Low-frequency noise in dc superconducting quantum interference devices below 1 K

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At temperatures below about 1 K, a series of dc superconducting quantum interference devices (SQUID's) exhibited an apparent flux noise with a spectral density scaling as $1/f^{\alpha}$, where $0.58 < \alpha < 0.80$. Typically, the magnitude of the noise increased as the temperature was lowered below 1 K, tending to flatten out at low temperatures with a value of 7 ± 3 $\mu\Phi_0\text{Hz}^{-1/2}$ at 1 Hz that was nearly independent of the parameters and materials of the SQUID's. Although a large number of hypothetical sources of the noise have been eliminated, the origin remains unidentified.

We consider a dc superconducting quantum interference device (SQUID) with a loop inductance L, and two Josephson tunnel junctions each with critical current I_0, capacitance C, and shunting resistance R. An incomplete figure of merit commonly used to characterize the sensitivity of the bare SQUID is the flux noise energy per unit bandwidth $S_\Phi/2L$, where $S_\Phi(f)$ is the spectral density of the equivalent magnetic flux noise. Here, Φ is the magnetic flux threading the SQUID. In the thermal noise limit one finds an optimized white noise flux energy of $12k_BT(LC)^{1/2}$ at temperature T for $\beta \equiv 2LI_0/\Phi_0 \approx 1$ and $\beta_c \equiv 2\pi I_0R^2C/\Phi_0 \approx 1$, where $\Phi_0 \equiv h/2e$ is the flux quantum.[1,2] By reducing the values of L and/or C several groups have achieved very low values of the flux noise energy of SQUID's in the ^{4}He temperature range.[3-6] However, in addition to white noise most dc SQUID's exhibit an excess low-frequency noise with a power spectrum scaling approximately as $1/f$, where f is the frequency. In the ^{4}He temperature range, Koch *et al.*[7] pointed out that this noise may originate in either critical current fluctuations or in an unidentified "flux noise". Unfortunately, in the case of either mechanism, reductions in L or C tend to increase the magnitude of the flux noise energy. In certain applications one requires very low noise at low frequencies, for example, the bar antenna for detecting gravity waves.[8] These requirements have prompted us to study the effects of reducing T as a means of improving the flux noise energy. This letter reports results obtained from 12 SQUID's at temperatures down to 90 mK.

Using a thin-film technology described in detail elsewhere[9] we have fabricated a variety of planar dc SQUID's in the configurations listed in Fig. 1 and Table I. The junctions were nominally $2\times2\ \mu\text{m}^2$ Nb-NbO$_x$-PbIn or Nb-NbO$_x$-Pb tunnel junctions and the shunts were 35-nm-thick AuCu films. We measured the noise in each SQUID(1) using a second dc SQUID(2) in a flux-locked loop[9] as shown in Fig. 2. Both SQUID's were biased at a voltage V (at low frequencies) by means of resistors r_1 and r_2 ($\ll R$) so that the output voltage V_0 was proportional to the current I in SQUID(1). By varying I_{b1} we measured the current-voltage (I-V) characteristics of SQUID(1). We measured the spectral density of the noise I, $S_I(f)$, using a spectrum analyzer and computed the flux noise energy $S_\Phi(f)/2L = S_I(f)/2LI_\Phi^2$, where $I_\Phi \equiv (\partial I/\partial\Phi)_v$. The two SQUID's were surrounded by a Nb shield that was mounted in a stainless-steel cell, usually filled with ^{4}He/^{3}He, bolted to the mixing chamber of a dilution refrigerator.

Figure 3 shows a typical flux noise spectrum for a device at 90 mK with $\Phi = \Phi_0/4$. For $f < 2$ kHz the spectrum exhibits excess noise with a power spectral density proportional to $1/f^\alpha$, where $\alpha = 0.66 \pm 0.08$. At higher frequencies the spectrum flattens, with a total flux noise energy of $15.4 \pm 1.5\hbar$ at 10 kHz. This noise contains a contribution from the electronics, which has the measured spectrum indicated by the dashed line in Fig. 3. The hatched area is our estimate of the SQUID noise after the electronics noise and $1/f^\alpha$ noise have been subtracted, and corresponds to $S_\Phi/2L = (4 \pm 2)\hbar$. The expected noise calculated in the thermal limit is $(3.5 \pm 0.4)\hbar$. Thus we conclude that self-heating of the resistive shunts did not raise their temperature to more than about 150 mK at the low operating voltages (typically 1.5 μV) used in the voltage-biased scheme.

One important property of the excess noise at the lowest temperatures is its dependence on Φ. As we varied I_Φ by changing the bias voltage and/or Φ we found the excess equivalent flux noise $S_\Phi(f) = S_I(f)/I_\Phi^2$ to be independent of I_Φ. For sufficiently small values of I_Φ (Φ near $\Phi_0/2$ or Φ_0) $S_I(f)$ became independent of I_Φ with a value one to two orders of magnitude smaller than at $\Phi = \Phi_0/4$, where I_Φ is a maximum. These results imply that the noise for Φ far from

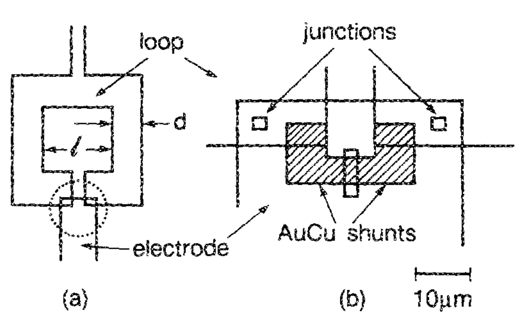

FIG. 1. (a) Configuration of dc SQUID; the portion in the dotted circle is shown enlarged in (b).

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TABLE I. Parameters and materials (loop/electrode) for SQUID's in configuration of Fig. 1.

Device	Metals	l (μm)	d (μm)	L (nH)
A1,A2,A3	Nb/PbIn	180	380	0.4
A4	Nb/Pb	180	380	0.4
A5	PbIn/Nb	200	400	0.4
B1	PbIn/Nb	400	50	1.0
C1	PbIn/Nb	100	50	0.2
C2	Pb/Nb	100	50	0.2
E1,E2	PbIn/Nb	100	10	0.26
F1	Nb/PbIn	100	2	0.36
G1	PbIn/Nb	50	10	0.1

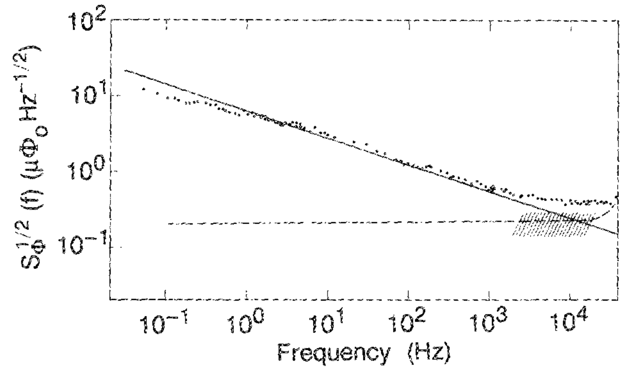

FIG. 3. Rms flux noise spectrum $S_\Phi^{1/2}(f)$ for SQUID $C1$ at 90 mK biased at $\Phi = \Phi_0/4$ with $R = 6\ \Omega$, and $2I_0 = 12.6\ \mu$A.

$\Phi_0/2$ or Φ_0 cannot be generated by SQUID(2), the measuring electronics, or by fluctuations in the inductance, resistance, or critical current of SQUID(1). On the other hand, this behavior is consistent with a fluctuating magnetic flux in the SQUID loop.

To investigate the origin of the excess noise we tested 11 more SQUID's with the parameters listed in Table I. In all of these devices we observed an excess flux noise at low temperatures with a power spectrum scaling as $1/f^\alpha$ where $0.58 < \alpha < 0.80$. Figure 4 shows $S_\Phi{}^{1/2}$ (1 Hz), the rms excess equivalent flux noise at 1 Hz, for all 12 SQUID's. Remarkably, the spread in $S_\Phi^{1/2}$ (1 Hz) at low temperatures is only about a factor of 2 despite enormous differences in the size of the SQUID's, the linewidth of the SQUID loops, the critical currents, and the loop materials. Furthermore, the values of $S_\Phi^{1/2}$ (1 Hz) in the range $1\ \mathrm{K} \leqslant T \leqslant 4.2\ \mathrm{K}$ vary by an order of magnitude. In particular the two lowest values were obtained from SQUID's $C1$ and $C2$, which have small Pb or PbIn loops. For $T > 1$ K, the noise in both cases was dominated by critical current fluctuations, and exhibited a spectral density that scaled as $1/f$. These two facts provide strong evidence that the flux noise observed at 100 mK has a different origin than the excess noise observed for $T > 1$ K.

From the behavior of the excess flux noise below 50 mK it is possible to eliminate several hypothetical sources, as summarized in Table II. We note that there are two modes of fluctuation of the SQUID parameters, symmetric and antisymmetric.[7] These two modes arise from fluctuations in the two critical currents I_{01} and I_{02}, in the two shunt resistors R_1 and R_2, or in the inductances of the two arms of the SQUID, L_1, and L_2. The antisymmetric modes usually produce flux-like noise whereas the symmetric modes do not. Most of the hypothetical noise sources in Table II can be unequivocably ruled out; a few, namely, noise from the substrate or mount and the motion of flux lines trapped in the SQUID, cannot be entirely dismissed but we consider them most unlikely in view of the lack of dependence of the noise on the materials used.

As we see in Table II, we have been able to exclude many hypothetical sources of the excess flux noise that we observe below 500 mK. However, one remaining candidate is a collection of fluctuating magnetic moments. Since the flux noise is independent of the area of the SQUID, this source would have to be *local*, that is, within a distance comparable to the dimensions of the SQUID. One possibility is an unidentified contaminant introduced by our processing technology. In this regard, we note that the increase in noise as the temperature is lowered below roughly 500 mK is reminiscent of the behavior of some spin-glass systems.[10]

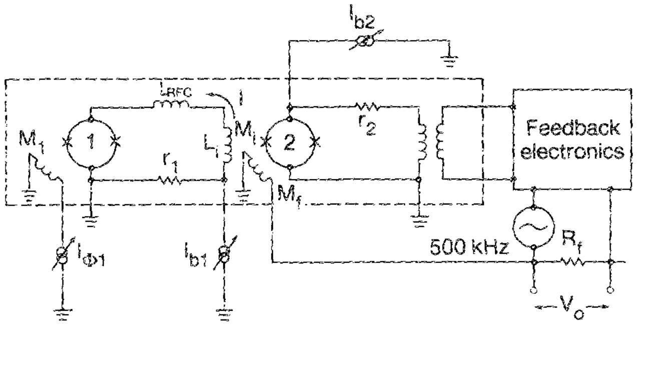

FIG. 2. Measurement scheme. Components within the dashed box are cooled by the dilution refrigerator, all other components are at room temperature. Typical parameters are $r_1 = 0.07\ \Omega$, $L_{RFC} = 12\ \mu$H, $M_1 = 30$ pH, $M_i = 6$ nH, $M_f = 100$ pH, $R_f = 5$ kΩ, $r_2 = 1\ \Omega$. $I_{\Phi1}$, I_{b1}, and I_{b2} are, respectively, the SQUID(1) flux bias, SQUID(1) current bias, and SQUID(2) current bias.

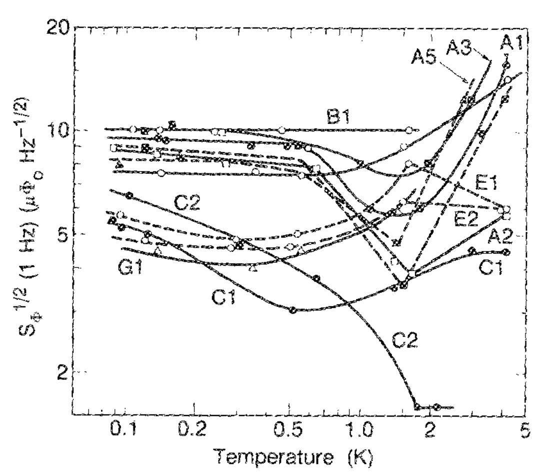

FIG. 4. Excess equivalent flux noise magnitude at 1 Hz vs the bath temperature for 12 devices.

TABLE II. Hypothetical sources of $1/f^{\alpha}$ noise and the property each would exhibit; in each case this property is not observed.

Hypothetical noise source	Properties of source
Noise from SQUID(2) or I_{b1}	Noise would not appear as flux noise
Noise from $I_{\Phi 1}$	Noise would depend on M_i
Symmetric fluctuations in I_{01} & I_{02}, R_1 & R_2, or L_1 & L_2	Noise would not appear as flux noise
Antisymmetric fluctuations in I_{01} and I_{02}	[a]
Antisymmetric fluctuations in L_1 and L_2	S_Φ would scale as I^2
Antisymmetric fluctuations in R_1 and R_2	S_Φ would scale as V^2
Fluctuations in external magnetic field	$S_\Phi^{1/2}$ would scale as SQUID area
Noise from substrate	Should depend on material
Noise from SQUID support	Should depend on material
Liquid helium in cell	Should change in absence of helium
Heating effects	Should depend on power dissipated
Motion of flux lines trapped in SQUID	Should depend on material

[a] To rule out antisymmetric fluctuations as the source of the noise we note that if I_{01} and I_{02} are very different the resulting noise would not be flux-like. Measurements on SQUID's with highly asymmetric critical currents showed flux-like noise, thereby excluding this source.

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[1] C. D. Tesche and J. Clarke, J. Low Temp. Phys. **29**, 301 (1977).
[2] J. J. P. Bruines, V. J. de Waal, and J. E. Mooij, J. Low Temp. Phys. **46**, 383 (1982).
[3] M. W. Cromar and P. Carelli, Appl. Phys. Lett. **38**, 723 (1981).
[4] R. F. Voss, R. B. Laibowitz, A. I. Broers, S. I. Raider, C. M. Knoedler, and J. M. Viggiano, IEEE Trans Magn. **MAG-17**, 395 (1981).
[5] D. J. Van Harlingen, R. H. Koch, and J. Clarke, Appl. Phys. Lett. **41**, 197 (1982).
[6] R. T. Wakai and D. J. Van Harlingen, Appl. Phys. Lett. **49**, 593 (1986).
[7] R. H. Koch, J. Clarke, W. M. Goubau, J. M. Martinis, C. M. Pegrum, and D. J. Van Harlingen, J. Low Temp. Phys. **51**, 207 (1983).
[8] M. Bassan, W. M. Fairbank, E. Mapoles, M. S. McAshan, P. F. Michelson, B. Moskowitz, K. Ralls, and R. C. Taber, *Proceedings of the Third Marcel Grossmann Meeting on General Relativity*, edited by H. Ning (North-Holland, New York, 1983), p. 667.
[9] F. Wellstood, C. Heiden, and J. Clarke, Rev. Sci. Instrum. **55**, 952 (1984).
[10] W. Reim, R. H. Koch, A. P. Malozemoff, M. B. Ketchen, and H. Maletta, Phys. Rev. Lett. **57**, 905 (1986).