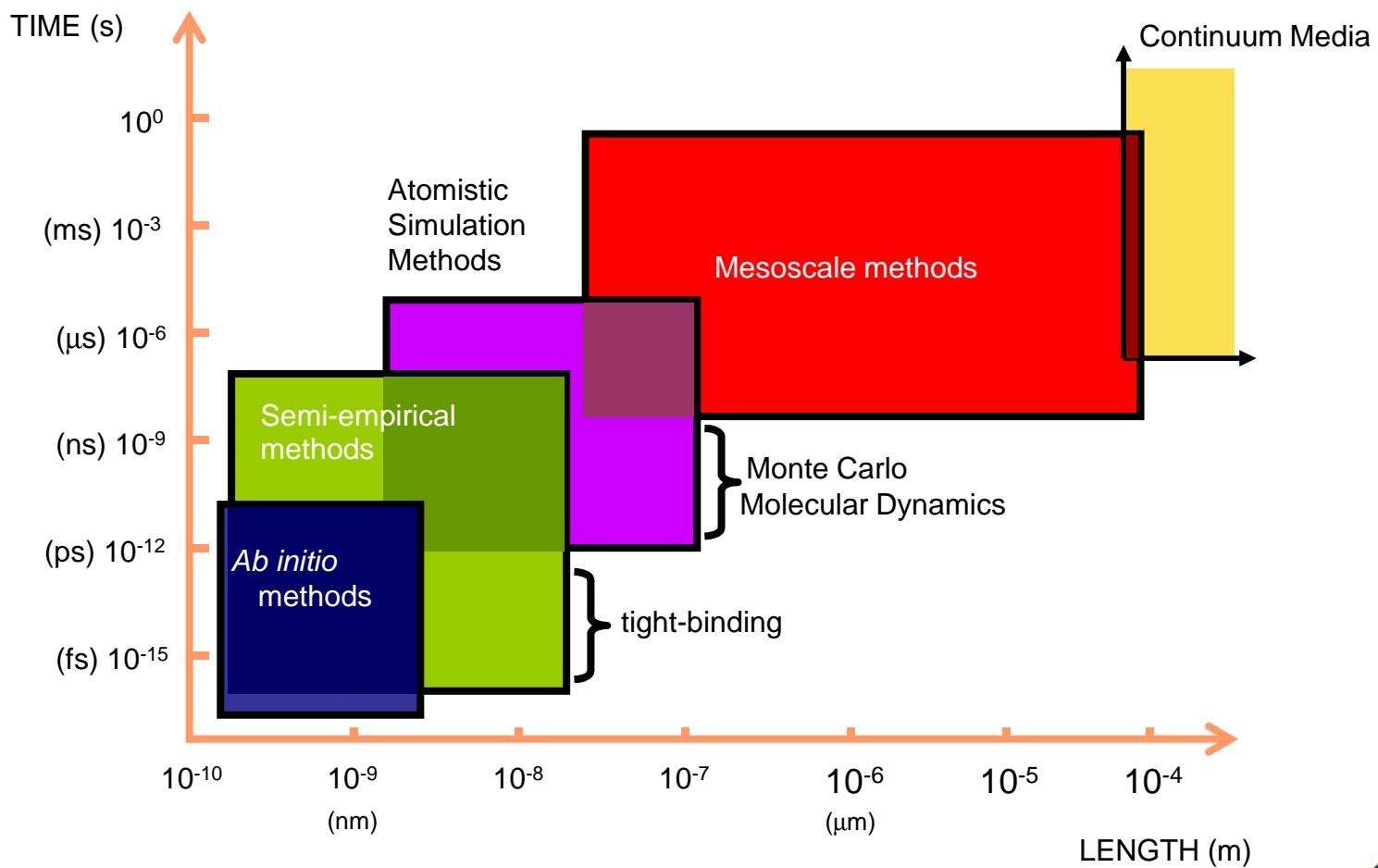
Ripple formation, transport & coarsening

$V = 0.07 \text{ cm/s}$

$\lambda = 6 \text{ cm}$

Theoretical approaches

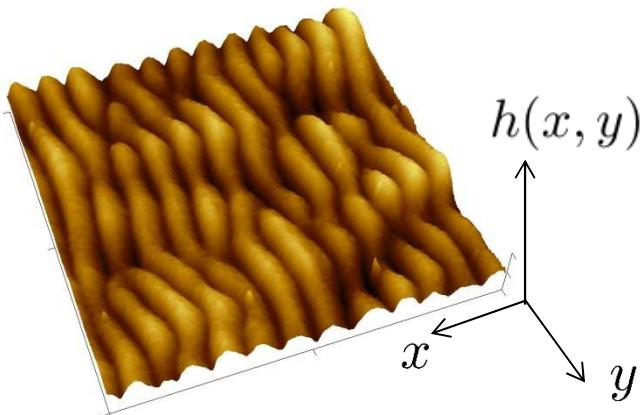
Material independence (universality) + time scales → Continuum models are a natural choice



Adapted from M. B. Nardelli et al. '04

Continuum descriptions of pattern formation:

- (Time) evolution equation for a few (scalar) degrees of freedom



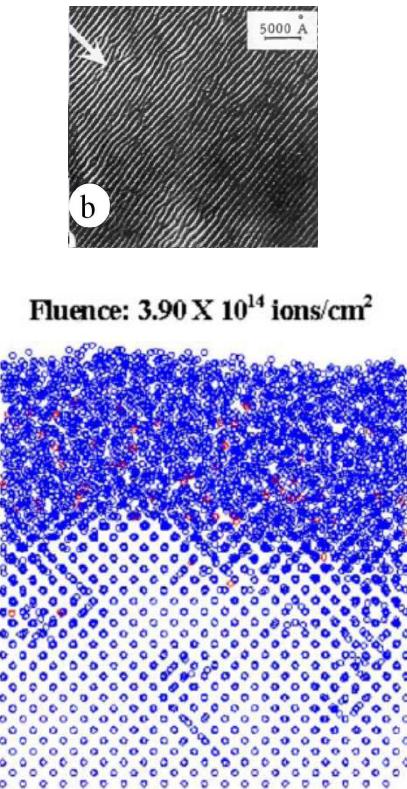
Height field $h(x,y,t)$

Material density $R(x,y,t)$

- Summarizes complex dynamics into a compact description
- Approximations required: long wavelength, small disturbances
- Computational advantages
- Rich mathematical properties (e.g. analytical results)
- Universality: Same nonlinear equation applies to very different systems

(Toy) reaction-diffusion, aka “hydrodynamic” model

N. Kalyanasundaram et al. MRC '08:0.7 keV Ar⁺ / Si (MD)

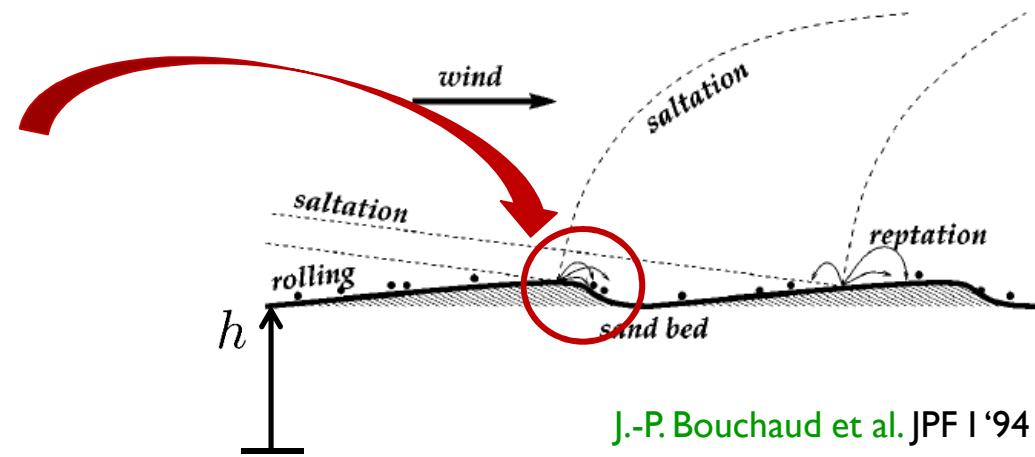
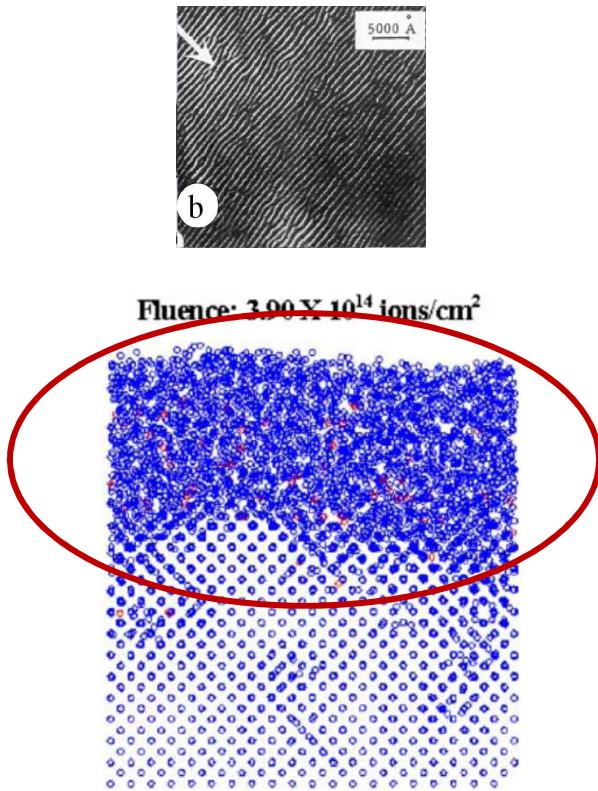


$h(\mathbf{r}, t)$: Height of target

$R(\mathbf{r}, t)$: Density of mobile surface species (defects, ...)

(Toy) reaction-diffusion, aka “hydrodynamic” model

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J.-P. Bouchaud et al. JPF I '94

$h(\mathbf{r}, t)$: Height of target

$R(\mathbf{r}, t)$: Density of mobile surface species (defects, ...)

Conservation laws

T.Aste & U.Valbusa Physica A '04

R.C. et al NIMB '11

Γ_{ex} = Rate for material leaving the immobile bulk \rightarrow sputtered/transported away

Γ_{qd} = Rate for mobile material to incorporate back into immobile bulk

$\phi = 1$: All eroded material is sputtered away

$\phi = 0$: All eroded material is *available to local redeposition*
 (*aeolian sand dunes* case: conserved # of reptating grains)

Constitutive equations (1/3): sputtering

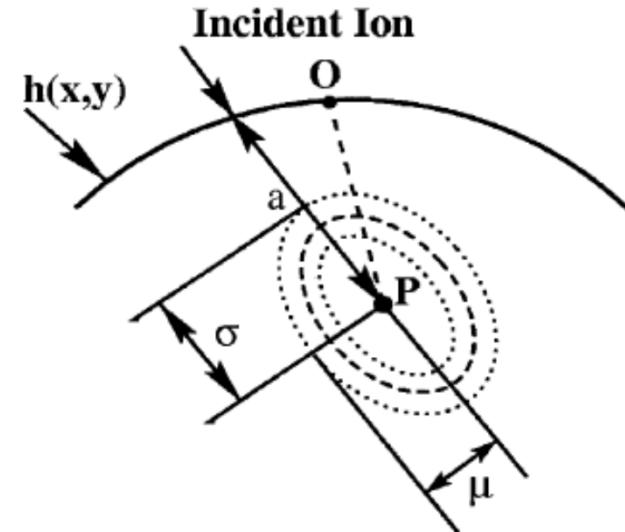
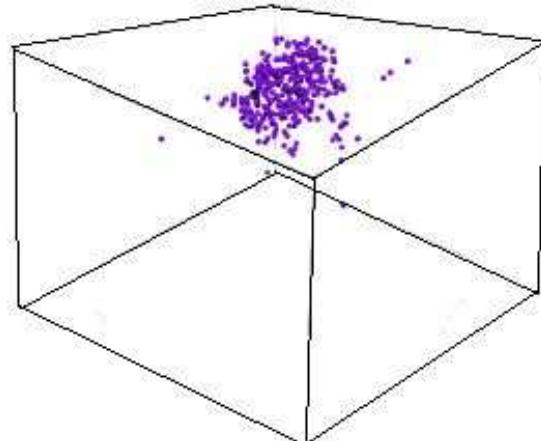
Energy deposition (linear cascade approximation)

P. Sigmund PR '69

R.M. Bradley & J.M.E. Harper JVSTA '88

$$E(r', z') \propto E e^{-\frac{(z'+a)^2}{2\sigma^2} - \frac{r'^2}{2\mu^2}}$$

Gaussian



Constitutive equations (1/3): sputtering

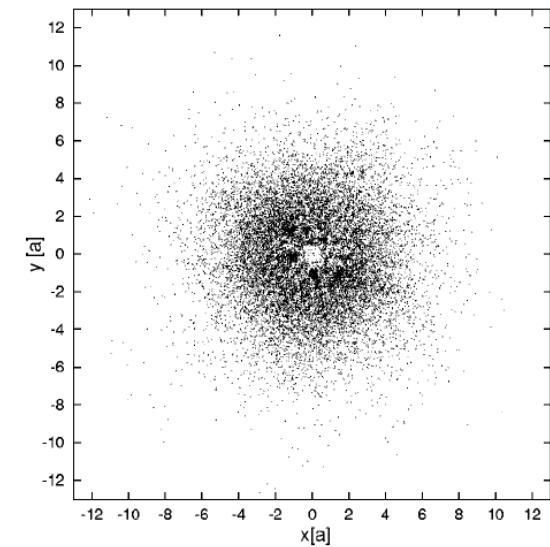
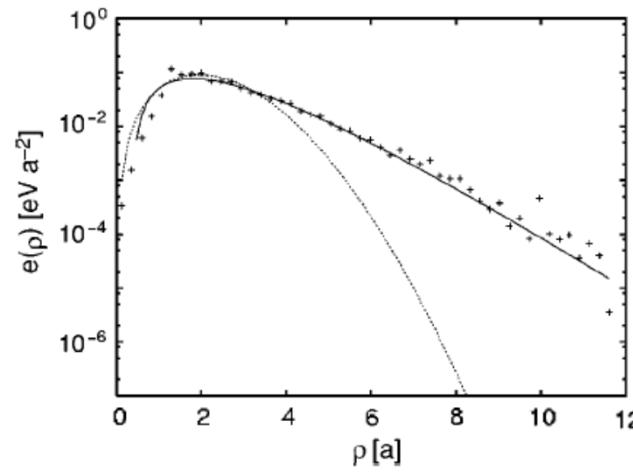
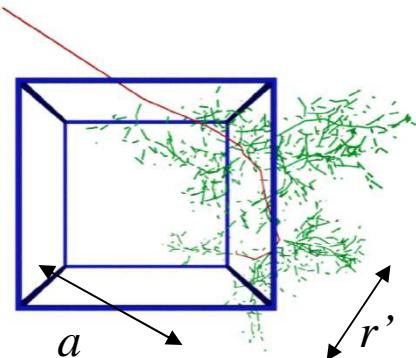
Energy deposition (linear cascade approximation)

M. Feix et al. PRB '05

BCA simulation Cu on Cu

Non-Gaussian

$$E(r', z') \propto E r'^2 e^{-r'/\mu} e^{-\frac{(z'+a)^2}{2\sigma^2}}$$



Constitutive equations (1/3): sputtering

Energy deposition (linear cascade approximation)

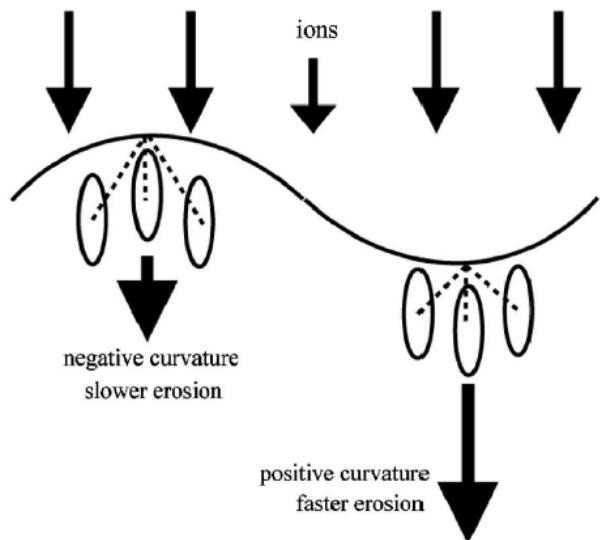
B. Davidovitch, M. Brenner &
M.J.Aziz PRB '07

Arbitrary

$$E(r', z') \propto e^{-g(r') - f(z')}$$

Constitutive equations (1/3): sputtering

Energy deposition (linear cascade approximation)



BH morphological instability:

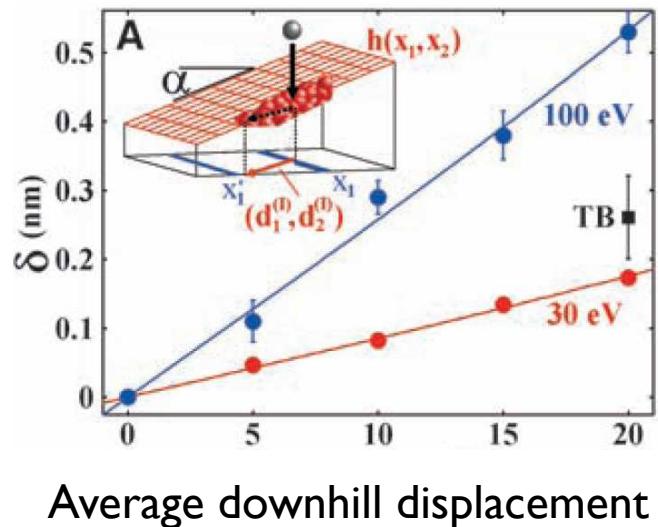
Surface troughs erode faster than surface protrusions

$$\Gamma_{ex} = \alpha_0 [1 + \alpha_{2x} \partial_x^2 h + \alpha_{2y} \partial_y^2 h + \dots]$$

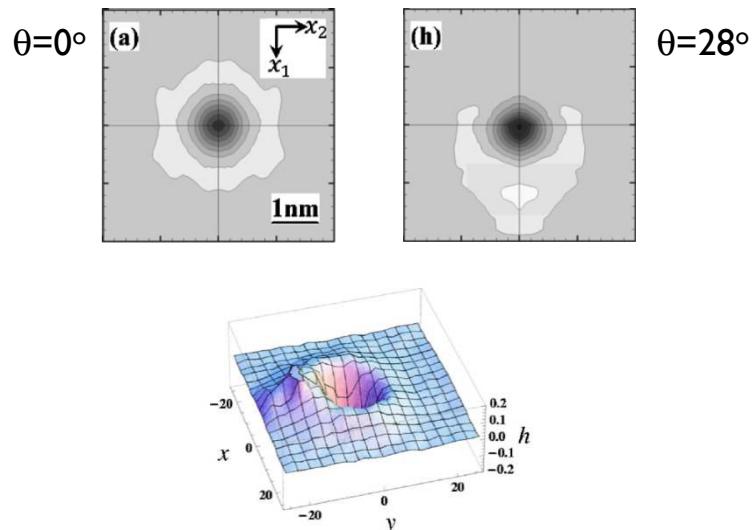
$$\alpha_0 \simeq 1 \text{ nm s}^{-1}$$

Constitutive equations (2/3): material redistribution

M. Moseler et al. Science '05:
C / ta-C films (MD)



N. Kalyanasundaram et al. APL '08:
0.5 keV Ar⁺ / Si (MD)



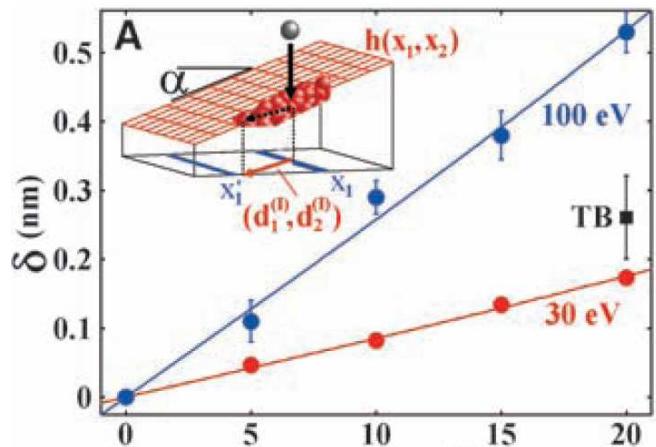
G. Carter & V. Vishnyakov PRB '96
B. Davidovitch, M. Brenner & M.J. Aziz PRB '07

Stabilizing effect of collisional rearrangements

$$\Gamma_{ex} = -\beta_0 [\beta_{2x} \partial_x^2 h + \beta_{2y} \partial_y^2 h + \dots]$$

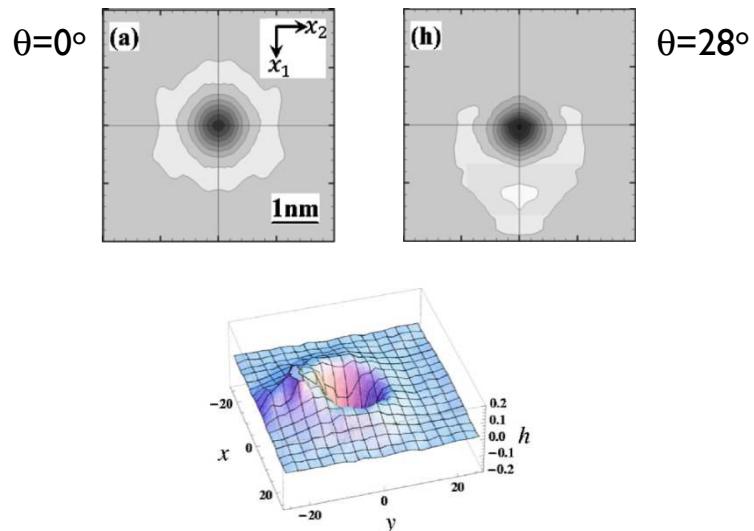
Constitutive equations (2/3): material redistribution

M. Moseler et al. Science '05:
C / ta-C films (MD)



Average downhill displacement

N. Kalyanasundaram et al. APL '08:
0.5 keV Ar⁺ / Si (MD)



G. Carter & V. Vishnyakov PRB '96

B. Davidovitch, M. Brenner & M.J. Aziz PRB '07

Stabilizing effect of collisional rearrangements

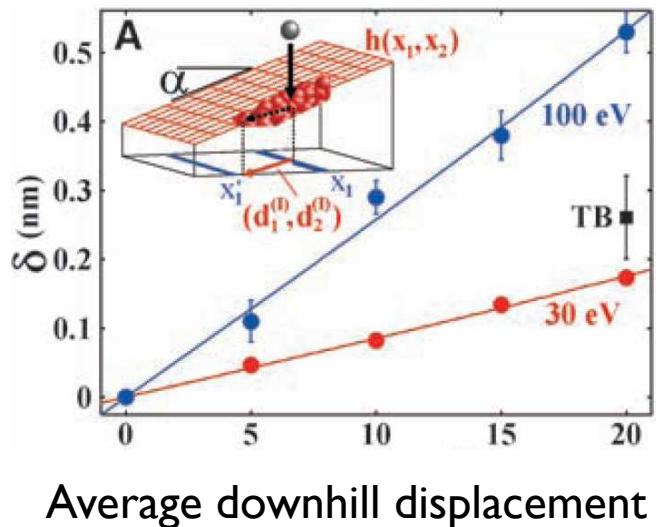
$$\Gamma_{ex} = -\beta_0 [\beta_{2x} \partial_x^2 h + \beta_{2y} \partial_y^2 h + \dots]$$

$$\beta_0 \propto \cos \varphi(x)$$

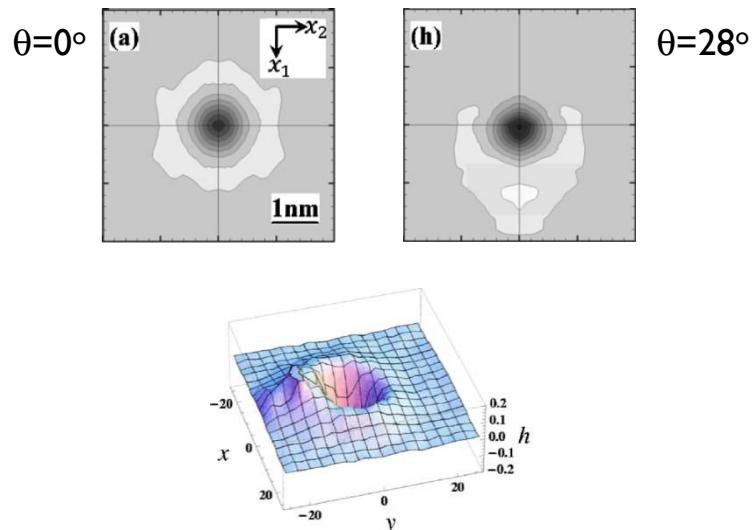
Corrected ion flux

Constitutive equations (2/3): material redistribution

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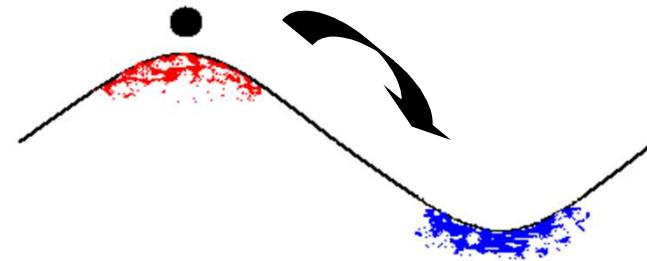
$$\beta_0 \propto \cos \varphi(x) \quad \sin \varphi(x)$$

Corrected ion flux

Momentum along surface

Constitutive equations (3/3): local redeposition

Smaller addition at surface maxima → available material to thermal & ion-induced diffusion



Gibbs-Thomson effect

$$\Gamma_{ad} = \gamma_0 [R - R_{eq}(1 - \gamma_{2x}\partial_x^2 h - \gamma_{2y}\partial_y^2 h)]$$

$$\gamma_0 \simeq 10^9 \text{ s}^{-1}$$

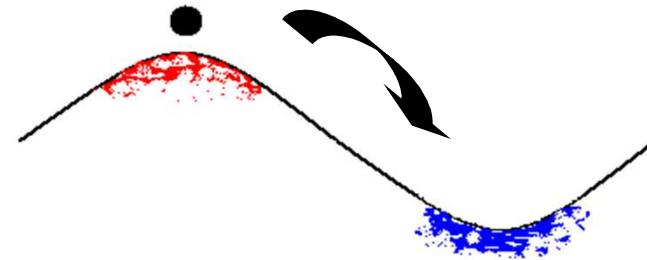
For $\Gamma_{er} = 0$ (ion beam switched off) and $\gamma_{2x} = \gamma_{2y}$

$$\left. \begin{array}{l} \frac{\partial h}{\partial t} = -DR_{eq}\gamma_2 \nabla^4 h \\ \frac{\partial h}{\partial t} = -\frac{D_s(T)n_s\Omega^2\gamma}{k_B T} \nabla^4 h \end{array} \right\} \begin{array}{lcl} D & \simeq & D_s(T) \\ R_{eq} & \simeq & n_s\Omega \\ \gamma_2 & \simeq & \frac{\gamma\Omega}{k_B T} \end{array}$$

W.W. Mullins
JAP '54

Constitutive equations (3/3): local redeposition

Smaller addition at surface maxima →
available material to
thermal & ion-induced diffusion



Gibbs-Thomson effect

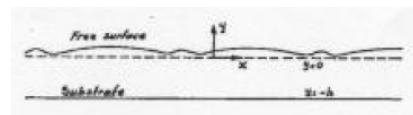
$$\Gamma_{ad} = \gamma_0 [R - R_{eq}(1 - \gamma_{2x}\partial_x^2 h - \gamma_{2y}\partial_y^2 h)]$$

$$\gamma_0 \simeq 10^9 \text{ s}^{-1}$$

In general, take $R_{eq} = R_{eq}^{(ion)} + R_{eq}^{(T)}$

Contribution to surface transport in the surface
confined viscous flow form

$$\frac{\partial h}{\partial t} \simeq -\frac{\gamma d^3}{\eta_s} \nabla^4 h + \dots$$



Analysis

$$\begin{aligned}\partial_t h &= -\Gamma_{ex} + \Gamma_{ad} \\ \partial_t R &= (1 - \phi) \Gamma_{ex} - \Gamma_{ad} - D \nabla^2 R\end{aligned}$$

Planar solution ($t \gg \gamma_0^{-1}$)

$$\begin{aligned}h_p(t) &= -(\phi \alpha_0) t \\ R_p &= R_{eq} + (1 - \phi) \epsilon\end{aligned}$$

$$\epsilon = \frac{\alpha_0}{\gamma_0 R_{eq}} \propto \frac{\text{excavation rate}}{\text{addition rate}} \simeq 10^{-10} - 10^{-5} \ll 1$$

Time scale separation:

Matter transport (R) relaxes much faster than morphological (h) changes occur

Linear stability

Dynamics of periodic perturbations to planar solution (= short times for flat initial condition!!)

$$h(t) = h_p(t) + h_0 e^{\omega_k t + i \mathbf{k} \cdot \mathbf{r}}$$

$$R(t) = R_p + R_0 e^{\omega_k t + i \mathbf{k} \cdot \mathbf{r}}$$

Linear dispersion relation: ($\epsilon \ll 1, k \ll 1$)

Linear stability

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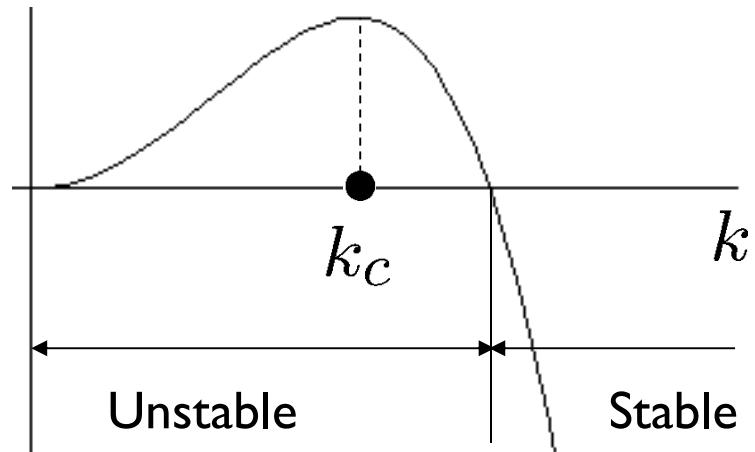
$$R(t) = R_p + R_0 e^{\omega_k t + i \mathbf{k} \cdot \mathbf{r}}$$

Linear dispersion relation: ($\epsilon \ll 1, k \ll 1$)

$$\omega_{\mathbf{k}} = \nu k^2 - \mathcal{K} k^4 \quad (\theta = 0) \quad \omega_k$$

The mode with $k = k_c$ sets in a characteristic length-scale

$$\lambda = 2\pi/k_c$$



Linear stability

Dynamics of periodic perturbations to planar solution (= short times for flat initial condition!!)

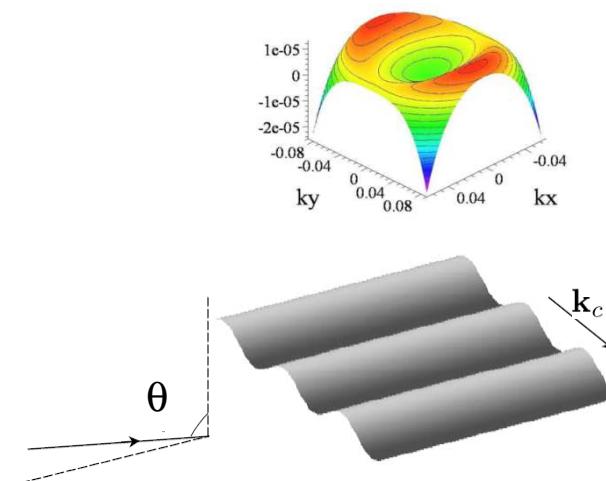
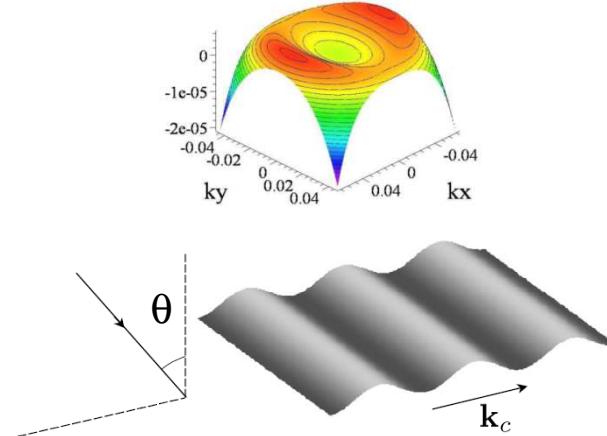
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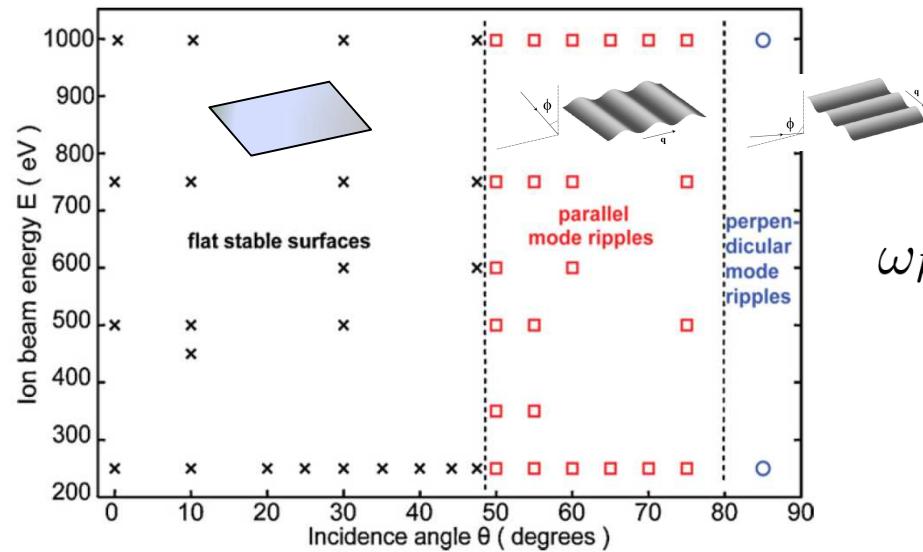
Linear dispersion relation: ($\epsilon \ll 1, k \ll 1$)

$$\omega_{\mathbf{k}} = \nu_x k_x^2 + \nu_y k_y^2 - \mathcal{K}_x k_x^4 - \mathcal{K}_y k_y^4 - 2\mathcal{K}_{xy} k_x^2 k_y^2 \quad (\theta \neq 0)$$

Ripple formation with $\lambda = 2\pi/k_c$



Ripple formation: (low E) morphologies for Ar^+ / Si

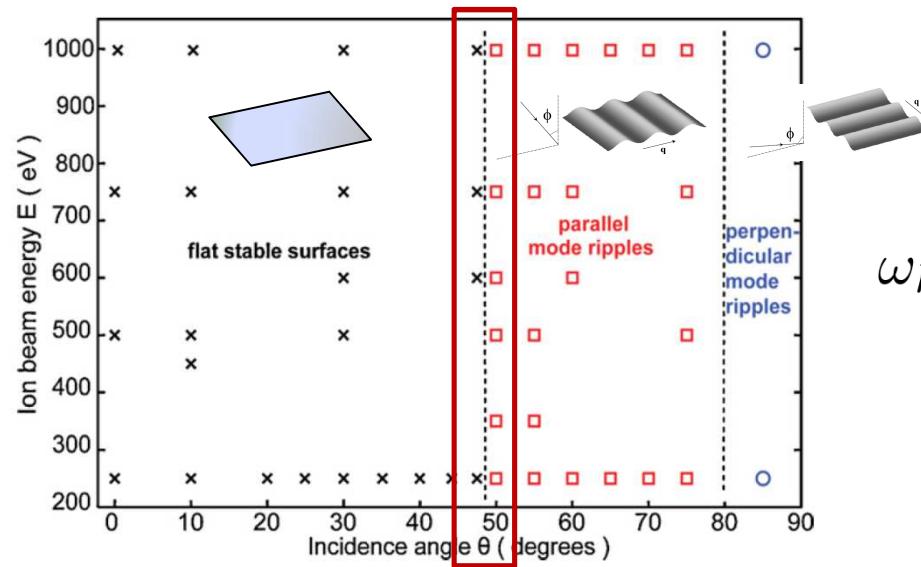


C. Madi & M. J. Aziz Appl. Surf. Sci. '12: Ar^+ / Si

1d system:

$$\omega_{k_x} = (-\beta \cos 2\theta + \nu_{BH}) k_x^2 - \frac{\gamma d^3}{\eta_s} k_x^4$$

Ripple formation: (low E) morphologies for Ar^+ / Si

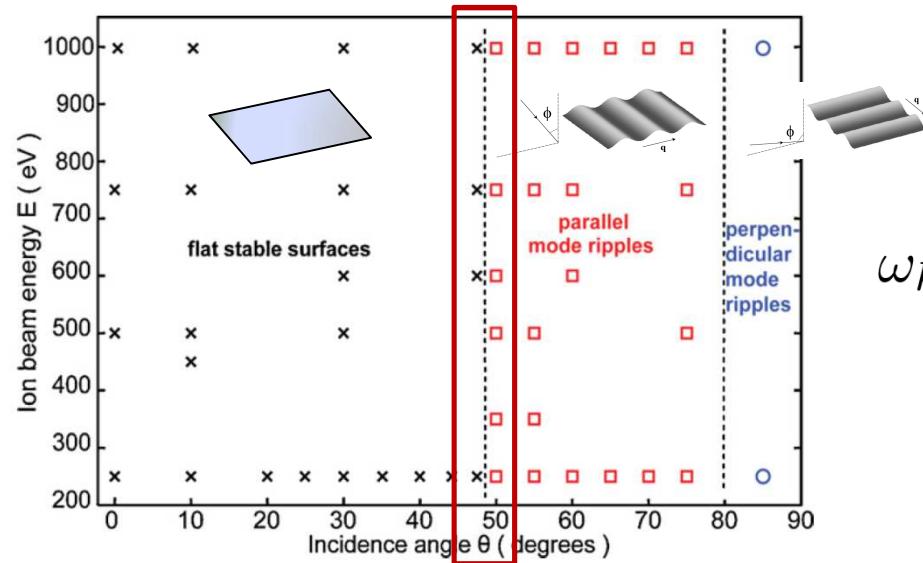


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$$\theta_c^{\text{exp}} \simeq 48^\circ$$

Ripple formation: (low E) morphologies for Ar^+ / Si



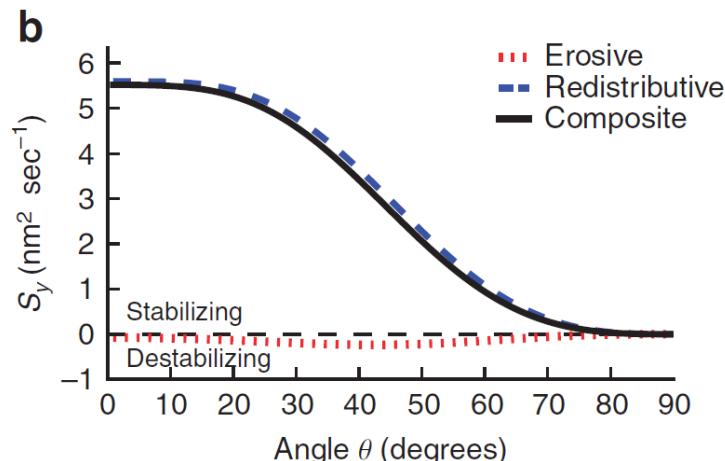
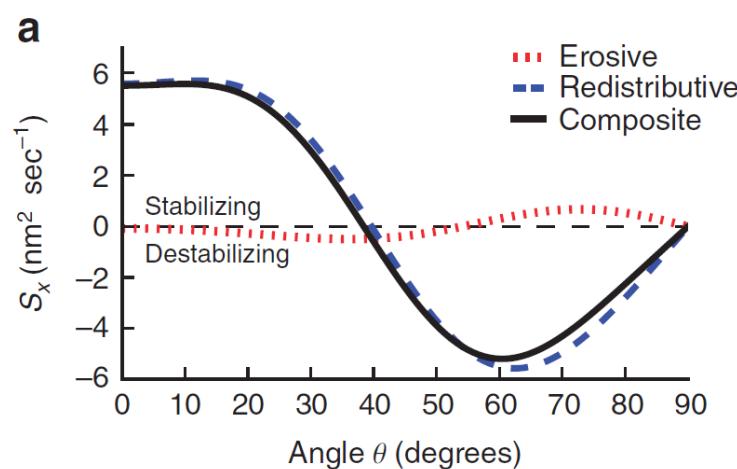
C. Madi & M. J. Aziz Appl. Surf. Sci. '12: Ar^+ / Si

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$$\theta_c^{\text{exp}} \simeq 48^\circ$$

$$\theta_c^{\text{theor}} = 45^\circ$$



Pattern properties within linear regime

- Ripple formation: wavelength dependence of system parameters E , θ , ...
- Ripple in-plane propagation
- Assumed flat initial condition; others possible: e.g. rippled initial surfaces
- Universality with respect to ion/target combination (?)
- Exponential amplitude growth
- Stationary wavelength

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Limitations

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Improvements:

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Improvements:

- Relax linear approximation → Non-linear models

Pattern properties within linear regime

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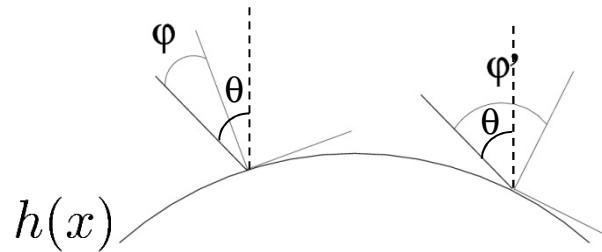
Limitations

Improvements:

- Relax linear approximation → Non-linear models
- Models with increased predictive power

Nonlinearities: effective height equation

Local angle of incidence is a non-linear function
of surface slopes:



T. C. Kim et al. PRL '04, '05

M. Castro, R. C., R. Gago & L. Vázquez PRL '05

J. Muñoz-García, R. C. & M. Castro JPCM '09

$$\cos \varphi = \frac{\cos \theta - (\partial_x h) \sin \theta}{\sqrt{1 + (\nabla h)^2}}$$

Nonlinearities: effective height equation

T. C. Kim et al. PRL '04, '05

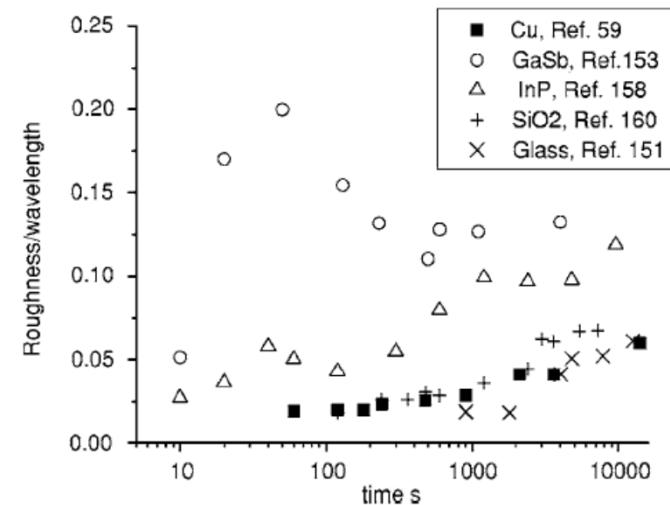
M. Castro, R. C., R. Gago & L. Vázquez PRL '05

J. Muñoz-García, R. C. & M. Castro JPCM '09

Local angle of incidence is a non-linear function
of surface slopes:

Small aspect ratios (ripples)

W. L. Chan & E. Chason JAP '07



Nonlinearities: effective height equation

T. C. Kim et al. PRL '04, '05

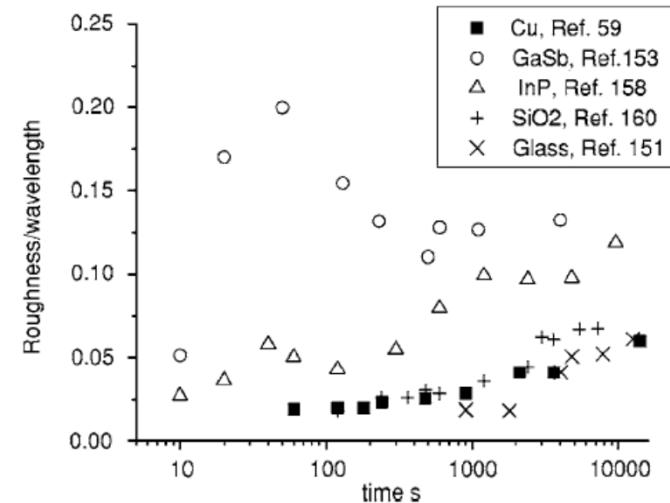
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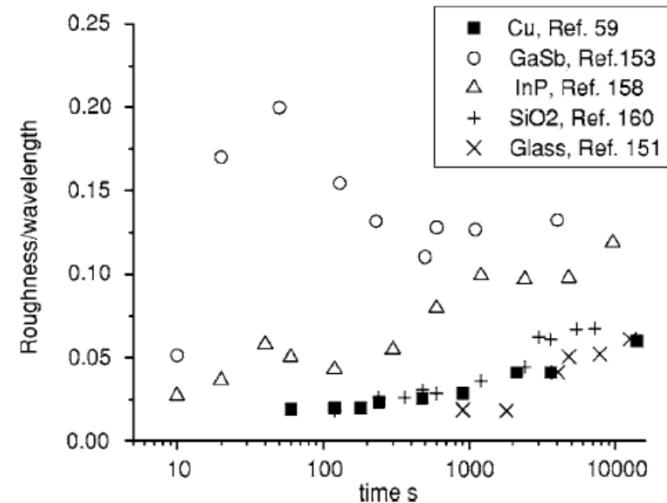
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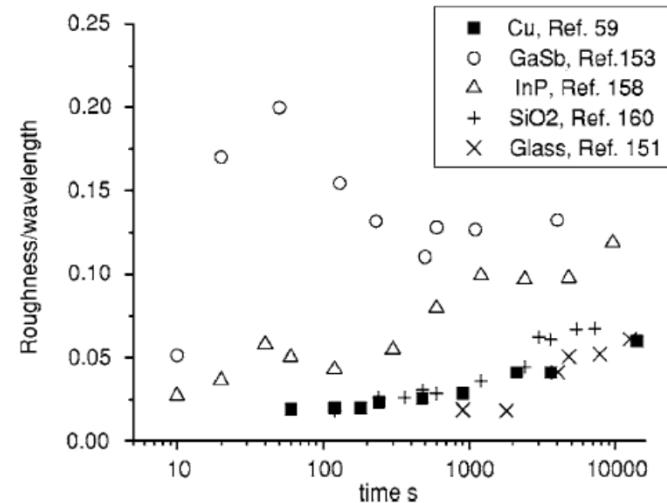
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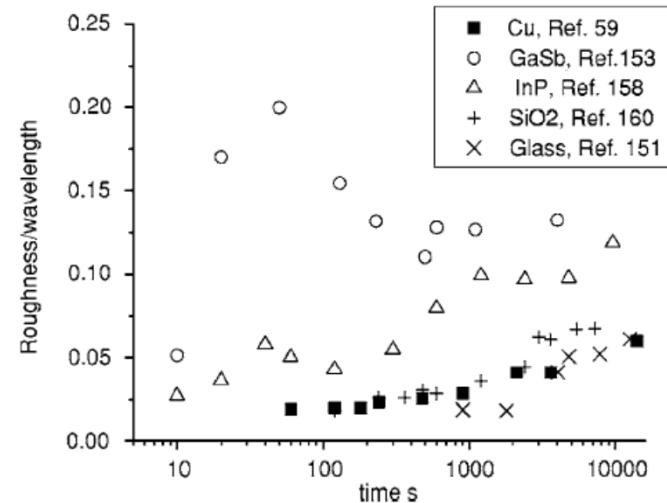
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Kuramoto-Sivashinsky (KS) equation

Extended Kuramoto-Sivashinsky equation

Nonlinear interface equation ($\theta = 0$)

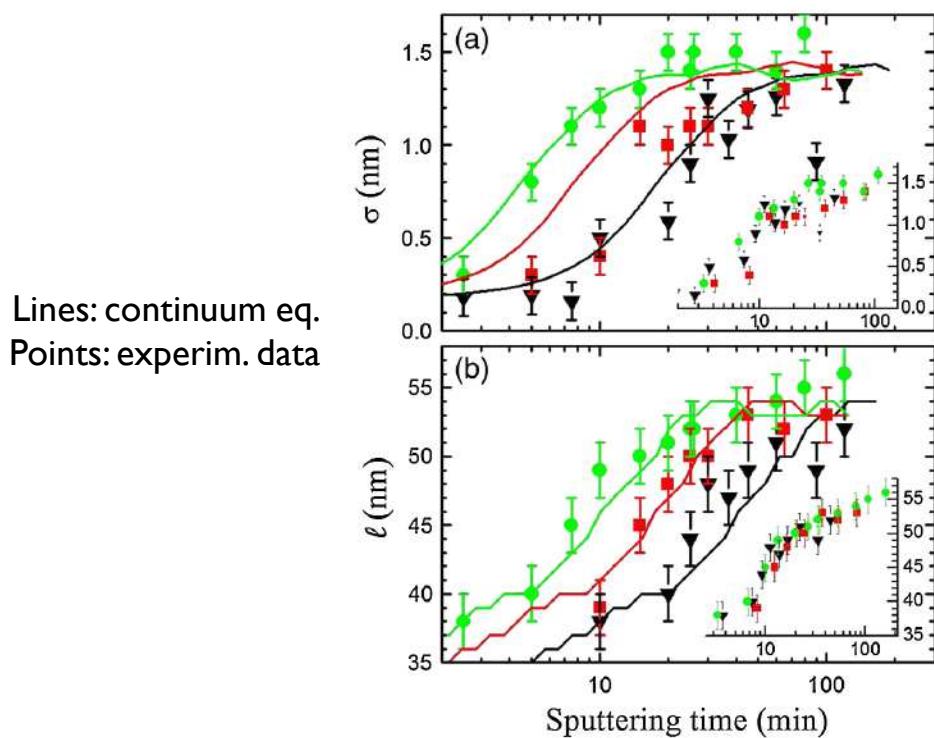
- Same linear dispersion relation: For flat initial conditions, dynamics are controlled by linear terms → linear wavelength selection
- In general, analytical solutions are not available → Numerical integration

Nonlinear interface equation ($\theta = 0$)

$$\partial_t h = -\nu \nabla^2 h - \mathcal{K} \nabla^4 h + \lambda^{(1)} (\nabla h)^2 - \lambda^{(2)} \nabla^2 (\nabla h)^2$$

Nonlinear interface equation ($\theta = 0$)

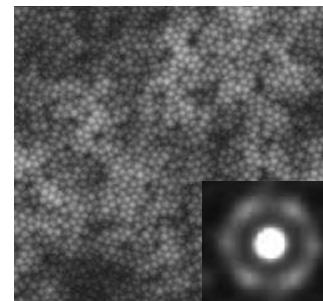
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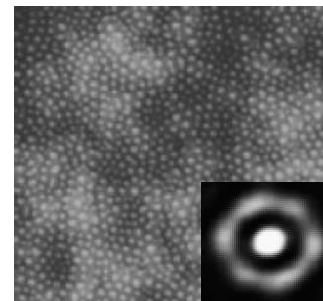
gisc

J. Muñoz-García et al. PRL '10: 1 keV Ar⁺ / Si

Late morphologies



Model



Experiment

M. Castro et al. PRL '05:
1.2 keV Ar⁺ / Si



Comments

- But no pattern formation at $\theta=0$ for monoelemental targets!!!
Role of impurities
Universality: eKS equation applies to many growth/erosion systems
- Extended KS equation does admit approximate analytical solutions:
4 independent measurements fix 4 equation coefficients

Comments

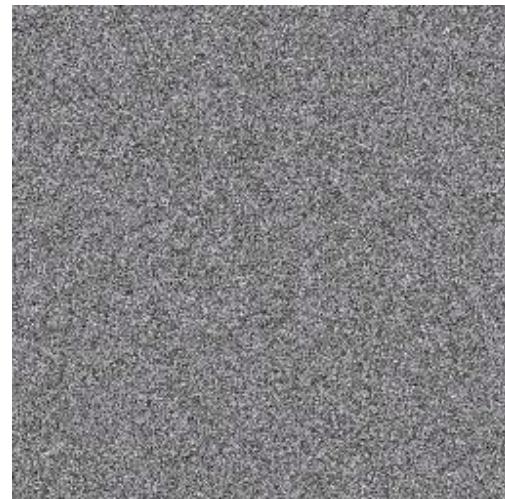
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- $\theta \neq 0$:

$$\begin{aligned} \partial_t h = & \gamma \partial_x h + \sum_{i=x,y} \left\{ -\nu_i \partial_i^2 h + \lambda_i^{(1)} (\partial_i h)^2 + \Omega_i \partial_i^2 \partial_x h \right. \\ & \left. + \xi_i (\partial_x h) (\partial_i^2 h) \right\} + \sum_{i,j=x,y} \left\{ -\mathcal{K}_{ij} \partial_i^2 \partial_j^2 h + \lambda_{ij}^{(2)} \partial_i^2 (\partial_j h)^2 \right\} \end{aligned}$$

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“Large” $\lambda_{i,j}^{(2)}$ vs $\lambda_i^{(1)}$

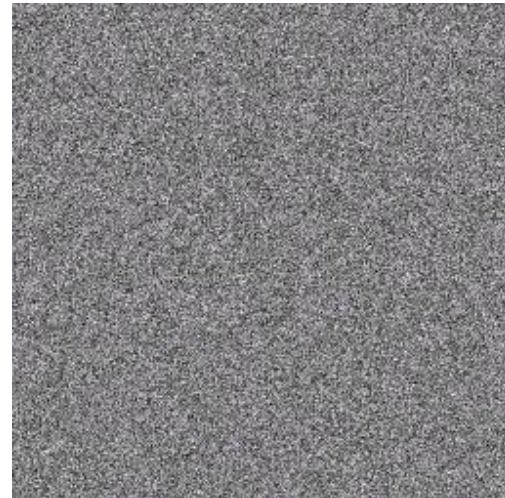


Numerical integration: top view

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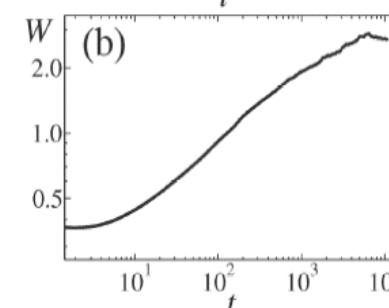
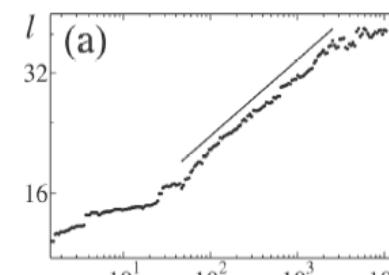
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Numerical integration: top view



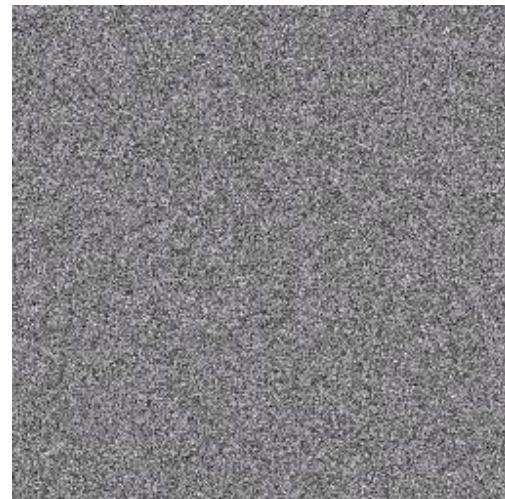
$$l \sim t^{0.19}$$



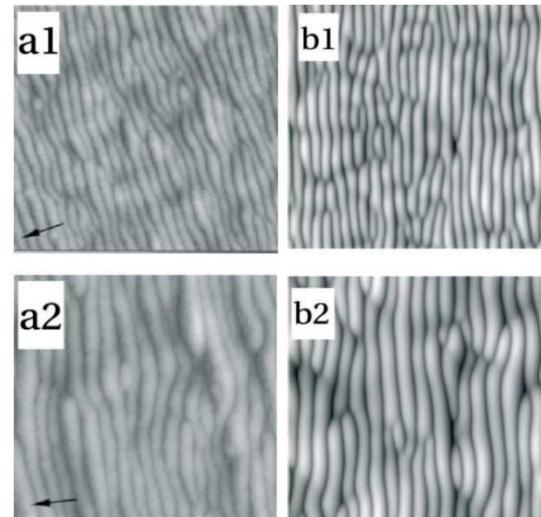
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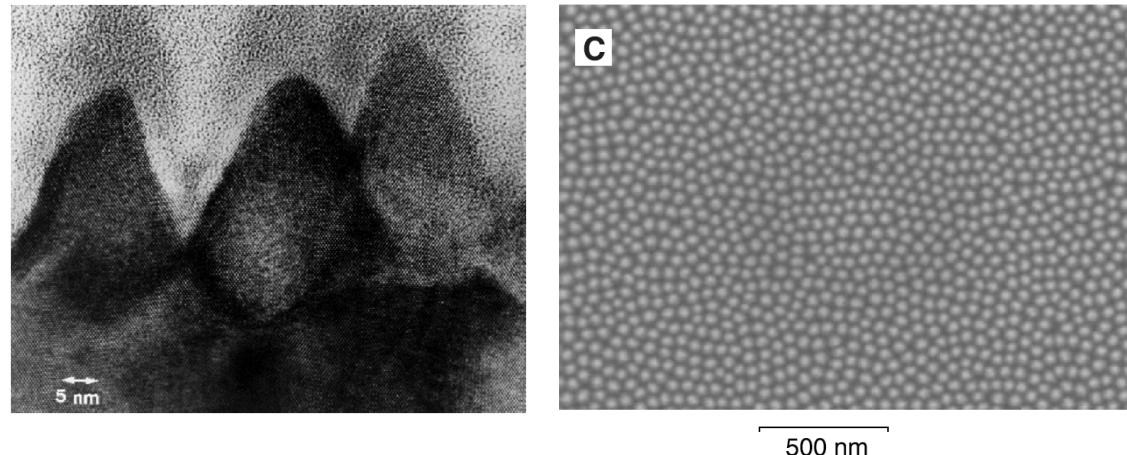


D. Flamm et al.
Appl. Surf. Sci. '01
R.C et al. EJPST '07



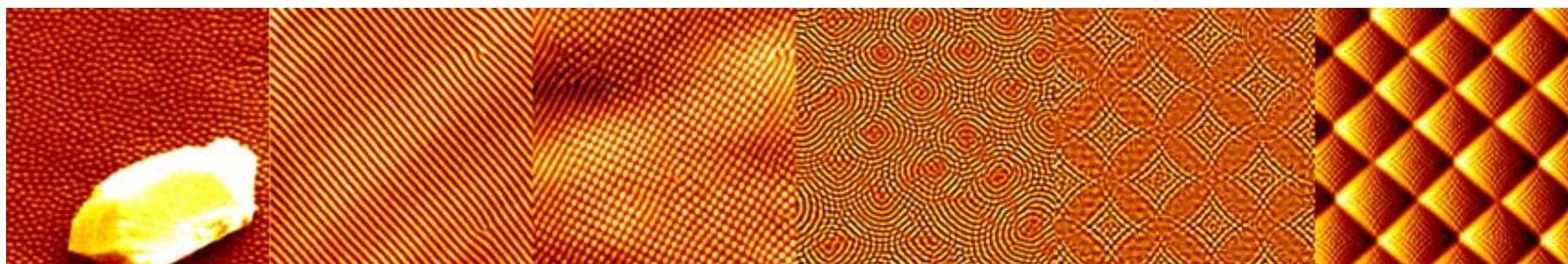
Presence of impurities/compound targets: a detour

Pattern formation at $\theta=0$ for compound targets and/or in the presence of metallic impurities



S. Facsko et al. Science '99:
0.4 keV Ar⁺ / GaSb

DFG Forschergruppe 845: Patterns on Si



<http://www.iom-leipzig.de/for845>

“Hydrodynamic” models of thin film growth

So-called *coupled growth equations* in the context of *epitaxial growth*

E.g. GaAs growth

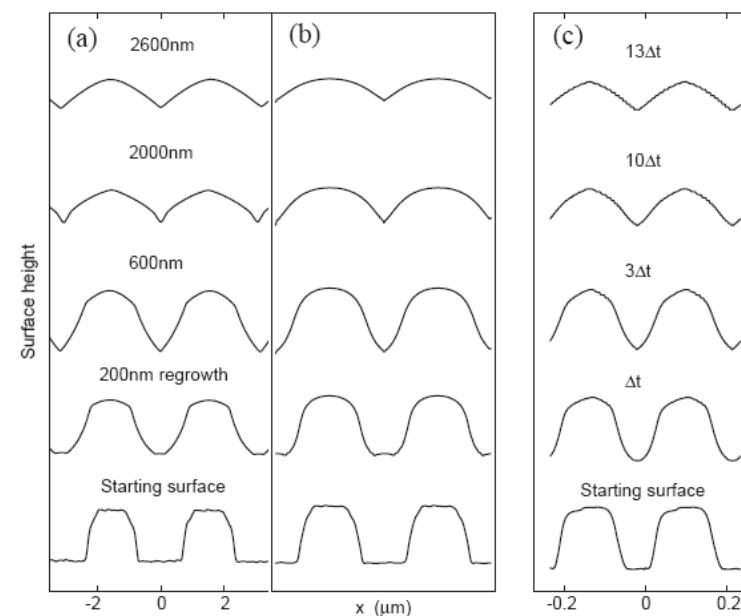
$$\begin{aligned} h(\mathbf{r}, t) : & \text{ Thin film height} & \partial_t n &= -\nabla \cdot \mathbf{J} + F - \partial_t h \\ n(\mathbf{r}, t) : & \text{ Adatom density} & \partial_t h &= Dn^2 + DnS - \chi S \\ S(\mathbf{r}, t) : & \text{ Step density} \\ & = (S_0^2 + (\nabla h)^2)^{1/2} \end{aligned}$$

(a) Experiment

(b) Reaction-diffusion

(c) kMC

A. Ballestad et al. JCG '04



IBS vs composition: conservation laws

Strained alloys: model composition explicitly

Growth systems (linear approximation)

B.J. Spencer, P.W. Voorhees & J. Tersoff PRB '01

$$\begin{aligned}\partial_t h &= \Omega [F_A + F_B - \nabla \cdot (\mathbf{J}_A + \mathbf{J}_B)] \\ \partial_t C + C \partial_t h &= \Omega (F_A - \nabla \cdot \mathbf{J}_A)\end{aligned}$$

Erosion systems

V. B. Shenoy, W. L. Chan & E. Chason PRL '07

$$\begin{aligned}\partial_t h &= -\Omega [\delta F_A + \delta F_B + \nabla \cdot (\mathbf{J}_A + \mathbf{J}_B)] \\ \partial_t \zeta &= \Omega [(c_b - 1)(\delta F_A + \nabla \cdot \mathbf{J}_A) + c_b(\delta F_B + \nabla \cdot \mathbf{J}_B)]\end{aligned}$$

$$\delta F_i = F Y_i (\zeta_i - c_i v_{\text{BH}})$$

Predictions from linear approximation

Predictions from linear approximation

BH-type instability → high-yield species at peaks

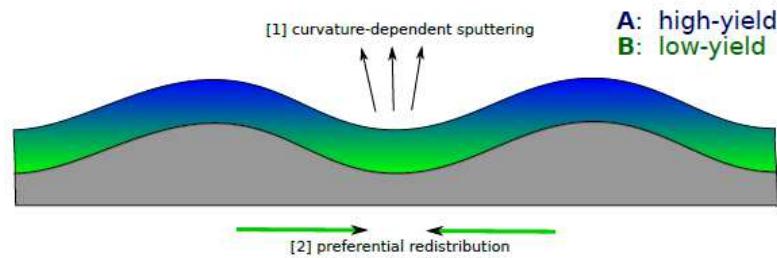
V. B. Shenoy, W. L. Chan & E. Chason PRL '07

R.M. Bradley & P. D. Shipman PRL '10

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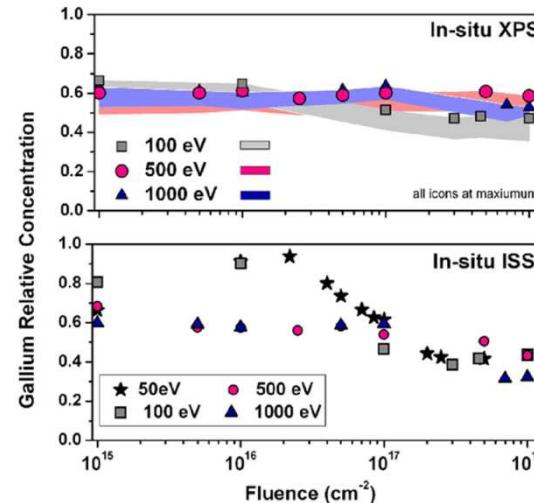
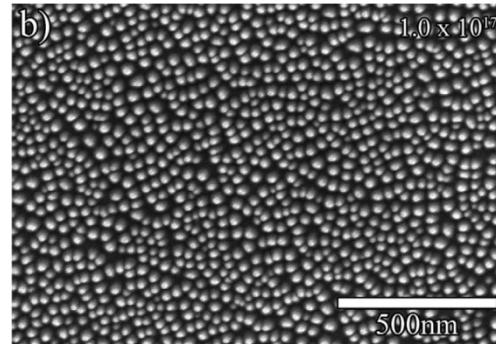
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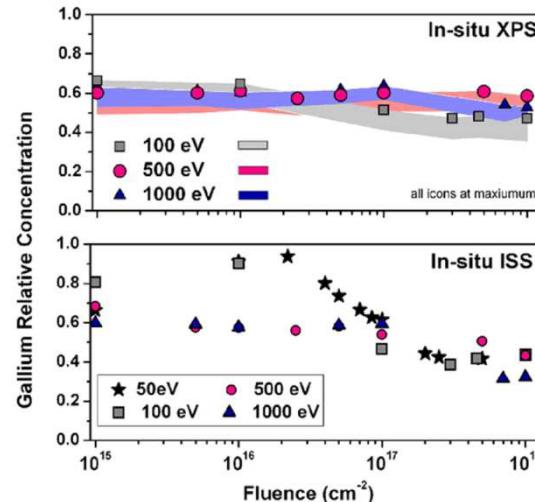
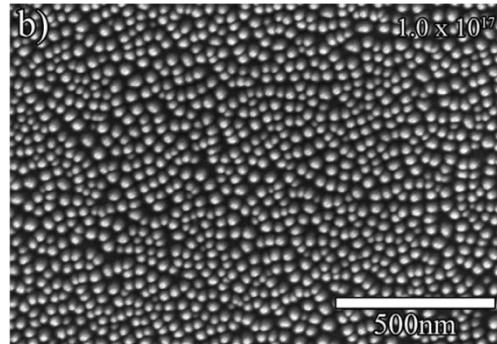
O. El-Atwani et al. JAP '11:
 Ar^+ / GaSb

Sb on top

Predictions from linear approximation

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V. B. Shenoy, W. L. Chan & E. Chason PRL '07
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Chemical instability → low-yield species at peaks

S. Le Roy et al. JAP '09

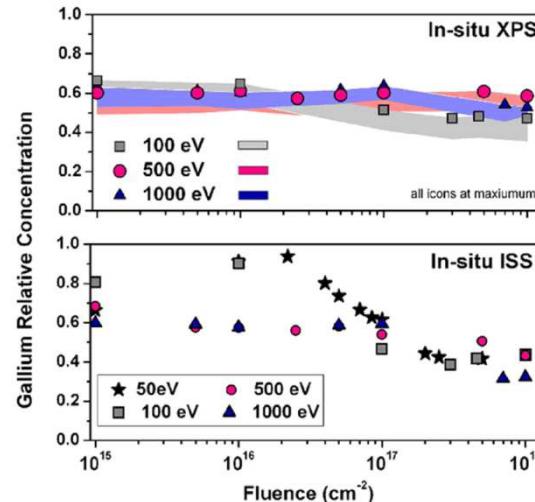
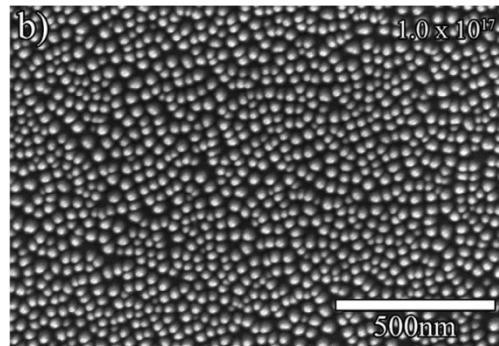
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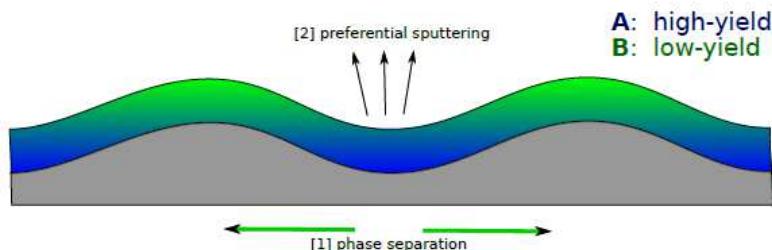


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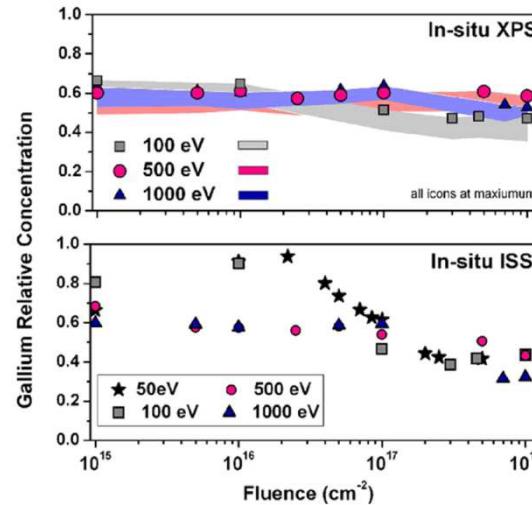
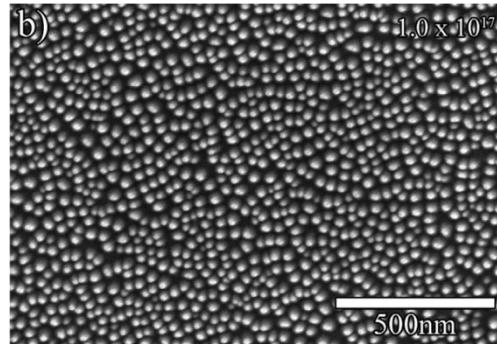


S.A. Norris arXiv:1205.6834

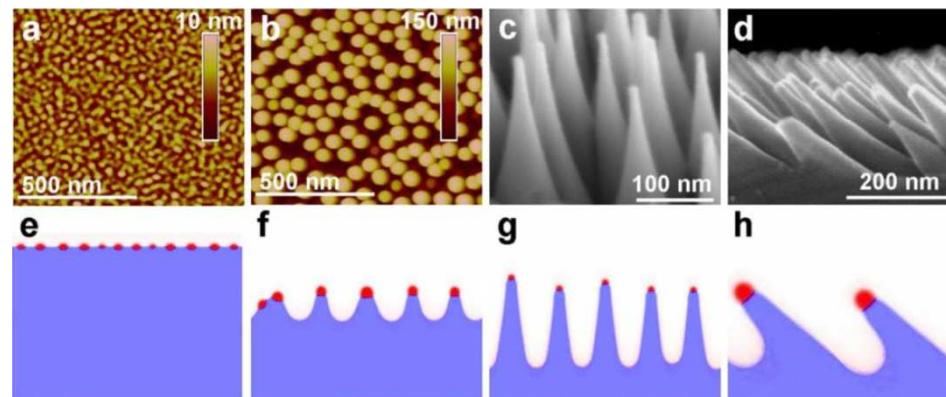
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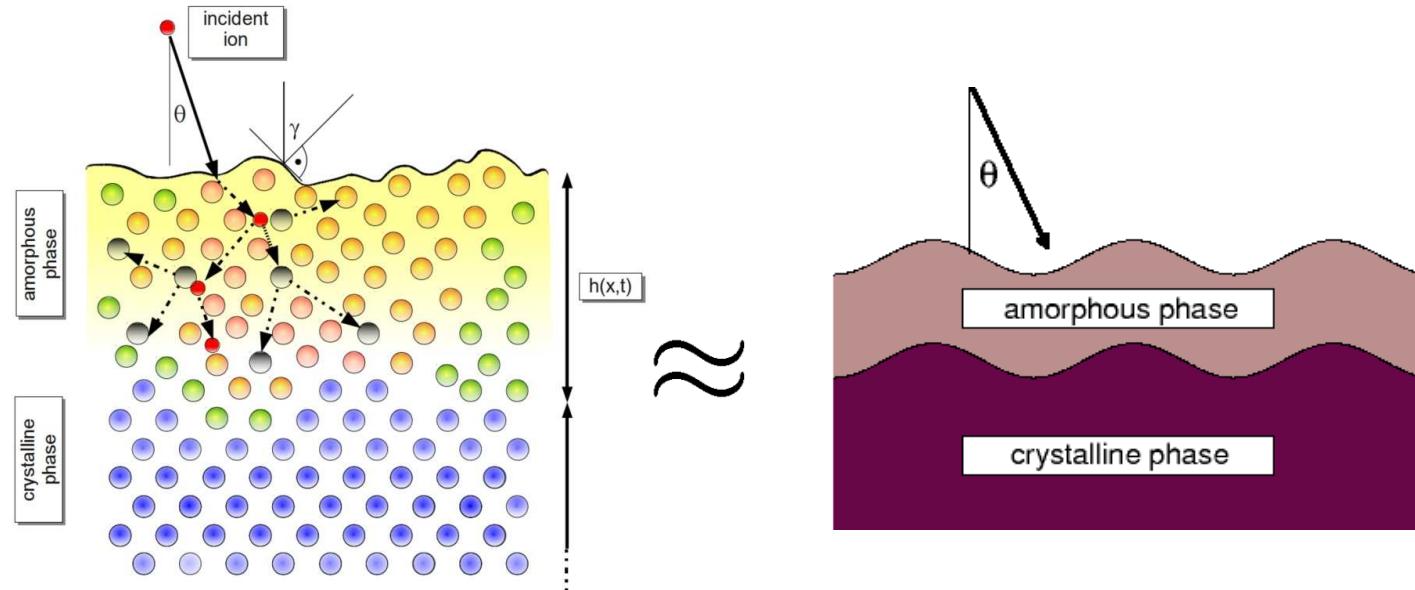
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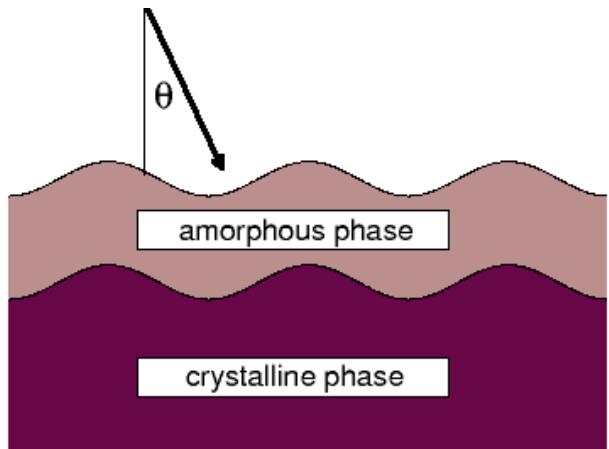
Ga on top

Back to basics: clean monoelemental targets

- Previous models depend on **undetailed (*ad-hoc*) parameters / mechanisms**
- **Strong simplifications** in formulation of reaction-diffusion models
- Upgrade: “**hydrodynamic**” model → hydrodynamic model
 - Viscous flow of full amorphous layer
 - Height dynamics as a by-product

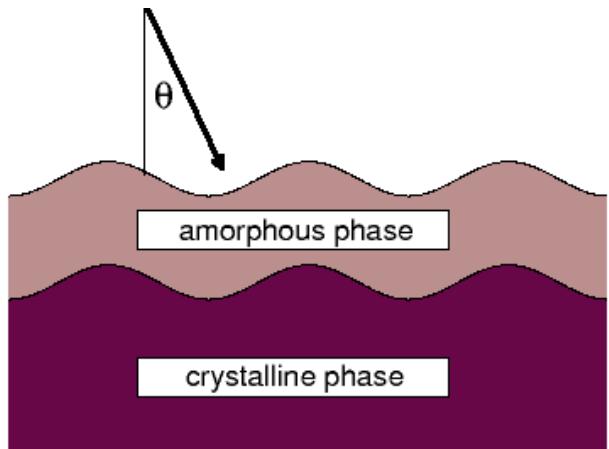


Hydrodynamic model



- Incompressible, highly viscous, Newtonian flow
- Ion-induced body force
- Stationary thickness of amorphous layer
- No-slip at a/c interface

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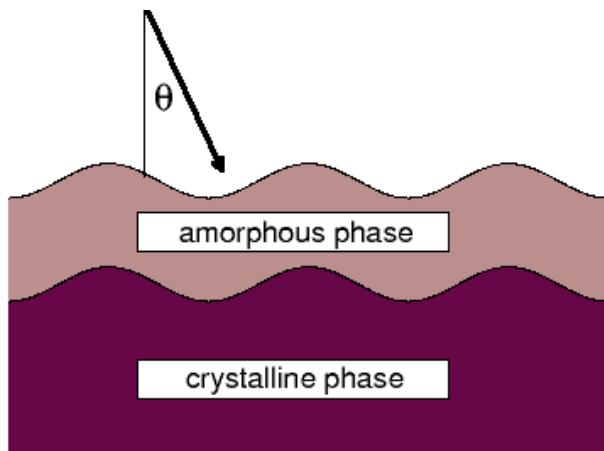
Layer flow:

$$\mathbf{0} = \mathbf{b} + \mu \nabla^2 \mathbf{V} - \nabla P$$



Ion-induced body force
= stress gradient

Hydrodynamic model



- Incompressible, highly viscous, Newtonian flow
- Ion-induced body force
- Stationary thickness of amorphous layer
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Layer flow:

$$\mathbf{0} = \mathbf{b} + \mu \nabla^2 \mathbf{V} - \nabla P$$



Ion-induced body force
= stress gradient

Boundary conditions (at free surface h):

$$\mathbf{n} \cdot \mathbf{T} \cdot \mathbf{n} = -\sigma \kappa + T_n^{\text{ext}}$$

$$\mathbf{n} \cdot \mathbf{T} \cdot \mathbf{t} = -\nabla \sigma \cdot \mathbf{t} + T_t^{\text{ext}}$$

$$\mathbf{n} \cdot \mathbf{T} \cdot \mathbf{q} = -\nabla \sigma \cdot \mathbf{q} + T_q^{\text{ext}}$$

Driving force: irradiation-induced stress

Ion-induced body force = stress gradient

Geometrical
correction to
local flux

$$\mathbf{b} \equiv \mathbf{f}_E \Psi(\theta - \gamma)$$

$$f_E \sim T^s/d$$

H.Windischmann JAP '87: Permanent displacements (linear cascade approx.)

$$T_s \sim E^{1/2}$$

C. C. Davis TSF '93: Partial relaxation of stress

Defect motion to free surface mediated by spike formation

$$T_s \sim E^{-7/6} \quad \text{for } E > 100 \text{ eV}$$

B. Abendroth et al. APL '07: High accuracy binary-collision simulations

Corrections to Davis' scaling for $E \gtrsim 2 \text{ keV}$

Results (linear stability)

$$h_k \propto e^{\omega_k t}$$

Purely erosive

$$\omega_k = -\frac{[k^2\sigma + f_E \partial_\theta (\sin(\theta)\Psi(\theta))] [-2dk + \sinh(2dk)]}{2k\mu (1 + 2d^2k^2 + \cosh(2dk))} + \overline{\omega}_k$$

Orchard's result for flow of arbitrarily thick amorphous layer in the absence of irradiation ($f_E = 0$)

Shallow layer limit $d \ll \lambda$:

$$\omega_k = -\frac{f_E \partial_\theta (\sin(\theta)\Psi(\theta)) d^3}{3\mu} k^2 - \frac{\sigma d^3}{3\mu} k^4 + \overline{\omega}_k$$

Surface-confined
irradiation-induced stress

Orchard's surface-confined
viscous flow

Dependences for ripple wavelength

$$\lambda = \frac{2\pi}{k_c}, \text{ where } \omega_k \text{ takes at } k_c \text{ its positive maximum}$$

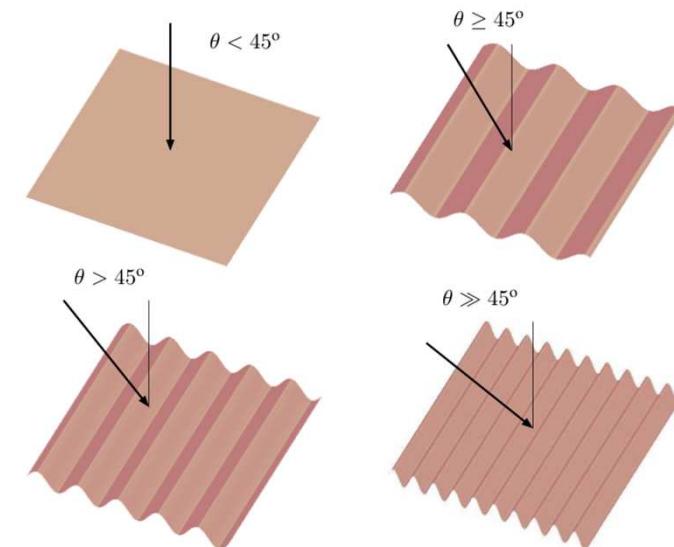
- Incidence angle θ :
Simplest choice for flux correction

$$\Psi(\theta) = \cos \theta \Rightarrow \lambda = 2\pi \left(\frac{2\sigma}{-f_E \cos(2\theta)} \right)^{1/2}$$

- Energy dependence:

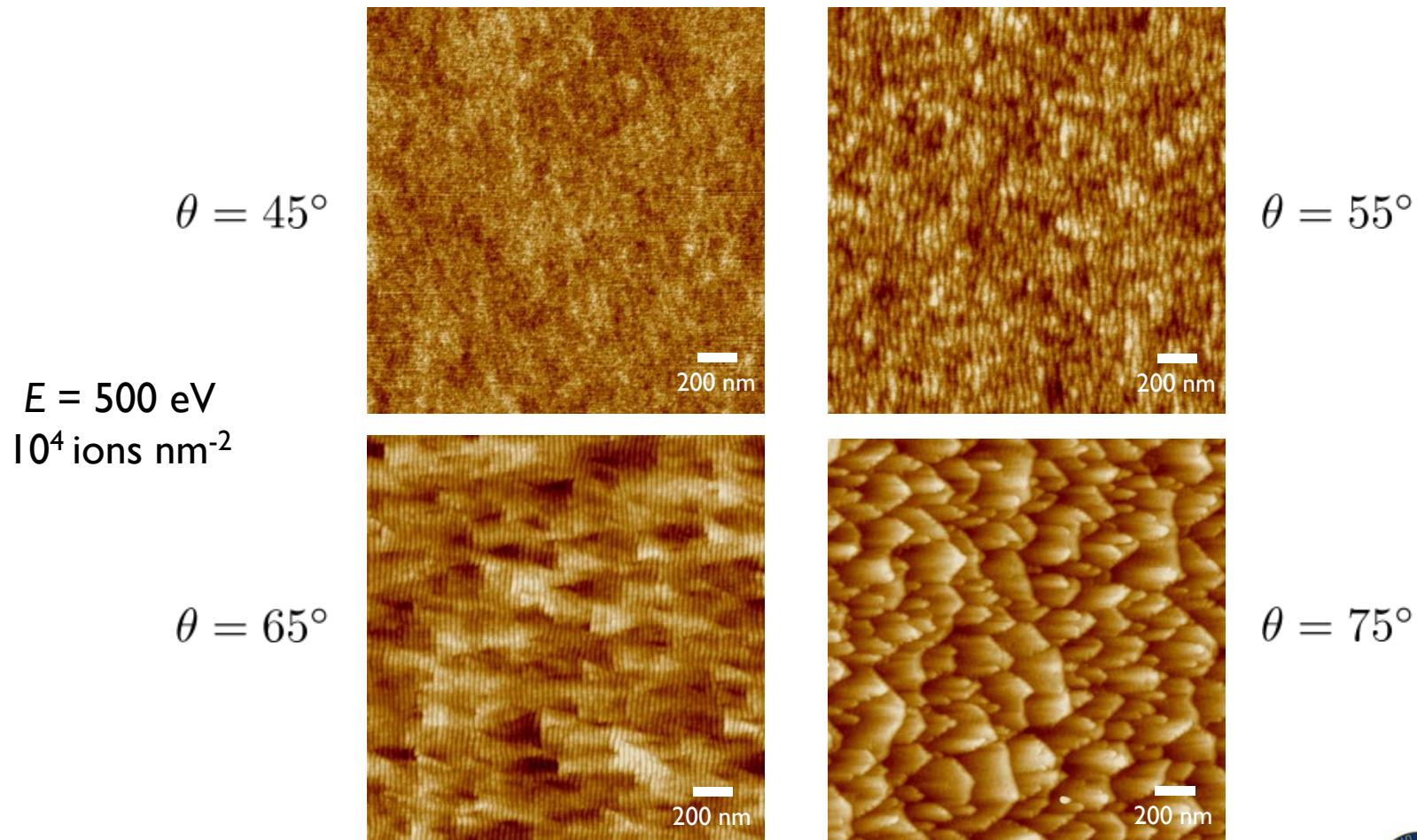
$$\left. \begin{array}{l} d \sim E^{2m}, \quad 1/3 \lesssim m \lesssim 1/2 \\ T^s \sim E^{-7/6} \end{array} \right\} \Rightarrow \lambda \sim E^{0.92-1.08}$$

Roughly linear dependence



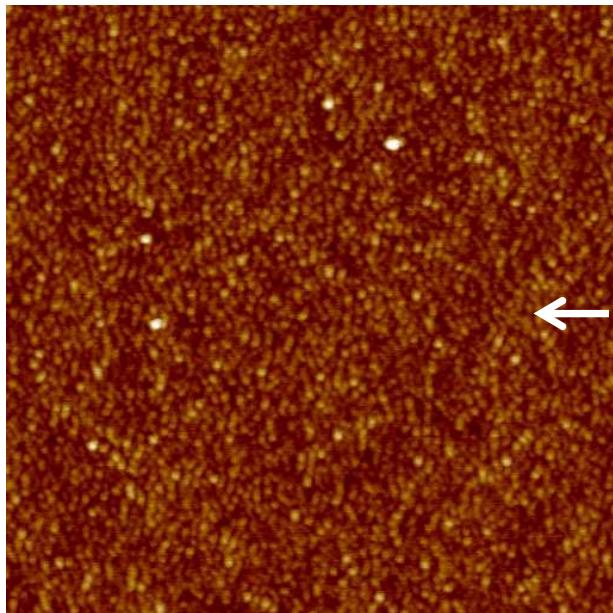
Comparison with experiments

$\text{Ar}^+ \rightarrow \text{Si}(100)$, $J = 63 \text{ ions nm}^{-2} \text{ s}^{-1}$, $300 \text{ eV} < E < 1.1 \text{ keV}$

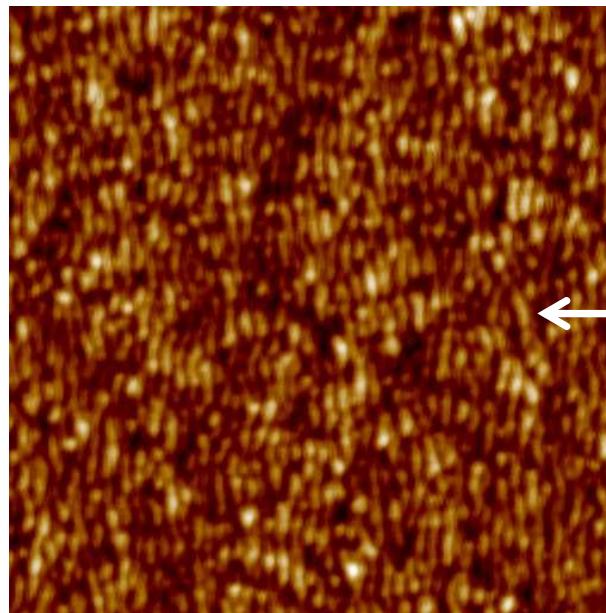


Faster dynamics at larger θ

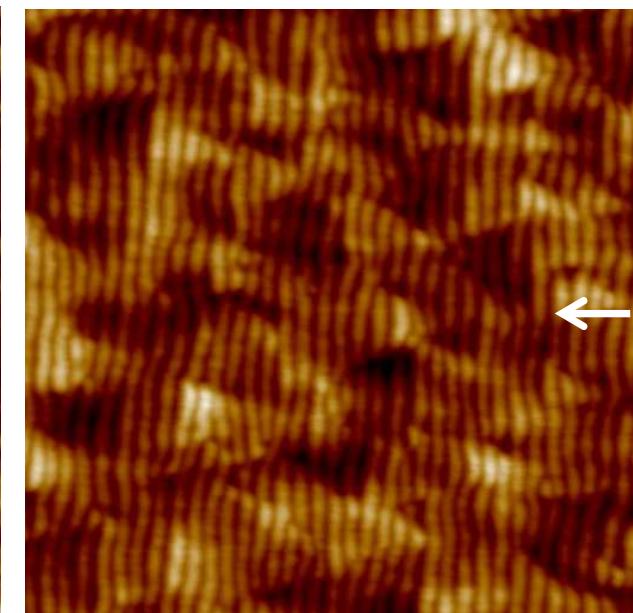
$\theta = 65^\circ \quad E = 500 \text{ eV} \quad 1 \times 1 \mu\text{m}^2$



$t = 3 \text{ min.}$



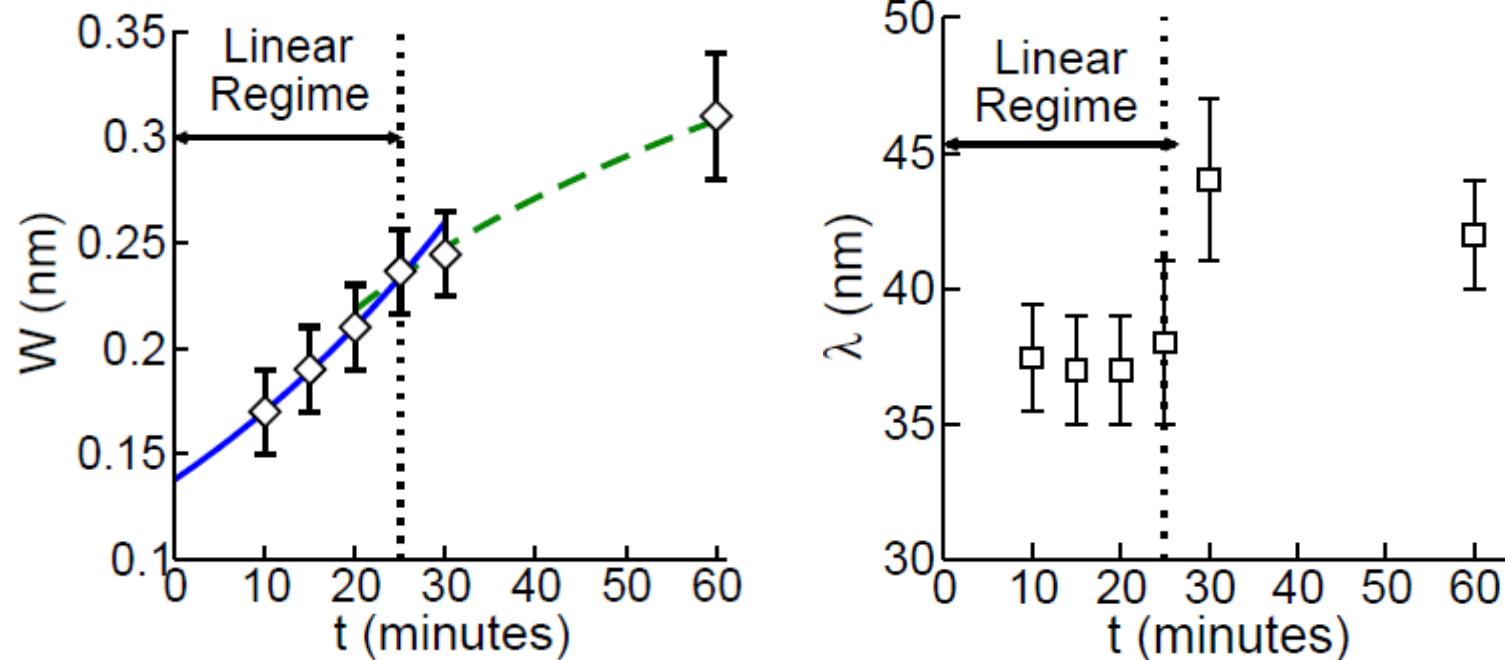
$t = 15 \text{ min.}$



$t = 90 \text{ min.}$

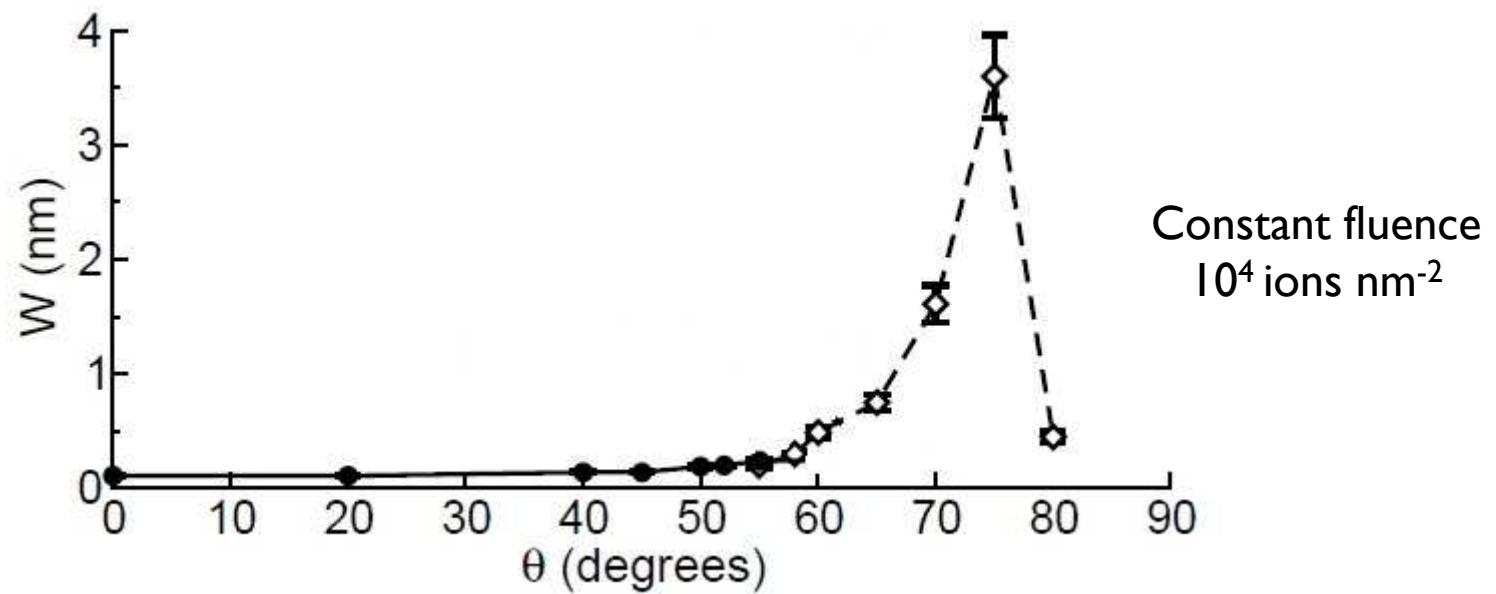
Roughness, wavelength vs fluence

$$E = 700 \text{ eV}, \theta = 55^\circ$$



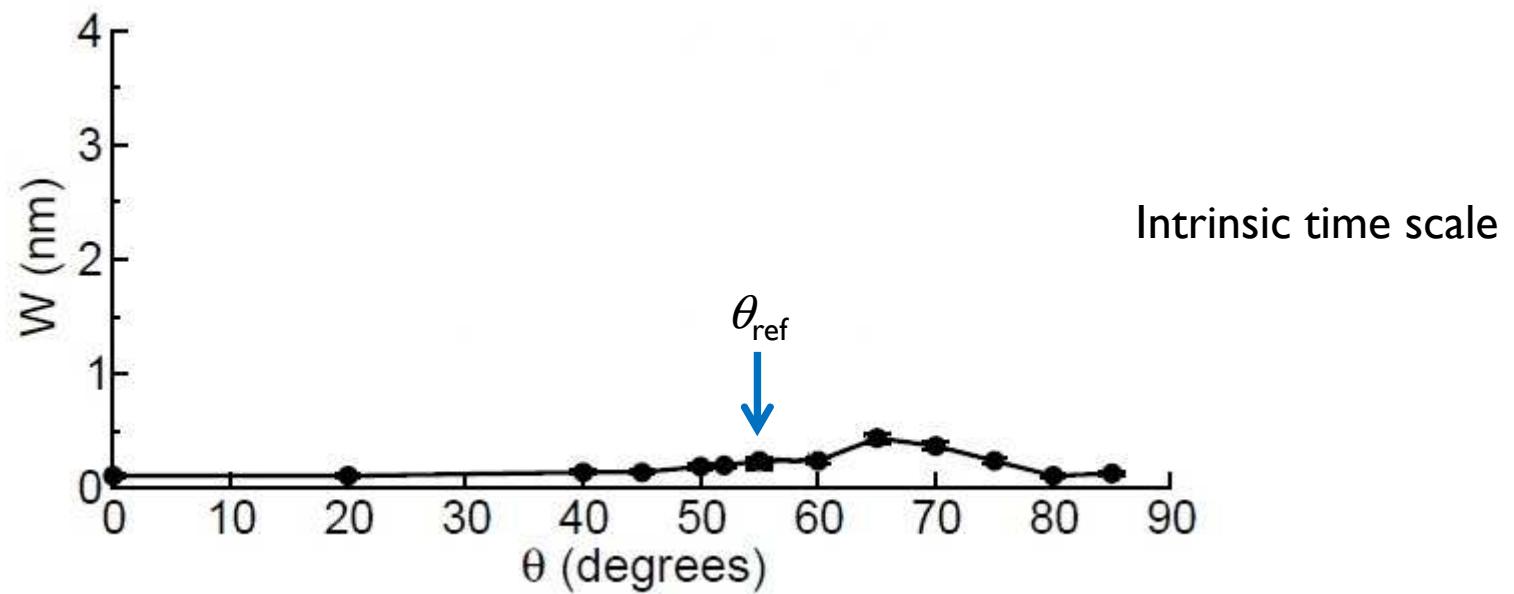
Roughness vs incidence angle

$E = 700 \text{ eV}$



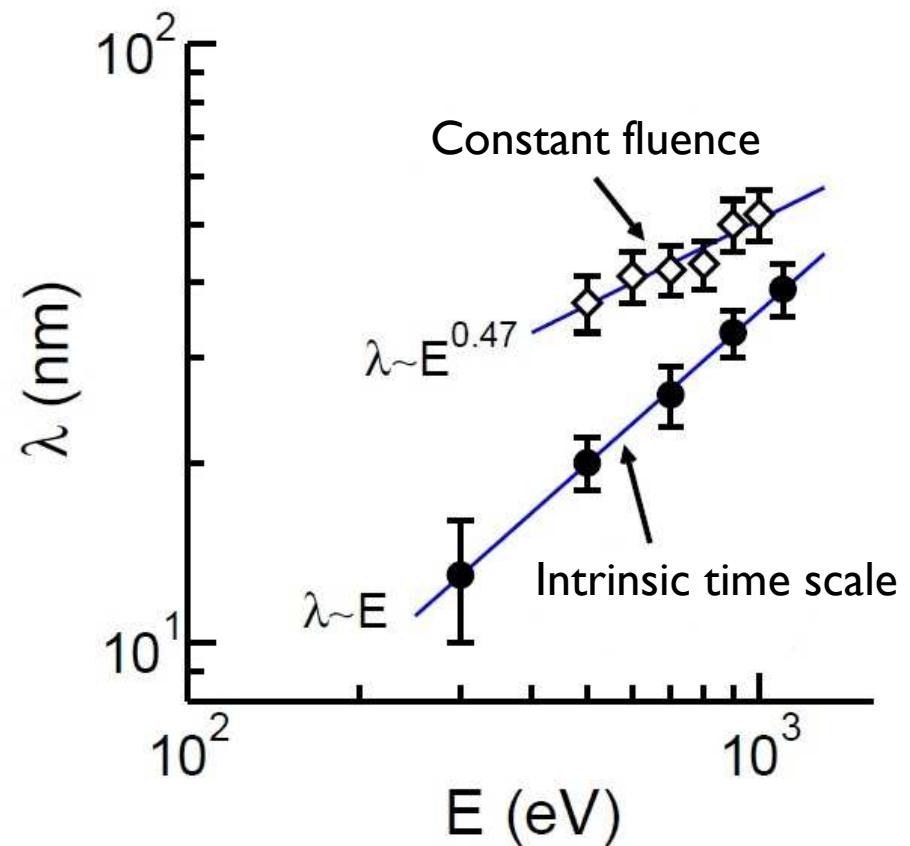
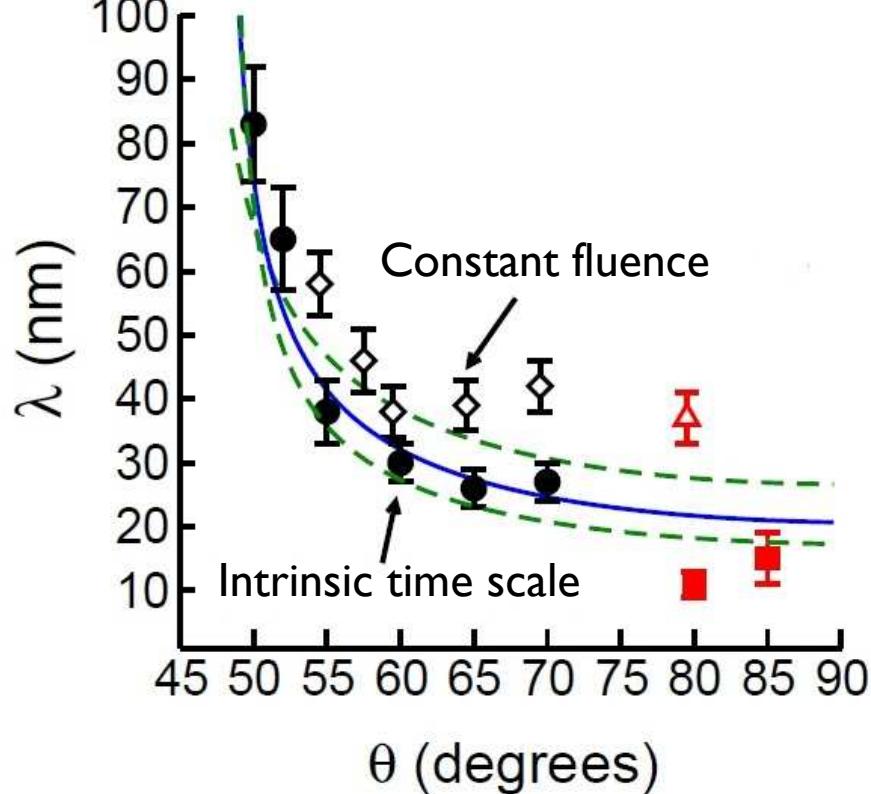
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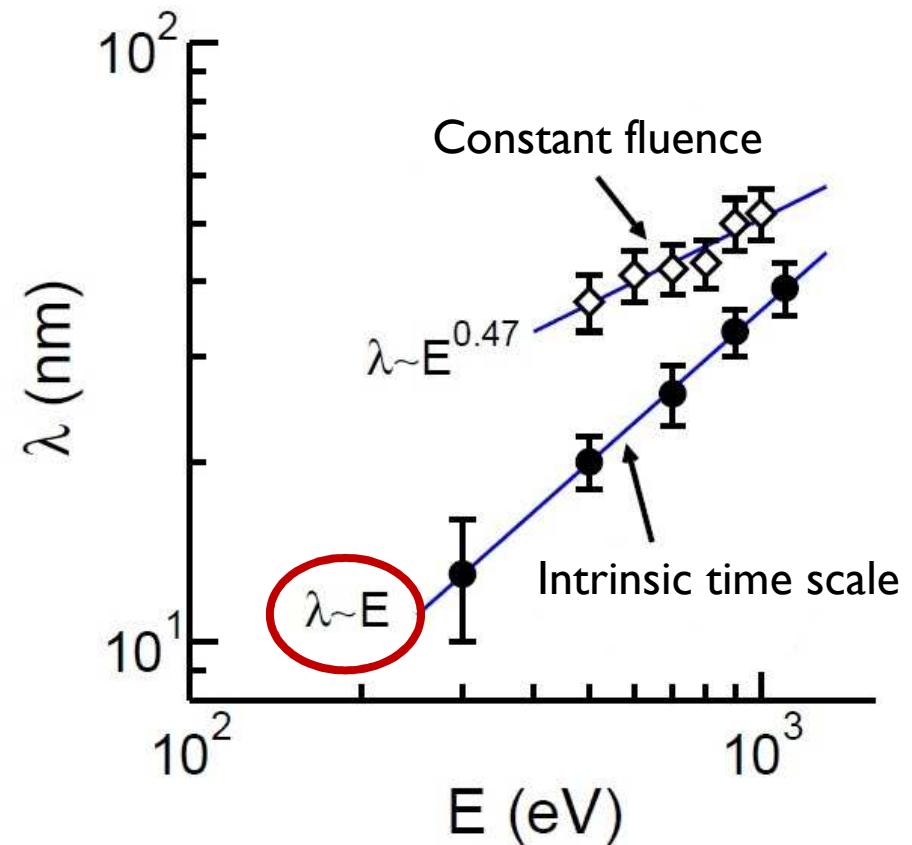
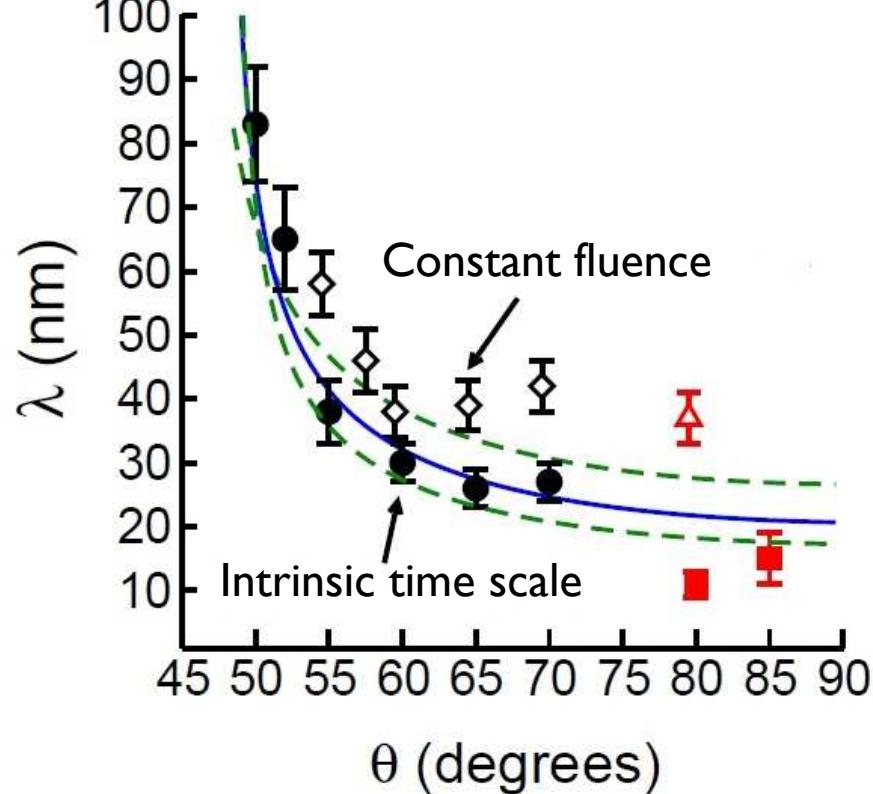


$$\tau(\theta, E) = \tau(\theta_{\text{ref}}, E_{\text{ref}}) \frac{J_{\text{exp}}(\theta_{\text{ref}}, E_{\text{ref}}) E_{\text{ref}}^{2m-7/3} \cos^2(2\theta_{\text{ref}})}{J_{\text{exp}}(\theta, E) E^{2m-7/3} \cos^2(2\theta)}$$

Wavelength vs incidence angle, energy



Wavelength vs incidence angle, energy



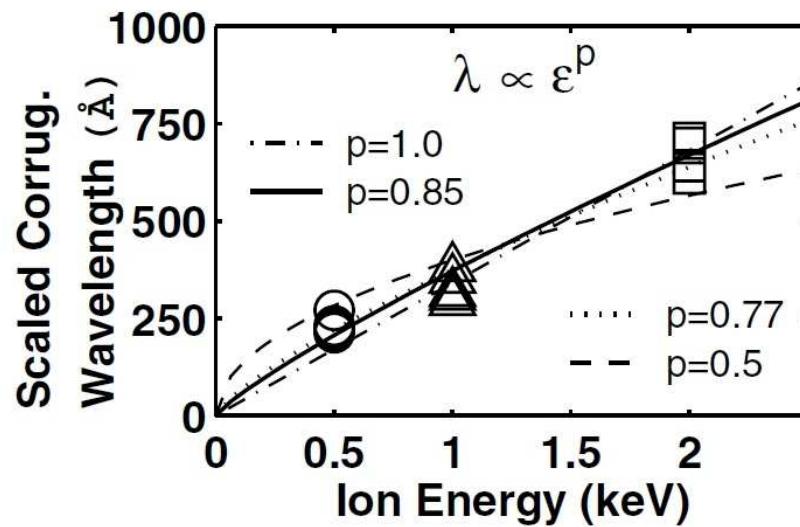
Other systems with a close-to linear $\lambda(E)$

Classically (BH), $\lambda(E) = 1/E^p$ for relaxation by e.g. thermal surface diffusion

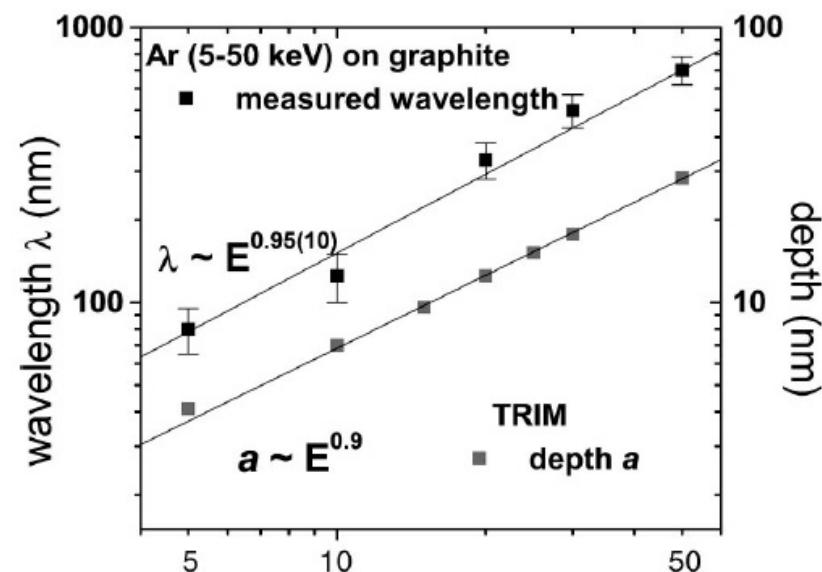
Ion-induced “transport” already advocated to explain $\lambda(E) = E^p$

M.A. Makeev, R.C. & A.-L. Barabási NIMB ‘02

C. C. Umbach et al. PRL ‘01: 10 keV Ar⁺ / SiO₂



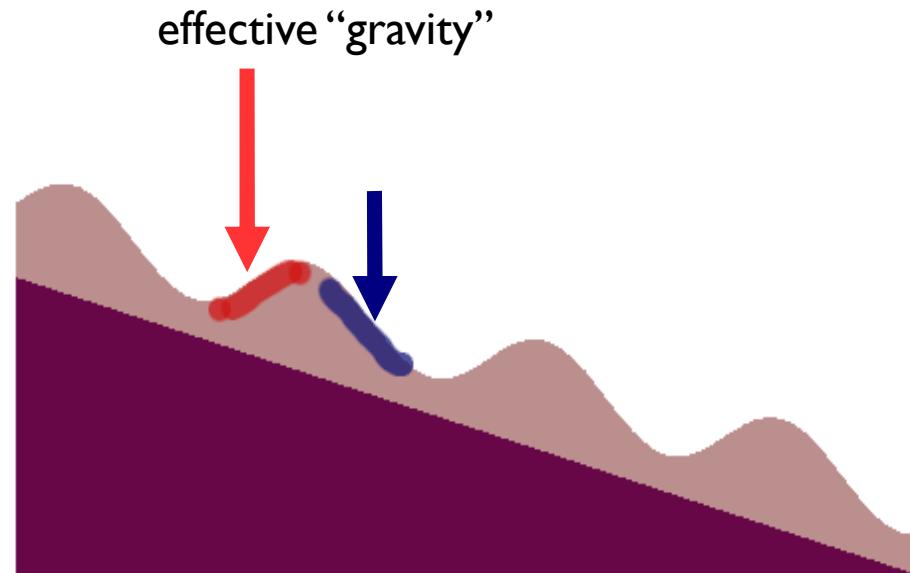
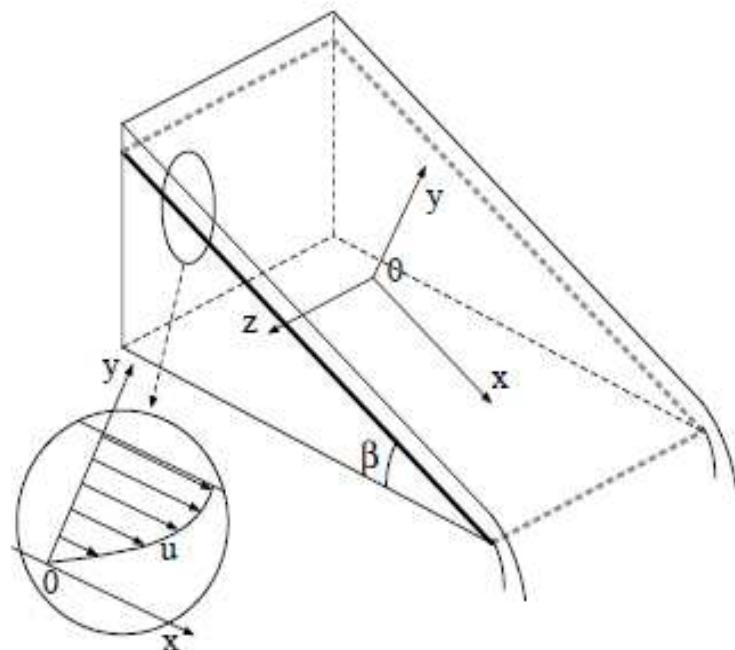
S. Habenicht PRB ‘01: 5-50 keV Ar⁺ / graphite



Non-erosive nature of morphological instability

Amorphization/stress gradient: **erosion-independent**

Analogy: **incompressible** fluid on inclined plane at angle θ ,
but here “gravity” is not constant



Above critical angle, space gradients in flow induce “bulging out”
of protuberances, due to incompressibility

Conclusions (specific to monoelemental targets)

- Non-erosive nature of morphological instability
- “Microscopic” driving as, partially healed, damage-induced stress
 - “Macroscopic” relaxation through viscous flow
- Agreement with experiments:
 - Angle dependence of λ
 - Energy dependence of λ ; common to other targets for which viscous flow has been advocated
- Generality of results (?): ion/target combination
- Understanding full anisotropy; transitions among patterns
- Further experiments / modeling needed

Conclusions (general to IBS)

- Continuum models still significant approach to IBS nanopatterning
- Future focus: quantitative approach, rather than universal properties
- Input from alternative types of modelling needed (MD, kMC)
- Many issues to be improved upon:
 - Constitutive laws need to be settled in different contexts:
 - Monoelemental vs compound targets
 - Impurities
 - Metallic vs semiconducting
 - Low / medium / high energy
 - ...
 - Role of noise: deterministic vs stochastic
 - Role of initial conditions: prepatterned targets
- ...



Nanoscale Pattern Formation at Surfaces

Copenhagen, Denmark, May 26-30, 2013



Second Announcement and Call for Papers

Abstract submission is now open for the 7th international workshop on

Nanoscale Pattern Formation at Surfaces,

to be held on 26-30 May 2013 at the Royal Academy of Sciences and Letters in Copenhagen.
You are invited to submit your abstract at

<http://www.nanopatterning2013.fotonik.dtu.dk/Abstracts.aspx> .

The server accepts only text. If you wish to submit a figure, send it by email to

sigmund@sdu.dk

Registration is open at

<http://www.nanopatterning2013.fotonik.dtu.dk/Fees%20and%20registration.aspx>



Coworkers and some overview references

Mario Castro Universidad Pontificia Comillas de Madrid

Javier Muñoz-García Universidad Carlos III de Madrid

Raúl Gago Instituto de Ciencia de Materiales de Madrid, CSIC

Luis Vázquez

- Lecture Notes on Nanoscale Science & Technology **5**, 323 (2009)
arXiv: 0706.2625
- NIMB **269**, 894 (2011)