Zener Enhancement of Quantum Tunneling in a Two-Level Superconducting Circuit

G. Ithier,[1] E. Collin,[1] P. Joyez,[1] D. Vion,[1] D. Esteve,[1] J. Ankerhold,[1,2] and H. Grabert[2]
[1]*Quantronics group, Service de Physique de l'Etat Condensé, DSM/DRECAM, CEA Saclay, 91191 Gif-sur-Yvette, France*
[2]*Physikalisches Institut, Albert-Ludwigs-Universität Freiburg, Hermann Herder Straße 3, 79104 Freiburg, Germany*
(Received 5 November 2004; published 10 February 2005)

We have investigated the macroscopic quantum tunneling (MQT) of the phase across a Josephson junction embedded in a superconducting circuit. This system is equivalent to a spin $1/2$ particle in a potential energy well. The MQT escape rate of such a particle was recently predicted to be strongly modified when a crossing of its inner Zeeman levels occurs while tunneling. In this regime, we observe a significant enhancement of the MQT rate and compare it to theory.

DOI: 10.1103/PhysRevLett.94.057004 PACS numbers: 74.50.+r, 74.78.–w, 85.25.Cp

The escape out of a potential well by quantum tunneling is ubiquitous in many areas of physics and chemistry [1]. The simplest model is that of a particle moving in a one-dimensional potential presenting a metastable minimum such that the escape rate is dominated by tunneling at sufficiently low temperature. The theoretical predictions of this model, including the effect of damping, were thoroughly tested for macroscopic quantum tunneling (MQT) of the phase in current-biased Josephson junctions [2]. Recently, the MQT problem was extended to the case when the particle has an additional spin 1/2-like degree of freedom [3], with a position dependent Zeeman splitting. A significant effect on tunneling has been predicted when this splitting happens to be suppressed at a certain point under the barrier, referred to below as the "crossing point," as sketched in Fig. 1. The calculated escape rate is strongly increased due to Zener flips between the spin states during tunneling. This theoretical work was motivated by experiments on the quantronium [4], a superconducting circuit implementing a two-level quantum system used as a qubit. The readout of this qubit is based on the MQT of a Josephson junction whose escape rate differs for the two qubit states. In this Letter, we present MQT rate measurements for a quantronium circuit initialized in its ground state prior to readout.

The quantronium circuit shown in Fig. 2 is based on a superconducting loop including a Cooper pair box (CPB) whose Josephson junction is split in two small junctions with Josephson energies $E_J(1 \pm d)/2$, delimiting an island with total capacitance C_Σ and charging energy $E_C = (2e)^2/2C_\Sigma$. When a phase difference γ across the series combination of the two junctions is imposed by an external magnetic field, the degree of freedom is only the number operator N of extra Cooper pairs on the island whose conjugate is the island phase δ. The Hamiltonian of this first subsystem is

$$h_{\mathrm{CPB}} = E_C(N - N_g)^2 - E_J\left[\cos\frac{\gamma}{2}\cos\delta + d\sin\frac{\gamma}{2}\sin\delta\right], \tag{1}$$

where the reduced gate charge $N_g = C_gV_g/2e$ is an external parameter. The energy spectrum of h_{CPB} is discrete, and the ground state with energy E_0 and the first excited state with energy E_1 define a quantum bit equivalent to a spin $1/2$. For $N_g = 1/2$, the energy separation $E_{01} = E_1 - E_0$ between these states has a minimum at $\gamma = \pi$, which vanishes with d, as shown in Fig. 2. In parallel with the two small junctions is a larger readout junction (RJ) with Josephson energy $\mathcal{E}_J \gg E_J$, effective capacitance $C_J \gg C_\Sigma$, and Cooper pair Coulomb energy $\mathcal{E}_C \ll \mathcal{E}_J$, biased with a current source I_b. The Hamiltonian of this second subsystem,

$$h_{\mathrm{RJ}} = \mathcal{E}_C q^2 - \mathcal{E}_J(\cos\theta + s\theta), \tag{2}$$

is that of a fictitious particle with mass $m = C_J/4e^2$, position θ, and conjugate momentum $q = Q/2e$, moving in a tilted cosine potential with tilt slope $s = I_b/I_0$, where $I_0 = \mathcal{E}_J/\varphi_0$ is the critical current of the junction, and $\varphi_0 = \Phi_0/2\pi$ with $\Phi_0 = h/2e$ the flux quantum. Notice that the magnetic field used to control γ can also slightly penetrate the readout junction and lower I_0. The plasma frequency is then $\omega_P = \omega_{P0}(1 - s^2)^{1/4}$ with $\omega_{P0} = (\varphi_0 C_J/I_0)^{-1/2}$, and the escape out of the well corresponds to the switching to a finite voltage state $V = \varphi_0\langle d\theta/dt\rangle$.

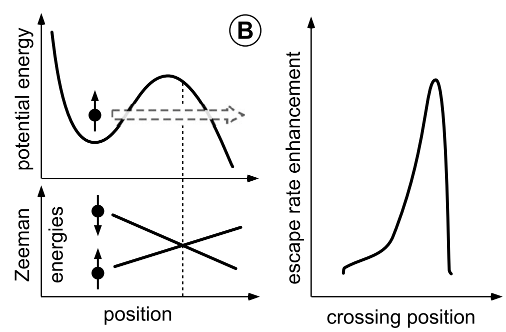

FIG. 1. The rate at which a particle escapes by quantum tunneling out of a metastable potential well has been predicted to strongly increase [3] when the particle carries a spin $1/2$ degree of freedom whose Zeeman energies are position dependent and cross in the barrier, so that the spin can flip while tunneling.

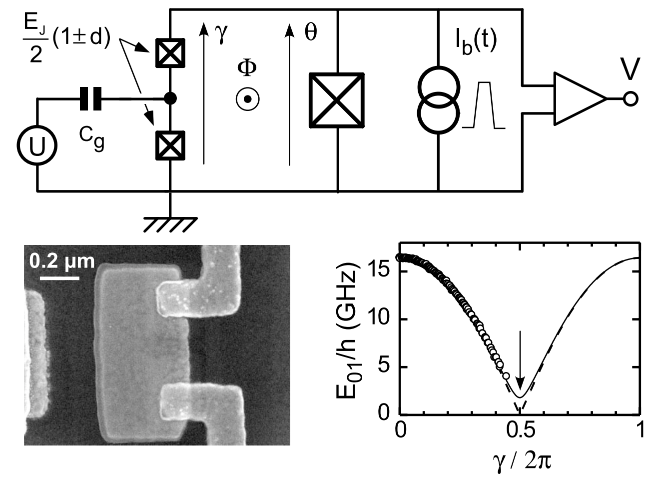

FIG. 2. Top: the quantronium circuit [4] is based on a split Cooper pair box (CPB) with charging energy E_C, Josephson energy E_J, and asymmetry d (see the text), controlled by a gate voltage U and a magnetic flux Φ. For readout, a larger Josephson junction with phase difference θ is biased by a current pulse I_b able to induce the switching to the voltage state. Bottom left: scanning electron micrograph of the island with the two small junctions. Bottom right: measured energy splitting E_{01} of the two lowest energy eigenstates of the CPB (dots), at $N_g = C_g U/2e = 1/2$, as a function of $\gamma = \theta + 2e\Phi/\hbar$. The dashed and solid lines are fits using $E_J = 0.655\ k_B$K and $E_C = 0.870\ k_B$K, with $d = 0$ and $d = 0.1$, respectively. At $\gamma = \pi$ (see the arrow), a level crossing or a small gap occurs depending on d.

The superconducting loop formed by the three junctions imposes the phase relation $\gamma = \theta + 2\pi\phi \pmod{2\pi}$, where $\phi = \Phi/\Phi_0$ is the reduced external magnetic flux applied to the loop. This relation couples the two subsystems and the Hamiltonian of the complete circuit, $H = h_{\mathrm{CPB}} + h_{\mathrm{RJ}}$, describes a spin 1/2 particle moving in a potential well.

At small bias $s \ll 1$, the phase θ, and consequently γ, can be seen as classical variables with negligible kinetic energies due to the large capacitance C_J. The fictitious particle can thus, depending on its spin state, be regarded as evolving in one of the two θ-dependent *adiabatic* potentials $E_k - \mathcal{E}_J(\cos\theta + s\theta)$, $k = 0, 1$. On the contrary, when s is close to 1, the switching occurs by MQT. When the reduced magnetic flux ϕ locates the crossing point $\theta_c = \pi(1 - 2\phi)$ within the barrier range, the Hamiltonian H is most conveniently represented [3] in the spin eigenstate basis at the minimum $\theta_{\min}$ of the lower adiabatic potential, i.e.,

$$H = \begin{pmatrix} \frac{q^2}{2m} + V_+(\theta) & \Delta(\theta) \\ \Delta(\theta)^* & \frac{q^2}{2m} + V_-(\theta) \end{pmatrix}. \qquad (3)$$

Here $V_\pm(\theta)$ denote two *diabatic* potentials, which are coupled by the off-diagonal element $\Delta(\theta)$. By construction, at $\theta_{\min}$, H is diagonal in spin space and the diabatic potential energies $V_\pm(\theta_{\min})$ coincide with the adiabatic ones. As long as $V_\pm$ are sufficiently separated so that $|V_- - V_+| \gg \Delta$, the spin is frozen and the particle tunnels through V_- by standard MQT at a rate $\Gamma_B = f_B \exp(-S_B/\hbar)$, with S_B the action of the bounce trajectory in the inverted potential [5]. When the crossing point θ_c lies within the barrier range, the MQT rate is $\Gamma_{\mathrm{tot}} = \Gamma_B + f_{\mathrm{flip}} \exp(-S_{\mathrm{flip}}/\hbar)$, where the Zener flip contribution involves the action S_{flip} along the flip bounce trajectory [3]. Since the ordinary and the flip contributions to the rate are exponentially sensitive to the shape of the barrier, a changeover from the V_- to the V_+ surface during tunneling may lead to a much smaller action $S_{\mathrm{flip}} < S_B$, and thus to a substantial rate enhancement.

Experimentally, the switching rate Γ is measured by repetitively initializing the quantronium in its ground state and applying then a trapezoidal current pulse with a rise sufficiently slow that the circuit follows adiabatically. The dimensionless peak value $s_{\max} = I_{\max}/I_0$ and the duration τ of this pulse are such that the switching to the voltage state occurs with a probability $p(s_{\max}) = 1 - \exp[-\Gamma(s_{\max})\tau]$. Practically, for each flux ϕ, p can be measured as a function of $s_{\max}$ (direct mode), or $s_{\max}$ can be adjusted to maintain p, and consequently Γ, at a constant value (feedback mode).

The actual sample on which measurements have been performed has been fabricated using Al evaporation and oxidation through a resist shadow mask patterned by e-beam lithography. The scanning electron micrograph of Fig. 2 shows the central part of this quantronium with the two small Josephson junctions formed by two fingers overlapping the island. The 0.67 μm^2 readout junction has an effective capacitance $C_J = 0.6 \pm 0.2$ pF dominated by a parallel on-chip interdigitated capacitor whose goal is to lower ω_P. An RC filter also in parallel with the junction limits its quality factor $2.5 < Q < 10$. The sample is placed in a shielding box thermally anchored to the mixing chamber of a dilution refrigerator with a 15 mK base temperature and is wired using carefully filtered lines. The rise time of the trapezoidal readout pulse is 50 ns and the plateau duration is either 100 or 200 ns. A switching event is detected by measuring the voltage V across the junction with a room temperature amplifier, and the switching probability p is determined by repeating the sequence a few 10^4 times at a rate between 10 and 50 kHz.

We have first inferred the parameters entering h_{CPB} from the increase of p when a resonant microwave pulse is applied to the gate at $I_b = 0$, just before the readout pulse. By fitting $E_{01}(N_g, \gamma)$, we deduce $E_C = 0.870\ k_B$K and $E_J = 0.655\ k_B$K (see Fig. 2). The gap E_{01} cannot be measured at its minimum $\gamma = \pi$ where the two quantronium states have vanishing loop currents. From the fit in the vicinity $\gamma \simeq \pi$, we deduce the upper bound $d \leq 0.13$, which implies $E_{01}(N_g = 1/2, \gamma = \pi) \leq 0.1\ k_B$K. Then, we follow a standard procedure [2,6] to determine $I_0(\phi)$ and to check that the MQT regime is reached at low temperature. For that purpose, p is measured as a function of I_b at a reference point R ($\phi_R = -0.22$) where the quantronium loop current is close to zero at the switching of the readout junction, and at $N_g = 0$ and $\gamma \simeq 0$ between $T = 15$ mK and $T = 200$ mK. The values of Γ obtained from p are fitted to the expression for thermal activation, which leads to an equivalent escape temperature $T_{\mathrm{esc}}(T)$ and to $I_0(\phi \simeq 0) = 445 \pm 20$ nA. As shown in the inset of Fig. 3, T_{esc} follows T at high temperature and saturates at a value $T_{\mathrm{esc}} = 35 \pm 2$ mK. Finally the switching current is measured in the feedback mode while sweeping the external flux over about $20\Phi_0$. These data lead to $I_0(\phi) = I_0(0) \times (1 - 0.005\phi^2)$.

To probe Zener flips, we also operate in the feedback mode by finding the value I_{60} of $I_{\max}$ that corresponds to $p = 60\%$. There, the slope $\partial p/\partial I_{\max}$ is the steepest and provides a maximal sensitivity to rate variations. For a readout pulse duration $\tau \simeq 100$ ns, the rate is $\Gamma_{60} \simeq 14.5$ MHz. Figure 3 shows I_{60} as a function of ϕ, for $N_g = 0$ and $N_g = 1/2$. At $N_g = 0$ (top panel), the dependence $I_{60}(\phi)$ is fitted from the standard MQT rate $\Gamma_B(I_{60}, \phi) = \Gamma_{60}$, taking into account the $I_0(\phi)$ dependence. The best fits are completely insensitive to d and give $\mathcal{E}_C = 0.0051\ k_B$K and $\mathcal{E}_J = 10.2\ k_B$K, thus fixing the junction parameters ($I_0 = 427$ nA, $\omega_{P0}/2\pi \simeq 7.5$ GHz).

At $N_g = 1/2$ (bottom panel), three ϕ regions have to be distinguished. The first region noted by A, where $\gamma \neq \pi$ anywhere in the barrier range, is also well described by the standard MQT expression. In a second region noted by B, where Zener flips are predicted to occur since $\gamma = \pi$ somewhere in the barrier, $I_{60}(\phi)$ deviates from the standard MQT prediction but is well fitted by the full theory, i.e., from $\Gamma_{\mathrm{tot}}(I_{60}, \phi) = \Gamma_{60}$. The optimal fit gives the remaining parameter $d = 3.0 \pm 0.1\%$, which corresponds to a minimum gap $E_{01} = 26\ k_B$mK. We have also plotted in

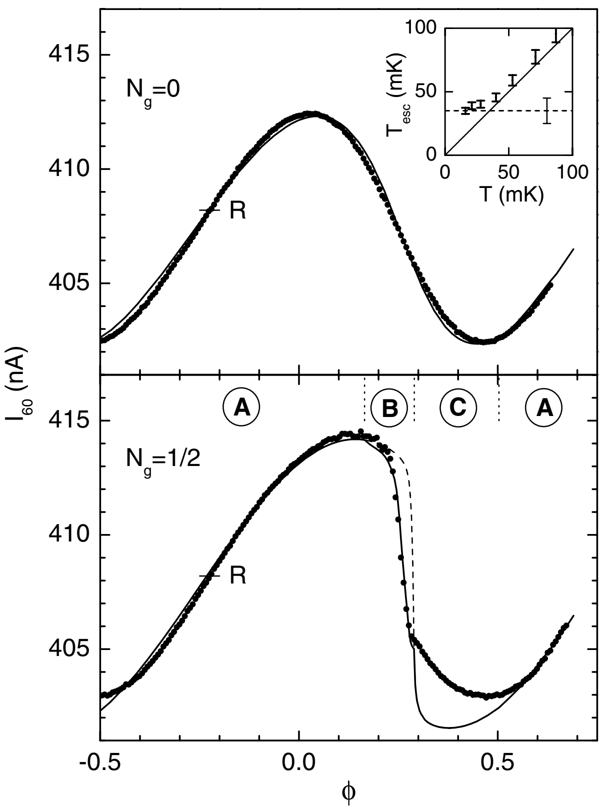

FIG. 3. Experimental (dots) and theoretical (lines) amplitudes I_{60} of 100 ns long current pulses giving a switching probability $p = 60\%$, as a function of the reduced flux ϕ at $N_g = 0$ (top) and $N_g = 1/2$ (bottom). The lines are best fits leading to $\mathcal{E}_C = 0.0051\ k_B$K, $\mathcal{E}_J = 10.2\ k_B$K for the readout junction, and to the CPB asymmetry $d = 3.0\%$. The solid (dashed) curve of the bottom panel is a fit calculated with (without) Zener flips. Three regions A, B, and C have to be distinguished (see the text), and R denotes a reference point. Inset: escape temperature T_{esc} calculated from escape rate measurements performed at R and at $N_g = 0$ with 200 ns long current pulses, as a function of the measured temperature T. The solid line corresponds to $T_{\mathrm{esc}} = T$, and the dashed line to the theoretical zero temperature limit.

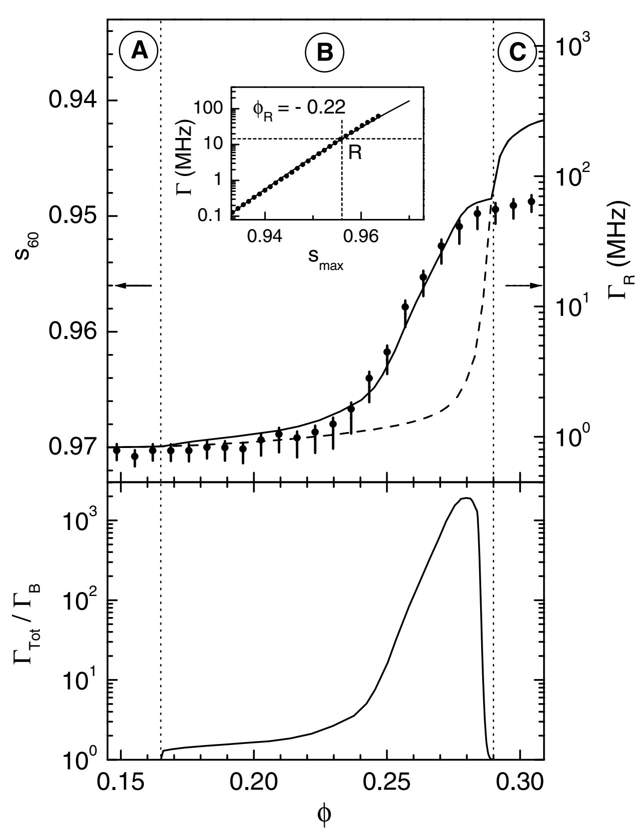

FIG. 4. Top panel: experimental (dots) and calculated (lines) values of $s_{60} = I_{60}/I_0$, as a function of the reduced flux ϕ, in region B. Error bars are mainly due to gate-charge noise. The curves are calculated with (solid line) and without (dashed line) Zener flips. The right vertical scale results from the conversion of s_{60} into a rate Γ_R according to the inset (see the text). Arrows indicate the reference point R. Inset: escape rate $\Gamma(s_{\max})$ measured (dots) and calculated (line) at $N_g = 1/2$ at the reference point R. Bottom panel: escape rate enhancement calculated at constant rate $\Gamma_{\mathrm{tot}}(\phi, s_{60}) = \Gamma_{60}$ by dividing Γ_{60} by the rate Γ_B that would be observed at the same s_{60} in the absence of Zener flips.

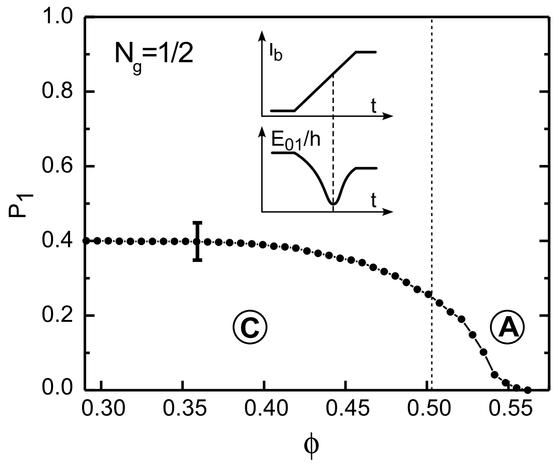

FIG. 5. Average population P_1 of the first excited state of the quantronium as a function of the reduced flux ϕ, calculated from the difference between the theoretical and experimental I_{60} in and close to region C, where the minimum of E_{01} is crossed during the current ramp, as depicted in the inset. The error bar indicates the systematic error on the plateau.

the top panel of Fig. 4 $s_{60}(\phi)$ together with both predictions. With the 0.4% experimental error bars on s_{60} that result principally from N_g noise, the experimental data agree well only with the prediction taking into account Zener flips. Although our experimental procedure has the advantage of maximizing the sensitivity to rate variations, it is more useful for the sake of clarity to convert the data into rates Γ_R that would have been measured at a constant reference value $s = s_R$. For that purpose, we use again the reference point R of region A where $s_R = 0.956$. The rate $\Gamma(\phi_R, s_{\max})$ is directly measured as a function of $s_{\max}$ and compared to the theoretical prediction, as shown in the inset of Fig. 4. With the value $Q \simeq 3.6$, both theoretical and experimental rates follow the same exponential variation with a precision on the rate better than 10%. Using the slope $K = \partial \log\Gamma/\partial s_{\max}$, s_{60} is converted into Γ_R according to $\log(\Gamma_R) = K \times (s_R - s_{60}) + \log(\Gamma_{60})$. Note that this conversion procedure is reliable because rate measurements performed at different ϕ show very similar behaviors. The top-right scale of Fig. 4 now shows that the tunneling rate Γ_R is increased by up to a factor of about 20 by Zener flips and that our error bars correspond to less than a factor of 2 in rates. This Zener flip effect can also be modeled by a tunneling rate enhancement ratio calculated at constant total rate Γ_{tot} rather than at constant s: The bottom panel of Fig. 4 shows that this theoretical ratio $\Gamma_{\text{tot}}(\phi, s_{60})/\Gamma_B(\phi, s_{60})$ with s_{60} chosen so that $\Gamma_{\text{tot}}(\phi, s_{60}) = \Gamma_{60}$, increases by up to 3 orders of magnitude.

A corollary effect of the Zener flip tunneling in region B is the large deviation between the theoretical prediction for MQT from the ground state and the experimental data in a third region C. In this region, the intersection of $V_\pm$ does occur in the well region *during* the rise of the bias current prior to the set-in of MQT. As a consequence, for the minimal E_{01} being of the order of the mean thermal energy per degree of freedom, at the crossing point all collective and microscopic degrees of freedom can excite the spin. In addition, conventional Zener transitions may also take place. Accordingly, the switching occurs by standard MQT from a statistical mixture of the spin states, the deeper excited potential well leading to a higher s_{60} as we indeed observe. Predicting quantitatively this effect would require an exact knowledge of all the environmental degrees of freedom and of their coupling to the quantronium, which is not available. So we have fitted the experimental s_{60} to a weighted average of the standard MQT rates in the two adiabatic potentials, which gives the population P_1 of the excited spin state inside and in the vicinity of region C. As shown in Fig. 5, P_1 is close to its thermal equilibrium saturation value $1/2$ on the left side of C, where the crossing point is traversed just before MQT sets in. On the contrary, P_1 decreases on the right side of the region, where the intersection appears at the foot of the current ramp so that the spin can relax again before the top of the pulse is reached.

To summarize, we have reported on the first observation of "Zener flip quantum tunneling," a general effect recently predicted, which consists of a large increase of the escape rate of a particle that tunnels out of a well when this particle has an inner spin degree of freedom undergoing a level crossing somewhere in the barrier. This phenomenon may be also of relevance for other physical systems, such as other superconducting qubit circuits [7] or recent realizations of atomic transport in optical lattices [8].

We acknowledge essential interactions in the Quantronics group, the technical help of P.F. Orfila, P. Sénat, and J.C. Tack, the support of the European project SQUBIT, of Yale University (DAAD 19-02-1-0044), and of DFG.

[1] *Quantum Tunneling in Condensed Media*, edited by Yu. Kagan and A.J. Leggett (Elsevier, Amsterdam, 1992).
[2] M.H. Devoret *et al.*, in Ref. [1], p. 313.
[3] J. Ankerhold and H. Grabert, Phys. Rev. Lett. **91**, 016803 (2003).
[4] D. Vion *et al.*, Science **296**, 886 (2002).
[5] A.O. Caldeira and A.J. Leggett, Ann. Phys. (N.Y.) **149**, 374 (1983).
[6] A. Cottet *et al.*, Physica (Amsterdam) **367C**, 197 (2002).
[7] I. Chiorescu *et al.*, Science **299**, 1869 (2003).
[8] O. Mandel *et al.*, Phys. Rev. Lett. **91**, 010407 (2003).