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High frequency satellites in resonant activation

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Abstract

The zero-voltage state of a current biased Josephson tunnel junction is metastable and the system can switch to the voltage state by thermal activation. In the presence of a microwave current the rate of escape is resonantly activated when the microwave frequency matches the plasma frequency [M.H. Devoret et al. Phys. Rev. Lett. 53 (1984) 1260]. In accordance with theoretical predictions [S. Linkwitz and H. Grabert, Phys. Rev. B 44 (1991) 11 901] we have measured high frequency satellites of the resonant activation peak.

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1. Introduction

The escape of a Brownian particle from a potential well is an important problem in non-equilibrium statistical mechanics with many applications in Physics and Chemistry [1]. To the thermal Langevin random force acting on the particle can be superimposed an external oscillatory force. When the frequency of the force is close to the frequency of the small oscillations at the bottom of the well, the escape rate out of the well is resonantly enhanced. This phenomenon is known as ''resonant activation'' [2,3]. A well controlled example of this effect is found in the system consisting of a current biased Josephson junction submitted to microwave irradiation. For this system the position coordinate of the particle corresponds to the phase difference across the junction, which, properly biased, can be trapped in the metastable zero-voltage state.

The escape rate enhancement is an intrinsically non-linear phenomenon because during the escape the particle undergoes a large-amplitude non-sinusoidal motion. Due to the difficulty of the problem, early theories by Fonseca and Grigolini [4], Larkin and Ovchinnikov [5] and finally Ivlev and Mel'nikov [6] had to resort to simplifying assumptions. Although these theories captured various important aspects of the problem, they were not able to predict the detailed shape of the resonance. However, a complete theoretical analysis is now available [7] which offers a good quantitative understanding of the various features of the resonance. An interesting and as yet untested prediction of this theory is the presence of satellite resonances at frequencies which are multiples of the frequency of the main resonance,

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the only one which has been explored experimentally so far. These resonances are expected on general grounds since an anharmonic oscillator can usually be pumped by an oscillatory force at harmonics of its fundamental frequency. In this paper, we report the results of an experiment aimed at testing this prediction. A key advantage of the Josephson junction system is that all the parameters entering in the theory can be well controlled experimentally. In our experiment, we have aimed at controlling in particular the damping of the particle which, at low enough temperatures, is due to the electromagnetic environment of the junction. In the following section we briefly review the theoretical predictions. In Section 3 we discuss the experimental apparatus and the measurement techniques. Finally, in Section 4 we present the results and our conclusions.

2. Theoretical predictions

We describe the Josephson junction in terms of a resistively shunted junction model generalized to arbitrary environmental impedances [4]. When biased with a constant current I, the junction can be modeled as a particle moving in a tilted washboard potential

$$U(\delta) = -\frac{I_0\Phi_0}{2\pi}\left(\cos\delta + \frac{I}{I_0}\delta\right), \tag{1}$$

where δ is the difference between the phases of the two superconductors on the two sides of the junction, I_0 is the junction critical current, and $\Phi_0 = h/2e$ is the flux quantum. The equation of motion for the phase difference δ takes the form

$$C\left(\frac{\Phi_0}{2\pi}\right)^2 \ddot{\delta}(t) + \left(\frac{\Phi_0}{2\pi}\right)^2 \int_{-\infty}^{t} \mathrm{d}s\, Y(t-s)\, \dot{\delta}(s) + \frac{\mathrm{d}U(\delta(t))}{\mathrm{d}\delta(t)} = \frac{\Phi_0}{2\pi}\left[I_\mathrm{N}(t) + I_\mathrm{m}(t)\right], \tag{2}$$

where C is the junction self-capacitance. The junction is shunted by leads of admittance $Y(\omega) = \int_0^\infty \mathrm{d}t\, \mathrm{e}^{-i\omega t}\, Y(t)$. The thermal noise current $I_\mathrm{N}(t)$ due to this electromagnetic environment at temperature T is Gaussian with zero mean and the correlation function $\langle I_\mathrm{N}(t) I_\mathrm{N}(s)\rangle = k_\mathrm{B} T Y(|t-s|)$. Finally, $I_\mathrm{m}(t) = I_\mathrm{m}\cos(\omega t + \phi)$ is the microwave current with frequency ω.

In the experiments I is close to I_0 and the potential may be approximated by a cubic potential characterized by the barrier height

$$\Delta U = \frac{2^{5/2}}{3}\frac{I_0\Phi_0}{2\pi}\left(1 - \frac{I}{I_0}\right)^{3/2} \tag{3}$$

and the frequency of small plasma oscillations at the bottom of the well

$$\omega_\mathrm{p} = \left(\frac{2\pi I_0}{\Phi_0 C}\right)^{1/2}\left(1 - \frac{I^2}{I_0^2}\right)^{1/4}. \tag{4}$$

In the absence of a driving microwave current ($I_\mathrm{m} = 0$) the escape from the zero-voltage state occurs via thermal activation over the barrier at a rate $\Gamma(0)$. We have measured the enhancement factor

$$\rho = \ln\frac{\Gamma(I_\mathrm{m})}{\Gamma(0)} \tag{5}$$

of the escape rate by the microwave current as a function of the microwave frequency ω. As described in the following section, we operate with a microwave power small enough for ρ to be proportional to I_m^2. The enhancement factor ρ can be calculated analytically for a large quality factor $Q = \omega_\mathrm{p} C/\mathrm{Re}[Y(\omega_\mathrm{p})]$ and small microwave power [7]. These limits are appropriate to the experiments. Introducing the dimensionless amplitude and frequency of the microwave force

$$A = \frac{I_\mathrm{m}}{\sqrt{\Delta U C \omega_\mathrm{p}^2}}, \quad F = \frac{\omega}{\omega_\mathrm{p}} \tag{6}$$

and the dimensionless temperature $\theta = k_\mathrm{B}T/\Delta U$, the result for ρ may be written as (cf. Eq. (86) in Ref. [7])

$$\rho = \frac{27\pi}{2}\frac{A^2}{\theta^2}\int_0^1 \mathrm{d}\varepsilon\,(1 - \mathrm{e}^{-\varepsilon/\theta}) \sum_{n=1}^{\infty} \frac{n q_n^2(\varepsilon)}{\lambda(\varepsilon)} \times \frac{1/2Q}{[1 - F/n\mu(\varepsilon)]^2 + 1/4Q^2}. \tag{7}$$

[4] For details of the experimental techniques and theoretical methods we refer to [2,3] and [7], respectively.

Here the integral is over the dimensionless energy $\varepsilon = E/\Delta U$ of states in the metastable well. The unperturbed oscillation of energy E may be written as

$$\delta(t) = \delta_0 \sum_{n=-\infty}^{\infty} q_n(\varepsilon) \exp\left[in\omega_p \mu(\varepsilon) t \right], \tag{8}$$

where

$$\delta_0 = 3 \times 2^{3/4}(1 - I/I_0)^{3/4}\left(1 - I^2/I_0^2\right)^{-1/4} \tag{9}$$

is the width of the barrier, which serves as a characteristic scale for δ. Eq. (8) defines the dimensionless oscillation frequency $\mu(\varepsilon)$ and the scaled amplitudes $q_n(\varepsilon)$ of the harmonic components of the oscillation. Finally, $\lambda(\varepsilon)$ is a dimensionless energy relaxation coefficient describing the average energy dissipated by the environment according to $\dot{E}_{\mathrm{diss}} = -[\Delta U \omega_p \mu(\varepsilon)/2\pi]\lambda(\varepsilon)$. Explicit expressions for the quantities $\mu(\varepsilon)$, $q_n(\varepsilon)$, and $\lambda(\varepsilon)$ are given in Eqs. (91), (92), and (95) in Ref. [7].

The result (7) gives the enhancement factor ρ as a function of the dimensionless frequency F. The main contribution to the integral over ε arises from the region near $\varepsilon = 0$. Noting [5] $\mu(0) = 1$, we see that the term with $n = 1$ describes the main resonance near $F = 1$, i.e. $\omega = \omega_p$. The terms with $n > 1$ describe satellite peaks the magnitude of which decreases rapidly with increasing n. However, all peaks are proportional to A^2 and thus of the same order in the microwave power.

3. Experimental apparatus and procedures

The practical realization of the junction and its coupling to a well-defined electromagnetic environment is shown in Fig. 1. The 18×22 μm Nb-NbOx-PbIn tunnel junction is patterned in a cross strip on a silicon chip (see inset). The junction is connected to a horn-shaped $Z_c = 50\,\Omega$ coplanar transmission line which is impedance matched to a coplanar line built from printed circuit board. A microwave absorbing block, referred to in the following as the ''load'', is pressed on this external line as close as possible from the chip. The junction is dc connected through this external line to a current source in parallel with a fast voltmeter. This wiring to room temperature electronics goes through a series of cold filters which prevent ambient noise from reaching the junction. A second line, opposite to the dc biasing line has been designed on the chip in order to excite the junction with a microwave current. This irradiation line is weakly coupled to the junction through a high-impedance capacitive element and contributes to the impedance seen by the junction in a negligible fashion. The microwave line between the microwave generator and the on-chip irradiation line contains several attenuators to make the frequency dependence of the microwave current amplitude at the junction as smooth as possible.

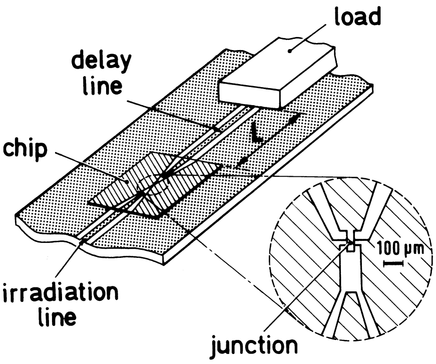

Fig. 1. Josephson junction (see inset) connected to a delay line and capacitively coupled to an irradiation line.

In the low-frequency range where the current source and voltmeter operate, the load and filters play no role in the biasing circuit and the junction experiences only a very high impedance. However, in the microwave range the portion of the line covered by the load behaves as a lossy transmission line. In this frequency range, the circuit shunting the junction can be modelled by a section of an ideal transmission line with characteristic impedance Z_c terminated by a resistor Z_t. The contribution of the filters to the impedance seen by the junction is made

[5] The small dispersive effect of the electromagnetic environment on $\mu(\varepsilon)$ (cf. Eq. (71) in Ref. [7]) may be disregarded here. If this effect is taken into account one finds $\mu(0) = 1 - \frac{1}{2}\mathrm{Im}\left[Y(\omega_p)\right]/C\omega_p$ which causes a small shift of the resonant activation peak. This shift becomes observable, however, if the environmental impedance is changed by varying the electric length l of the apparatus described in Section 3.

negligible by the attenuation due to the load. The ideal transmission line is also characterized by its propagation velocity v and electric length l. Microwave measurements of the printed circuit line with the load yielded $Z_t = 30 \pm 3\Omega$. The parameters Z_c, v and l are functions of the chip layout and can be estimated: $Z_c = 50 \pm 2\Omega$, $v = 1.1 \pm 0.1 \times 10^8$ m/s and $l = 6 \pm 1$ mm. In terms of these parameters, the environmental admittance reads

$$Y(\omega) = \frac{1}{Z_c} \frac{1 + a\exp(-i\omega u)}{1 - a\exp(-i\omega u)}, \qquad (10)$$

where $a = (Z_c - Z_t)/(Z_c + Z_t) \sim 0.25$ and $u = 2l/v \sim 5.5 \times 10^{-11}$ s. The ensemble of the chip, load, and filters was placed in a pumped helium bath the temperature of which was regulated to better than 2 mK.

The lifetime of the junction in its zero-voltage state was determined by square-wave modulating the bias current and recording the times τ that elapsed between the leading edge of each pulse and the consecutive voltage rise signalling the switching of the junction to its voltage state. We recorded typically 10^4 events in 10 s, and computed Γ from $\Gamma^{-1} = \langle \tau \rangle$. By measuring Γ as a function of the bias current I we determined $I_0 = 23.47 \pm 0.2$ μA, the critical current of the junction. The enhancement of the escape rate due to microwaves was measured by recording the microwave generator power P needed to double Γ. Apart from a microwave transmission factor, this constitutes a measurement of ρ.

4. Results and discussion

In Fig. 2 we plot the measurement of P as a function of the microwave frequency $f = \omega/2\pi$. Three resonances are observed. The main one is the well-studied resonant activation curve from which we extract the junction capacitance C. All parameters entering in the theory of Section 2 are now known. The theoretical prediction containing no adjustable parameter is plotted as a continuous line in Fig. 2. This theoretical curve includes a correction which accounts for the frequency dependence of the capacitive coupling between the irradiation line and the junction. There is good agreement between the

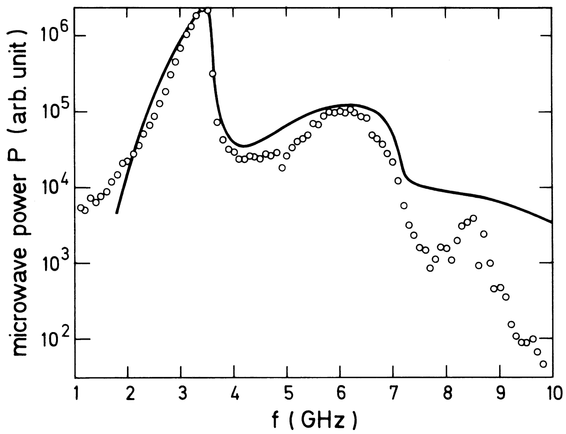

Fig. 2. The measured microwave power needed to double the decay rate is shown as a function of the microwave frequency. The full line gives the theoretical prediction calculated from Eq. (7).

experimental and the theoretical results in the middle part of the frequency range. The shape and relative amplitude of the secondary resonance at approximately twice the plasma frequency is remarkably well captured by the theory. We believe that the deviations which are observed in the outer sections are due to the crudeness of the model we have used to describe the electromagnetic environment of the junction. The assumption of a frequency independent resistance as a model for the load is probably too simplified.

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