Hot-electron limitation to the sensitivity of the dc superconducting quantum interference device

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The noise energy of conventional thin-film dc superconducting quantum interference devices (SQUIDs) flattened out as the operating temperature was lowered below 140 mK. We attribute this saturation to the heating of the electrons in the resistive shunts by the SQUID bias current. This "hot-electron effect" is a general property of normal metals at low temperatures and arises from the limited rate at which the electrons can transfer energy to phonons. The temperature of the electrons, and hence the noise energy of the SQUIDs, was reduced by a factor of about 3 by attaching large volume "cooling fins" to each shunt.

In the classical limit, the noise energy per unit bandwidth of a dc superconducting quantum interference device (SQUID) is well predicted by[1,2]

$$S_\Phi(f)/2L \simeq 9k_B TL/R \simeq 16k_B T(LC)^{1/2} \qquad (1)$$

at frequencies f above the $1/f$ noise region, provided $\beta = 2LI_0/\Phi_0$ is of the order of unity, and $\beta_c = 2\pi I_0 R^2 C/\Phi_0 \lesssim 1$. In Eq. (1), $S_\Phi(f)$ is the spectral density of the equivalent flux noise in the SQUID, T is the temperature, L is the inductance of the loop, Φ_0 is the flux quantum, and I_0, C, and R are the critical current, capacitance, and shunt resistance of each of the two Josephson tunnel junctions. Equation (1) is based on the assumption that the noise originates in the Nyquist noise of the resistive shunts, which are at a temperature T. The noise energy may be lowered by reducing L, C, or T, and it is generally believed that the quantum limit will be reached when $S_\Phi/2L \simeq \hbar$.[3,4] However, we re-emphasize here that quantum mechanics does not place a rigorous limit on the value of $S_\Phi/2L$, which is a measure only of the voltage noise across the SQUID, but rather on the noise temperature of a SQUID as an amplifier, which involves both the voltage noise and the current noise[5] in the SQUID loop.

Several groups have achieved a noise energy of a few $\hbar$ by reducing L or C.[6-9] However, a SQUID with a very low value of L ($\simeq 1$ pH) is of limited application because it cannot be efficiently coupled to an input coil with a useful inductance, say 1 μH. Furthermore, although one can reduce the self-capacitance of tunnel junctions by making them of small area, in practice, parasitic capacitance, such as that associated with the input coil, appears to set a lower bound on C. Thus to achieve the quantum limit in a SQUID which can be efficiently coupled to an inductance of 1 μH, one may have to operate the SQUID at temperatures below 1 K. A quantum limited SQUID at 1 kHz would be useful, for example, as the preamplifier in a Weber bar gravity wave antenna[10,11] cooled to the millikelvin temperature range.

In previous measurements[12] on SQUIDs, we found that the white noise energy scaled as T as we lowered the temperature to about 140 mK, but remained constant as the bath temperature was reduced further. In this letter, we show that electron heating in the resistive shunts of such SQUIDs does not allow the electron temperature to decrease below about 140 mK, no matter how cold the refrigerator. However, the addition of "cooling fins" to the shunts enables the shunts to be cooled to about 50 mK.

The resistive shunts of a typical dc SQUID are constructed from normal metal thin films. In operation, a SQUID is biased at a fixed voltage or current, and the resulting power P is dissipated as heat in these thin-film resistors. The power is supplied initially to the electrons in the metal, and is transmitted to the underlying insulating substrate by phonons. In the thin-film shunt at low temperatures, because the film thickness is much less than the wavelength of a thermal phonon, the temperature of the phonons in the metal, T_p, is held very close to the refrigerator temperature by the substrate, which, in our case, is an oxidized Si wafer. On the other hand, the level of the white noise in the SQUID is determined through Eq. (1) by the temperature of the electrons in the metal shunts, T_e, which in turn is determined by the rate at which the electrons can transfer energy to the phonons. The resulting equivalent thermal resistance between the electrons and the phonons, R_{ep}, has been estimated by Little,[13] and measured by Anderson and Peterson[14] in bulk Cu, Roukes *et al.*[15] in Cu thin films, and by ourselves in AuCu thin films.[16] Using a calculation based on electron-phonon scattering, we have shown under certain simplifying assumptions that the electrons lose energy to the phonons at a rate[16,17]

$$P = \Sigma\Omega(T_e^5 - T_p^5), \qquad (2)$$

where Ω is the volume of the metal, $\Sigma = 0.524\alpha^*\gamma$, γ is the electronic heat capacity per unit volume, and $\alpha^* T_e^3$ is the thermally averaged electron-phonon scattering rate.[18] From Eq. (2) one can show that the thermal resistance between the electrons and the phonons is $R_{ep} = 1/(5\Sigma\Omega T_e^4)$. Thus for the small volumes typical of thin-film shunts and at low temperatures, R_{ep} becomes large.

In separate experiments, we used a dc SQUID to measure the Nyquist noise in thin films of AuCu as a function of dissipated power. From the level of the Nyquist noise and the film resistance, we deduced the electron temperature. We found excellent agreement with the prediction of Eq.

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(2), $T_e = (T_0^5 + P/\Sigma\Omega)^{1/5}$, with $\Sigma = (2.4 \pm 0.6) \times 10^9$ W $\mathrm{K}^{-5}\,\mathrm{m}^{-3}$, and where we have set $T_p = T_0$, the bath temperature.[16,17] For comparison, from Refs. 15 (thin-film Cu) and 14 (bulk Cu), we can deduce values of Σ for Cu which are factors of about 1.25 and 2 times smaller, respectively, than our own value in AuCu. Setting $T = T_e$ in Eq. (1), we predict that the noise energy in a SQUID at a bath temperature T_0 will be given approximately by

$$\frac{S_\Phi}{2L} \simeq 9 k_B L \frac{(T_0^5 + P/\Sigma\Omega)^{1/5}}{R}. \tag{3}$$

In a further set of experiments, we investigated the effect of locally heating the electrons, and showed that the electron temperature can be substantially reduced by adding a "cooling fin," consisting of a large volume of normal metal.[16,17] Hot electrons from the small heated region diffuse out into the large surrounding region, and there lose energy to the phonons and the cooler electrons, while cooler electrons from the surrounding regions diffuse into the heated region. The electrical power is thus delivered to a larger number of electrons, thereby producing a smaller rise in the electron temperature in the locally heated region. Electrons within an inelastic diffusion length of the heated region can contribute to the cooling. At 20 mK in our AuCu thin films we estimate this length to be of the order of 2 mm, so that rather large thin-film cooling fins can be used.

Figure 1(a) shows the configuration of the four SQUIDs discussed here; all have an inductance of 5×10^{-10} H, and differ only in the construction of the resistive shunts.

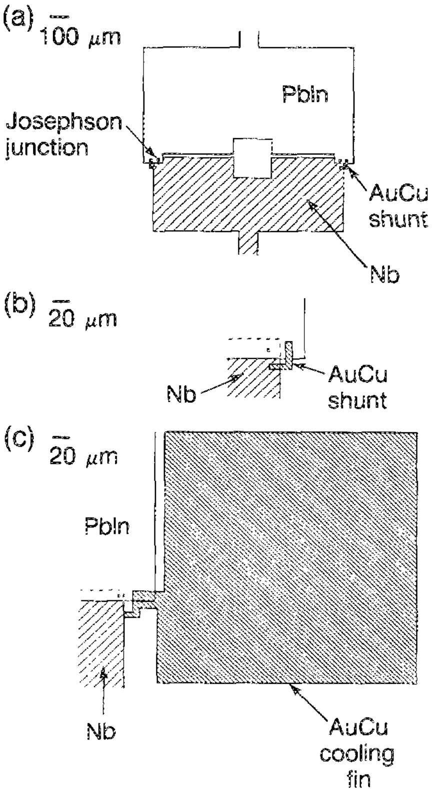

FIG. 1. (a) Configuration of SQUIDs for devices $D1$ and $D2$. (b) Detail of shunts for $D1$ and $D2$, and (c) detail of shunt and cooling fin for $M1$ and $M2$.

TABLE I. Parameters for the SQUIDs. The parameters A, Ω, and R are the area, volume, and resistance of the shunts and cooling fins; $\beta \equiv 2LI_0/\Phi_0$ and $\beta_c \equiv 2\pi I_0 R^2 C/\Phi_0$, where L is the inductance of the SQUID, I_0 and C are the critical current and capacitance of each junction, and Φ_0 is the flux quantum; P is the power dissipated by the SQUID; and $T_{\min} = (P/\Sigma\Omega)^{1/5}$, where $\Sigma = 2.4 \times 10^9$ W $\mathrm{K}^{-5}\,\mathrm{m}^{-3}$.

SQUID	A (mm^2)	Ω (μm^3)	R (Ω)	β	β_c	P (pW)	$T_{\min}$ (mK)
$D1$	9.6×10^{-4}	2.9×10^1	6	1.6	0.17	5.4	151
$D2$	9.6×10^{-4}	2.9×10^1	6	1.1	0.12	3.8	140
$M1$	1.54×10^{-1}	4.6×10^3	6	1.4	0.15	5.5	55
$M2$	1.54×10^{-1}	1.4×10^5	8	1.5	0.30	20.0	36

We chose this somewhat unusual configuration for the SQUID loop because the spectral density of the ensuing $1/f$-like noise at low temperatures had a steeper slope than that of other SQUIDs that we studied. As a result, we were able to determine the white noise at a lower frequency. The junctions are $2 \times 2\ \mu\mathrm{m}^2$, Nb-NbO$x$-PbIn window junctions formed using a procedure described elsewhere.[12] Figure 1(b) shows the configuration of the Au (25 wt. % Cu) shunts on the first two devices, $D1$ and $D2$. Their dimensions are typical of those of other SQUIDs we have tested; their area A and volume Ω are listed in Table I, together with the values of R, β, and β_c. The devices were fabricated with $\beta_c \ll 1$ to reduce resonant structure, which causes a considerable degradation of the SQUID performance at low temperatures. The SQUIDs were mounted on a dilution refrigerator, and were isolated from external noise by a Cu mesh screened room, μ-metal shields, a superconducting Nb shield, and electrical filters on the bias lines. We measured the electrical characteristics and noise (from 0.2 Hz to 25 kHz) in each SQUID using a second dc SQUID at the same temperature.[12]

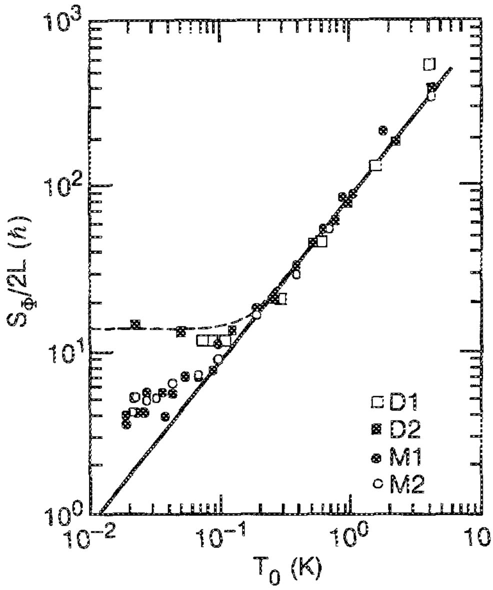

FIG. 2. Flux noise energy vs the refrigerator temperature T_0 for four SQUIDs. Solid line is the prediction of Eq. (1), and the dashed line is the prediction of Eq. (3) for the parameters of SQUID $D2$.

Figure 2 shows the noise energy $S_\Phi(f)/2L$ versus the temperature of the refrigerator, T_0. The SQUIDs are voltage biased (at low frequencies) between 1 and 4 μV, and a quasistatic flux near $(2n+1)\Phi_0/4$ (n is an integer) is applied. To obtain the white noise level of each SQUID, we have subtracted the $1/f$ noise originating in the device itself and the noise generated by the measuring system. At high temperatures, the noise energies of SQUIDs $D\,1$ and $D\,2$ scale with T_0, and are in excellent agreement with the prediction of Eq. (1). However, in each case, $S_\Phi/2L$ flattens out abruptly at about 140 mK, and remains more or less constant as the temperature of the refrigerator is lowered to about 70 mK in the case of $D\,1$ and 20 mK in the case of $D\,2$. The corresponding minimum noise energy in these SQUIDs is about 15 $\hbar$. The observed abrupt transition from linear behavior to a constant value is quite inconsistent with the presence of an external additive source of noise, which would produce a much smoother transition.

Using our measured value of Σ and Eq. (2), we can calculate the minimum temperature of the electrons in the shunts of the SQUIDs $D\,1$ and $D\,2$, $T_{\min} = (P/\Sigma\Omega)^{1/5}$. The results, 151 and 140 mK, respectively, are in close agreement with the observed temperature at which the noise in the SQUIDs levels out. We also note that the abrupt flattening of the noise energy as a function of temperature is well fitted by Eq. (3), as is shown by the dashed line in Fig. 2.

We fabricated a second pair of SQUIDs, $M\,1$ and $M\,2$, in which a large cooling fin was connected to each shunt. The configuration of the fin is shown in Fig. 1(c), and the relevant parameters are listed in Table I. The volumes of the metal in the shunts (including the cooling fins) for $M\,1$ and $M\,2$ are 160 and 4800 times greater, respectively, than that of SQUIDs $D\,1$ and $D\,2$. The measured noise energies of $M\,1$ and $M\,2$ are shown in Fig. 2. The scatter in the data at low temperatures is greater for $M\,1$ than for $M\,2$ because the measuring SQUID preamplifiers had 20-turn and 50-turn input coils, respectively. Consequently, the subtraction of the noise from the measuring system represented a smaller correction for $M\,2$ than for $M\,1$. We observe that the noise for both SQUIDs begins to level off at about 50 mK, in reasonable agreement with the prediction of $T_{\min}$ in Table I, reaching a minimum value of about $(5.0 \pm 0.6)\hbar$ at 20 mK. We note that because of resonant structure on the current-voltage characteristic for $M\,2$ (for which $\beta_c \simeq 0.3$, a factor of 2 higher than the value of $M\,1$) it was necessary to operate it at a substantially higher power dissipation that the other devices.

These results show rather dramatically the effects of electron heating in limiting the sensitivity of dc SQUIDs in the white noise region at low temperatures. We have shown that the effective electron temperature and hence the noise energy can be reduced by a factor of roughly 3 by attaching cooling fins to the shunt resistors. Nonetheless, we expect these hot-electron effects to play a crucial role in limiting the noise energy of any dc SQUID operated at low temperatures. Although the value of $T_{\min}$ depends on the volume of the shunts, the power being dissipated in the SQUID, and the shunt material, since $T_{\min}$ scales as $(P/\Sigma\Omega)^{1/5}$ it is relatively insensitive even to large variations in these parameters. The noise energies of most SQUIDs currently used should thus be expected to flatten out between 100 and 200 mK, unless large volume "cooling fins" are used. Conversely, we would not expect to see such effects at temperatures much above this range for typical SQUIDs. For example, very recently, Awschalom *et al.*[19] found that the noise energy scaled with T for temperatures greater than about 300 mK, and did not report any flattening out. This is in agreement with our earlier work,[12] where no flattening out was observed above 150 mK.

In conclusion, we point out that the flattening of the noise energy as the temperature is lowered is also characteristic of the crossover from the classical regime to the quantum regime, and one may have to exercise considerable care to distinguish quantum effects from hot-electron effects. Finally, hot-electron effects are not confined to the resistive shunts of SQUIDs, but will occur in any normal metal system which is operated at a temperature below $T_{\min}$.

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